Multifractality and the Estimation of Extreme Rainfall

D. Veneziano and A. Langousis
Dept. of Civil and Environmental Engineering, MIT

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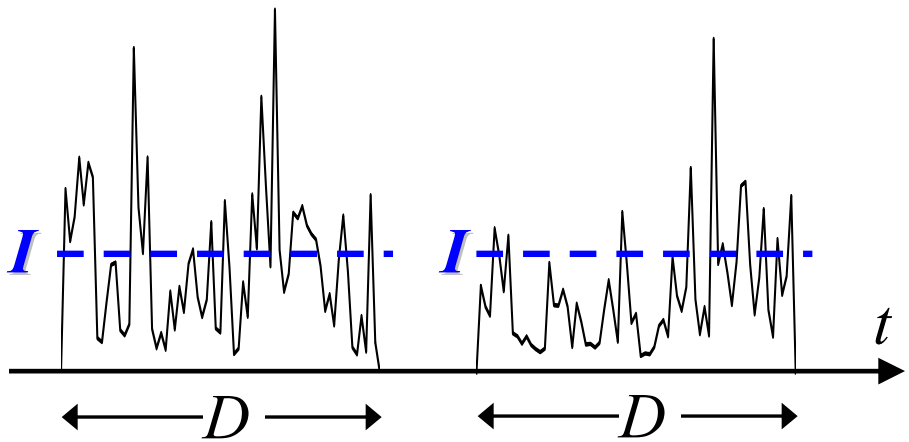
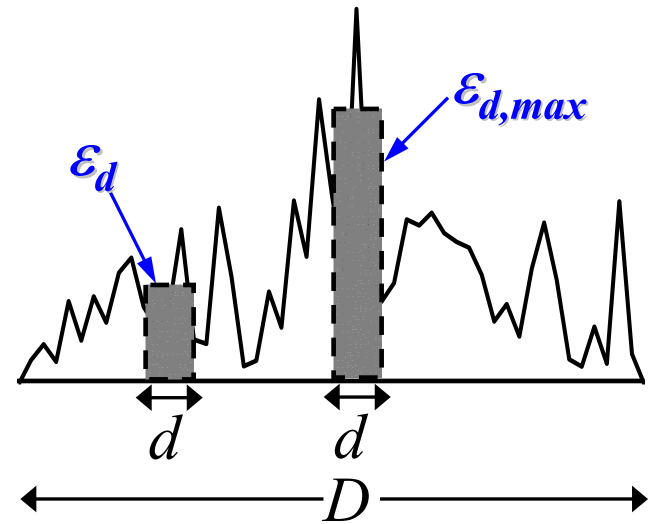
1. Rainfall as a sequence of *iid* multiplicative cascades





