

## **Boundary Layer Model for Moving Tropical Cyclones**

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## Abstract

We propose a simple theoretical model for the boundary layer (BL) of moving tropical cyclones (TCs). The model estimates the horizontal and vertical wind velocity fields from a few TC characteristics: the maximum tangential wind speed  $V_{max}$ , the radius of maximum winds  $R_{max}$ , and Holland's  $B$  parameter away from the surface boundary where gradient balance is approximately valid, in addition to the storm translation velocity  $V_t$ , the surface drag coefficient  $C_D$ , and the vertical diffusion coefficient of the horizontal momentum  $K$ .

The model is based on Smith's (1968) formulation for stationary (axisymmetric) tropical cyclones. Smith's model is first extended to include storm motion and then solved using the momentum integral method. The scheme is computationally very efficient and is stable also for large  $B$  values and fast-moving storms.

Results are compared to those from other studies (Shapiro 1983; Kepert 2001) and validated using the Fifth-Generation Pennsylvania State University/NCAR Mesoscale Model (MM5). We find that Kepert's (2001) BL model significantly underestimates the radial and vertical fluxes, whereas Shapiro's (1983) slab-layer formulation produces radial and vertical winds that are a factor of about two higher than those produced by MM5. The velocity fields generated by the present model are consistent with MM5 and with tropical cyclone observations.

We use the model to study how the symmetric and asymmetric components of the wind field vary with the storm parameters mentioned above. In accordance with observations, we find that larger values of  $B$  and lower values of  $R_{max}$  produce horizontal and vertical wind profiles that are more peaked near the radius of maximum winds. We also find that, when cyclones in the northern hemisphere move, the vertical and storm-relative radial winds intensify at the right-front quadrant of the vortex, whereas the storm-relative tangential winds are more intense in the left-front region. The asymmetry is higher for faster moving TCs and for higher surface drag coefficients  $C_D$ .

## 1. Introduction

Tropical cyclones (TCs) are a particular class of rotating low-pressure systems that develop over tropical and subtropical waters. The systems have a warm-core, a well-organized convection, and cyclonic surface wind circulation (Anthes 1982; Landsea 2000).

Empirical observations (La Seur and Hawkins 1963; Hawkins and Rubsam 1968; Holland 1980; Willoughby 1990, 1991; Vickery et al. 2000; among others) show that in the altitude range from 2-3km to about 10km, the tangential winds are in approximate gradient balance and the radial inflow is negligible. Based on earlier work by Schloemer (1954) and Myers (1957), Holland (1980) used a symmetric pressure distribution to derive the tangential gradient wind  $V_{gr}$ , as a function of distance  $R$  from the TC center. His result, which we refer to here as Holland's wind profile, is

$$V_{gr}(R) = V_{max} \sqrt{(R_{max}/R)^B \exp[1-(R_{max}/R)^B]} \quad (1)$$

where  $V_{max}$ ,  $R_{max}$ , and  $B$  are TC-specific constants. The tangential velocity  $V_{gr}$  increases with  $R$  to a maximum  $V_{max}$  at  $R = R_{max}$  (usually referred to as the radius of maximum winds). For  $R \gg R_{max}$ ,  $V_{gr}$  has an approximately power-law decay with distance, with exponent  $-B/2$ . According to Willoughby and Rahn (2004),  $B$  varies in the range  $[1, 2]$  with typical values around 1.4.

Inside the TC boundary layer (BL) (within approximately 1-2km from the surface), frictional stresses are important and result in an inward net force that drives low-level convergence. Consequently, the horizontal and vertical wind fields are strongly coupled and Eq. (1) does not apply. Horizontal convergence drives the vertical winds, which are maximum at the top of the boundary layer near the radius of maximum winds  $R_{max}$  (e.g. Kepert 2001 and Kepert and Wang 2001).

Since the convergence of moisture inside the BL is of major importance for the maintenance, evolution and destructive potential of TCs (Emanuel 1986, 1989; Renno and Ingersoll 1996), a number of studies (Myers and Malkin 1961; Chow 1971; Shapiro 1983; Kepert 2001) have focused on developing theoretical models for the boundary layer of moving TCs. These models derive the radial and tangential winds inside the boundary layer from an assumed radial profile of the tangential wind velocity under gradient balance, for example the profile in Eq. (1), and from suitable surface boundary conditions.

Section 2 reviews these BL models and their limitations. Section 3 describes our proposed model by giving the governing equations (an extension of the equations of Smith 1968) and discussing their numerical solu-

tion. In Section 4, we compare model results with earlier models and with simulations using the Fifth-Generation Pennsylvania State University/NCAR Mesoscale Model (MM5). Section 5 shows how the calculated winds depend on various storm parameters. Conclusions are stated in Section 6.

## 2. Review of Boundary Layer Models

The focus of this review is on BL models for moving tropical cyclones, but studies of stationary TCs that are relevant to what follows are also mentioned.

Boundary layer models differ mainly in their treatment of altitude  $Z$  and the surface boundary conditions. In one of the earlier studies of moving TCs, Myers and Malkin (1961) used a Lagrangian parcel trajectory approach to study the horizontal winds inside the BL. The authors assume that the frictional drag force is proportional to the square of the wind speed with equal tangential and radial components. Another (implicit) assumption is that the velocity of the background flow is zero rather than equal to the translation velocity of the TC. A finding of the study is that, when a TC in the northern (southern) hemisphere moves, the radial convergence is maximum at the right-front (left-front) quadrant of the vortex and the location of this maximum rotates anticyclonically as the translation velocity  $V_t$  increases.

Based on the work of Chow (1971), Shapiro (1983) approximated the boundary layer of a moving TC by a slab of constant depth  $H = 1\text{km}$ . The horizontal momentum equations are formulated in cylindrical coordinates that translate with the vortex and then averaged in the vertical direction. This results in a system of two partial differential equations (PDEs) that are solved numerically for the vertically averaged tangential  $\bar{V}(R, \theta)$  and radial  $\bar{U}(R, \theta)$  wind velocity as a function of radius  $R$  and the azimuth  $\theta$  relative to the direction of TC motion. Contrary to Myers and Malkin (1961), Shapiro's (1983) formulation assumes that the frictional drag force is parallel to the surface-relative flow and its magnitude is proportional to the square of the composite surface-relative wind velocity. Although the two studies use different formulations for the friction-induced convergence, they both produce maximum convergence at the right-front quadrant of vortices in the northern hemisphere. However, in Shapiro's model the location of the maximum does not depend on the translation velocity, whereas in Myers and Malkin's (1961) analysis it does.