National Technical University of Athens

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Use of Bayesian techniques in hydroclimatic prognosis

PhD thesis defence

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Long horizons of prediction within a stochastic framework

- The General Circulation Models (GCMs) are used to predict the climate.
  - Numerical representations of the climate system and deterministic.
  - Example of projection of the surface temperature to the year 3000 (!) (IPCC 2007 p.823).
- Many criticisms regarding the validity of the results.
  - Negligible hindcast properties (e.g. Koutsoyiannis et al. 2008; Anagnostopoulos et al. 2010; Fyfe et al. 2013).
  - Cannot predict the regional climate (e.g. Handorf and Dethloff 2012; Scafetta 2013).
  - Do not model adequately the climate (e.g. Spencer and Braswell 2011; McNider et al. 2012; Stevens and Bony 2013).
- **Long horizons** of prediction are inevitably associated with **high uncertainty**, whose quantification relies on the long-term **stochastic properties of the processes** (Koutsoyiannis 2010).

Long-term persistence in predicting the climate

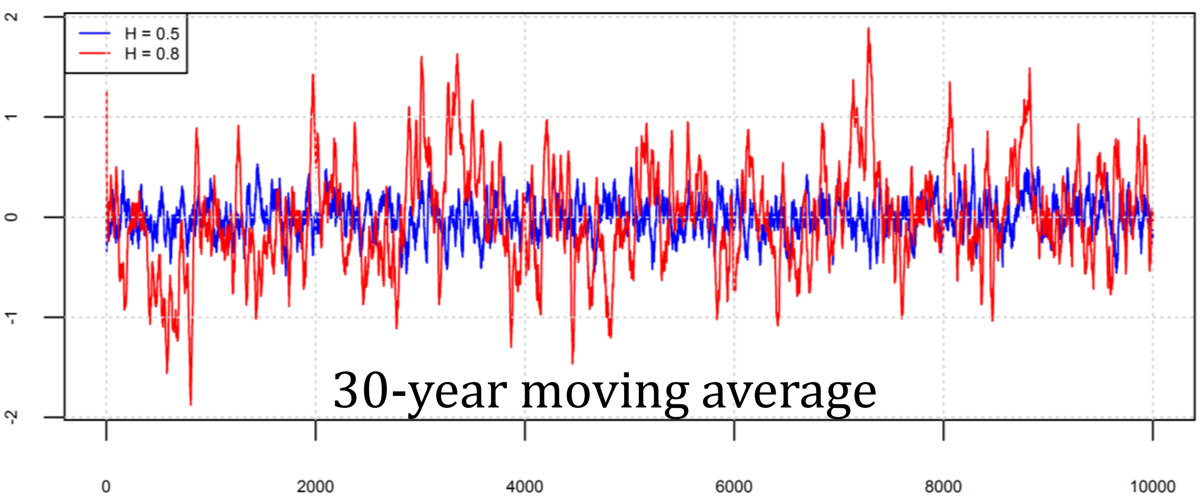
- Long-term persistence or Hurst-Kolmogorov (HK) behaviour defined by (Beran 1994, p.42)

$$\lim_{k\to\infty} \rho_k \,/\, (c\, k^{-a}) = 1,\ 0 < a < 1,\ 0 < c$$

- HK behaviour modelled by HK process (HKp) (Multivariate normal).

$$\rho_k = |k + 1|^{2H} / 2 + |k - 1|^{2H} / 2 - |k|^{2H},\ k = 0, 1, \dots,\ H := 1 - a / 2$$

- Useful model for geophysical time series (Hurst 1951), not artificial and parsimonious (Koutsoyiannis 2015), stationary (Koutsoyiannis 2006), result of 2nd law of thermodynamics (Koutsoyiannis 2011).



Explains bigger variations
Same μ and σ, but
$H = 0.5$
$H = 0.8$

A Bayesian framework on the prediction of climate

- We assume that there is a record of n observations $\boldsymbol{x}_{1:n} = (x_1,\dots,x_n)^{\mathrm{T}}$
- We define the random variable $\underline{\boldsymbol{x}}_{1:n}$

$$\underline{\boldsymbol{x}}_{1:n} := (\underline{x}_1,\dots,\underline{x}_n)^{\mathrm{T}}, \underline{\boldsymbol{x}}_{1:n} \sim f(\boldsymbol{x}_{1:n}|\boldsymbol{\theta})$$

- A parametric statistical model consists of $\boldsymbol{x}_{1:n}$ and $\underline{\boldsymbol{x}}_{1:n}$ (Robert 2007, p.7).
- We model the uncertainty of the parameter $\boldsymbol{\theta}$ using a probability distribution π, called prior distribution.

$$\underline{\boldsymbol{\theta}} \sim \pi(\boldsymbol{\theta})$$

- A Bayesian statistical model consists of $\boldsymbol{x}_{1:n}$, $\underline{\boldsymbol{x}}_{1:n}$ and π (Robert 2007, p.9).

Posterior distribution

$$\pi(\boldsymbol{\theta}|\boldsymbol{x}_{1:n}) = \frac{f(\boldsymbol{x}_{1:n}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})}{\int f(\boldsymbol{x}_{1:n}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})\mathrm{d}\boldsymbol{\theta}}$$

Prediction

$$g(\boldsymbol{y}|\boldsymbol{x}_{1:n}) = \int g(\boldsymbol{y}|\boldsymbol{\theta},\boldsymbol{x}_{1:n})\pi(\boldsymbol{\theta}|\boldsymbol{x}_{1:n})\mathrm{d}\boldsymbol{\theta}$$

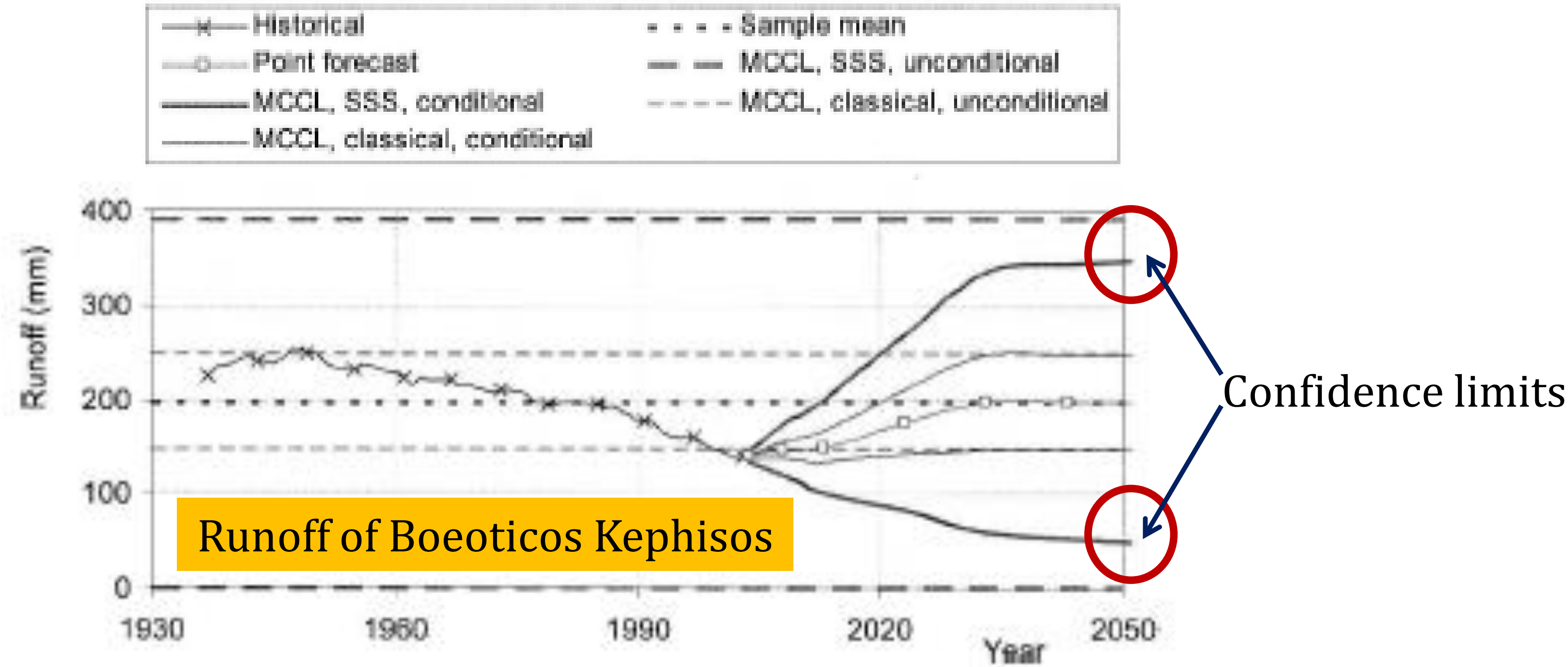
- Computations using analytical expressions or simulation.

Objectives and research questions

- Use of stochastic models instead of the state-of-the-art GCMs.
- We provide some tools towards the development of a stochastic framework for the prediction of hydroclimatic variables.
- We examine topics such as:
  - The estimation of the parameters of the model.
  - The uncertainty of the estimation of the parameters.
  - The incorporation of this uncertainty in the prediction uncertainty.
- Research questions:
  - How can the uncertainty in the estimation of the parameters be integrated in the uncertainty of the prediction?
  - How can the data be used for the prediction?
  - Which is an appropriate framework to gain from available information from deterministic models?

A first typical statistical approach for climate prediction

- First attempt to predict the climate by Koutsoyiannis et al. (2007).
- Estimation of confidence limits for a quantile (e.g. $\mu + 2\sigma$), using a heuristic Monte Carlo confidence interval algorithm.
- A HKp was used to model the observed time series.