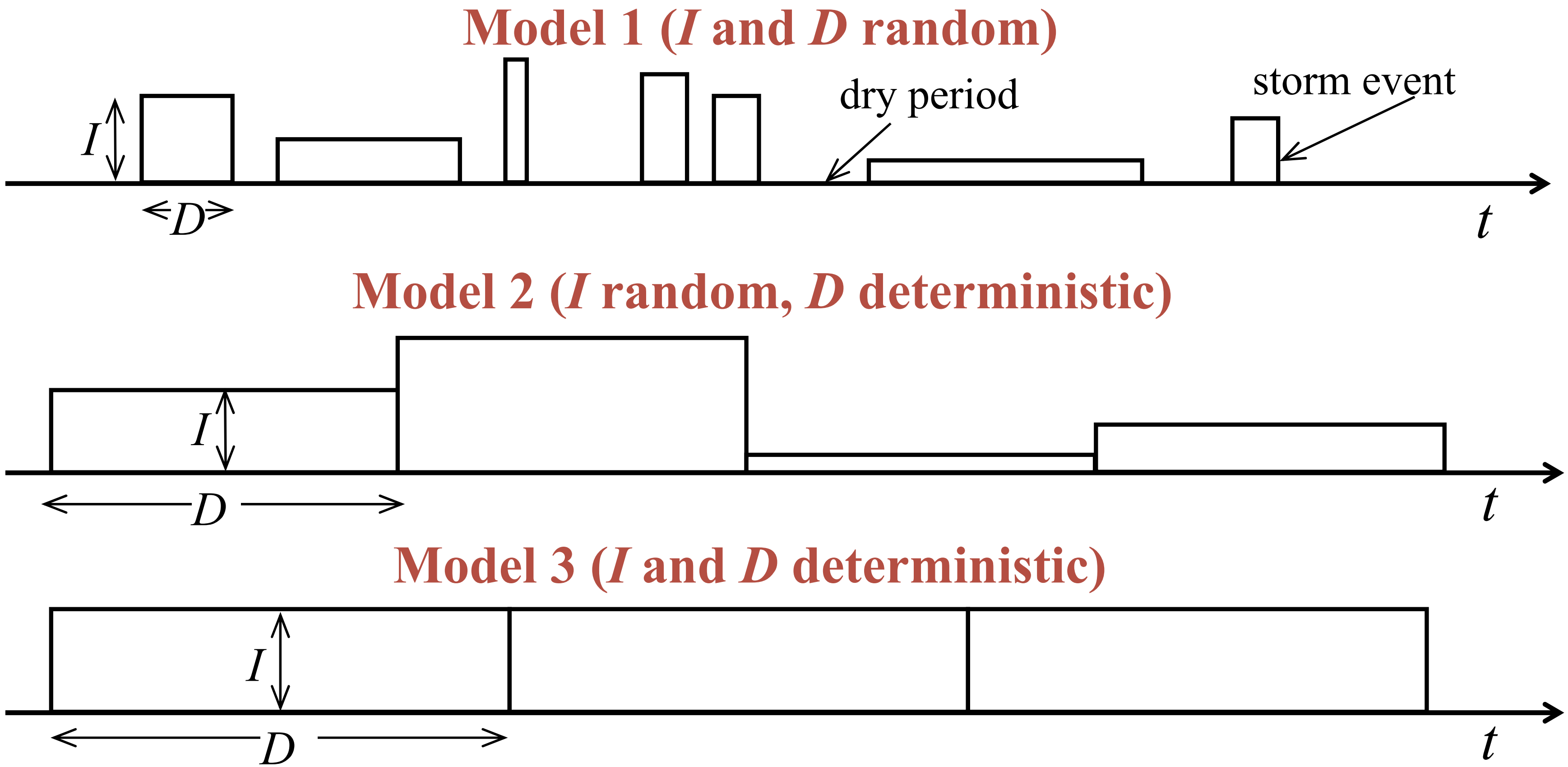
- **Tail properties of ε_d**

  $\Rightarrow$ asymptotic behavior of IDF curves (or $\varepsilon_{d,T}$) as $d \to 0$ and $T \to \infty$

- **Distribution of ε_d or $\varepsilon_{d,max}$**

  $\Rightarrow$ actual values of $\varepsilon_{d,T}$

2. More realistic rainfall models with MF "interiors"



- **Fitting models to data/IDF calculation**
- **Comparison with empirical IDF curves**

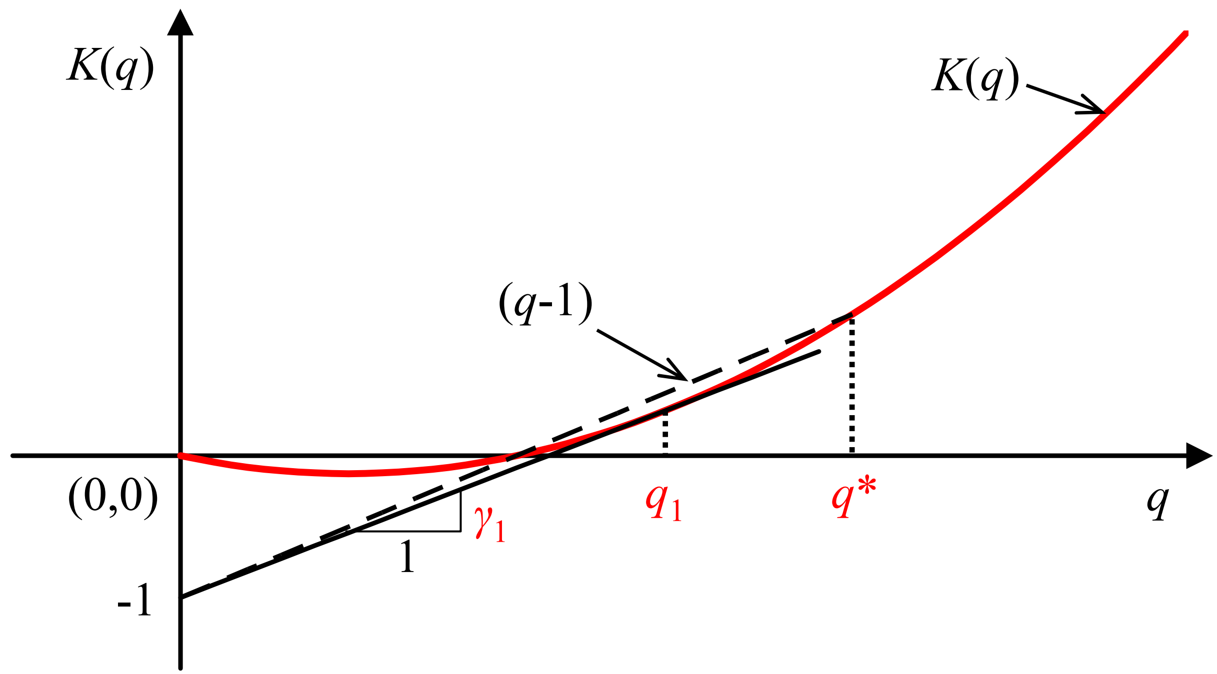
3. Conclusions

ε_d: tail properties

- **Rough limits:** $P[\varepsilon_d \geq (D/d)^{\gamma}] \sim (D/d)^{-C(\gamma)}$, as $d \to 0$

