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Plenary lecture

Hydrology and Change

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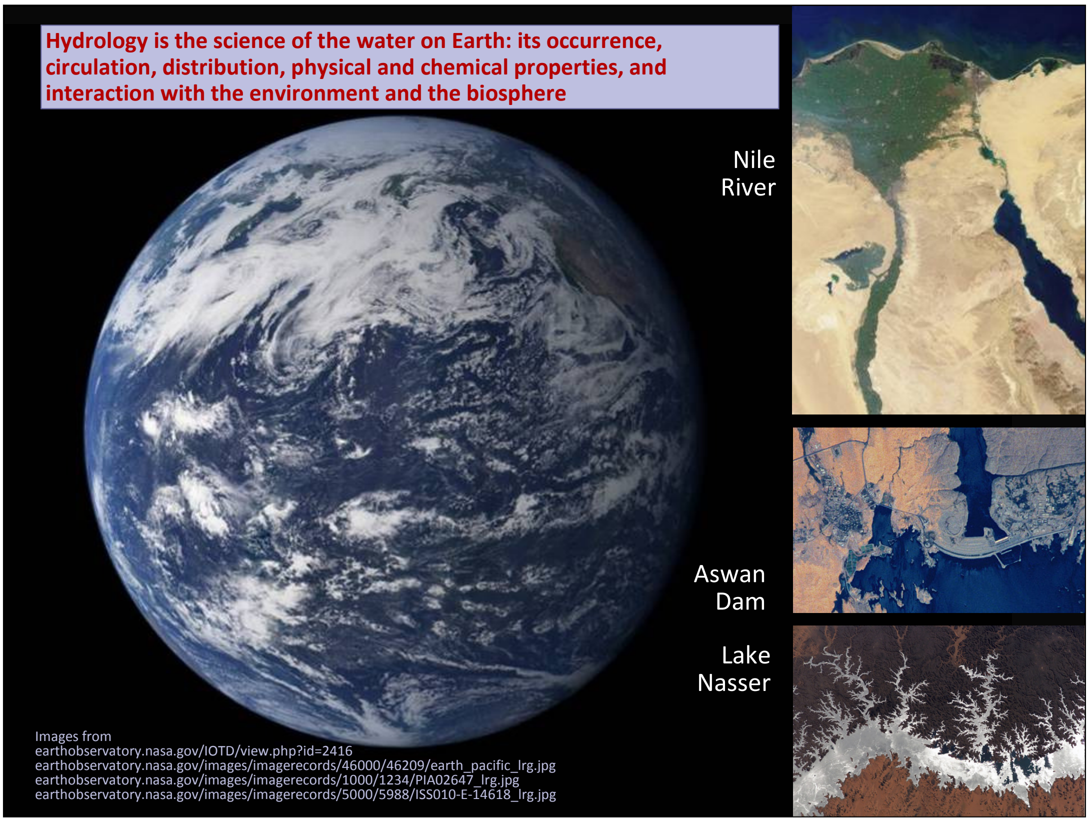
Presentation available online: itia.ntua.gr/1135/

I would like to thank the President and the officers of IUGG for the invitation to present this lecture, which is a great honour for me. I also thank all of you for attending it.

Hydrology is my field and its interplay with change is the subject of my talk.



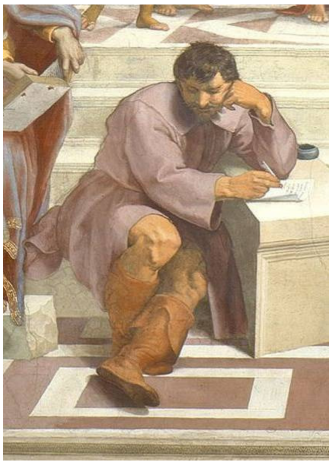
As illustrated in this slide, hydrology is the science of water on Earth. Its domain spans several temporal and spatial scales. Of course, large hydrological systems, like the Nile River are of huge importance. The image of the Nile also highlights the connection of water with life: the blue colour of the Nile is not seen, but it is seen the green of the life it gives rise to. I will refer to the Nile later. The spot on the Aswan dam on the Nile wants to emphasize the strong link of hydrology with engineering, and through this, with the improvement of diverse aspects of our live and civilization. The next spot on the Lake Nasser, upstream of the Aswan dam, indicates that the intervention of humans on Nature can really be beautiful.

I hope you can agree that the title of my talk "Hydrology and Change" harmonizes with a large body of literature, books, conferences, scientific papers and news stories, all of which scream about change. Changing planet, changing world, changing ocean, changing climate... Has our scientific community and our society only recently understood that things change?

Raphael's "**School of Athens**" (1509–1510; Apostolic Palace, Vatican City; en.wikipedia.org/wiki/School_of_Athens)

In reality, change has been studied very early, at the birth time of science and philosophy. All of these Greek philosophers you see in Raphael's "School of Athens" had something to say about change. I will only refer to two of them, the central figure of the painting, Aristotle, and this lonely figure in front, Heraclitus.

Heraclitus (*ca*. 540-480 BC)

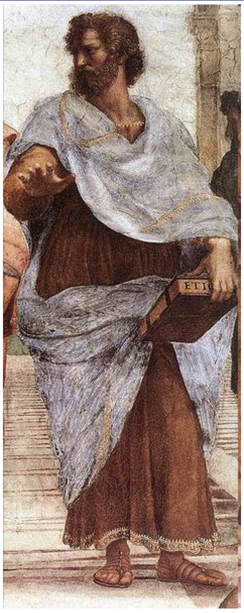


Heraclitus (figured by Michelangelo) in Raphael's School of Athens; en.wikipedia.org/wiki/Heraclitus

- *Πάντα ῥεῖ*
  Everything flows
  - Alternative versions
    - *Τὰ ὄντα ἰέναι τε πάντα καὶ μένειν οὐδέν* [from Plato's Cratylus, 401d]
      All things move and nothing remains still
    - *Πάντα χωρεῖ καὶ οὐδὲν μένει* [ibid, 402,a]
      Everything changes and nothing remains still
- *Δὶς ἐς τὸν αὐτὸν ποταμὸν οὐκ ἂν ἐμβαίης* [from Plato's Cratylus, 402a]
  You cannot step twice into the same river

Heraclitus summarized the dominance of change in a few famous aphorisms. Panta rhei--Everything flows. And: You cannot step twice into the same river (the second time you step into, it is no longer the same river). Notice the simple hydrological notions he uses to describe change: Flow and River.

Aristotle (384-322 BC) in Meteorologica



Aristotle in Raphael's "School of Athens"; en.wikipedia.org/wiki/ Aristotle

Change

- *ὅτι οὔτε ὁ Τάναϊς οὔτε ὁ Νεῖλος ἀεὶ ἔρρει, ἀλλ' ἦν ποτε ξηρὸς ὁ τόπος ὅθεν ῥέουσιν· τὸ γὰρ ἔργον ἔχει αὐτῶν πέρας, ὁ δὲ χρόνος οὐκ ἔχει. ... ἀλλὰ μὴν εἴπερ καὶ οἱ ποταμοὶ γίγνονται καὶ φθείρονται καὶ μὴ ἀεὶ οἱ αὐτοὶ τόποι τῆς γῆς ἔνυδροι, καὶ τὴν θάλατταν ἀνάγκη μεταβάλλειν ὁμοίως. τῆς δὲ θαλάττης τὰ μὲν ἀπολειπούσης τὰ δ' ἐπιούσης ἀεὶ φανερὸν ὅτι τῆς πάσης γῆς οὐκ ἀεὶ τὰ αὐτὰ τὰ μέν ἐστιν θάλαττα τὰ δ' ἤπειρος, ἀλλὰ* ***μεταβάλλει τῷ χρόνῳ πάντα*** [I.14, 353a 16]

  Neither the Tanais *[River Don in Russia]* ***nor the Nile have always been flowing, but the region in which they flow now was once dry****: for their life has a bound, but time has not… But if* ***rivers are formed and disappear*** *and the same places were not always covered by water, the sea must change correspondingly. And if the sea is receding in one place and advancing in another it is clear that* ***the same parts of the whole earth are not always either sea or land****, but that* ***all changes in course of time***

Conservation of mass within the hydrological cycle

- *ὥστε οὐδέποτε ξηρανεῖται· πάλιν γὰρ ἐκεῖνο φθήσεται καταβὰν εἰς τὴν αὐτὴν τὸ προανελθόν* [II.3, 356b 26]

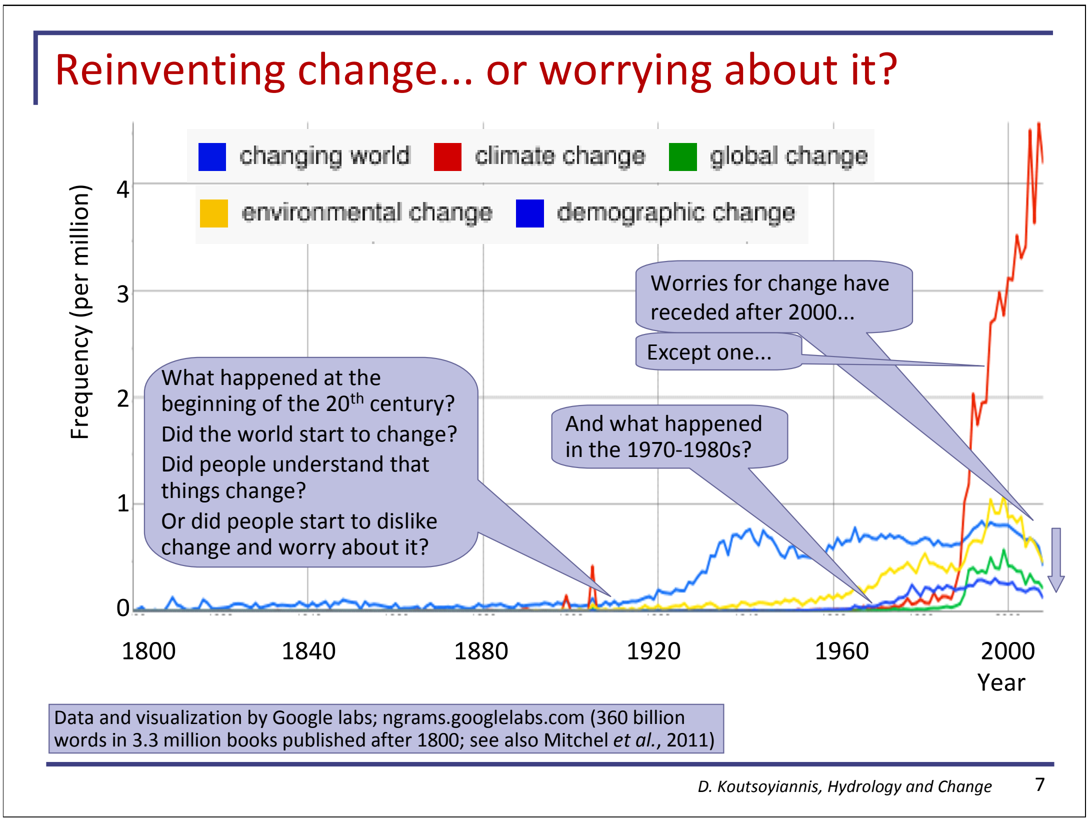
  Thus, [the sea] will never dry up; for [the water] that has gone up beforehand will return to it