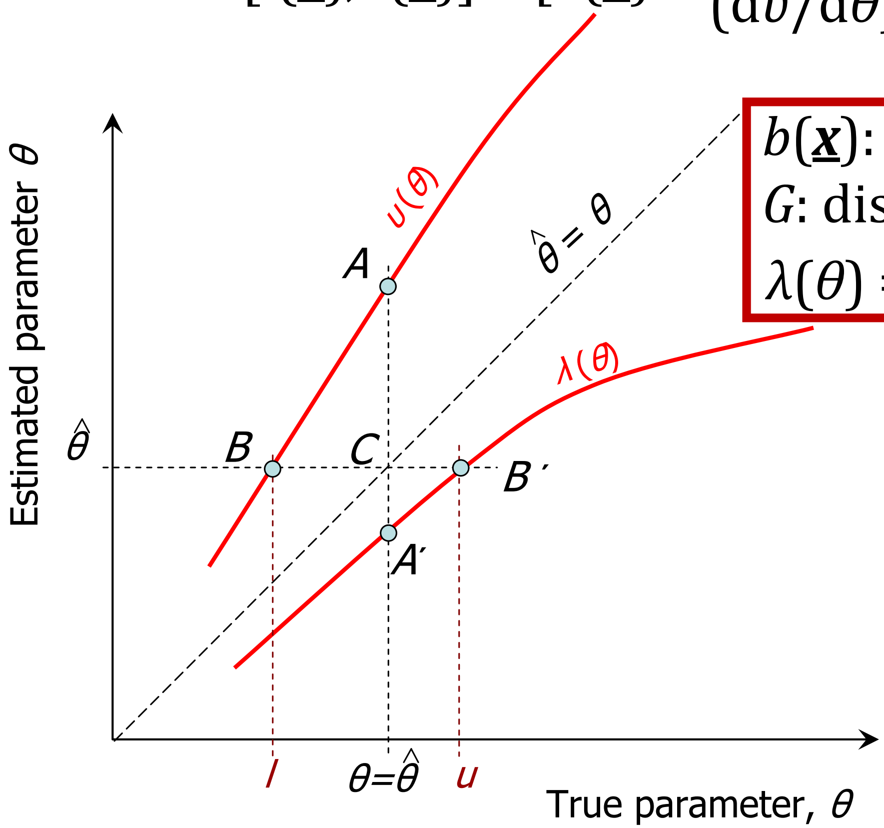


Source: Koutsoyiannis et al. (2007)

The Monte Carlo Confidence Interval (MCCI) algorithm

- Confidence interval estimate $[l,u]$ as a result of the inversion of a hypothesis test about the parameter θ (Casella and Berger 2001, p.385).
- Results in an approximate $1 - \alpha$ confidence interval (see the ABC triangle).

$$[l(\underline{\boldsymbol{x}}),u(\underline{\boldsymbol{x}})] = \left[b(\underline{\boldsymbol{x}}) + \frac{b(\underline{\boldsymbol{x}}) - \upsilon(b(\underline{\boldsymbol{x}}))}{(\mathrm{d}\upsilon/\mathrm{d}\theta)_{|\theta = b(\underline{\boldsymbol{x}})}}, b(\underline{\boldsymbol{x}}) + \frac{b(\underline{\boldsymbol{x}}) - \lambda(b(\underline{\boldsymbol{x}}))}{(\mathrm{d}\lambda/\mathrm{d}\theta)_{|\theta = b(\underline{\boldsymbol{x}})}}\right]$$



$b(\underline{\boldsymbol{x}})$: θ estimator
G: distribution function of $b(\underline{\boldsymbol{x}})$
$\lambda(\theta) = G^{-1}(\alpha/2|\theta)$ and $\upsilon(\theta) = G^{-1}(1 - \alpha/2|\theta)$

Triangle ABC

$$\frac{\upsilon(\hat{\theta}) - \hat{\theta}}{\hat{\theta} - l} = \frac{\mathrm{CA}}{\mathrm{CB}} \approx \left(\frac{\mathrm{d}\upsilon}{\mathrm{d}\theta}\right)_{|\theta = \hat{\theta}}$$

Properties of the algorithm

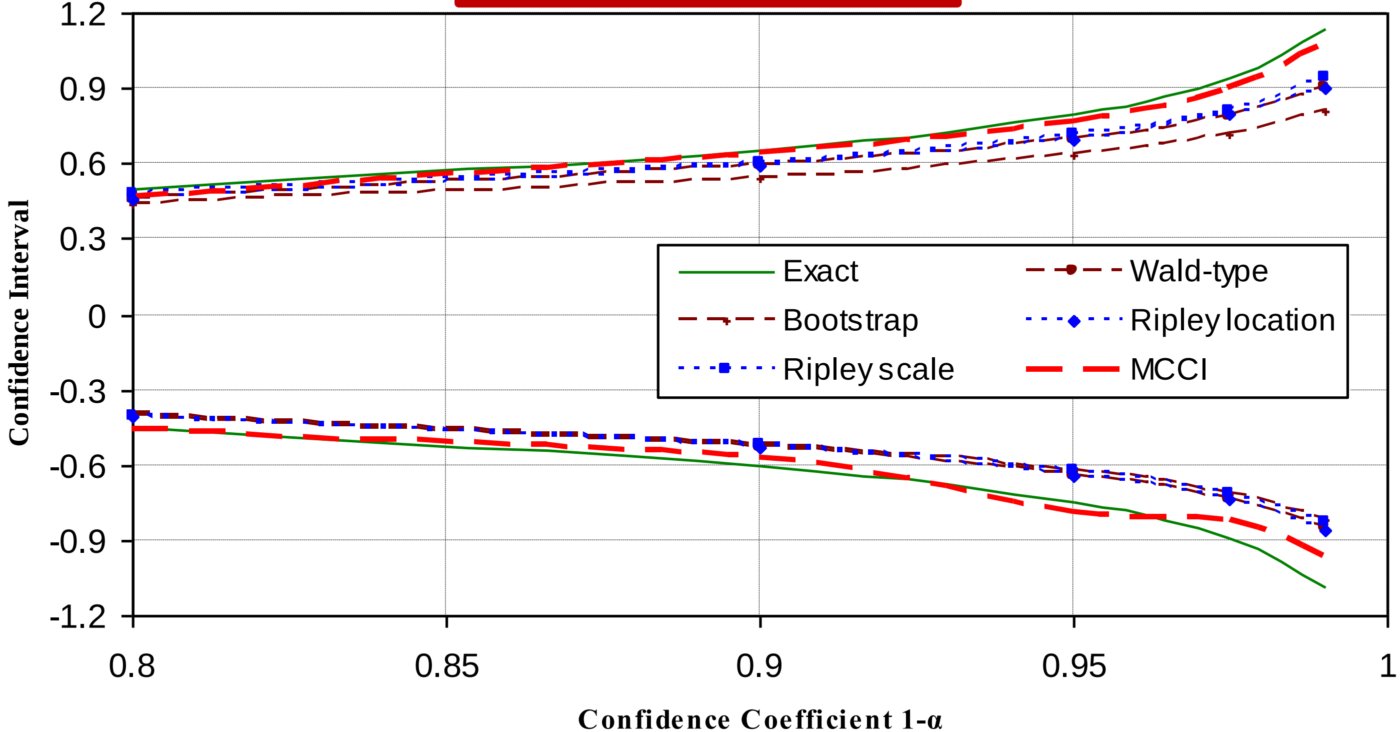
- General algorithm (simultaneously advantage and disadvantage).
- Exact for location and scale families.
- **Asymptotically equivalent** to Wald-type intervals.
- Applied using a maximum likelihood estimator.
- Unknown quantities are computed with simulations.
- Heuristic expansion for **multi-parameter** probability distributions.
- Still remains asymptotically equivalent to Wald-type intervals.

Proved in Tyralis et al. (2013).

Application to a sample from a normal distribution

- Simulated sample (size $n = 10$) from a normal distribution.
- Estimation of $1 - \alpha$ confidence intervals of μ for $1 - \alpha \in [0.80, 0.99]$, using general methods.
- The MCCI algorithm performs better compared to the other algorithms (closest to the exact confidence intervals).