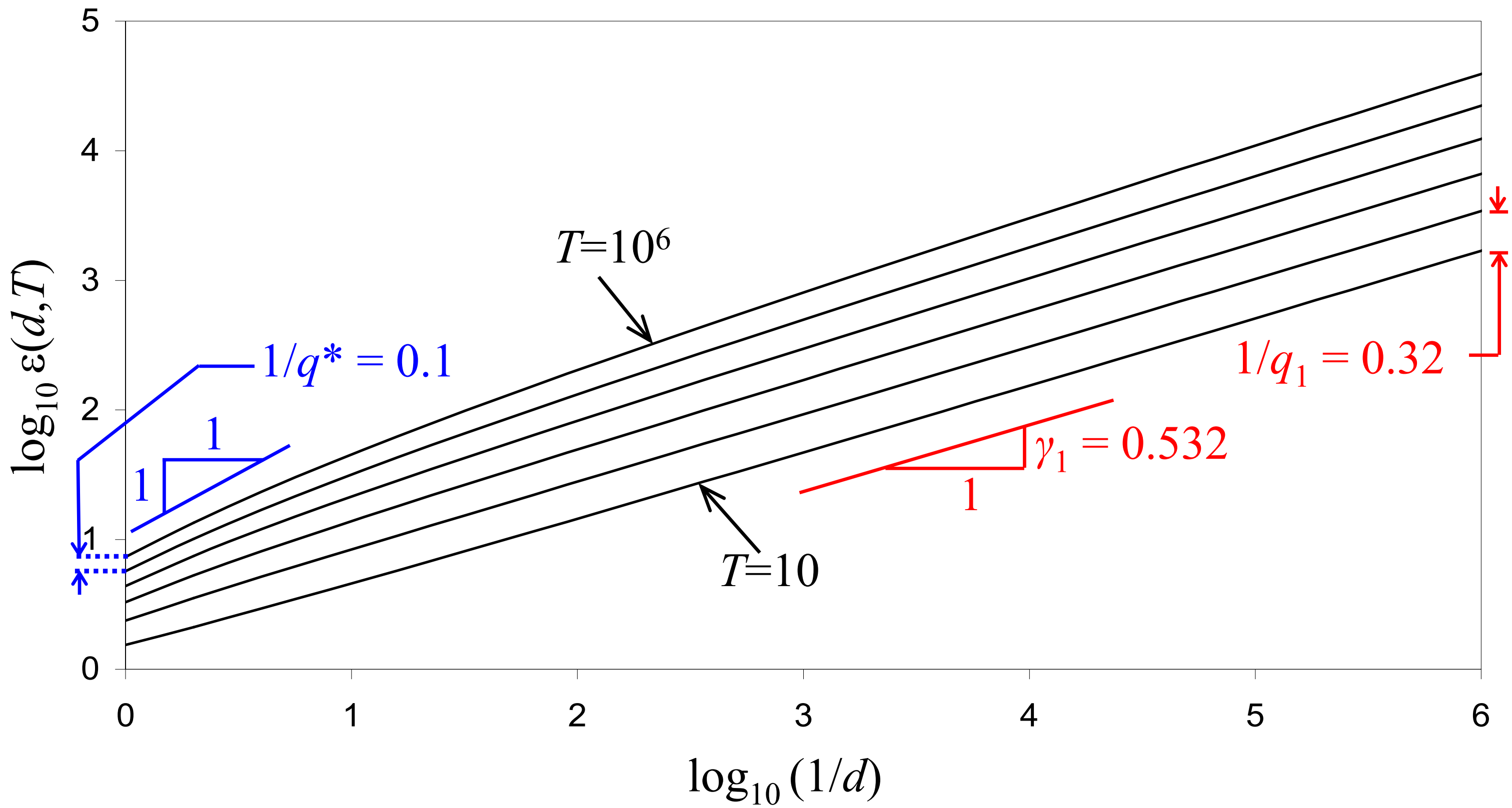
$$\varepsilon_{d,T} \sim \begin{cases} (D/d)^{\gamma_1}\, T^{1/q_1}, & \text{for } d \to 0 \text{ and } T \text{ finite} \\ (D/d)\, T^{1/q^*}, & \text{for } d \text{ finite and } T \to \infty \end{cases}$$

Moment scaling function



•Illustration

$$\varepsilon_{d,T} \sim \begin{cases} (D/d)^{\gamma_1}\, T^{\,1/q_1}, & \text{for } d \to 0 \text{ and } T \text{ finite} \\ (D/d)\, T^{\,1/q^*}, & \text{for } d \text{ finite and } T \to \infty \end{cases}$$



• Refined limits for ε_d

Bare (b)

$$P[\varepsilon_{d,b} \geq (D/d)^{\gamma}] \approx \left\{ 2\pi \frac{[C_b'(\gamma)]^2}{C_b''(\gamma)} \ln(D/d) \right\}^{-1/2} (D/d)^{-C_b(\gamma)}$$

Dressed ($\varepsilon_d = \varepsilon_{d,b} Z$)

$$P[\varepsilon_d \geq (D/d)^{\gamma}] \approx \begin{cases} P[\varepsilon_{d,b} \geq (D/d)^{\gamma}]\ E\left[Z^{C'(\gamma)}\right], & \gamma \leq \gamma^* \\ P[Z \geq (D/d)^{\gamma}]\ E\left[\varepsilon_{d,b}{}^{C'(\gamma)}\right], & \gamma > \gamma^* \end{cases}$$