- *κἂν μὴ κατ' ἐνιαυτὸν ἀποδιδῷ καὶ καθ' ἑκάστην ὁμοίως χώραν, ἀλλ' ἔν γέ τισιν τεταγμένοις χρόνοις ἀποδίδωσι πᾶν τὸ ληφθέν* [II.2, 355a 26]

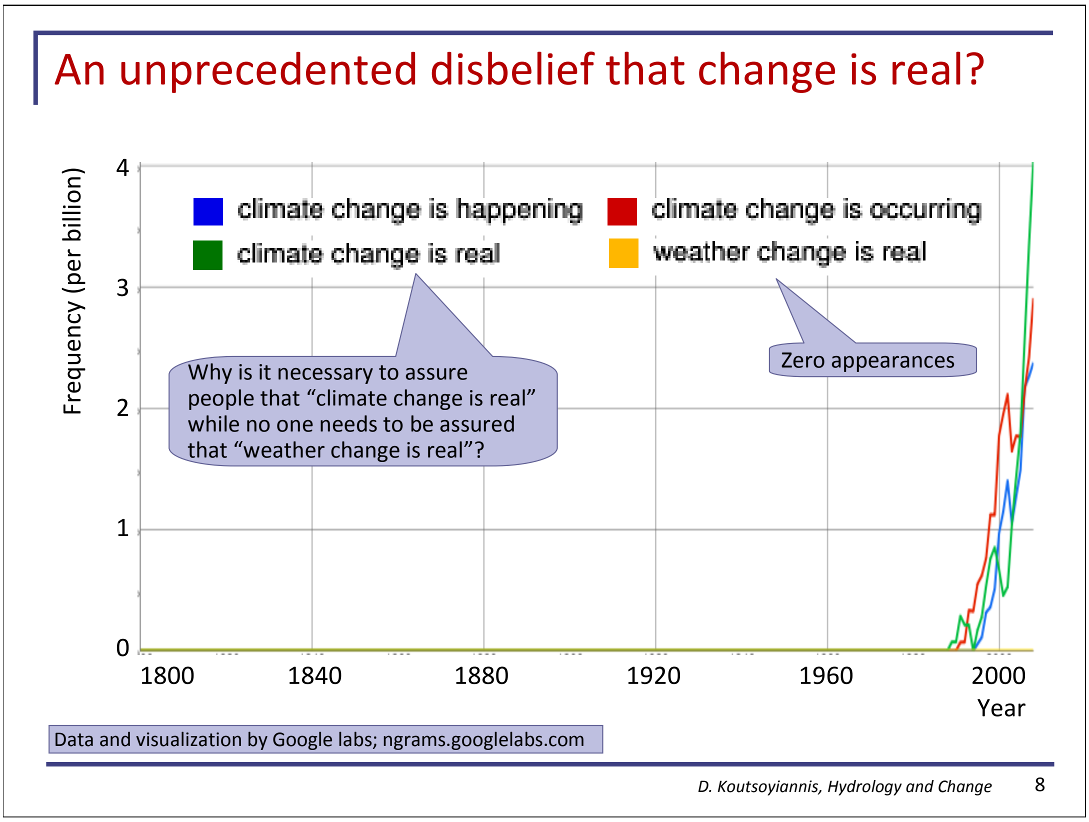
  Even if the same amount does not come back every year or in a given place, yet in a certain period all quantity that has been abstracted is returned

I found it amazing that Aristotle had understood the scale and extent of change much better than some contemporary scholars do: Neither the Tanais, nor the Nile have always been flowing, but the region in which they flow now was once dry. Rivers are formed and disappear. The same parts of the whole earth are not always either sea or land. All changes in course of time.

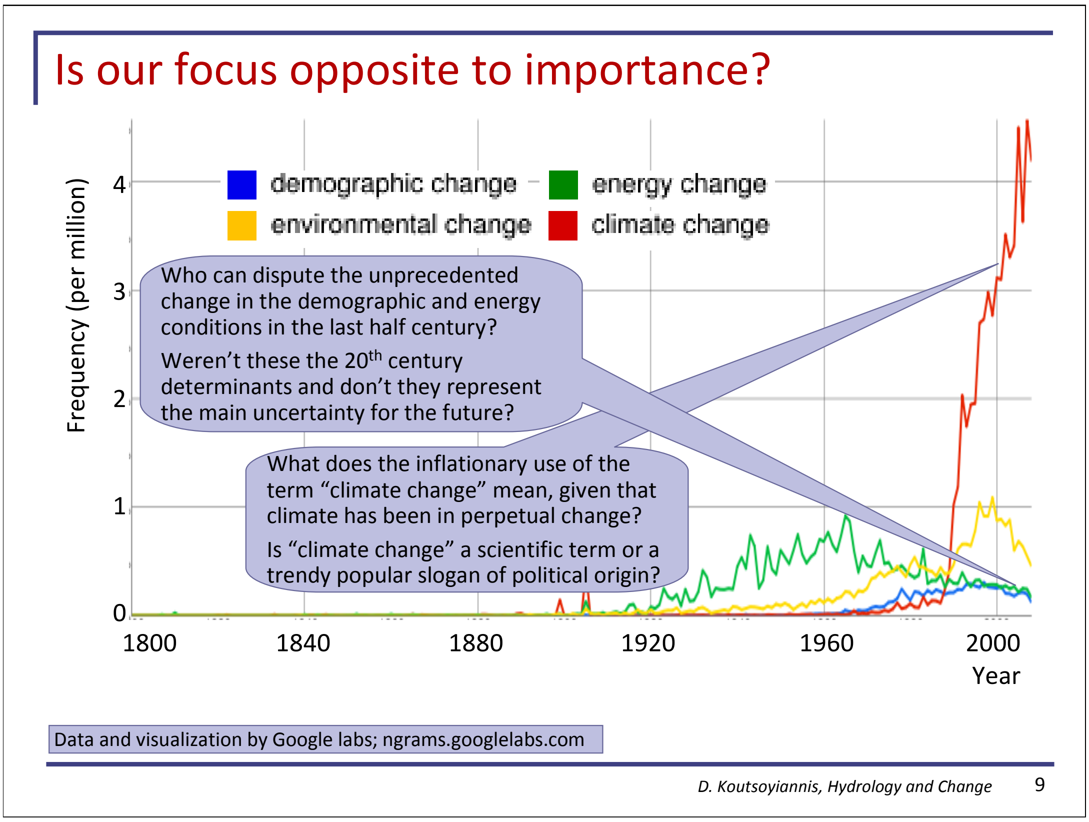
He also understood and neatly expressed the conservation of mass within the hydrological cycle.



Let us come back to the present and try to investigate our perception of change in quantified terms. To this aim, I found the information provided recently by the Google Labs very useful. This information is the frequency per year of each word or phrase in some millions of books. Here you see that at about 1900 people started to speak about a “changing world” and a little later about “environmental change”. “Demographic change”, “climate change” and “global change” “start” at 1970s and 1980s. In my view, the intense use of these expressions reflects worries for change. So, all worries have receded after 2000, except one: climate change.

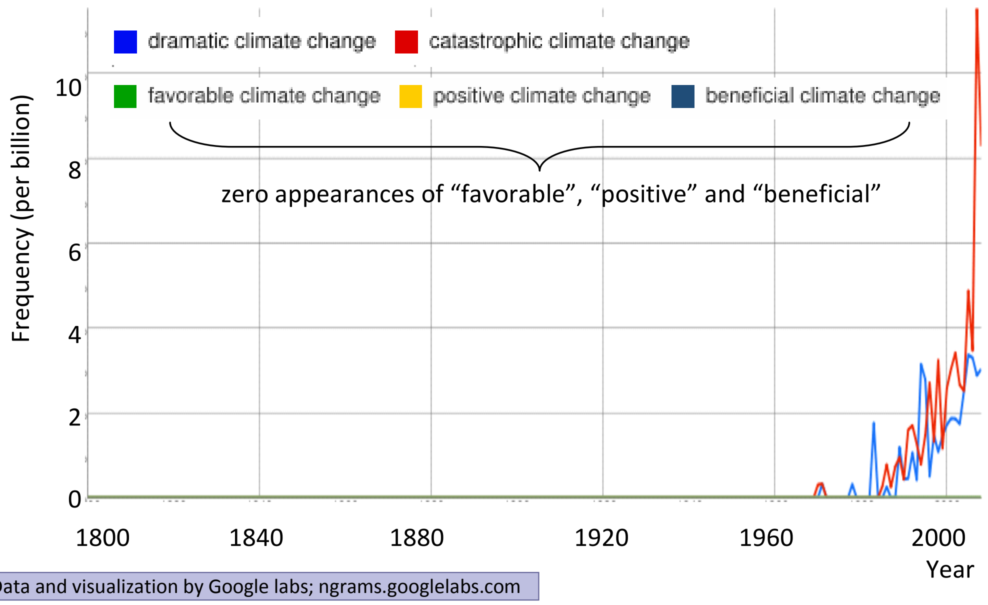


I have made a lot of searches of this type using Google labs but I do not have the time to discuss them in detail. Anyhow, this is the funniest: The statements “climate change is happening”, “... is occurring”. “... is real” contain the same truth as “weather change is real”. You can speculate why the former has been in wide use, while the latter has not.



You can also think and speculate whether or not the more frequent use of a phrase indicates higher importance or the opposite.

Is change only negative?