$\hat{\mu} = 0.026, \hat{\sigma} = 1.023$

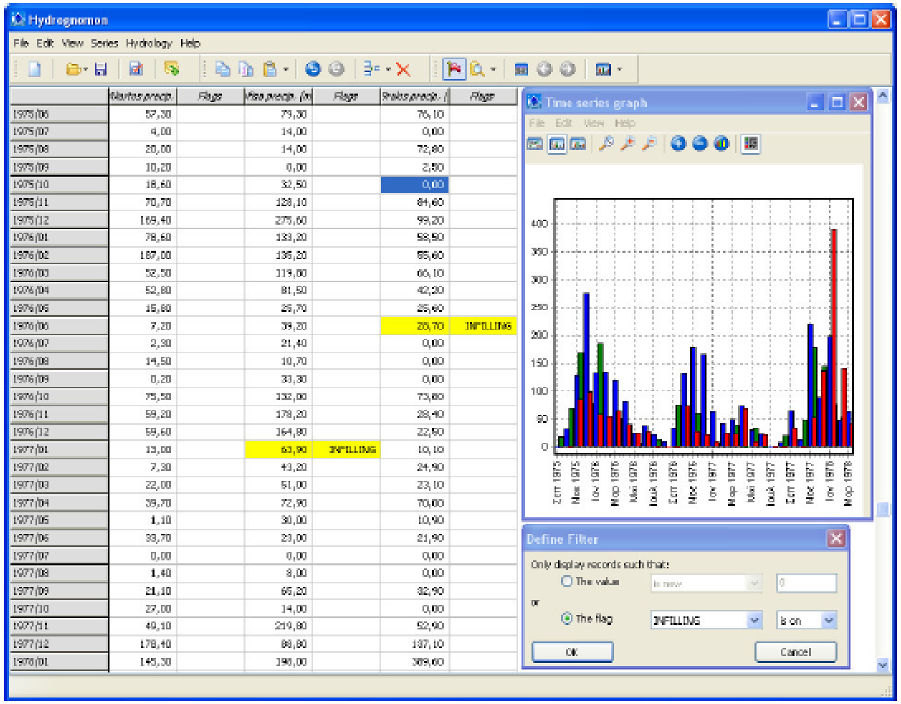


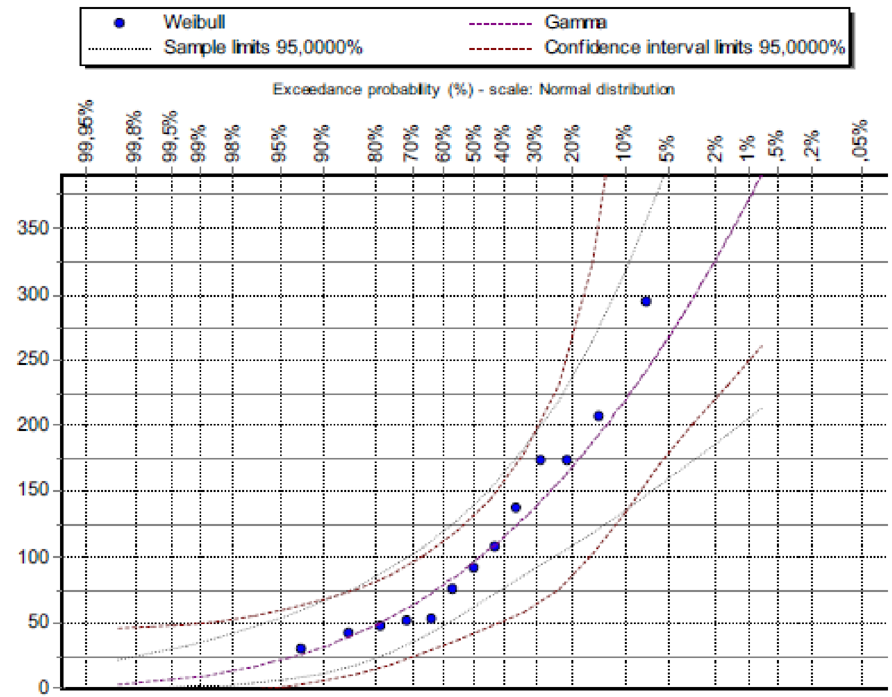
Comparison of coverage probabilities

Coverage probabilities when calculating 0.975 confidence intervals for all methods (with ranks in parentheses, rank 1 is assigned to the method of best performance)

Case	Distribution	Parameter	Approximate	Ripley location	Ripley scale	Wald-type	Bootstrap	MCCI
1	Exponential	Scale		0.889 (5)	0.977 (2)	0.975 (1)	0.916 (4)	0.966 (3)
2	Normal	Location		0.946 (3)	0.946 (3)	0.947 (2)	0.931 (5)	0.968 (1)
3	Normal	Percentile		0.919 (4)	0.929 (2)	0.929 (2)	0.867 (5)	0.973 (1)
4	Gamma	Scale	0.753	0.923 (5)	0.976 (1)	0.940 (4)	0.957 (3)	0.974 (1)
5	Gamma	Shape	0.976	0.948 (5)	0.972 (2)	0.978 (2)	0.956 (4)	0.974 (1)
6	Weibull	Scale	0.971	0.969 (3)	0.970 (2)	0.966 (4)	0.965 (5)	0.973 (1)
7	Weibull	Percentile	0.971	0.968 (3)	0.970 (1)		0.961 (4)	0.969 (2)
	mean rank			4.000	1.857	2.500	4.286	$\boxed{1.429}$

Implementation in software package “Hydrognomon”





User interface

Monthly precipitation in Zographou campus (November 1993-2006). The sample is modelled by a gamma distribution

Maximum Likelihood Estimator of the HKp parameters

- The MCCI estimates confidence intervals for parameters.
- Instead the prediction requires estimation of confidence regions for random variables.
- This can be accomplished in a Bayesian framework.
- First step: Estimation of the parameters using a Maximum Likelihood Estimator (MLE). $\boldsymbol{R}$ is the correlation matrix, thus it is function of H.

Probability density and likelihood function (parameters: μ, σ, H)

$$f(\boldsymbol{x}_{1:n}|\boldsymbol{\theta}) = (2\pi)^{-n/2}\,|\sigma^2\,\boldsymbol{R}_{[1:n]\,[1:n]}|^{-1/2}\exp[(-1/2\sigma^2)\,(\boldsymbol{x}_{1:n}-\mu\,\boldsymbol{e}_n)^{\mathrm{T}}\,\boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\,(\boldsymbol{x}_{1:n}-\mu\,\boldsymbol{e}_n)]$$

$$l(\boldsymbol{\theta}|\boldsymbol{x}_{1:n}) = (2\pi)^{-n/2}\,|\sigma^2\,\boldsymbol{R}_{[1:n]\,[1:n]}|^{-1/2}\exp[(-1/2\sigma^2)\,(\boldsymbol{x}_{1:n}-\mu\,\boldsymbol{e}_n)^{\mathrm{T}}\,\boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\,(\boldsymbol{x}_{1:n}-\mu\,\boldsymbol{e}_n)]$$

First maximize $g_1(H)$

$$g_1(H):=-\frac{n}{2}\ln\Big[\Big(\boldsymbol{x}_{1:n}-\frac{\boldsymbol{x}^{\mathrm{T}}_{1:n}\,\boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\,\boldsymbol{e}_n}{\boldsymbol{e}^{\mathrm{T}}_n\,\boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\,\boldsymbol{e}_n}\boldsymbol{e}_n\Big)^{\mathrm{T}}\boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\Big(\boldsymbol{x}_{1:n}-\frac{\boldsymbol{x}^{\mathrm{T}}_{1:n}\,\boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\,\boldsymbol{e}_n}{\boldsymbol{e}^{\mathrm{T}}_n\,\boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\,\boldsymbol{e}_n}\boldsymbol{e}_n\Big)\Big]-\frac{1}{2}\ln(|\boldsymbol{R}_{[1:n]\,[1:n]}|)$$

Then substitute to obtain the estimates of μ and σ

$$\hat{\mu} = \frac{\boldsymbol{x}^{\mathrm{T}}_{1:n}\,\hat{\boldsymbol{R}}^{-1}_{[1:n]\,[1:n]}\,\boldsymbol{e}_n}{\boldsymbol{e}^{\mathrm{T}}_n\,\hat{\boldsymbol{R}}^{-1}_{[1:n]\,[1:n]}\,\boldsymbol{e}_n} \qquad \hat{\sigma} = \sqrt{\frac{(\boldsymbol{x}_{1:n}-\hat{\mu}\,\boldsymbol{e}_n)^{\mathrm{T}}\,\hat{\boldsymbol{R}}^{-1}_{[1:n]\,[1:n]}\,(\boldsymbol{x}_{1:n}-\hat{\mu}\,\boldsymbol{e}_n)}{n}}$$

Least Squares based on Variance (LSV) estimator

- A second estimator (LSV) was also developed (Tyralis and Koutsoyiannis (2011).
- Based on the Least Squares Based on Standard Deviation (LSSD, Koutsoyiannis 2003), but using analytic expressions instead of simulation.

Step

1 $$\underline{x}_t^{(\kappa)} := (1/\kappa) \sum_{l=(t-1)\kappa+1}^{t\kappa} \underline{x}_l \qquad \underline{s}_n := \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (\underline{x}_i - \underline{x}_1^{(n)})^2}$$

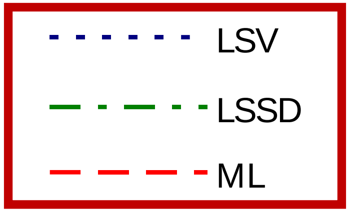
2 $$\mathrm{E}[\underline{s}_n^2] = \frac{n - n^{2H-1}}{n-1} \sigma^2$$ See Beran (1994 p.9)

3 $$\mathrm{E}[\underline{s}_n^{2(\kappa)}] = \frac{(n/\kappa) - (n/\kappa)^{2H-1}}{(n/\kappa) - 1} \kappa^{2H} \sigma^2$$ Due to the self-similarity properties of the HKp

4 Minimize the $\mathrm{er}^2(\sigma,H)$ to estimate H and σ

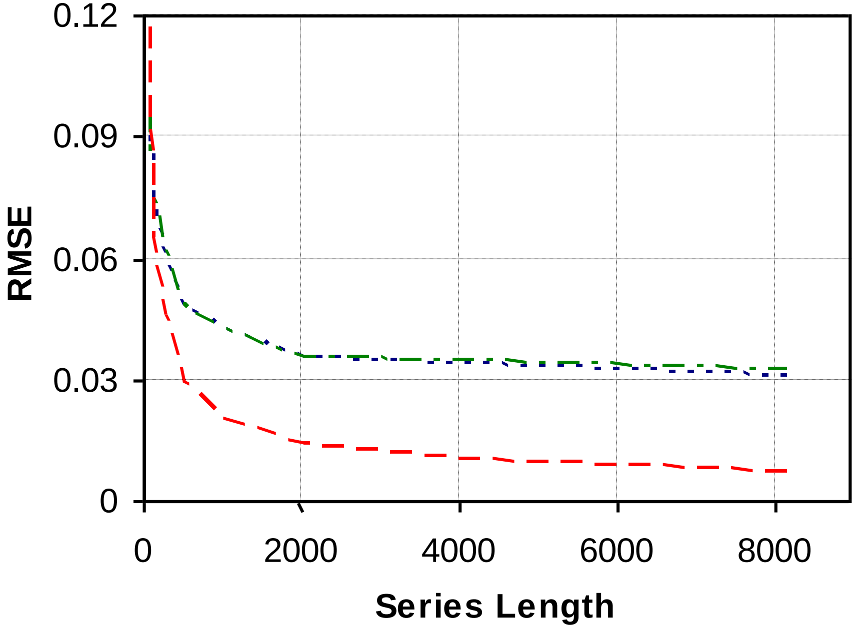
$$\mathrm{er}^2(\sigma, H) := \sum_{\kappa=1}^{\kappa'} \frac{[\mathrm{E}[\underline{s}_n^{2(\kappa)}] - s_n^{2(\kappa)}]^2}{\kappa^p}, \ \kappa' = [n/10]$$