⇒ **Refined limits for IDF**

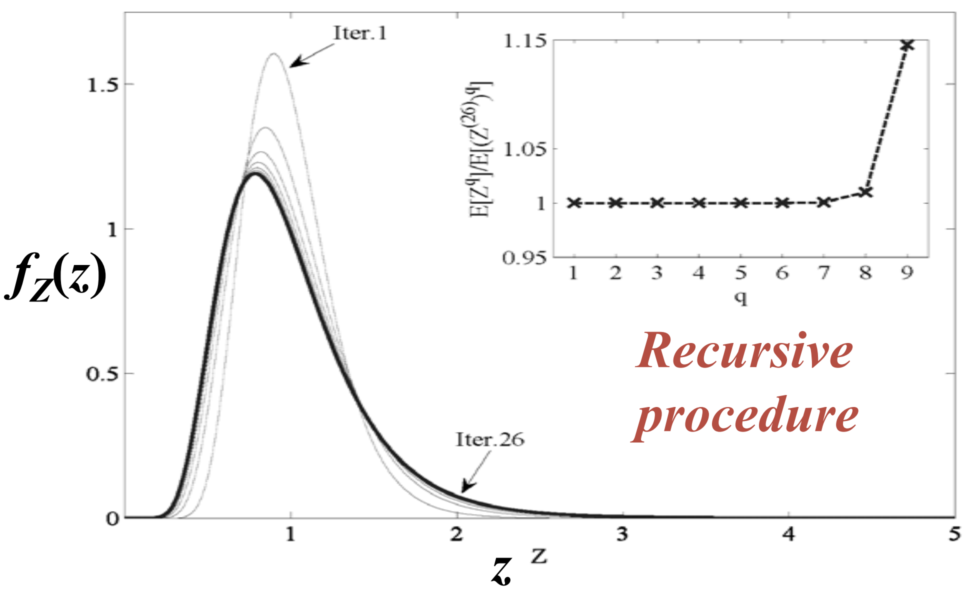
⇒ Extension to **space-time rainfall** (IDAF curves and ARF)

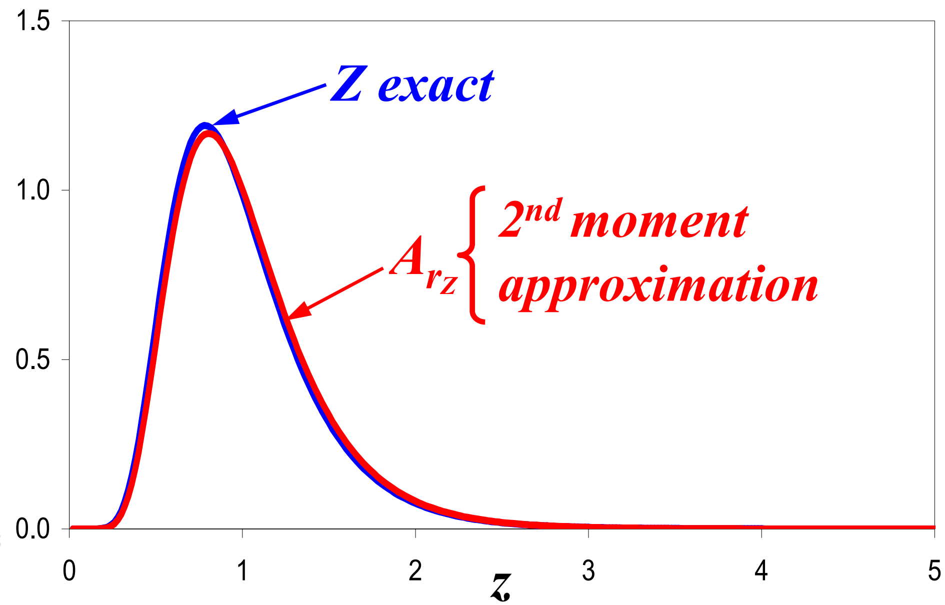
IDF from distribution of $\varepsilon_d = \varepsilon_{d,b} Z$

$\varepsilon_{d,b} = \prod_{i=1}^{n} A_i$, where $A_1, \ldots, A_n$ = independent copies of the cascade generator A