Data and visualization by Google labs; ngrams.googlelabs.com

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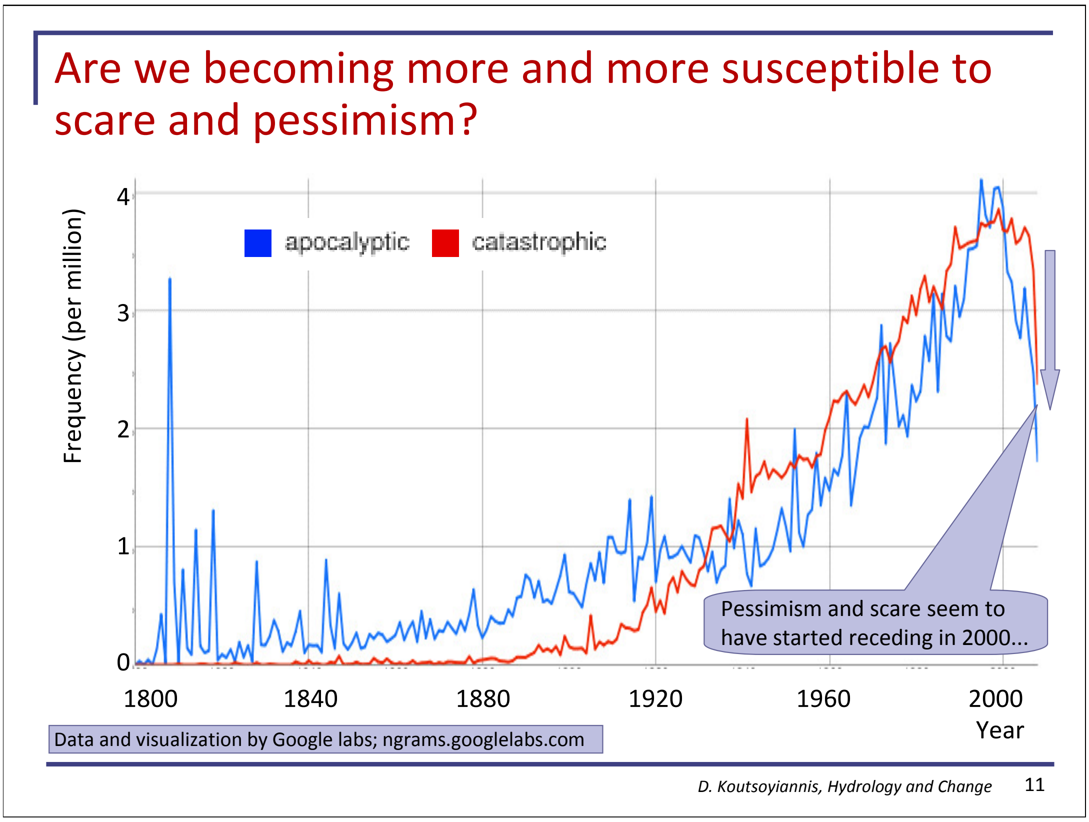
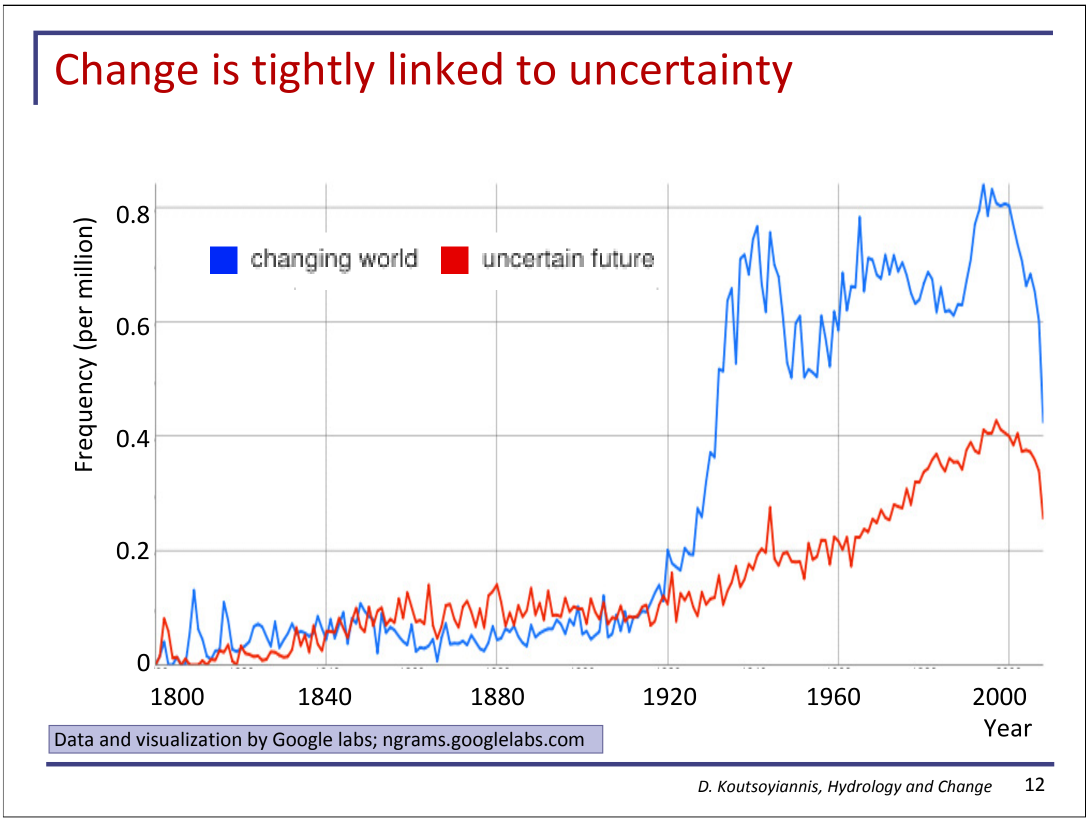
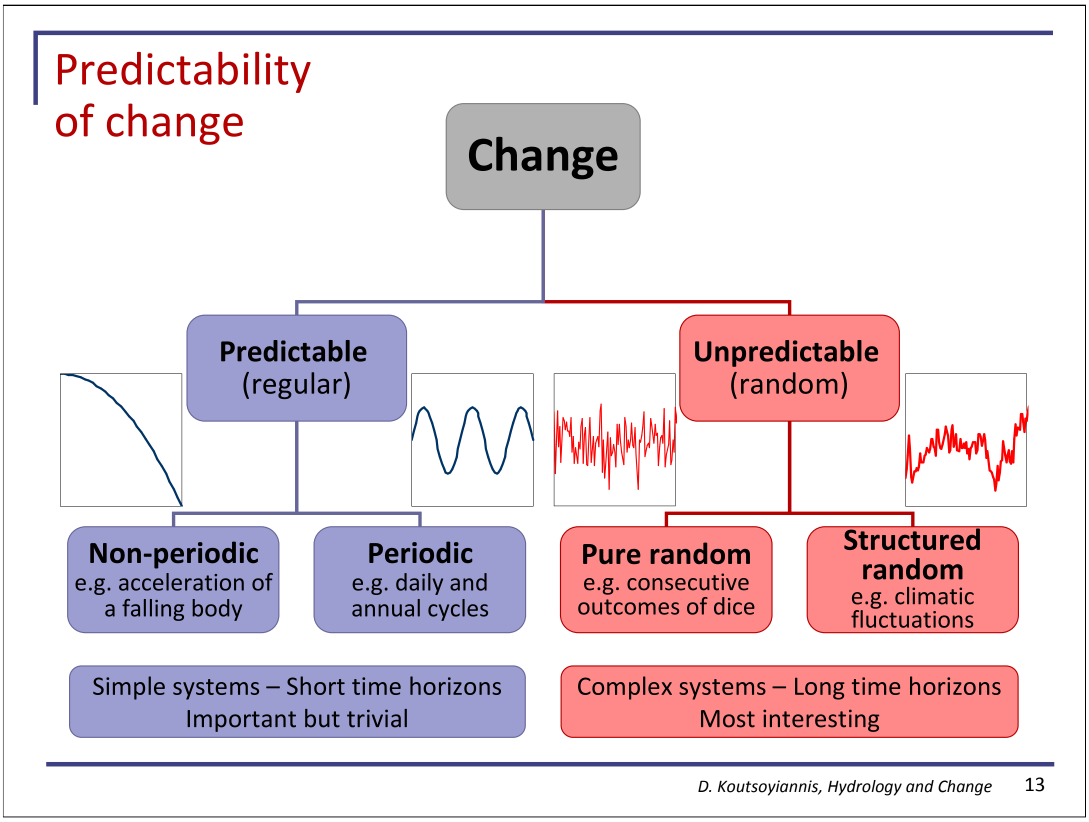
You can further think whether, according to conventional wisdom, change can only be dramatic and catastrophic.



Even removing “climate” or “climate change” from our phrases, we can see that our society may like scare and pessimism, as testified by the more and more frequent use of words like “apocalyptic” and “catastrophic”.



In any case, I think that there is no doubt that the world is changing and, also, that the future is uncertain. Change is tightly linked to uncertainty.



This hierarchical chart wants to say that in simple systems the change is regular. The regular change can be periodic or aperiodic. Whatever it is, using equations of dynamical systems, regular change is predictable at short time horizons. But this type of change is rather trivial. More interesting are the more complex systems at long time horizons, where change is unpredictable in deterministic terms, or random. Pure randomness, like in classical statistics, where different variables are identically distributed and independent, is sometimes a useful model, but in most cases it is inadequate. A structured randomness should be assumed instead. As we will see in the next slides, the structured randomness is enhanced randomness, expressing enhanced unpredictability of enhanced multi-scale change.

What is randomness?

- Common dichotomous view:
  - Natural process are composed of two different, usually additive, parts or components—deterministic (signal) and random (noise)
  - Randomness is cancelled out at large scales and does not produce change; only an exceptional forcing can produce a long-term change
- My view (explained in Koutsoyiannis, 2010):
  - Randomness is none other than unpredictability
  - Randomness and determinism coexist and are not separable
  - Deciding which of the two dominates is simply a matter of specifying the time horizon and scale of the prediction
  - At long time horizons (where length depends on the system) **all is random**

Heraclitus (Fragment 52)
Αἰών παῖς ἐστι παίζων πεσσεύων· παιδός ἡ βασιληίη
Time is a child playing, throwing dice; the ruling power is a child's
Vs. **Einstein** (in a letter to Max Born in 1926)
I am convinced that He does not throw dice

This die is from 580 BC (photo from the Kerameikos Ancient Cemetery Museum, Athens)

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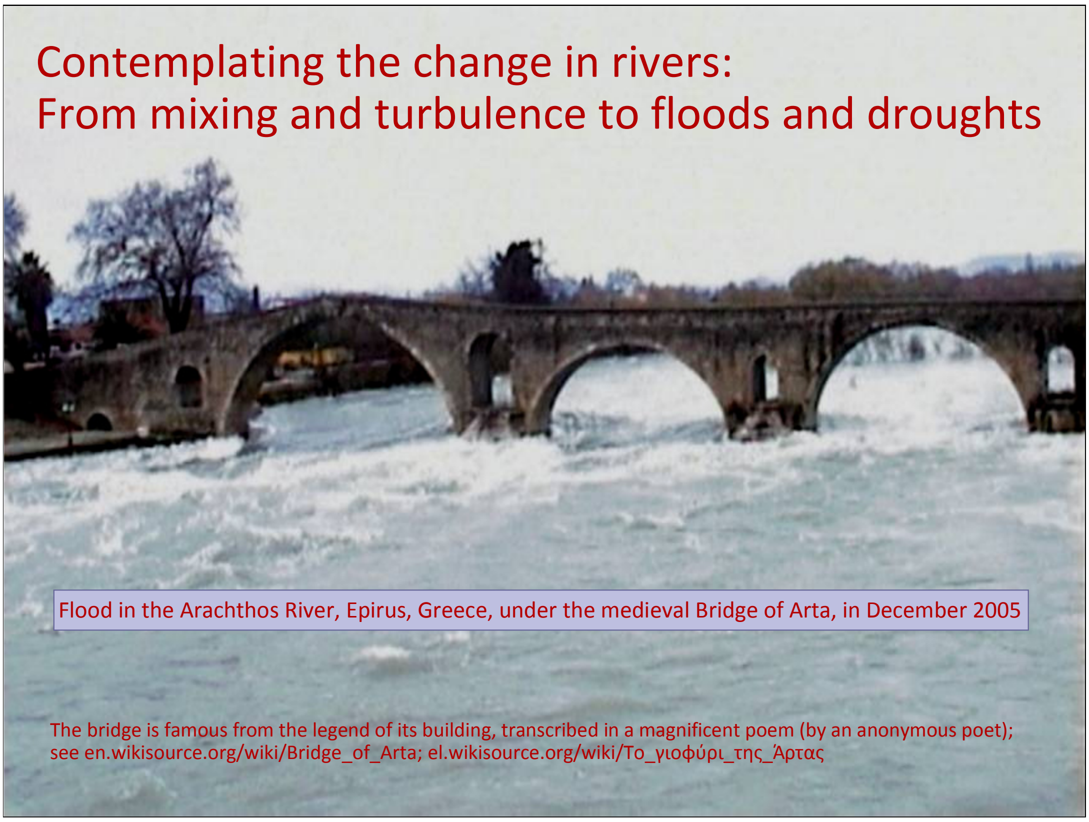
But what is randomness? According to the common dichotomous view, natural process are composed of two different, usually additive, parts or components—deterministic (signal) and random (noise). Such randomness is cancelled out at large scales and does not produce change. In this view, only an exceptional forcing can produce a long-term change.