Comparison of Root Mean Squared Errors (RMSEs)

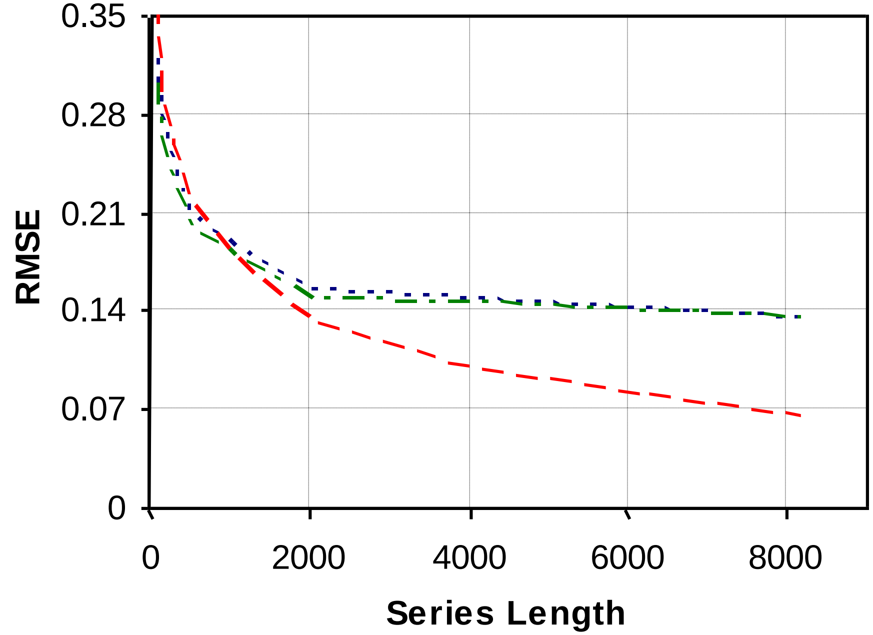


$H = 0.95$
$\sigma = 1$

RMSE of H estimators



RMSE of σ estimators

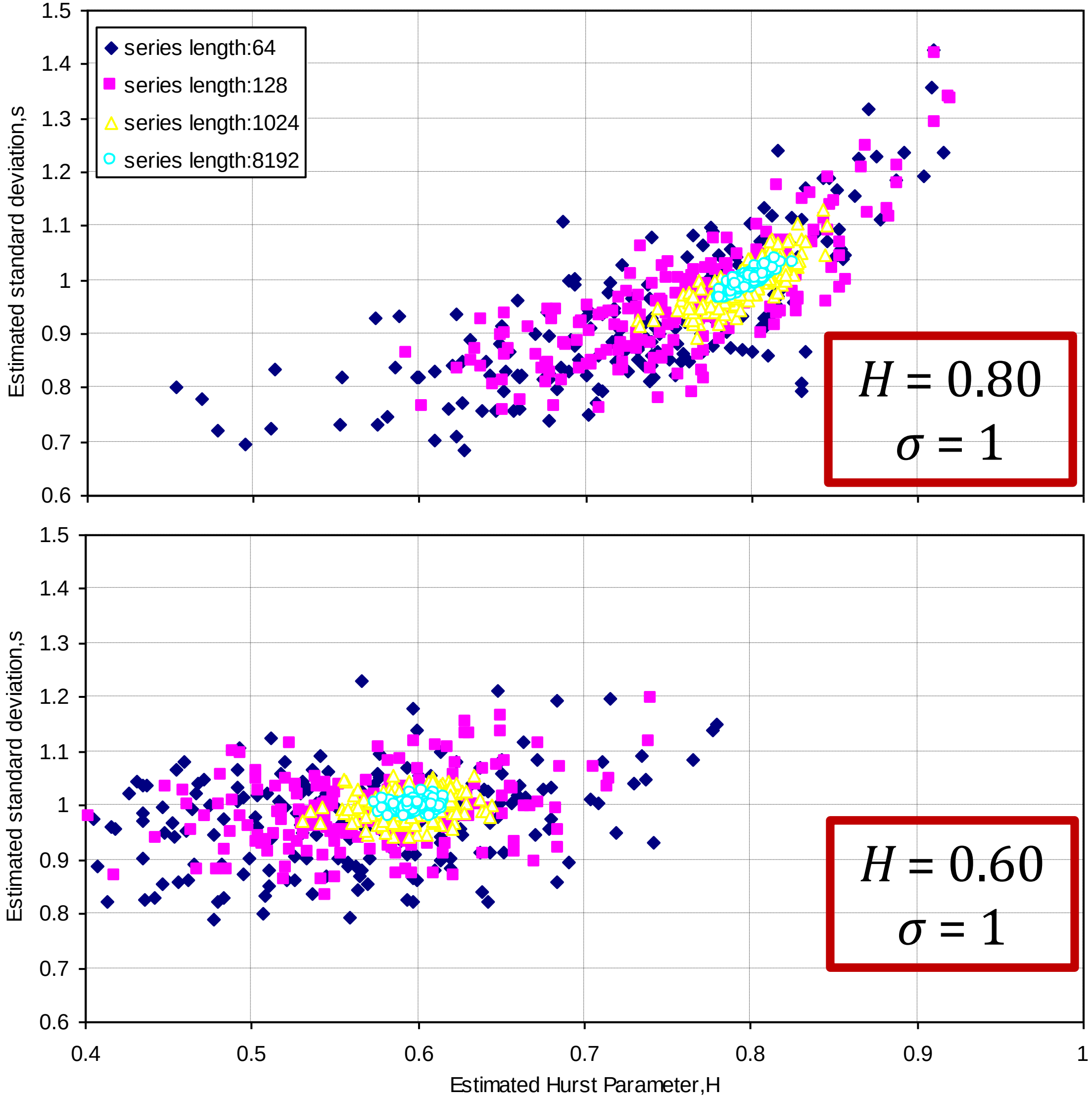


Comparison between the three estimators

- 200 independent realizations
- 8 192 long
- τ: standard deviation of the sample containing the estimated H's.
- Additionally the three estimators, seem to be more accurate compared to other estimators of the literature (see Tyralis and Koutsoyiannis 2011).

Estimation method		Nominal H			
		0.6	0.7	0.8	0.9
ML	$\hat{H}$	0.599	0.700	0.799	$\boxed{0.899}$
	τ	0.008	0.007	0.008	0.007
	RMSE	0.008	0.007	0.008	$\boxed{0.007}$
LSSD	$\hat{H}$	0.599	0.699	0.799	0.892
	τ	0.011	0.011	0.015	0.015
	RMSE	0.011	0.012	0.015	0.017
LSV	$\hat{H}$	0.599	0.700	0.800	$\boxed{0.895}$
	τ	0.009	0.008	0.011	0.014
	RMSE	0.009	0.008	0.011	$\boxed{0.015}$

Investigation of parameters orthogonality



σ and H are not orthogonal (definition of orthogonality in Cox and Reid 1987).

This is also proved in Tyralis and Koutsoyiannis (2011).

Numerous publications that calculate the standard deviation by the classical statistical estimator.

Posterior distribution of the parameters

- f is the multivariate normal distribution function .
- $\boldsymbol{R}$ is the correlation matrix, thus it is function of $\boldsymbol{\varphi}$.
- $\boldsymbol{\varphi}$ could be equal to H, or other parameter (depends on the kind of correlation).

$$f(\boldsymbol{x}_{1:n}|\boldsymbol{\theta}) = (2\pi)^{-n/2}\, |\sigma^2 \boldsymbol{R}_{[1:n]\,[1:n]}|^{-1/2} \exp[(-1/2\sigma^2)\,(\boldsymbol{x}_{1:n} - \mu\, \boldsymbol{e}_n)^{\mathrm{T}}\, \boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\,(\boldsymbol{x}_{1:n} - \mu\, \boldsymbol{e}_n)]$$

$\boldsymbol{\theta} := (\mu, \sigma^2, \boldsymbol{\varphi})$ $\quad \pi(\boldsymbol{\theta}) \propto 1/\sigma^2$

Prior distribution

Step	Posterior distribution mixture
1	$\pi(\boldsymbol{\varphi}\|\boldsymbol{x}_{1:n}) \propto \|\boldsymbol{R}_{[1:n]\,[1:n]}\|^{-1/2}\, [\boldsymbol{e}_n^{\mathrm{T}}\, \boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\, \boldsymbol{e}_n \cdot \boldsymbol{x}_{1:n}^{\mathrm{T}}\, \boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\, \boldsymbol{x}_{1:n} - (\boldsymbol{x}_{1:n}^{\mathrm{T}}\, \boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\, \boldsymbol{e}_n)^2]^{-(n-1)/2}\, (\boldsymbol{e}_n^{\mathrm{T}}\, \boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\, \boldsymbol{e}_n)^{n/2-1}$
2	$\underline{\sigma}^2\|\boldsymbol{\varphi}, \boldsymbol{x}_{1:n} \sim \text{Inv-gamma}\{(n-1)/2, [\boldsymbol{e}_n^{\mathrm{T}}\, \boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\, \boldsymbol{e}_n \cdot \boldsymbol{x}_{1:n}^{\mathrm{T}}\, \boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\, \boldsymbol{x}_{1:n} - (\boldsymbol{x}_{1:n}^{\mathrm{T}}\, \boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\, \boldsymbol{e}_n)^2]/(2\, \boldsymbol{e}_n^{\mathrm{T}}\, \boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\, \boldsymbol{e}_n)\}$
3	$\underline{\mu}\|\sigma^2, \boldsymbol{\varphi}, \boldsymbol{x}_{1:n} \sim N[(\boldsymbol{x}_{1:n}^{\mathrm{T}}\, \boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\, \boldsymbol{e}_n)/(\boldsymbol{e}_n^{\mathrm{T}}\, \boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\, \boldsymbol{e}_n), \sigma^2/(\boldsymbol{e}_n^{\mathrm{T}}\, \boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\, \boldsymbol{e}_n)]$