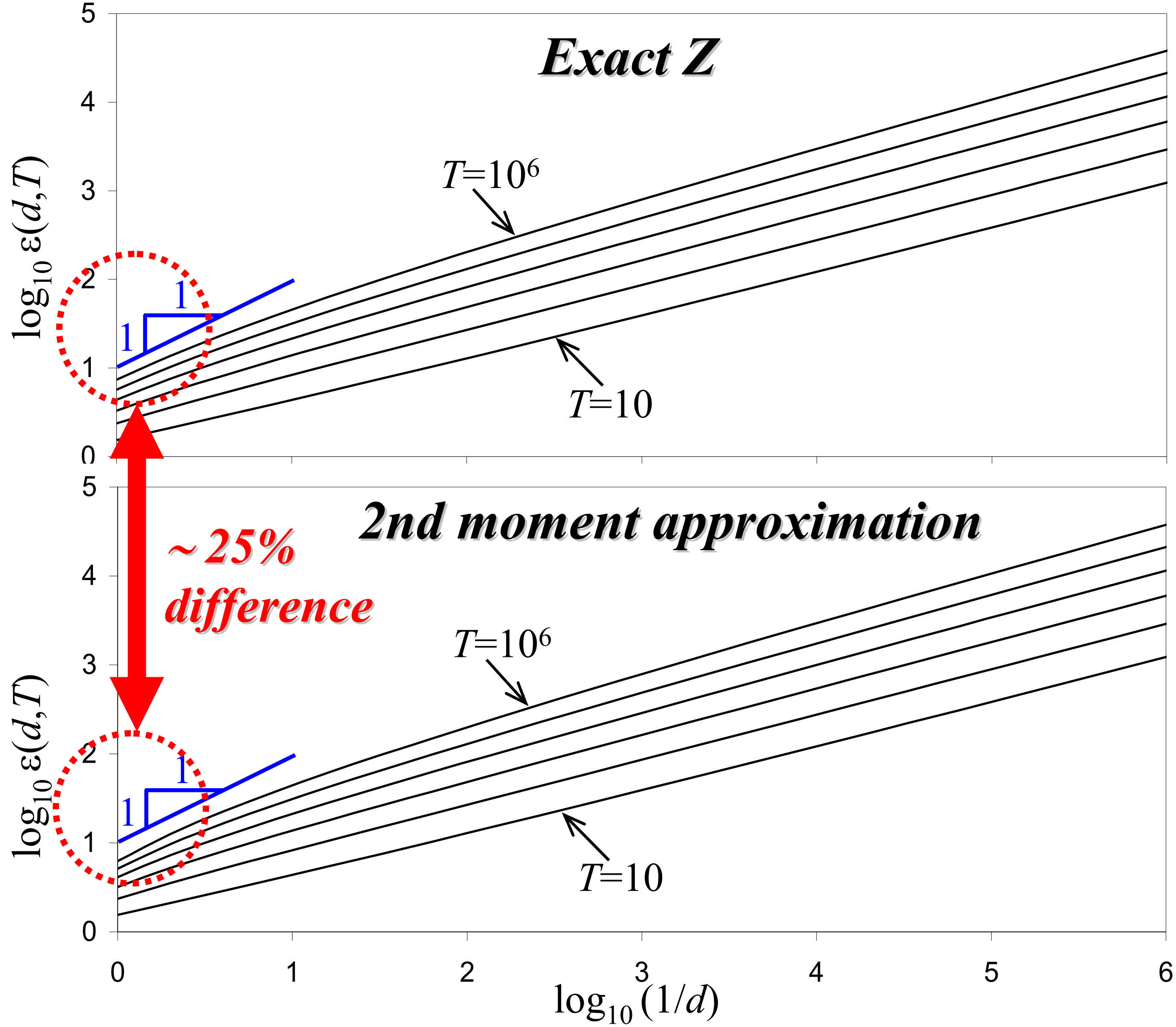
F_Z from

- recursive numerical procedure (exact)
- analytical approximation, e.g. $Z \approx A_{r_Z}$ with r_z to match some moment of Z





Illustration



Distribution of $\varepsilon_{d,max}$

- **Exact Iterative Procedure**

$$Z = \varepsilon_{D,max} \rightarrow \varepsilon_{(D/2),max} \rightarrow \varepsilon_{(D/4),max} \rightarrow \cdots \rightarrow \varepsilon_{d,max}$$

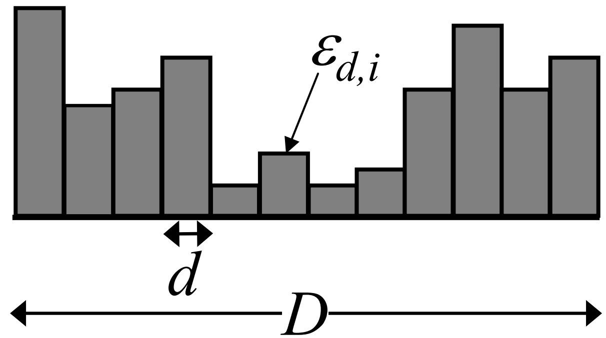
Convolutions needed to evaluate F_Z and in each step