My view (which I explain in a recent paper in HESS) is different from this. Randomness is none other than unpredictability. Randomness and determinism coexist and are not separable. Deciding which of the two dominates is simply a matter of specifying the time horizon and scale of the prediction. At long time horizons all is random.

This view has been initially proposed by Heraclitus, who said that: Time is a child playing, throwing dice; I illustrate his aphorism with a picture of a die from his epoch (580 BC). As you can see, in this case there is not much change in the dice themselves for several millennia.

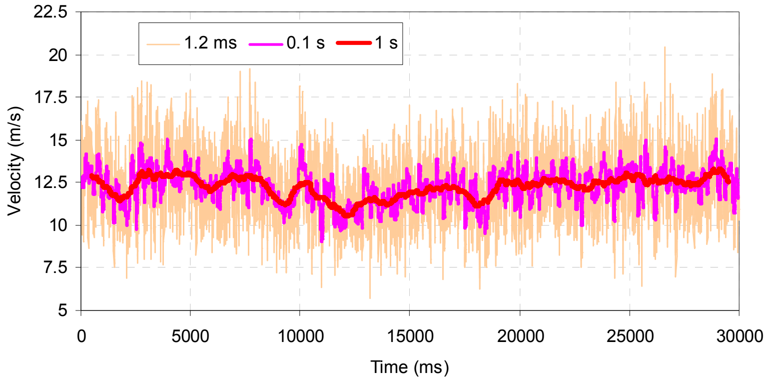


Let us now return to rivers and flows. The photo is from a flood in a famous medieval bridge close to my homeland. Several phenomena can help us contemplate the change: From mixing and turbulence to floods and droughts.

Turbulence: macroscopic motion at millisecond scale

- Laboratory measurements of nearly isotropic turbulence in Corrsin Wind Tunnel (section length 10 m; cross-section 1.22 m by 0.91 m) at a high-Reynolds-number (Kang *et al.*, 2003)
- Measurements by X-wire probes; Sampling rate of 40 kHz, here aggregated at 0.833 kHz—each point is the average of 48 original values

When I meet God, I am going to ask him two questions: Why relativity? And why turbulence? I really believe he will have an answer for the first
(attributed to **Werner Heisenberg** or, in different versions, to **Albert Einstein** or to **Horace Lamb**)



Data downloaded from www.me.jhu.edu/meneveau/datasets/Activegrid/M20/H1/m20h1-01.zip

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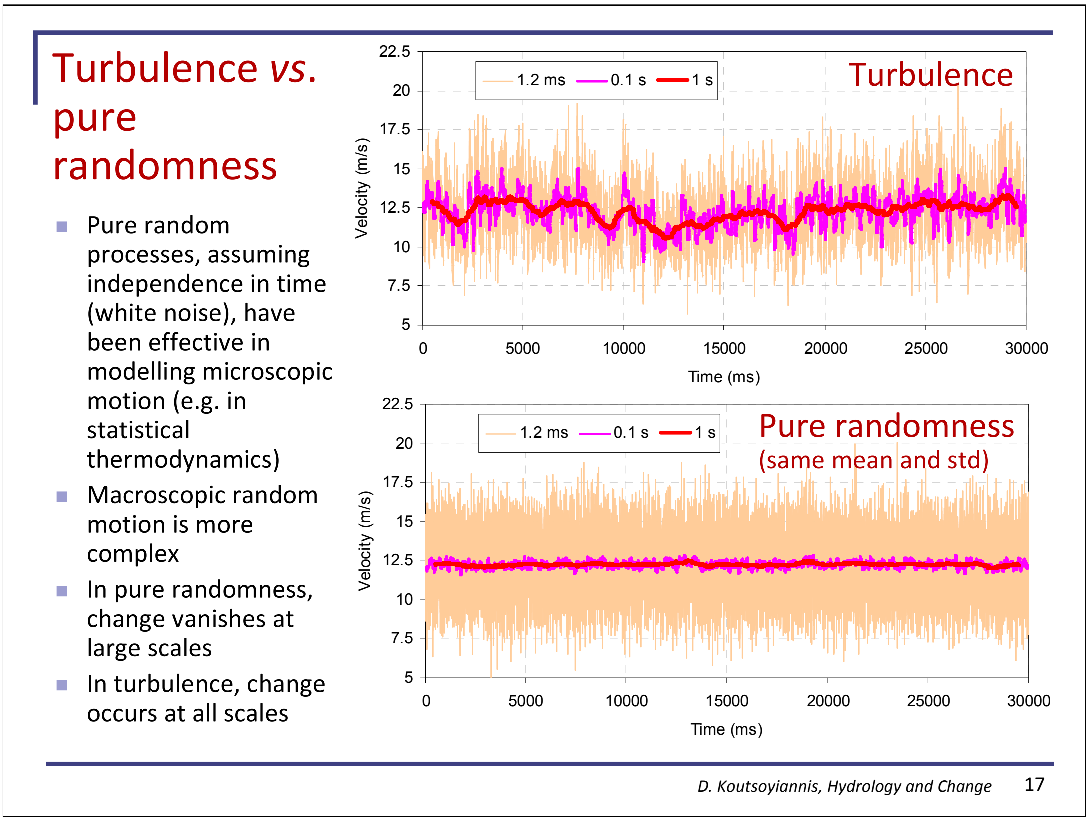
Let us start with turbulence. Turbulence is a very complex phenomenon; according to this saying attributed to Heisenberg or to Einstein, even God may not have answers about it. My view is quite different. Turbulence is God's dice game and He wants us to be aware that He plays His dice game without having to wait too much: we can see it even in time scales of milliseconds. We can see it visually, just looking at any flow, but nowadays we can also take quantified measurements at very fine time scales. In the graph each measurement represents the average velocity every 1.2 milliseconds from an experiment performed by a group of very kind scientists cited on the slide who made the data available on line at the address seen in the slide. Each value is in fact the average of 48 original data values. Averages at time scales of 0.1 and 1 second are also plotted.



To understand the change encapsulated in turbulence we can compare our graph of turbulent velocity measurements with pure random noise with the same statistical characteristics at the lowest time scale. The differences are substantial. At scales 0.1 and 1 second the pure randomness does not produce any change. In turbulence, change is evident even at these scales.

A river viewed at different time scales—from seconds to millions of years

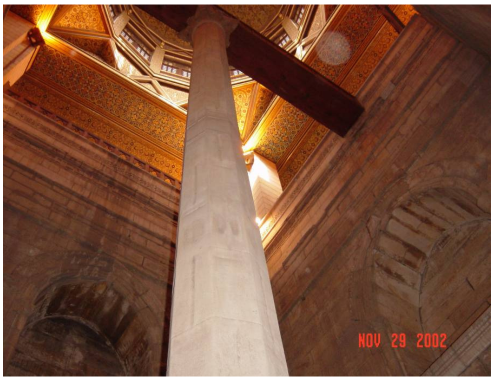
- Next second: the hydraulic characteristics (water level, velocity) will change due to turbulence
- Next day: the river discharge will change (even dramatically, in case of a flood)
- Next year: The river bed will change (erosion-deposition of sediments)
- Next century: The climate and the river basin characteristics (e.g. vegetation, land use) will change
- Next millennia: All could be very different (e.g. the area could be glacialized)
- Next millions of years: The river may have disappeared

- None of these changes will be a surprise
- Rather, it would be a surprise if things remained static
- These changes are not predictable
- Most of these changes can be mathematically modelled in a stochastic framework admitting stationarity!

So, contemplating the flow in a river, and the river itself, we may imagine several changes. Next second: the hydraulic characteristics, like water level and velocity, will change due to turbulence. Next day: the river discharge will change—even dramatically, in the case of a flood. Next year: The river bed will change because of erosion and deposition of sediments. Next century: The climate and the river basin characteristics, such as vegetation and land use, will change. Next millennia: All could be very different; for example the area could be glacialized. Next millions of years: The river may have disappeared.

None of these changes will be a surprise. Rather, it would be a surprise if things remained static. Nonetheless, these changes are not predictable. And amazingly, as we will see, most of these changes can be mathematically modelled in a stochastic framework admitting stationarity! Here we should distinguish the term “change” from “nonstationarity” which has been recently been used in statements like “a nonstationary world”. The world is neither stationary nor nonstationary. The world is real and unique. It is our models that can be stationary or nonstationary as the latter terms need the notion of an ensemble (e.g. of simulations) to apply.

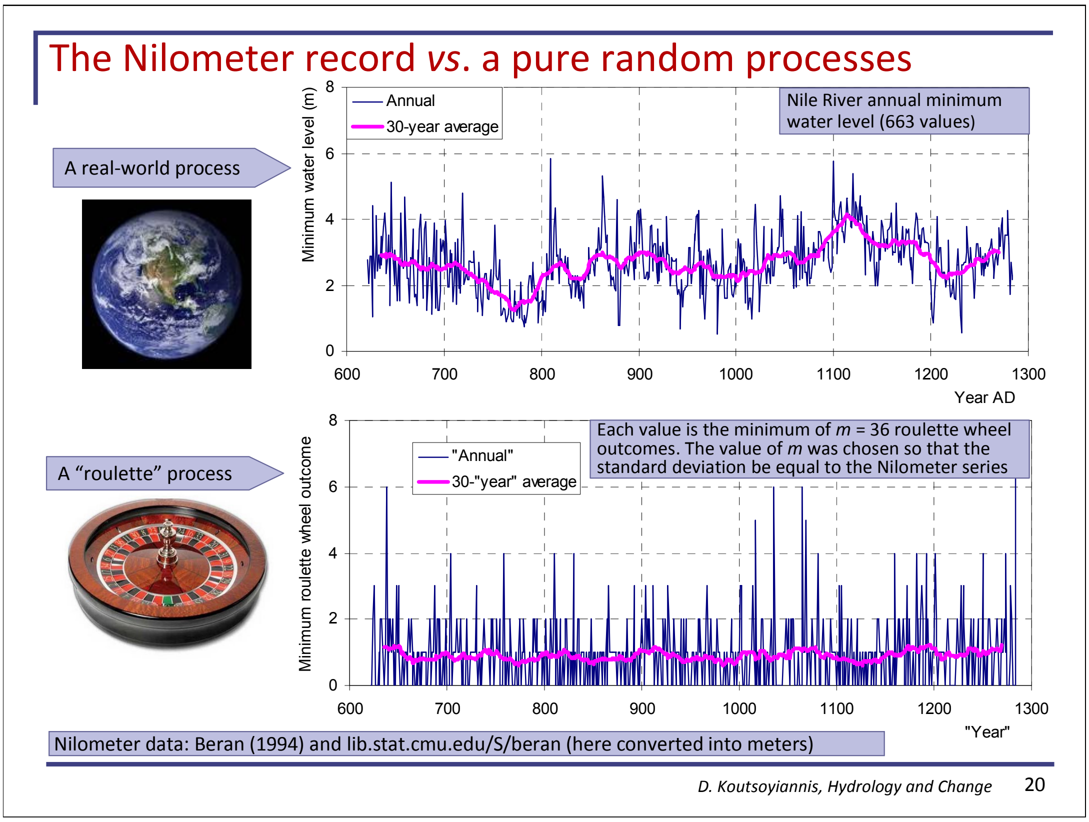
The Roda Nilometer and over-centennial change