Posterior distribution of the truncated process parameters

Truncated multivariate normal in the interval $[a,b]^n$

$$f(\boldsymbol{x}_{1:n}|\boldsymbol{\theta}) \propto \exp[(-1/2\sigma^2)\,(\boldsymbol{x}_{1:n} - \mu\,\boldsymbol{e}_n)^{\mathrm{T}}\,\boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\,(\boldsymbol{x}_{1:n} - \mu\,\boldsymbol{e}_n)]\ \mathrm{I}_{[a,b]^n}(x_1, \ldots, x_n)$$

Gibbs sampler

$$\pi(\mu|\sigma^2, \boldsymbol{\varphi}, \boldsymbol{x}_{1:n}) \propto \exp\{-[\mu - (\boldsymbol{x}^{\mathrm{T}}_{1:n} \cdot \boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\,\boldsymbol{e}_n)/(\boldsymbol{e}^{\mathrm{T}}_n\,\boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\,\boldsymbol{e}_n)]^2/(2\sigma^2/\boldsymbol{e}^{\mathrm{T}}_n\,\boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\,\boldsymbol{e}_n)\}\ \mathrm{I}_{[a,b]}(\mu)$$

$$\underline{\sigma}^2|\mu, \boldsymbol{\varphi}, \boldsymbol{x}_{1:n} \sim \text{Inv-gamma}\{n/2,\ (\boldsymbol{x}_{1:n} - \mu\,\boldsymbol{e}_n)^{\mathrm{T}}\,\boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\,(\boldsymbol{x}_{1:n} - \mu\,\boldsymbol{e}_n)/2\}$$

$$\pi(\boldsymbol{\varphi}|\,\mu, \sigma^2, \boldsymbol{x}_{1:n}) \propto |\boldsymbol{R}_{[1:n]\,[1:n]}|^{-1/2} \exp[-\,(\boldsymbol{x}_{1:n} - \mu\,\boldsymbol{e}_n)^{\mathrm{T}}\,\boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\,(\boldsymbol{x}_{1:n} - \mu\,\boldsymbol{e}_n)/2\sigma^2]$$

Posterior predictive distribution and variable of interest

The posterior predictive distribution for a horizon of length m

$$f(\boldsymbol{x}_{(n+1):(n+m)}|\boldsymbol{\theta},\boldsymbol{x}_{1:n}) = (2\pi\sigma^2)^{-m/2}\,|\boldsymbol{R}_{m|n}|^{-1/2}\exp[(-1/2\sigma^2)\cdot$$

$$(\boldsymbol{x}_{(n+1):(n+m)} - \boldsymbol{\mu}_{m|n})^{\mathrm{T}}\,\boldsymbol{R}^{-1}_{m|n}\,(\boldsymbol{x}_{(n+1):(n+m)} - \boldsymbol{\mu}_{m|n})]$$

where

$$\boldsymbol{\mu}_{m|n} = \mu\boldsymbol{e}_m + \boldsymbol{R}_{[(n+1):(n+m)]\,[1:n]}\,\boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\,(\boldsymbol{x}_{1:n} - \mu\boldsymbol{e}_n)$$

$$\boldsymbol{R}_{m|n} = \boldsymbol{R}_{[(n+1):(n+m)]\,[(n+1):(n+m)]} - \boldsymbol{R}^{\mathrm{T}}_{[1:n]\,[(n+1):(n+m)]}\,\boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\,\boldsymbol{R}_{[1:n]\,[(n+1):(n+m)]}$$

- Similar solution for the case of truncation

Variable of interest

$$\underline{x}_{t(30)} := (1/30)\left(\sum_{l=t-29}^{n} x_l + \sum_{l=n+1}^{t} \underline{x}_l\right),\ t = n+1, \ldots, n+29 \text{ and}$$

$$\underline{x}_{t(30)} := (1/30)\sum_{l=t-29}^{t} \underline{x}_l,\ t = n+30, n+31, \ldots$$

Asymptotic behaviour

The posterior predictive distribution for a horizon of length *l*, *m* times ahead

$$f(\boldsymbol{x}_{(n+m+1):(n+m+l)}|\boldsymbol{\theta},\boldsymbol{x}_{1:n}) = (2\pi\sigma^2)^{-l/2}|\boldsymbol{R}_{l|n}|^{-1/2} \cdot$$

$$\exp[(-1/2\sigma^2)(\boldsymbol{x}_{(n+m+1):(n+m+l)} - \boldsymbol{\mu}_{l|n})^{\mathrm{T}}\boldsymbol{R}_{l|n}^{-1}(\boldsymbol{x}_{(n+m+1):(n+m+l)} - \boldsymbol{\mu}_{l|n})]$$

$$\boldsymbol{\mu}_{l|n} = \mu\boldsymbol{e}_l + \boldsymbol{R}_{[(n+m+1):(n+m+l)]\,[1:n]}\,\boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\,(\boldsymbol{x}_{1:n} - \mu\boldsymbol{e}_n)$$

$$\boldsymbol{R}_{l|n} = \boldsymbol{R}_{[(n+m+1):(n+m+l)]\,[(n+m+1):(n+m+l)]} - \boldsymbol{R}^{\mathrm{T}}_{[1:n]\,[(n+m+1):(n+m+l)]}\,\boldsymbol{R}^{-1}_{[1:n]\,[1:n]}\,\boldsymbol{R}_{[1:n]\,[(n+m+1):(n+m+l)]}$$

When $m \to \infty$ (asymptotic behaviour)

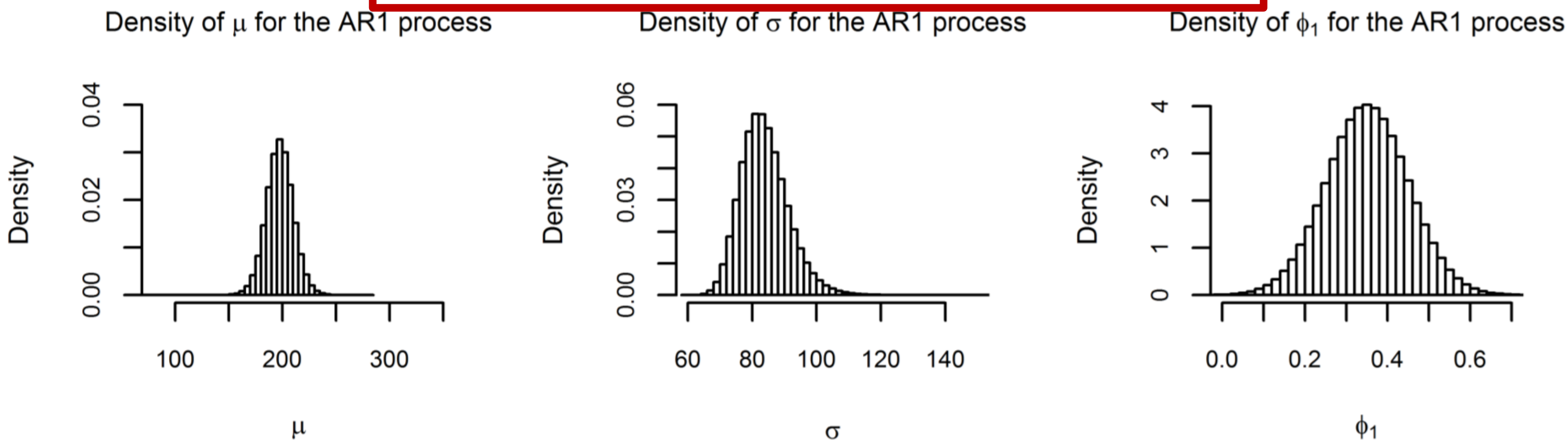
$$\boldsymbol{\mu}_{l|n} = \mu\boldsymbol{e}_l$$

$$\boldsymbol{R}_{l|n} = \boldsymbol{R}_{[1:l]\,[1:l]}$$

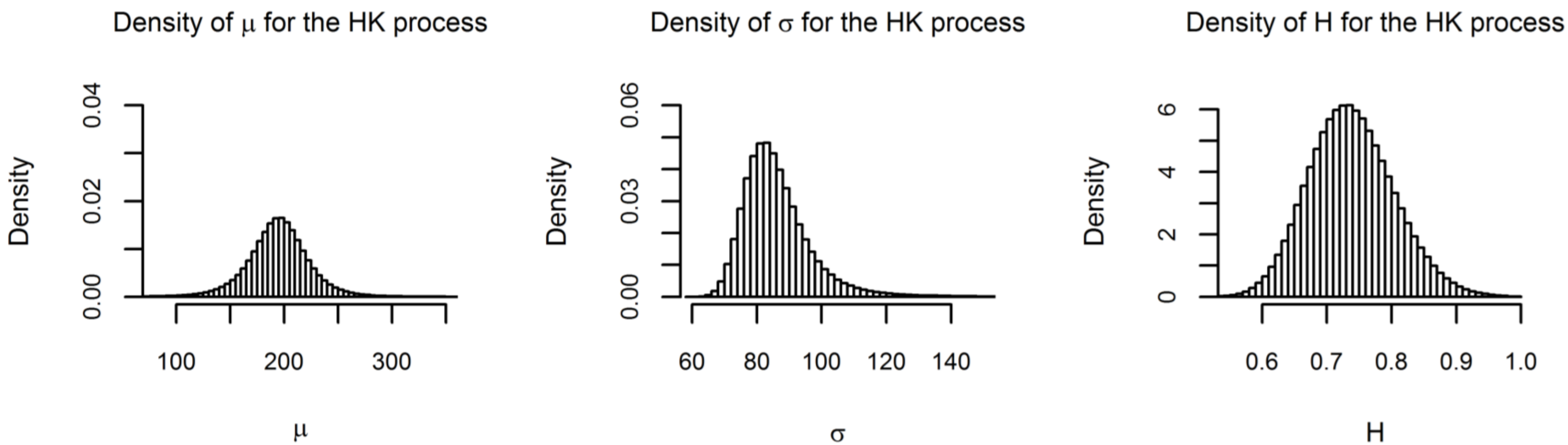
Proofs of the results of slides 17-20 can be found in Tyralis and Koutsoyiannis (2014)