- **Approximations**

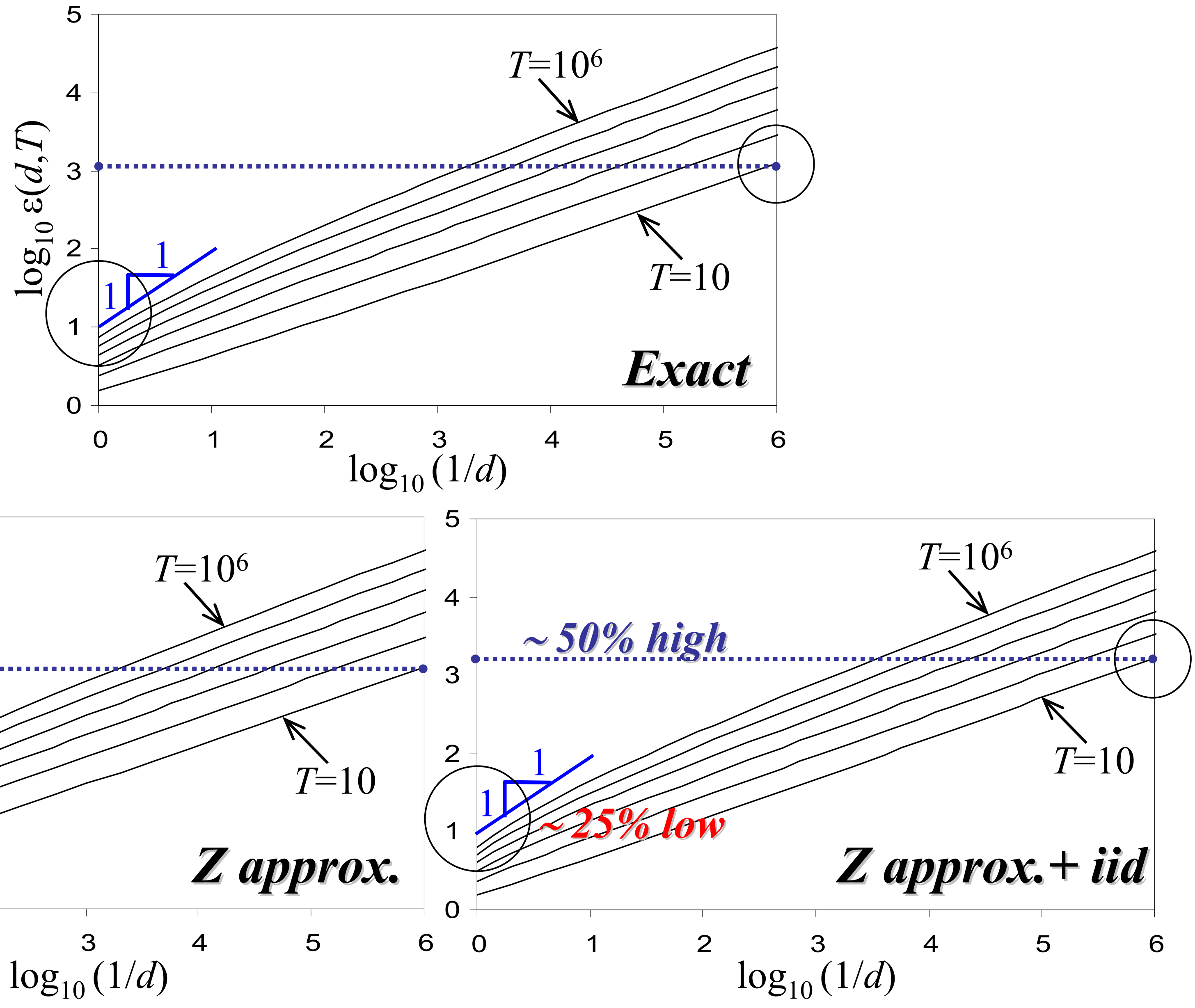
(a) $Z \approx A_{r_Z}$

(b) Assume independence among $\varepsilon_{d,i}$ densities

(c): (a) + (b)

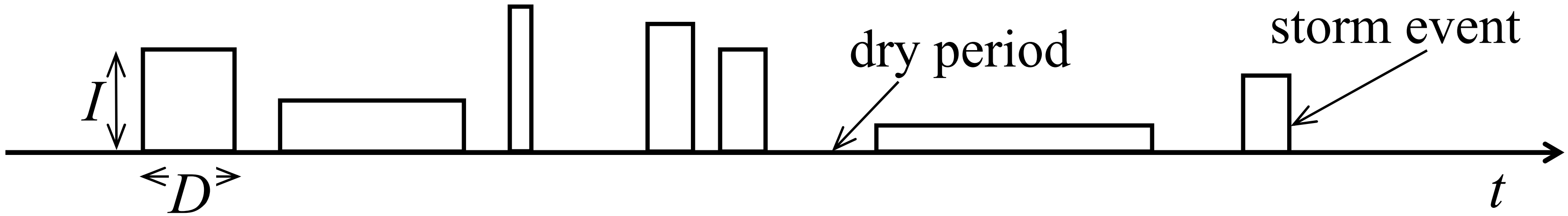


Illustration

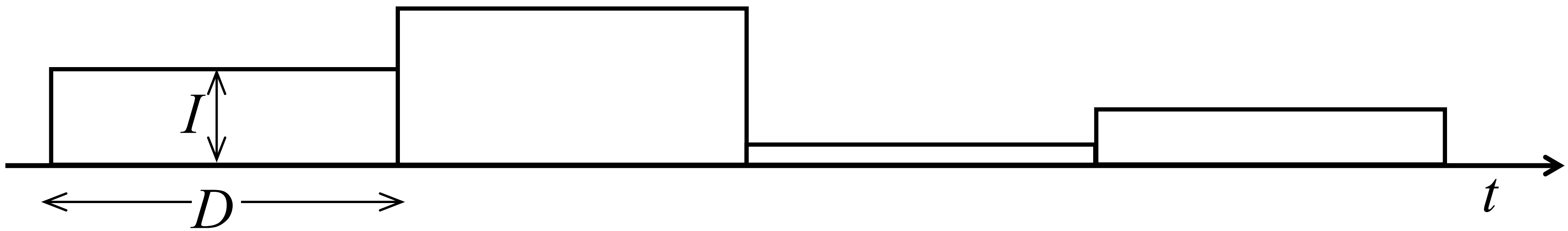


Other Rainfall Models

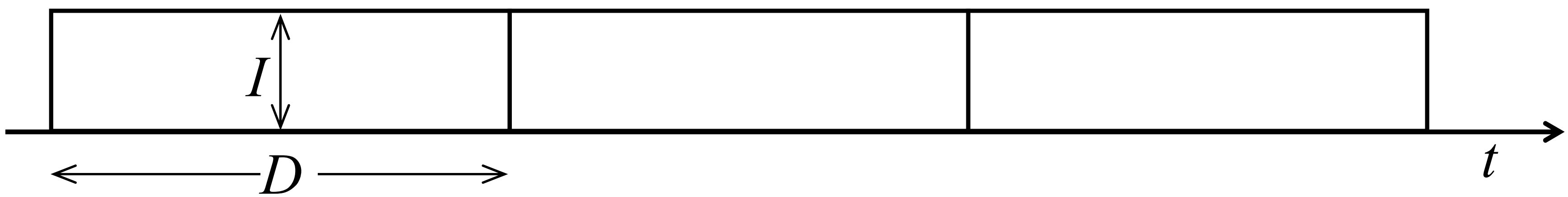
Model 1 (*I* and *D* random)



Model 2 (*I* random, *D* deterministic)



Model 3 (*I* and *D* deterministic)