- The Roda Nilometer as it stands today; water entered and filled the Nilometer chamber up to river level through three tunnels
- In the centre of the chamber stands a marble octagonal column with a Corinthian crown; the column is graded and divided into 19 cubits (a cubit is slightly more than half a meter) and could measure floods up to about 9.2 m
- A maximum level below the 16th mark could portend drought and famine; a level above the 19th mark meant catastrophic flood

(Credit: Aris Georgakakos)

Are these changes just imagination or speculation? Certainly not. The information abounds on the earth's crust and that is why Aristotle was able to understand the extent of change. In addition, we have records of quantified change. The longest instrumental record is that of the water level of the Nile River. It was taken at the Roda Nilometer near Cairo and extends for more than six centuries. As you see, the measurements were taken at a robust structure, much more elaborate than today's measuring devices.



The upper graph shows the annual minimum water level of the Nile for 663 years: from 622 to 1284 AD. May I repeat that it is not a proxy, but an instrumental record. Apart from the annual values, 30-year averages are also plotted. These we commonly call climatic values. Due to the large extent of the Nile basin, these reflect the climate evolution of a very large area in the tropics and subtropics. Notice that around 780 AD the climatic value was 1.5 meters, while at 1110 AD it was 4 meters. 2.5 times higher.

In the lower panel we can see a simulated series from a roulette wheel which has equal variance as the Nilometer. Despite equal "annual" variability, the roulette wheel produces a static "climate".

Hurst’s (1951) seminal paper

- The motivation of Hurst was the design of the High Aswan Dam on the **Nile** River
- However the paper was theoretical and explored numerous data sets of diverse fields
- Hurst observed that: *Although in random events **groups of high or low values** do occur, **their tendency to occur in natural events is greater**. This is the main difference between natural and random events*

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2447

LONG-TERM STORAGE CAPACITY OF RESERVOIRS

BY H. E. HURST[1]

WITH DISCUSSION BY VEN TE CHOW, HENRI MILLERET, LOUIS M. LAUSHEY, AND H. E. HURST.

SYNOPSIS

A solution of the problem of determining the reservoir storage required on a given stream, to guarantee a given draft, is presented in this paper. For example, if a long-time record of annual total discharges from the stream is available, the storage required to yield the average flow, each year, is obtained by computing the cumulative sums of the departures of the annual totals from the mean annual total discharge. The range from the maximum to the minimum of these cumulative totals is taken as the required storage.

Obstacles in the dissemination and adoption of Hurst’s finding:

- Its direct connection with reservoir storage
- Its tight association with the Nile
- The use of a complicated statistic (the rescaled range)

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The first who noticed this behaviour, and in particular the difference of pure random processes and natural processes, was the British hydrological engineer Hurst who worked in Egypt. His motivation was the design of the High Aswan Dam on the Nile River. This is the first page of his seminal paper. He gave the emphasis on the clustering or grouping of similar events in natural processes.

Kolmogorov (1940)

- Kolmogorov studied the stochastic process that describes the behaviour to be discovered a decade later in geophysics by Hurst
- The proof of the existence of this process is important, because several researches, ignorant of Kolmogorov's work, regarded Hurst's finding as inconsistent with stochastics and as numerical artefact

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1940. Volume XXVI, № 2

MATHEMATIK

WIENERSCHE SPIRALEN UND EINIGE ANDERE INTERESSANTE KURVEN IM HILBERTSCHEN RAUM

Von A. N. KOLMOGOROFF, Mitglied der Akademie

Wir werden hier einige Sonderfälle von Kurven betrachten, denen meine vorhergehende Note «Kurven im Hilbertschen Raum, die gegenüber einer einparametrigen Gruppe von Bewegungen invariant sind» (1) gewidmet ist.

Unter einer Ähnlichkeitstransformation im Hilbertschen Raum H werden wir eine beliebige ... $\neq x$ der Punkte, die auf derselben ... liegen, übergeht.

Satz 6. *Die Funktion* $B_\xi(\tau_1, \tau_2)$, *die der Funktion* $\xi(t)$ *der Klasse* $\mathfrak{A}$ *entspricht, kann in der Form*

$$B_\xi(\tau_1, \tau_2) = c[|\tau_1|^\gamma + |\tau_2|^\gamma - |\tau_1 - \tau_2|^\gamma]$$

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- Kolmogorov's work did not become widely known
- The process was named by Kolmogorov "Wiener's Spiral" (Wienersche Spiralen) and later "Self-similar process", or "fractional Brownian motion" (Mandelbrot and van Ness, 1968); here it is called the **Hurst-Kolmogorov (HK) process**

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Strikingly, earlier the Soviet mathematician and physicist Kolmogorov, a great mind of the 20th century who gave the probability theory its modern foundation and also studied turbulence, had proposed a mathematical process that has the properties to be discovered 10 years later by Hurst in natural processes. Here we call this the Hurst-Kolmogorov (or HK) process, although it is more widely known with other names.

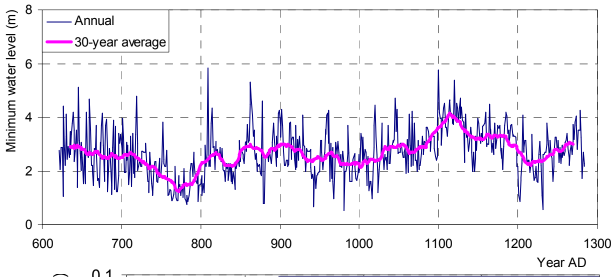
The climacogram: A simple statistical tool to quantify the change across time scales

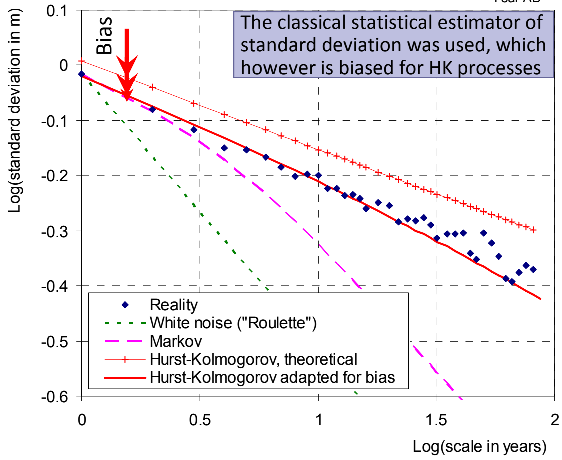
- Take the Nilometer time series, $x_1, x_2, \ldots, x_{663}$, and calculate the sample estimate of standard deviation $\sigma^{(1)}$, where the superscript (1) indicates time scale (1 year)
- Form a time series at time scale 2 (years):
  $x^{(2)}_1 := (x_1 + x_2)/2, x^{(2)}_2 := (x_3 + x_4)/2, \ldots, x^{(2)}_{331} := (x_{661} + x_{662})/2$
  and calculate the sample estimate of standard deviation $\sigma^{(2)}$
- Form a time series at time scale 3 (years):
  $x^{(3)}_1 := (x_1 + x_2 + x_3)/3,, \ldots, x^{(3)}_{221} := (x_{661} + x_{662} + x_{663})/3$
  and calculate the sample estimate of standard deviation $\sigma^{(3)}$
- Repeat the same procedure up to scale 66 (1/10 of the record length) and calculate $\sigma^{(66)}$
- The **climacogram** is a logarithmic plot of standard deviation $\sigma^{(k)}$ *vs*. scale k
- If the time series x_i represented a pure random process, the climacogram would be a straight line with slope –0.5 (the proof is very easy)
- In real world processes, the slope is different from –0.5, designated as $H - 1$, where H is the so-called Hurst coefficient ($0 < H < 1$)
- The scaling law $\sigma^{(k)} = \sigma^{(1)} / k^{1-H}$ defines the **Hurst-Kolmogorov (HK) process**
- High values of H (> 0.5) indicate **enhanced change** at large scales, else known as **long-term persistence**, or strong **clustering** (grouping) of similar values

The mathematics to describe the HK process are actually very simple as shown in this slide and the calculations can be made in a common spreadsheet.

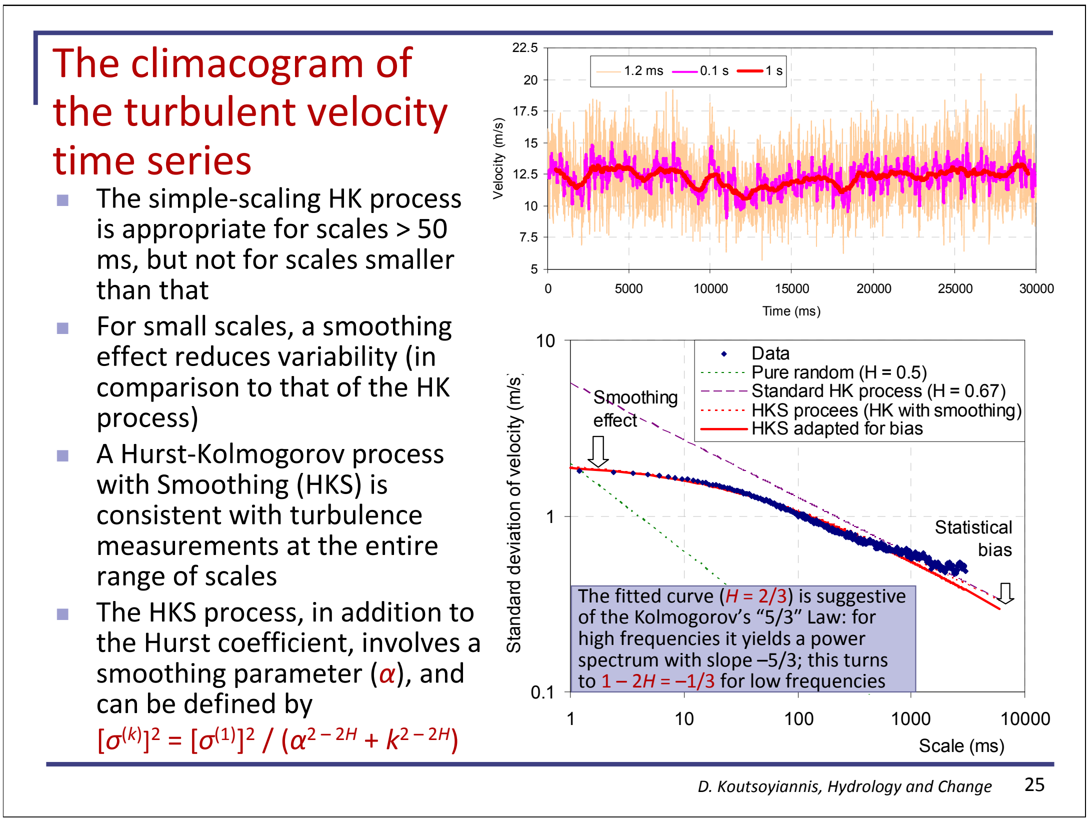
The climacogram of the Nilometer time series