Runoff of Boeoticos Kephisos

Posterior distribution of the parameters

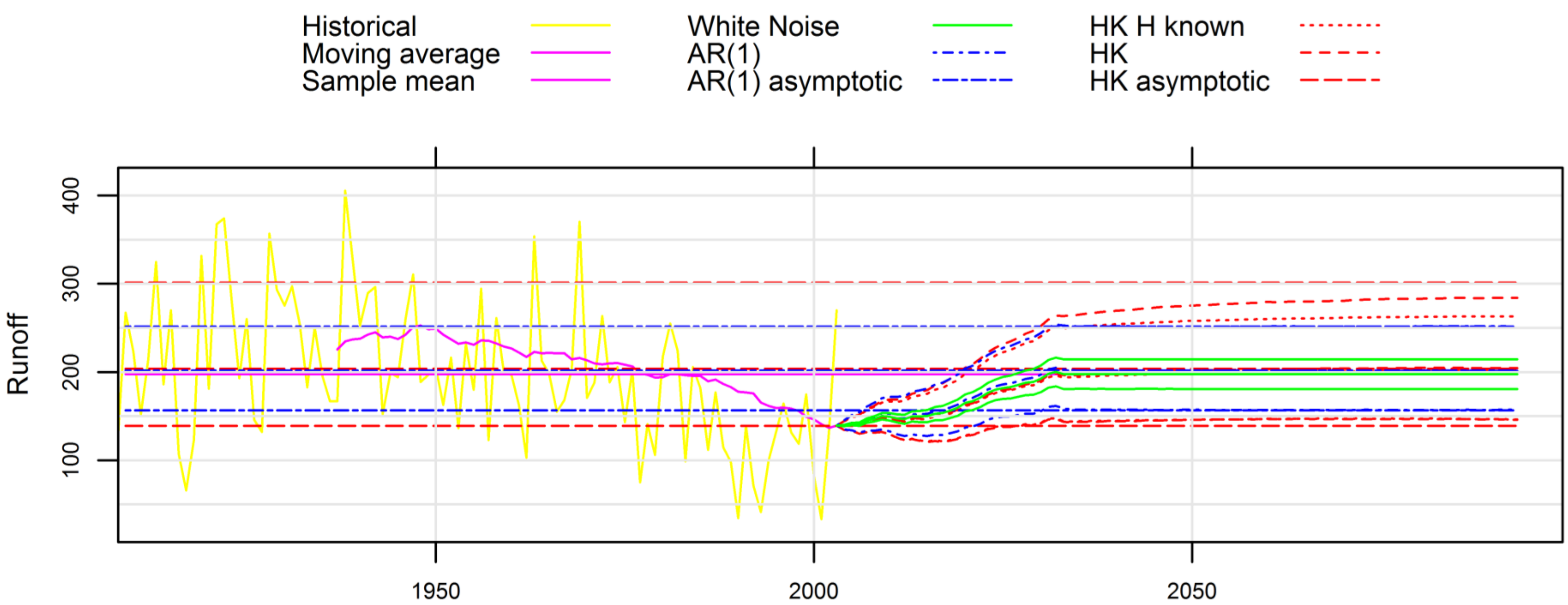


μ | σ | φ_1 or H



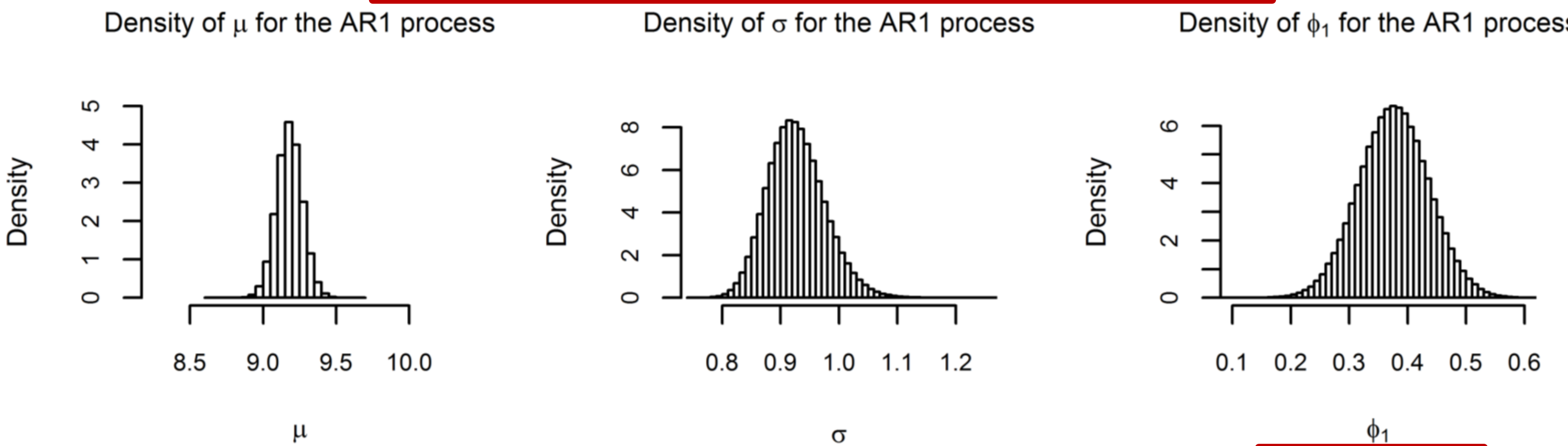
Prediction of runoff of Boeoticos Kephisos

- 95% confidence region of the 30-year moving average.
- Truncation case.
- Six examined cases.

