ONE PAGE DOCUMENTS: VERIFY MATHEMATICAL FORMULAS

DIMENSIONS

If F:$\mathbb{R}^n \rightarrow \mathbb{R}$ then

- the grad F or ∇F belongs to $\mathbb{R}^n$.
- the Laplace operator ($\nabla \cdot \nabla F$, $\nabla^2 F$ or ΔF) belongs to $\mathbb{R}$.
- the derivative of any order over any variable of F belongs to $\mathbb{R}$.
- the Hessian of F is a n-by-n matrix.

If $\boldsymbol{F}$:$\mathbb{R}^n \rightarrow \mathbb{R}^m$ and P: $\mathbb{R}^m \rightarrow \mathbb{R}$ then

- the Jacobian of $\boldsymbol{F}$ or J$\boldsymbol{F}$ is a m-by-n matrix
- manipulations of these functions follow either the rules of matrix algebra (e.g. J$\boldsymbol{F}^{\mathrm{T}}$ ∇P yields a vector with n elements) or the rules of dot product (e.g. $\boldsymbol{F} \cdot \nabla P$ yields a real number) or the rules of cross product (e.g. $\boldsymbol{F} \times \boldsymbol{F}$ yields a vector with m elements).

If $\boldsymbol{F}$:$\mathbb{R}^n \rightarrow \mathbb{R}^n$ then

- the $\nabla \boldsymbol{F}$ is a n-by-n matrix.
- the div $\boldsymbol{F}$ or $\nabla \cdot \boldsymbol{F}$ belongs to $\mathbb{R}$.
- the $\nabla \cdot \nabla \boldsymbol{F}$ and $\boldsymbol{F} \cdot \nabla \boldsymbol{F}$ belongs to $\mathbb{R}^n$

If $\boldsymbol{A}$ is a n-by-m matrix, $\boldsymbol{B}$ is a m-by-n matrix, $\boldsymbol{b}$ is vector with n elements and $\boldsymbol{c}$ is a vector m elements then

- $\boldsymbol{A}\,\boldsymbol{c}$ is a vector with n elements
- $\boldsymbol{A}\,\boldsymbol{c} + \boldsymbol{b}$ is a vector with n elements
- $\boldsymbol{A}\,\boldsymbol{B}$ is a n-by-n matrix

UNITS

Integration

- The units of the interval (surface or area of integration) should be equal to the units and the dimensions of the "with respect to" variable (i.e. the dx).
- The units of the resulting function are equal to the units of the integrated function multiplied by the units of the "with respect to" variable.

Differentiation

- The units of the n^{th} derivative of a function F are the units of the F divided by the units of differentiated variable raised in the power of n.

Statistics

- The units of standard deviation and mean value are the same with the units of the random variable.
- The units of the variance are the square of the units of the random variable.
- The units of the covariance of two variables is the product of the units of the two variables.
- The coefficient of variation and the correlation coefficient are dimensionless.

EXTREME VALUES

Check behaviour of a function for extreme values. If $F(x,y)$ is expected to return values in [a,b] the followings should also return a value in [a, b]:

- lim F with $x \rightarrow \pm\infty$
- $F(x, 0)$
- lim F with $y \rightarrow \pm\infty$
- $F(0, y)$

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References: http://iahs.info/Publications-News/Other-publications/Guidelines-for-the-use-of-units-symbols-and-equations-in-hydrology.do