- The Hurst-Kolmogorov process seems consistent with reality
- The Hurst coefficient is *H* = 0.89 (Similar *H* values are estimated from the simultaneous record of maximum water levels and from the modern, 131-year, flow record of the Nile flows at Aswan)
- Essentially, the Hurst-Kolmogorov behaviour, depicted in the climacogram, manifests that long-term changes are much more frequent and intense than commonly perceived and, simultaneously, that the future states are much more uncertain and unpredictable on long time horizons than implied by pure randomness





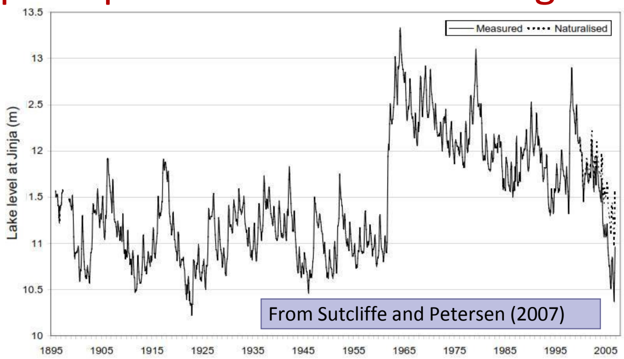
Let us see this applied to the Nilometer time series. The -0.5 slope corresponds to white noise (a pure random process). The reality departs substantially from this and is consistent with the HK behaviour with H = 0.89. Essentially, the Hurst-Kolmogorov behaviour manifests that long-term changes are much more frequent and intense than commonly perceived and, simultaneously, that the future states are much more uncertain and unpredictable on long time horizons than implied by pure randomness. Unfortunately, in literature the high value of H is typically interpreted as long-term or infinite memory. This is a bad and incorrect interpretation and the first who pointed this out was the late Vit Klemes back in 1974 (later he became the president of IAHS). So, a high value of H indicates enhanced multi-scale change and has nothing to do with memory.

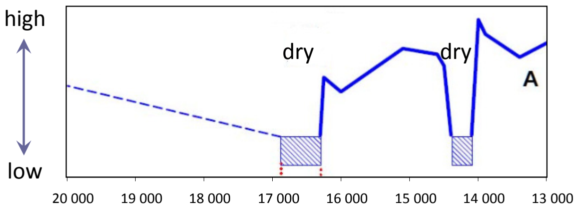


Coming back to the turbulent velocity time series, we observe that its climacogram shows a structure more complex than the simple HK process. Again, there is significant departure from the pure random process. For all time scales, the slope is milder than -0.5 and asymptotically suggests a Hurst coefficient of 2/3.

Are reconstructions of past hydroclimatic behaviours consistent with the perception of enhanced change?

- Lake Victoria is the largest tropical lake in the world (68 800 km^2) and is the headwater of the White Nile
- The contemporary record of water level (covering a period of more than a century) indicates huge changes
- Reconstructions of water level for past millennia from sediment cores (Stager *et al.*, 2011) suggest that the lake was even dried for several centuries

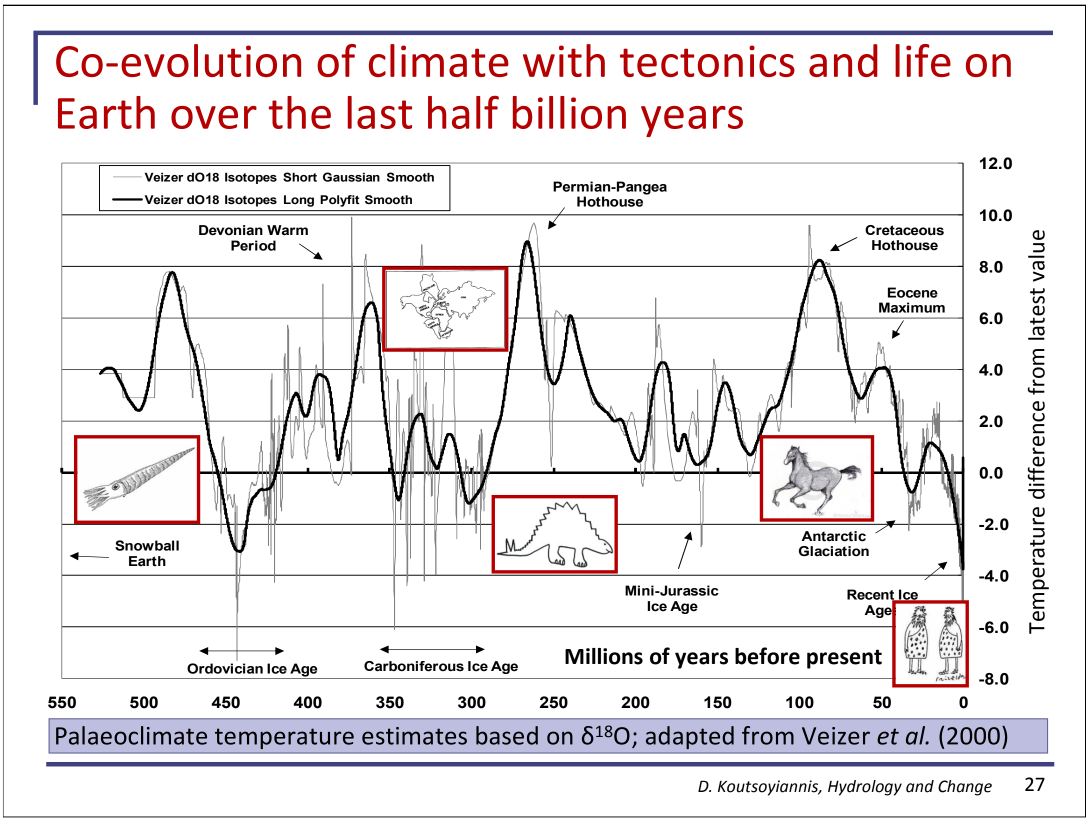




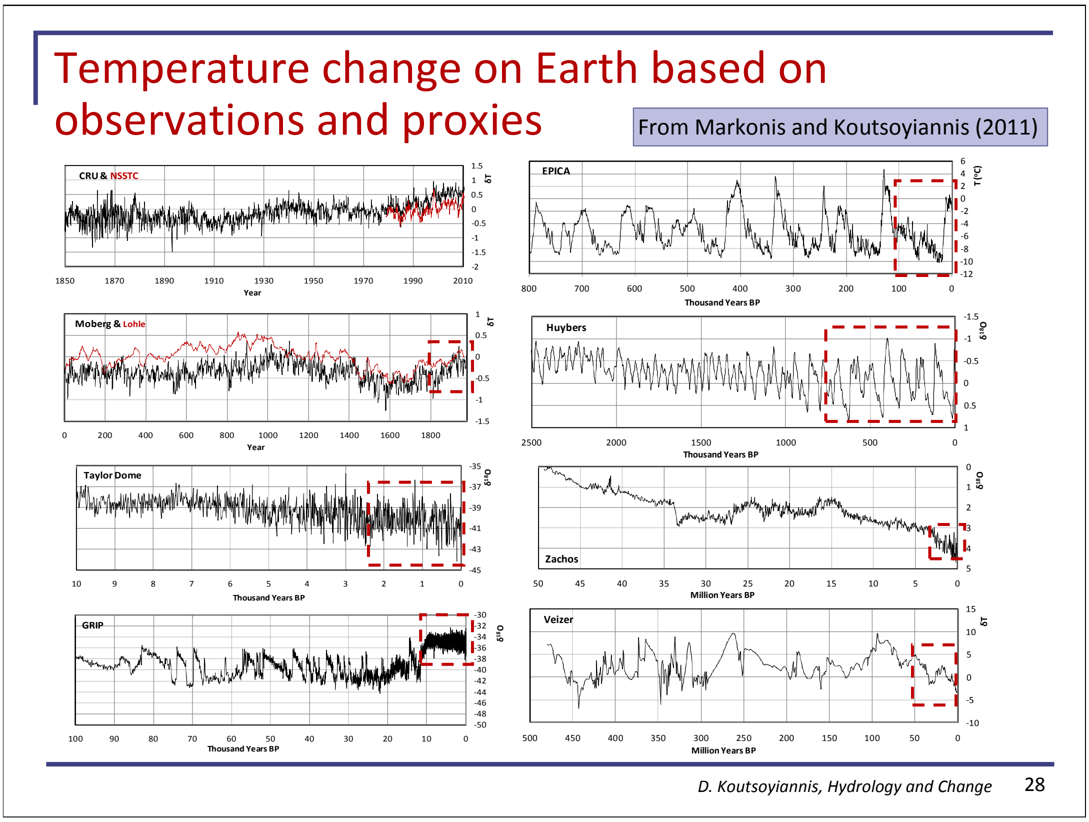
Adapted from Stager *et al.* (2011)

These two examples, the Nilometer and the turbulence, gave us an idea of enhanced change and enhanced uncertainty over long time scales. Here there is another example from hydrology. Lake Victoria is the largest tropical lake in the world and is also associated with the Nile. The contemporary record of water level (from a recent paper in Hydrological Sciences Journal) covers a period of more than a century and indicates huge changes.