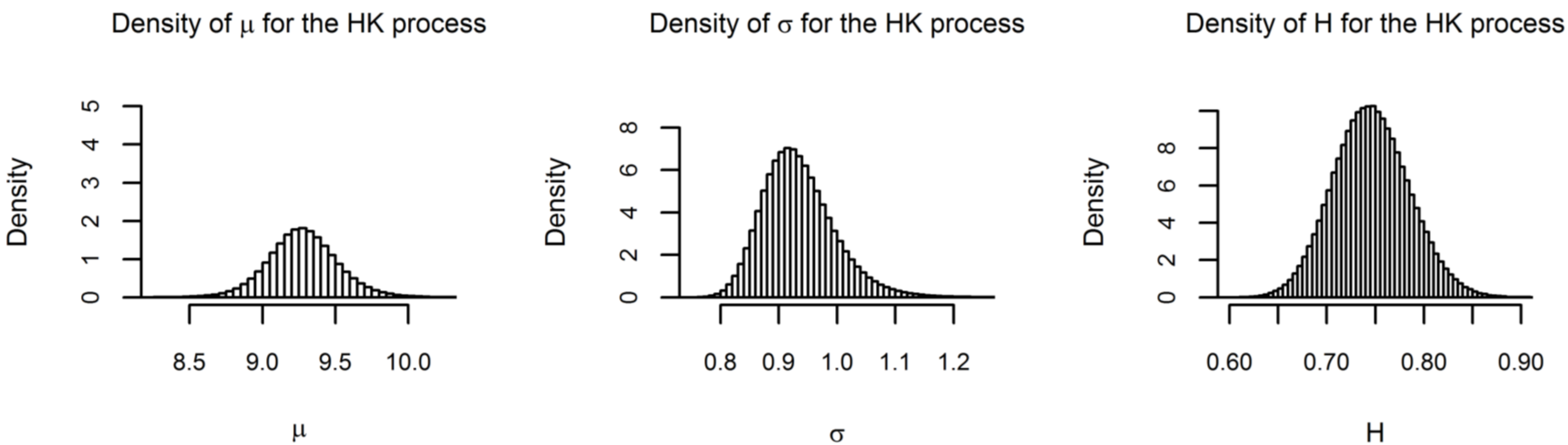


Temperature in Berlin

Posterior distribution of the parameters

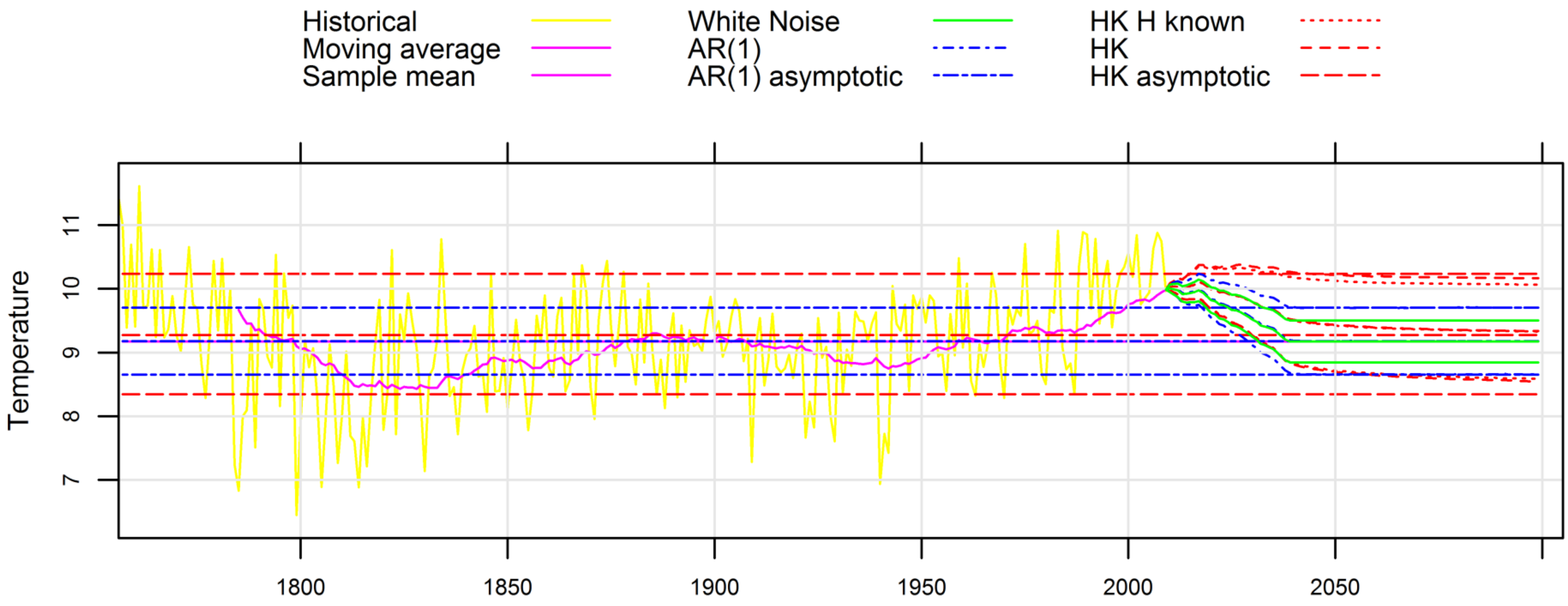


μ σ φ_1 or H



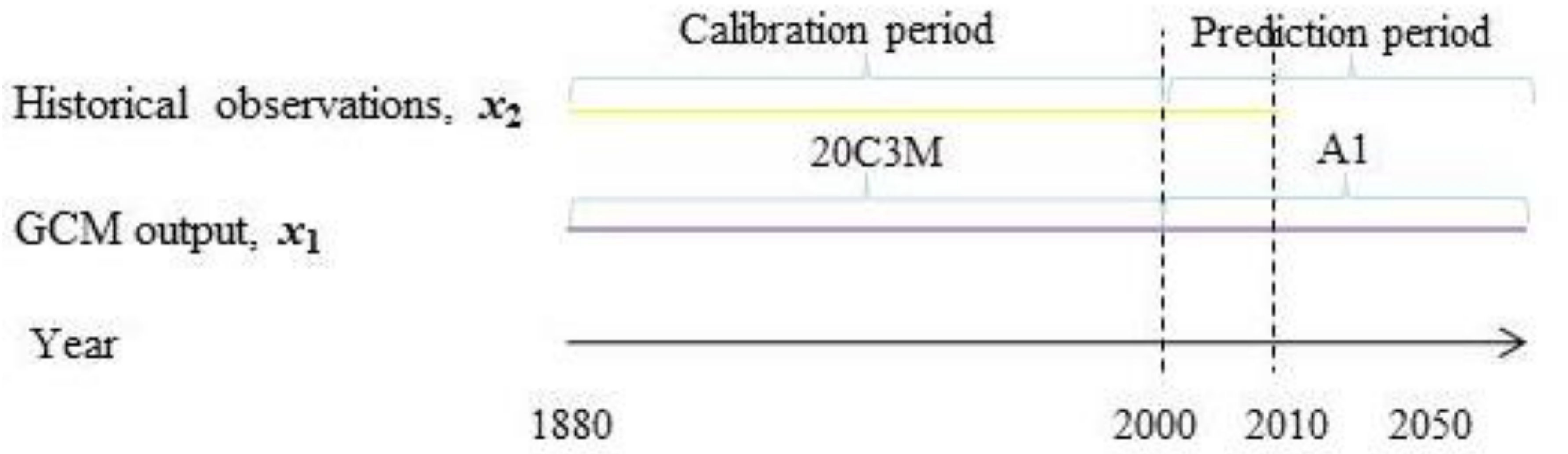
Prediction of temperature in Berlin

- 95% confidence region of the 30-year moving average.
- Truncation is not needed.
- Six examined cases.



The Bayesian joint probability (BJP) modelling approach

- Idea: Use of the Bayesian joint probability (BJP) modelling approach (Wang et al. 2009 and Pokhrel et al. 2013, for seasonal forecasting of streamflows at multiple sites).
- Modelling of $\boldsymbol{x}_1$ (deterministic model output) and $\boldsymbol{x}_2$ (historical observations) as two correlated stochastic processes.



The bivariate HKp

- Modelling of $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ as a bivariate HKp.
- Previously applied to White Noise and Markovian processes.

Definition

$\{\underline{x}_{1t}\}$ and $\{\underline{x}_{2t}\}$, $t = 1, 2, \ldots$: two HKps with parameters (μ_1, σ_1, H_1) and (μ_2, σ_2, H_2) respectively.

Then $\{\underline{\boldsymbol{x}}_t = (\underline{x}_{1t}, \underline{x}_{2t})\}$, $t = 1, 2, \ldots$ is a well-balanced HKp if (Amblard et al. 2012)

$$w_{ij}(k) := \rho_{i,j}\, |k|^{H_i + H_j},\ \rho_{i,i} = 1,\ \rho_{i,j} = \rho_{j,i} = \rho,\ \{i,j\} \in \{\{1,2\},\{1,2\}\}$$

$$\gamma_{ij}(k) := \mathrm{Cov}[\underline{x}_{it}, \underline{x}_{j\,t+k}] = (1/2)\, \sigma_i\, \sigma_j\ \left(w_{ij}(k-1) - 2\, w_{ij}(k) + w_{ij}(k+1) \right)$$

$$\rho^2 \leq \frac{\Gamma(2H_1+1)\, \Gamma(2H_2+1)\, \sin(\pi H_1)\, \sin(\pi H_2)}{\Gamma^2(H_1+H_2+1)\, \sin^2(\pi(H_1+H_2)/2)}$$

Prediction

- Maximum likelihood estimate of the 7 parameters of the bivariate HKp.
- Posterior prediction using the estimates of the parameters.

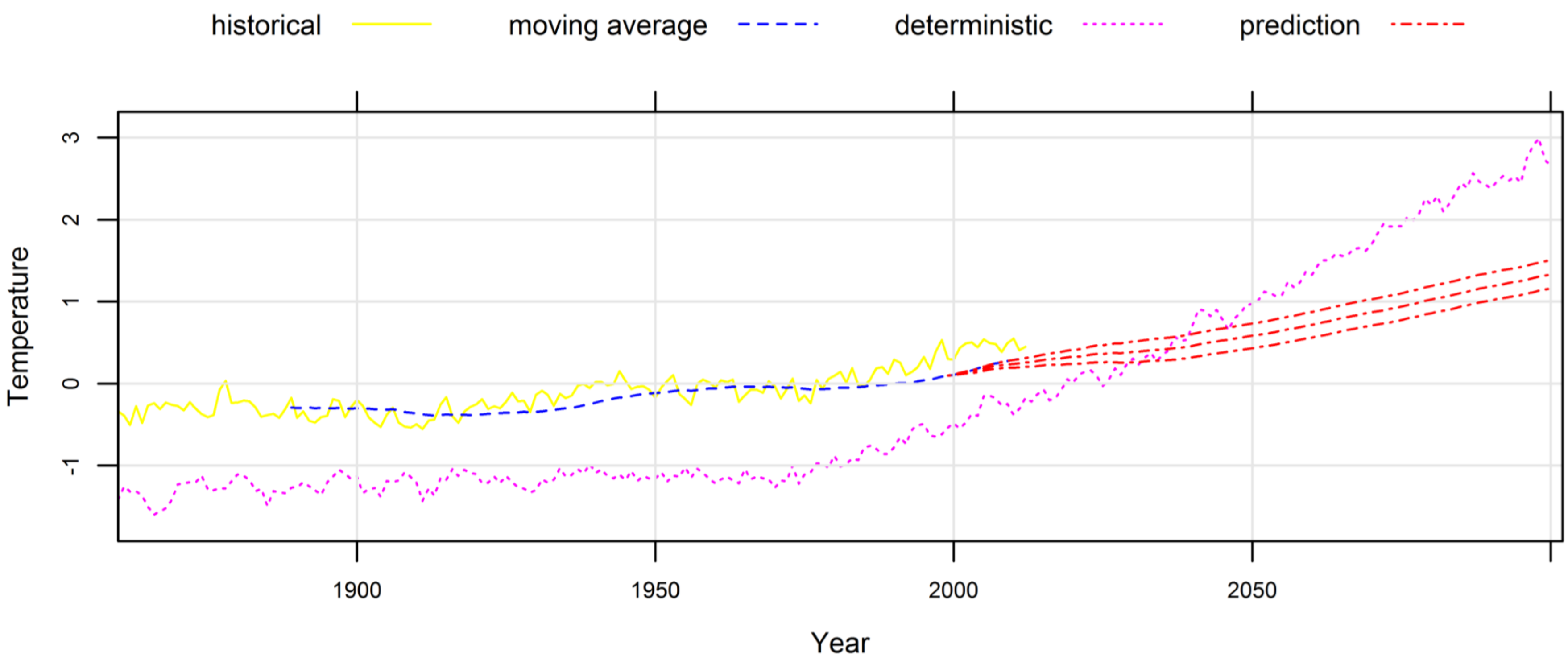
Simulation of $f(\boldsymbol{x}_{2\,(n+1):(n+m)}|\boldsymbol{x}_{1\,1:(n+m)},\boldsymbol{x}_{2\,1:n},\boldsymbol{\theta})$

$$f(\boldsymbol{x}_{2\,(n+1):(n+m)}|\boldsymbol{x}_{1\,1:(n+m)},\boldsymbol{x}_{2\,1:n},\boldsymbol{\theta}) = (2\pi\sigma^2)^{-m/2}\,|\boldsymbol{R}_{m|n}|^{-1/2}\,\exp[(-1/2\sigma^2)\cdot$$
$$(\boldsymbol{x}_{2\,(n+1):(n+m)} - \boldsymbol{\mu}_{m|n})^{\mathrm{T}}\,\boldsymbol{R}_{m|n}^{-1}\,(\boldsymbol{x}_{2\,(n+1):(n+m)} - \boldsymbol{\mu}_{m|n})]$$

Proofs of the results can be found in
Tyralis and Koutsoyiannis (2015)