Fitting Models to Data

- **Model 1**
  - **Identify storms** and find their rate λ
  - **Distribution of *D*** (3-par. gamma)
  - **Distribution of (*I*|*D*)** (shifted LN, tail-fitted)
  - $K(q)$ within storms
  ($1 < q \leq 3 \Rightarrow$ LN, $C_1 = 0.1$, same for $\neq D, I$, storm types)
- **Model 2**
  - ***D*, distribution of *I*** (tail-fitted)
  - $K(q)$ (β-LN, $0 \leq q \leq 3$)
- **Model 3**
  - ***D* and *I*** (= mean rain rate)
  - $K(q)$ (β-LN, $0 \leq q \leq 3$)

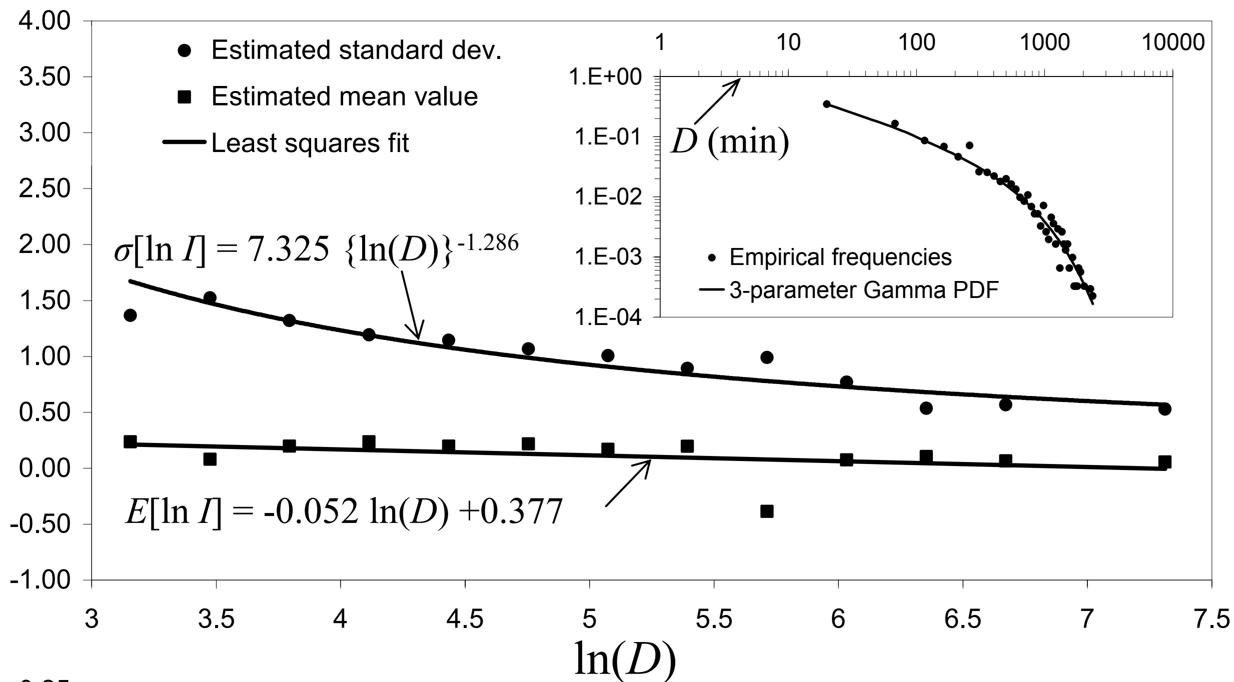
Note:

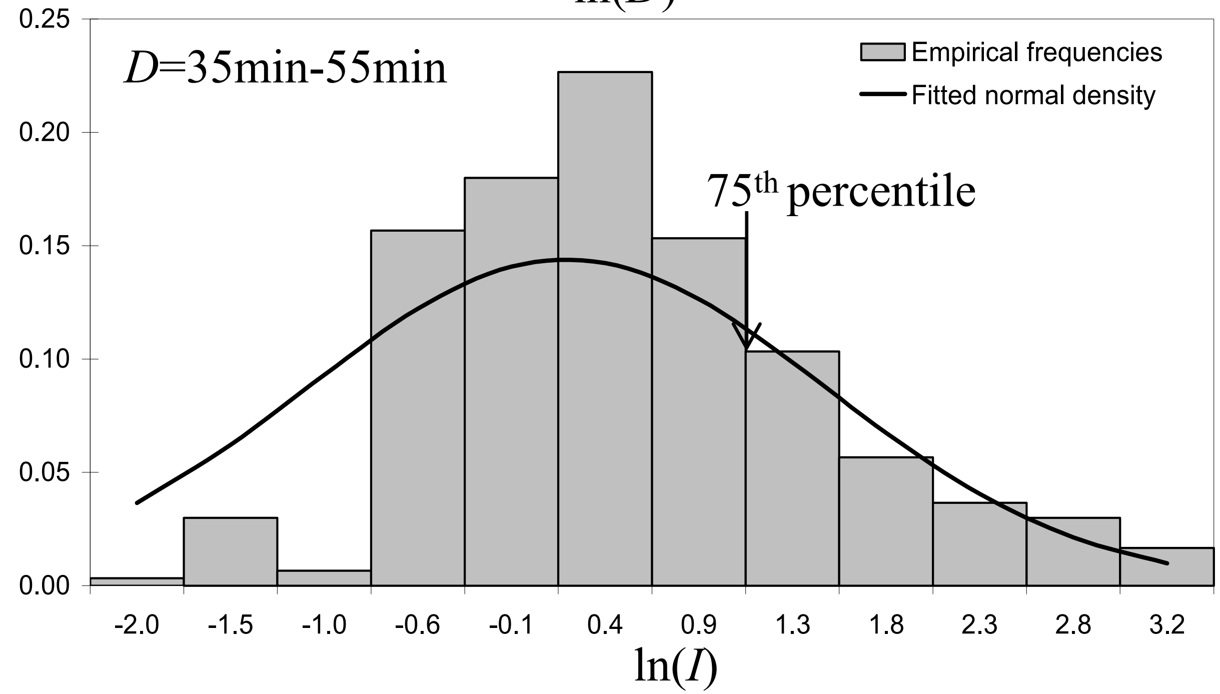
D is different in Models 2 and 3

$K(q)$ is different in different models

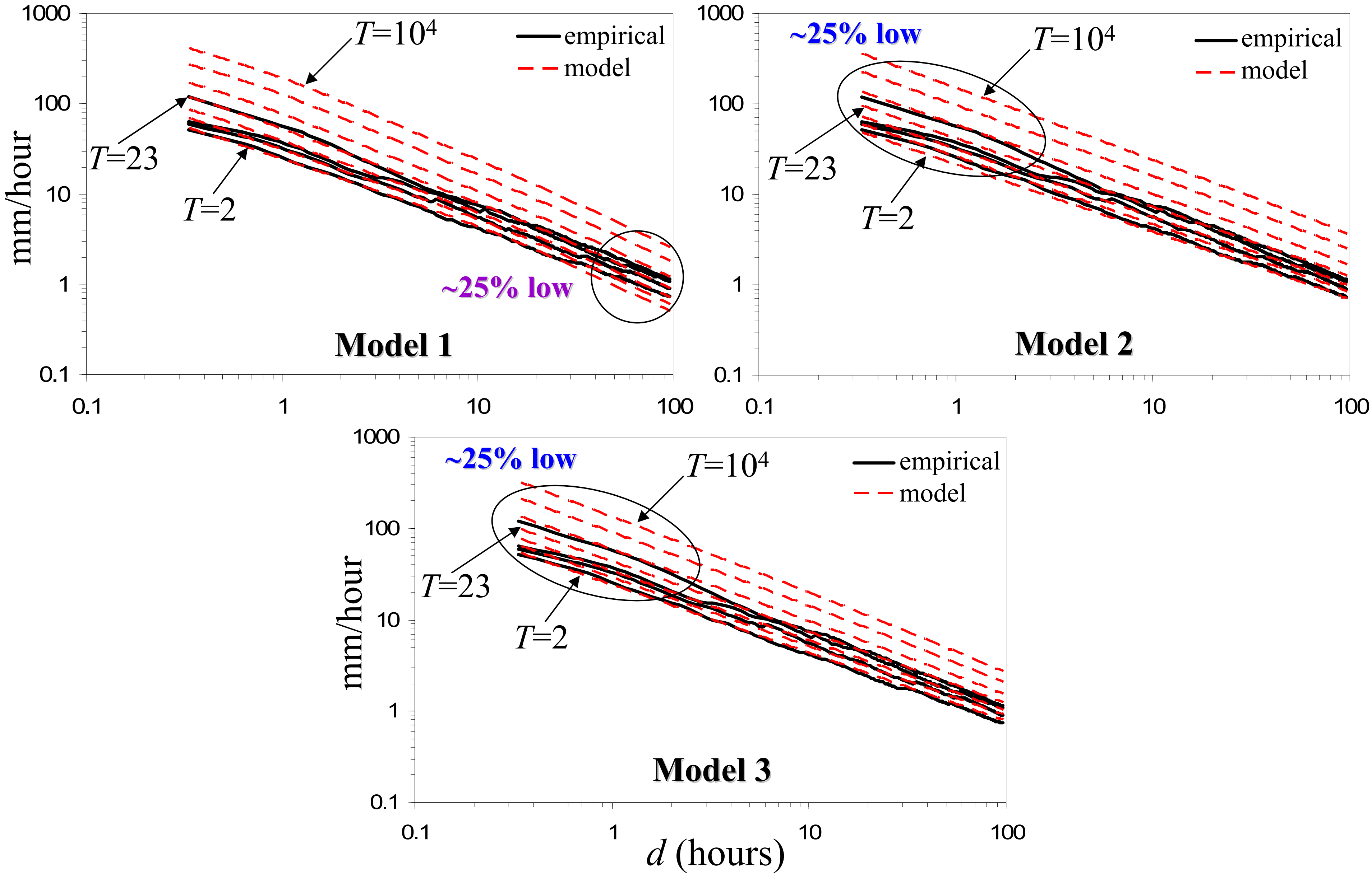
Model 1

(fit to Florence data)





Comparison with Empirical IDF



Conclusions

- **Several approaches and rainfall models** to estimate rainfall extremes (IDF, IDAF, ARF)

- **Approaches: from rough IDF scaling limits to actual values based on marginal and extreme distributions**

  - **IDF scaling** for $d \to 0$ more relevant in practice than for $T \to \infty$

  - **IDF values**: similar when using ε_d or $\varepsilon_{d,max}$. Z may be approximated

- **Models: from simple *iid* cascades to more complex models with multifractality limited to storm interiors**

  - **more detailed models** needed for storm mixtures

  - **F_I or $F_{(I|D)}$ must be tail-fitted**

  - **$K(q)$ fitted** using moments of order below $q_1 \approx 3$

  - **$K(q)$ similar** for all storm durations, intensities and types (universality?)