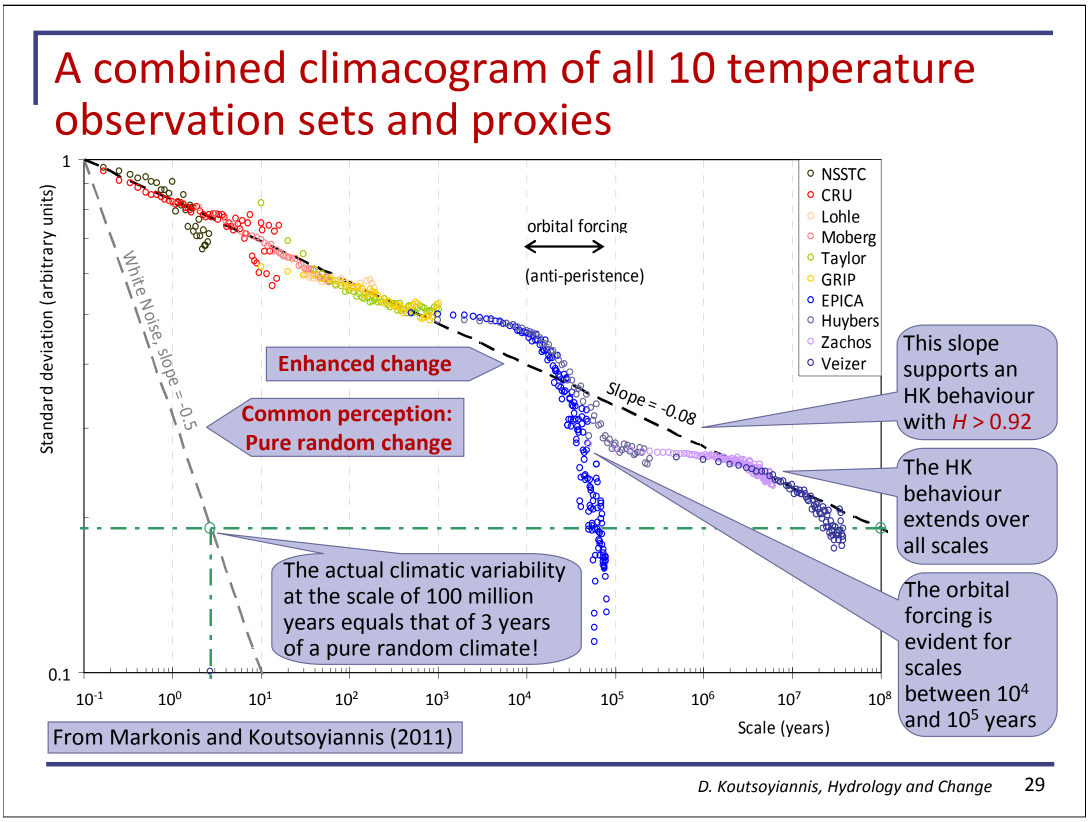
Reconstructions of water level for past millennia from sediment cores—from a brand new study cited in the slide—suggest that the lake was even dried for several centuries at about 14 and 16 thousand years before present.



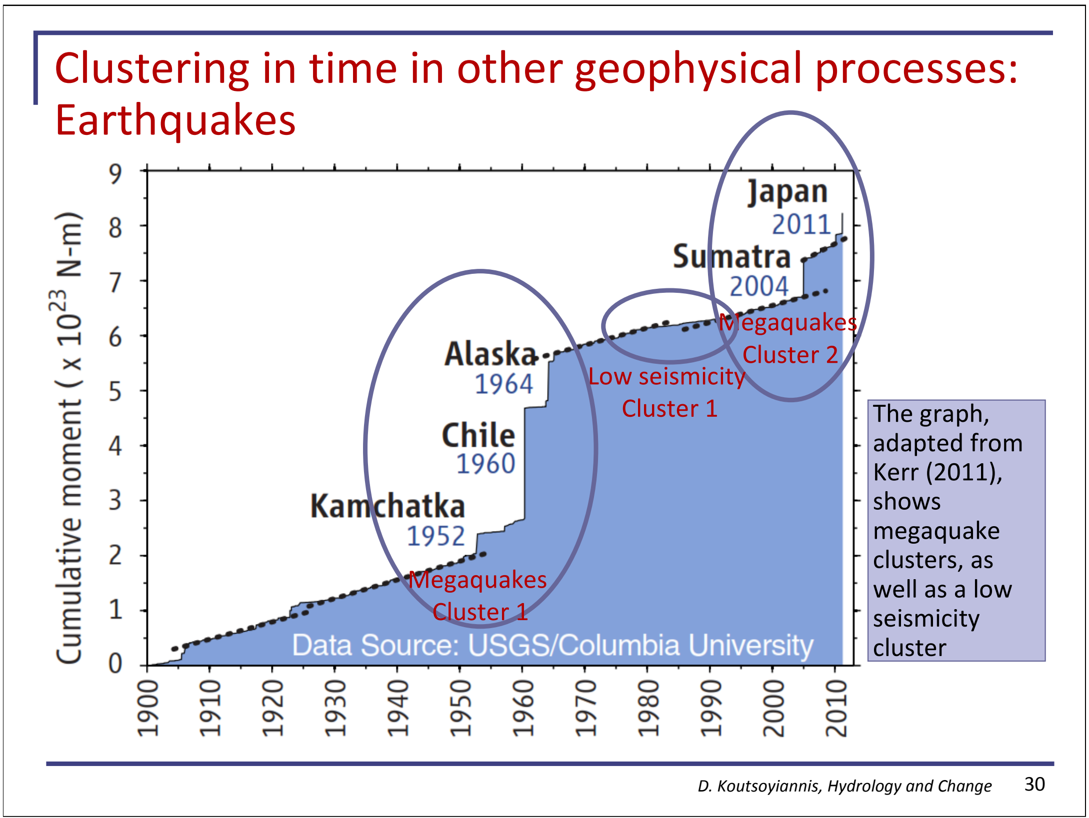
Of course, all these examples reflect changes of the climate on Earth. This graph depicts perhaps the longest palaeoclimate temperature reconstruction and is based on the ^{18}O isotope. It goes back to more than half a billion years. The change has been prominent. Today we are living in an interglacial period of an ice age. We can observe that in the past there were other ice ages, where glacial and integlacial periods alternated, as well as ages with higher temperatures free of ice. The graph also shows some landmarks providing links of the co-evolution of climate with tectonics (Pangaea) and life on Earth.



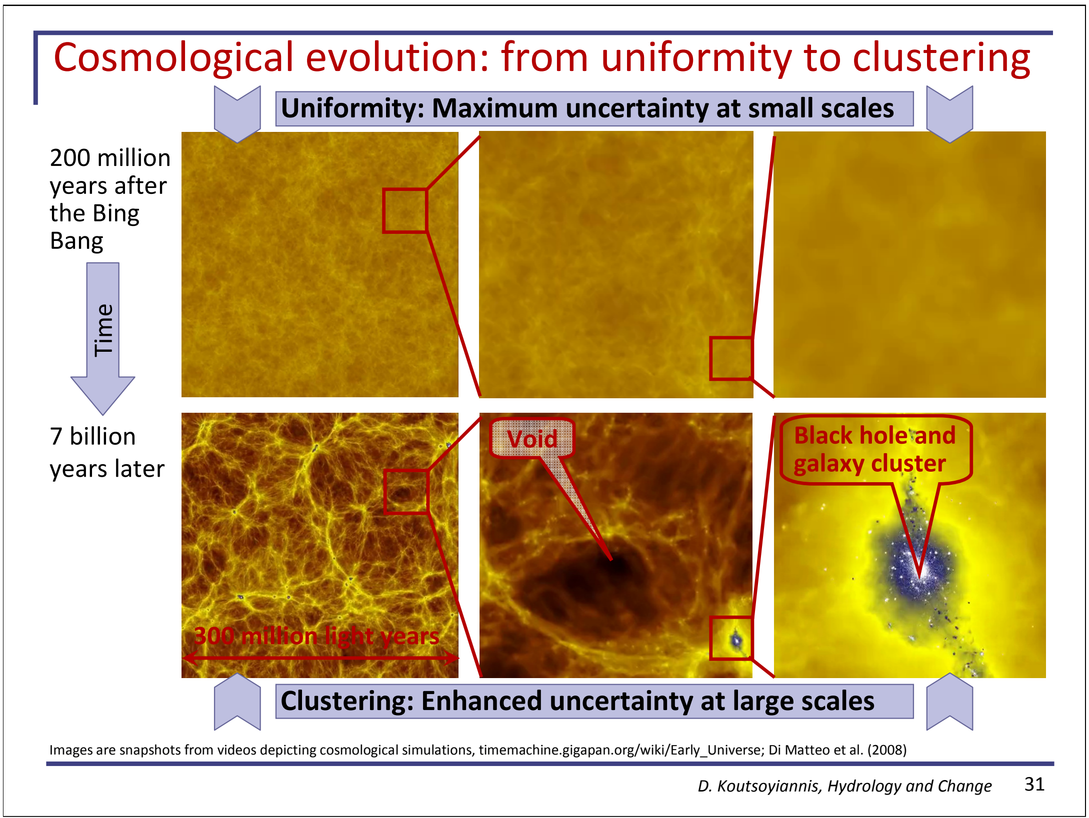
These eight graphs depict 10 data series related to the evolution of the climate, specifically temperature, on Earth. The first graph is made from instrumental records over the last 160 years and satellite observations over the last 30 years. The last panel is that of the previous slide. The red squares provide the links of the time period of each proxy with the one before.



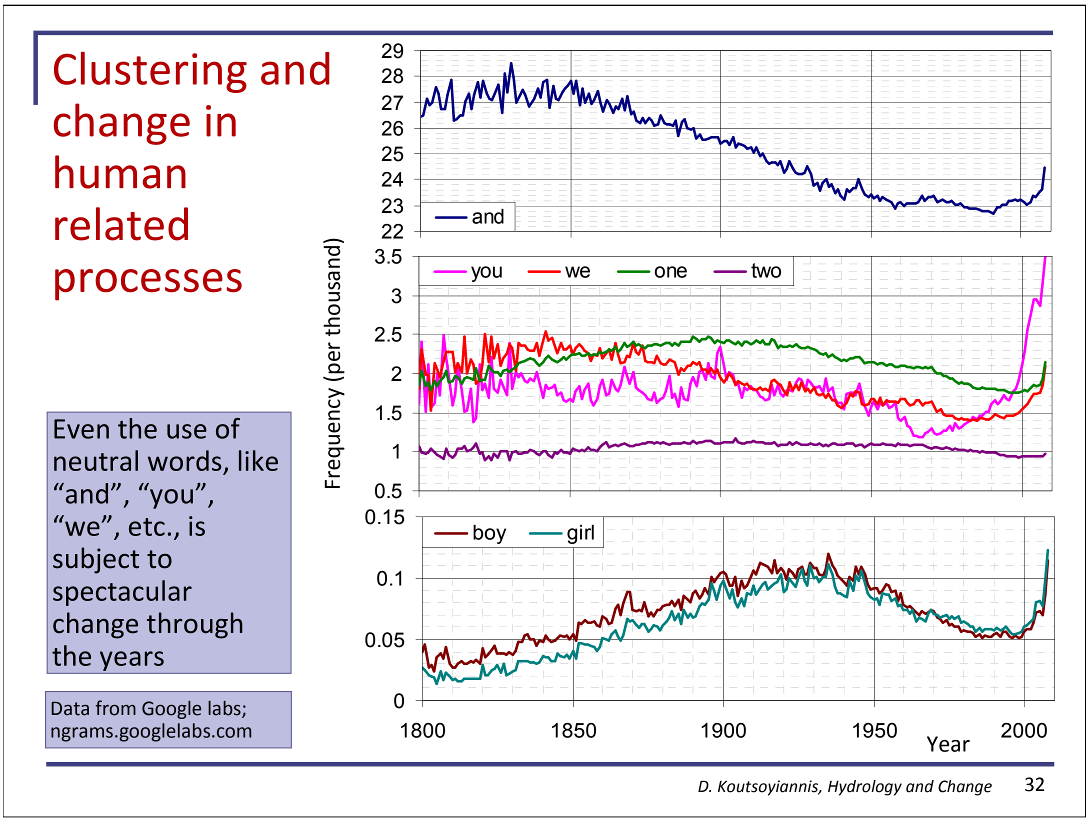
Now, if we superimpose, the climacograms of the 10 time series, after appropriate translation of the logarithm of standard deviation, but without modifying the slopes, we get this impressive overview for time scales spanning almost 9 orders of magnitude—from 1 month to nearly 100 million years. First we observe that the real-climate climacogram departs spectacularly from that of pure randomness. Second, the real climate is approximately consistent with an HK model with a Hurst coefficient greater than 0.92. Third, there is a clear deviation of the simple scaling HK law for scales between 10 and 100 thousand years, exactly as expected due to the Milancovich cycles acting at these time scales. Overall, the actual climatic variability at the scale of 100 million years equals that of about 2.5-3 years of a pure random climate! This dramatic effect should make us understand the inappropriateness of classical statistical thinking for climate. It should also help us understand what enhanced change and enhanced unpredictability are. My PhD student Yiannis Markonis and I are preparing a publication of this with further explanations (hopefully it will be published somewhere).