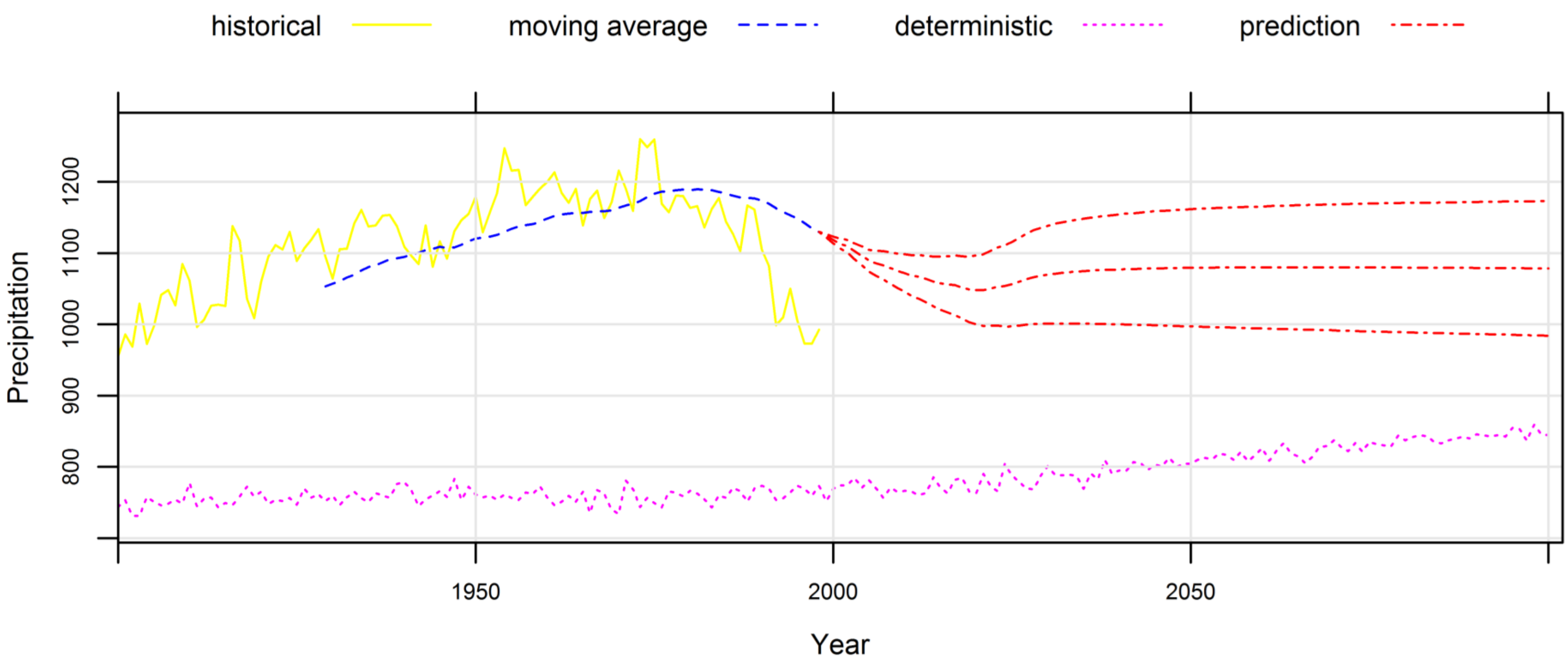
Example of temperature prediction

- 95% confidence region for the 30-year moving average.
- Historical time series: CRU combined land [CRUTEM4] and marine temperature anomalies.
- Deterministic model: A1B scenario of the UKMO HadGEM1 model.



Example of rainfall prediction

- 95% confidence region for the 30-year moving average.
- Historical time series: CRU precipitation over land areas.
- Deterministic model: A1B scenario of the ECHO-G model.



Contributions

- Investigation of the properties of a general Monte Carlo algorithm, to estimate confidence intervals.
- Construction of a novel estimator of the HKp parameters.
- Investigation of the properties of the MLE, the LSV and other estimators of the parameters of the HKp.
- Development of a Bayesian statistical model for climate prediction.
- Application of the Bayesian statistical model to temperature, rainfall and runoff time series.
- Incorporation of information from deterministic models, to improve the Bayesian statistical model's prediction (the second framework).
- Application of the second framework to temperature and rainfall data and GCM outputs.
- The Bayesian statistical model performs well.
- The information obtained from the GCMs is negligible.

Recommendations for further research and limitations

- Assignment of informative prior distribution to the HKp parameters.
- Derivation of the MLE for the multivariate HKp.
- Bayesian expansion of the second framework.
- Application to more datasets.
- Is HKp suitable to model geophysical time series?

References

Amblard PO, Coeurjolly JF, Lavancier F, Philippe A (2012) Basic properties of the multivariate fractional Brownian motion. arXiv:1007.0828v2

Anagnostopoulos GG, Koutsoyiannis D, Christofides A, Efstratiadis A, Mamassis N (2010) A comparison of local and aggregated climate model outputs with observed data. Hydrological Sciences Journal 55(7):1094-1110. doi:10.1080/02626667.2010.513518

Beran J (1994) Statistics for Long-Memory Processes. Chapman & Hall/CRC, New York

Casella G, Berger RL (2001) Statistical Inference, second edition. Duxbury Recourse Center, Pacific Grove

Cox DR, Reid N (1987) Parameter Orthogonality and Approximate Conditional Inference. Journal of the Royal Statistical Society Series B (Methodological) 49(1):1-39

Fyfe JC, Gillet NP, Zwiers FW (2013) Overestimated global warming over the past 20 years. Nature Climate Change 3:767-769. doi:10.1038/nclimate1972

Handorf D, Dethloff K (2012) How well do state-of-the-art atmosphere-ocean general circulation models reproduce atmospheric teleconnection patterns?. Tellus A 64:19777. doi:10.3402/tellusa.v64i0.19777

Hurst HE (1951) Long term storage capacities of reservoirs. Transactions of the American Society of Civil Engineers 116:776-808 (published in 1950 as Proceedings Separate no.11)

IPCC (2007) Climate Change 2007: The Physical Science Basis. Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change. Solomon S, Qin D, Manning M, Chen Z, Marquis M, Averyt KB, Tingor M, Miller HL (eds). Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA

Koutsoyiannis D (2003) Climate change, the Hurst phenomenon, and hydrological statistics. Hydrological Sciences Journal 48(1):3–24. doi:10.1623/hysj.48.1.3.43481

Koutsoyiannis D (2006) Nonstationarity versus scaling in hydrology. Journal of Hydrology 324(1-4):239–254. doi:10.1016/j.jhydrol.2005.09.022

Koutsoyiannis D (2010) A random walk on water. Hydrology and Earth System Sciences 14:585–601. doi:10.5194/hess-14-585-2010

Koutsoyiannis D (2011) Hurst-Kolmogorov dynamics as a result of extremal entropy production. Physica A: Statistical Mechanics and its Applications 390(8):1424–1432. doi:10.1016/j.physa.2010.12.035

Koutsoyiannis (2015) Generic and parsimonious stochastic modelling for hydrology and beyond. Hydrological Sciences Journal. doi:10.1080/02626667.2015.1016950

References

Koutsoyiannis D, Efstratiadis A, Georgakakos KP (2007) Uncertainty Assessment of Future Hydroclimatic Predictions: A Comparison of Probabilistic and Scenario-Based Approaches. Journal of Hydrometeorology 8(3):261-281. doi:10.1175/JHM576.1

Koutsoyiannis D, Efstratiadis A, Mamassis N, Christofides A (2008) On the credibility of climate predictions. Hydrological Sciences Journal 53(4):671-684. doi:10.1623/hysj.53.4.671

McNider RT, Steeneveld GJ, Holtslag AAM, Pielke Sr RA, Mackaro S, Pour-Biazar A, Walters J, Nair U, Christy J (2012) Response and sensitivity of the nocturnal boundary layer over land to added longwave radiative forcing. Journal of Geophysical Research: Atmospheres 117(D14). doi:10.1029/2012JD017578

Pokhrel P, Robertson DE, Wang QJ (2013) A Bayesian joint probability post-processor for reducing errors and quantifying uncertainty in monthly streamflow predictions. Hydrology and Earth Systems Sciences 17:795-804. doi:10.5194/hess-17-795-2013

Robert CP (2007) The Bayesian Choice: From Decision-Theoretic Foundations to Computational Implementation, second edition. Springer. New York

Scafetta N (2013) Solar and Planetary Oscillation Control on Climate Change: Hind-Cast, Forecast and a Comparison with the CMIP5 GCMS. Energy & Environment 24(3-4):455-496. doi:10.1260/0958-305X.24.3-4.455

Spencer RW, Braswell WD (2011) On the Misdiagnosis of Surface Temperature Feedbacks from Variations in Earth's Radiant Energy Balance. Remote Sensing 3(8):1603-1613. doi:10.3390/rs3081603

Stevens B, Bony S (2013) What Are Climate Models Missing?. Science 340(6136):1053-1054. doi:10.1126/science.1237554

Tyralis H, Koutsoyiannis D (2011) Simultaneous estimation of the parameters of the Hurst-Kolmogorov stochastic process. Stochastic Environmental Research & Risk Assessment 25(1):21-33. doi:10.1007/s00477-010-0408-x

Tyralis H, Koutsoyiannis D (2014) A Bayesian statistical model for deriving the predictive distribution of hydroclimatic variables. Climate Dynamics 42(11-12):2867-2883. doi:10.1007/s00382-013-1804-y

Tyralis H, Koutsoyiannis D (2015) On the prediction of persistent processes using the output of deterministic models. To be submitted

Tyralis H, Koutsoyiannis D, Kozanis S (2013) An algorithm to construct Monte Carlo confidence intervals for an arbitrary function of probability distribution parameters. Computational Statistics 28(4):1501-1527. doi:10.1007/s00180-012-0364-7

Wang QJ, Robertson DE, Chiew FHS (2009) A Bayesian joint probability modeling approach for seasonal forecasting of streamflows at multiple sites. Water Resources Research 45(W05407). doi:10.1029/2008WR007355