But the characteristics of the HK behaviour are not limited to hydrology and climate. This graph, adapted from a recent article in the Science Magazine, but with my own interpretation, shows how mega-earthquakes and periods of low seismicity cluster in time. That is, the HK behaviour is universal...

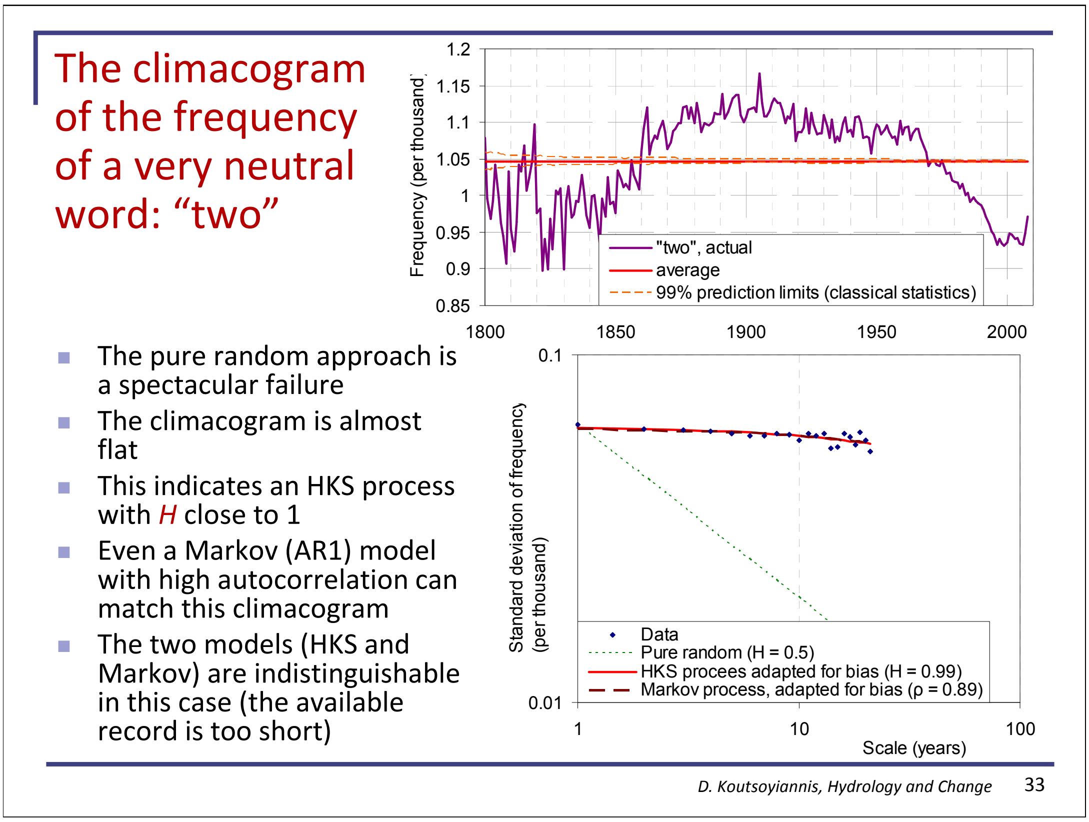


... and since it is universal, we should find it in the big picture of the universe itself. These snapshots taken from videos depicting cosmological simulation indicate how the universe evolved from the initial uniform soup to clustered structures, where the clusters are either voids or stars, galaxies and black holes.



Moreover, the HK behaviour is not a characteristic of the physical universe only, but also of mental universes constructed by humans. Here I return to the Google Labs to examine frequencies of simple and unsophisticated words. I chose very neutral and ingenuous words like “and”, “you”, “we”, “one”, “two”, “boy”, “girl”. One might expect that the frequency of using these words should be static, not subject to change. But the reality is impressively different as shown in these graphs.

The most neutral and less changing seems to be the world “two”, so let us examine it further...



A detailed plot will again reveal change different from pure randomness. The climacogram will indicate that the change is consistent with the HK behaviour with a Hurst coefficient very close to 1.

Seeking an explanation of long-term change: Entropy

- Definition of **entropy** of a random variable $\underline{z}$ (adapted from Papoulis, 1991)

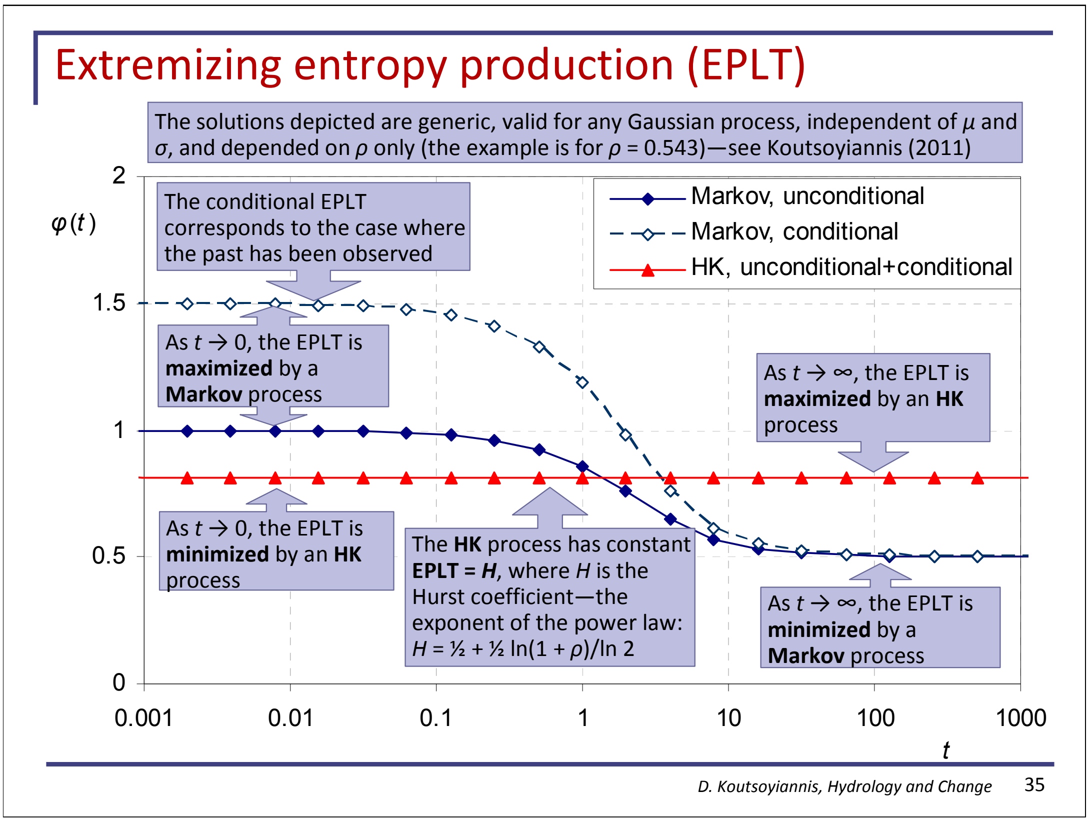
  $\Phi[\underline{z}] := E[-\ln[f(\underline{z})/l(\underline{z})]] = -\int_{-\infty}^{\infty} f(z) \ln[f(z)/l(z)]\, \mathrm{d}z$ [dimensionless]

  where $f(z)$ the probability density function, with $\int_{-\infty}^{\infty} f(z)\, \mathrm{d}z = 1$, and $l(z)$ a Lebesgue density (numerically equal to 1 with dimensions same as in $f(z)$)
- Definition of **entropy production** for the stochastic process $\underline{z}(t)$ in continuous time t (from Koutsoyiannis, 2011)

  $\Phi'[\underline{z}(t)] := \mathrm{d}\Phi[\underline{z}(t)] / \mathrm{d}t$ [units T^{-1}]
- Definition of **entropy production in logarithmic time** (**EPLT**)

  $\varphi[\underline{z}(t)] := \mathrm{d}\Phi[\underline{z}(t)] / \mathrm{d}(\ln t) \equiv \Phi'[\underline{z}(t)]\, t$ [dimensionless]
- Note 1: Starting from a stationary stochastic process $\underline{x}(t)$, the cumulative (nonstationary) process $\underline{z}(t)$ is defined as $\underline{z}(t) := \int_0^t \underline{x}(\tau)\, \mathrm{d}\tau$; consequently, the discrete time process $\underline{x}_i^{\Delta} := \underline{z}(i\Delta) - \underline{z}((i-1)\Delta)$ represents stationary intervals (for time step Δ in discrete time i) of the cumulative process $\underline{z}(t)$
- Note 2: For any specified t and any two processes $\underline{z}_1(t)$ and $\underline{z}_2(t)$, an inequality relationship between entropy productions, such as $\Phi'[\underline{z}_1(t)] < \Phi'[\underline{z}_2(t)]$ holds also true for EPLTs, e.g. $\varphi[\underline{z}_1(t)] < \varphi[\underline{z}_2(t)]$

Is there a physical explanation of this universal behaviour? In my opinion, yes, and it relies on entropy. Entropy is none other than a measure of uncertainty and its maximization means maximization of uncertainty. When time is involved, the quantity that we should extremize is entropy production, a derivative of entropy.



As this graph depicts, the HK process appears to extremize entropy production and the Hurst coefficient has a clear physical meaning: It is the entropy production in logarithmic time. The details can be found in my recent paper in Physica A.

Concluding remarks

- The world exists only in change
- Change occurs at all time scales
- Change is hardly predictable in deterministic terms
- Humans are part of the changing Nature—but change is hardly controllable by humans (fortunately)
- Hurst-Kolmogorov dynamics is the key to perceive multi-scale change and model the implied uncertainty and risk
- Hydrology has greatly contributed in discovering and modelling change—however, lately, following other geophysical disciplines, it has been affected by 19th-century myths of static or clockwork systems, deterministic predictability (*cf*. climate models) and elimination of uncertainty
- A new change of perspective is thus needed in which change and uncertainty are essential parts

Both classical physics and quantum physics are indeterministic
Karl Popper (in his book "Quantum Theory and the Schism in Physics")

The future is not contained in the present or the past
W. W. Bartley III (in Editor's Foreward to the same book)

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