

1. Abstract

Multi-step ahead forecasting is of practical interest for the operation of hydropower reservoirs. We conduct several large scale computational experiments using both streamflow data and simulated time series to provide generalized results concerning the variation over time of the error values in multi-step ahead forecasting. In more detail, we apply several popular forecasting methods to each time series as explained subsequently. Each time series is split into a fitting and a testing set. We fit the models to the former set and we test their forecasting performance in the latter set. Lastly, we compute the error and the absolute error at each time step of the forecast horizon for each test and carry out a statistical analysis on the formed data sets. Furthermore, we perform a sensitivity analysis on the length of the fitting set to examine how it affects the results.

2. Introduction

- The available methodologies for time series forecasting regarding the forecasting horizon can be classified as one- and multi-step ahead forecasting. There are five strategies for multi-step ahead forecasting, namely the recursive, direct, DirRec, MIMO and DIRMO (Taieb et al. 2012, Bontempi et al. 2013).
- Multi-step ahead forecasting is far more challenging than one-step ahead forecasting.
- Multi-step ahead forecasting is a common practice in hydrology (e.g. Cheng et al. 2008, Valipour et al. 2013, Papacharalampous 2016, Papacharalampous et al. 2017b) and beyond, while it is of particular importance for the operation of hydropower reservoirs (e.g. Coulibaly et al. 2000, Ballini et al. 2001) and, by extension, for the energy industry, especially if we consider that hydropower is a form of energy both reliable and sustainable.
- Herein, we conduct:
 - several large scale computational experiments based on simulations to provide generalized results on the error evolution in multi-step ahead forecasting
 - a multiple-case study using monthly time series of streamflow to highlight important facts, which exhibit greater interest when presented using real-world data

3. Methodology outline

- We conduct **6 large-scale simulation experiments** (SE_1a, SE_1b, SE_2a, SE_2b, SE_3a, SE_3b).
- Within each of the latter we simulate an adequate number of time series according to linear models of stationary stochastic processes, which are widely used for the modelling of hydrological processes. The simulated time series are of 150 or 350 values.
- We additionally conduct a **multiple-case study**, which is composed by **92 single-case studies** using monthly streamflow data.
- Some basic information about the time series used in the present study are provided in **4**.
- We apply several popular forecasting methods (see **5**) on the time series.
- Regarding the application of the forecasting methods, we split each time series into a **fitting** and a **testing set**. The latter is the last 50 values for the simulation experiments and the last 12 values for the multiple-case study.
- We fit the models to the fitting set and make predictions corresponding to the testing set using the **recursive** multi-step ahead forecasting method. Next, we calculate the errors and the absolute errors at each time step of the forecast horizon.
- Within the simulation experiments we carry out a statistical analysis on the formed data sets and we present the results accordingly.
- As regards the real-world time series, the fitting set is used after deseasonalization, which is performed using a multiplicative model of time series decomposition, while the seasonality is subsequently added to the predicted time series. This specific practice is suggested for the improvement of the forecast quality (Taieb et al. 2012).
- We present the results of the multiple-case study in a qualitative form to facilitate the detection of systematic patterns.

4. Time series

a) Simulated time series

- We simulate time series according to the ARFIMA(p,d,q) model. Although this specific modelling is accompanied by certain problems (Koutsoyiannis 2016), it is considered rather satisfying for the present study and has been widely applied in the literature (e.g. Montanari et al. 1997).
- We use the fracdiff.sim algorithm of the fracdiff R package (Fraily et al. 2012) to simulate 2 000 time series within each simulation experiment according to the following table:

Simulation experiment	Simulated process	Time series length
SE_1a	ARFIMA(0,0.30,0)	150
SE_1b		350
SE_2a	ARFIMA(1,0.30,0)	150
SE_2b		350
SE_3a	ARFIMA(0,0.30,1)	150
SE_3b		350

b) Real-world time series

- We use **92 monthly time series of streamflow**, which originate from catchments in Australia (Peel et al. 2000). We use the deseasonalized time series for the application of the forecasting methods.
- To describe the long-term persistence of the deseasonalized time series we estimate their **Hurst parameter H** using the mleHK algorithm of the HKprocess R package (Tyralis 2016), which implements the maximum likelihood method (Tyralis and Koutsoyiannis 2011).
- The parameter H ranges in the interval (0,1). The larger it is the larger the long-range dependence of the Hurst - Kolmogorov stochastic process, which is widely used for the modelling of geophysical processes instead of the ARFIMA(0, d ,0) model.
- The estimated values range between 0.56 and 0.99 with a mean value of 0.78.

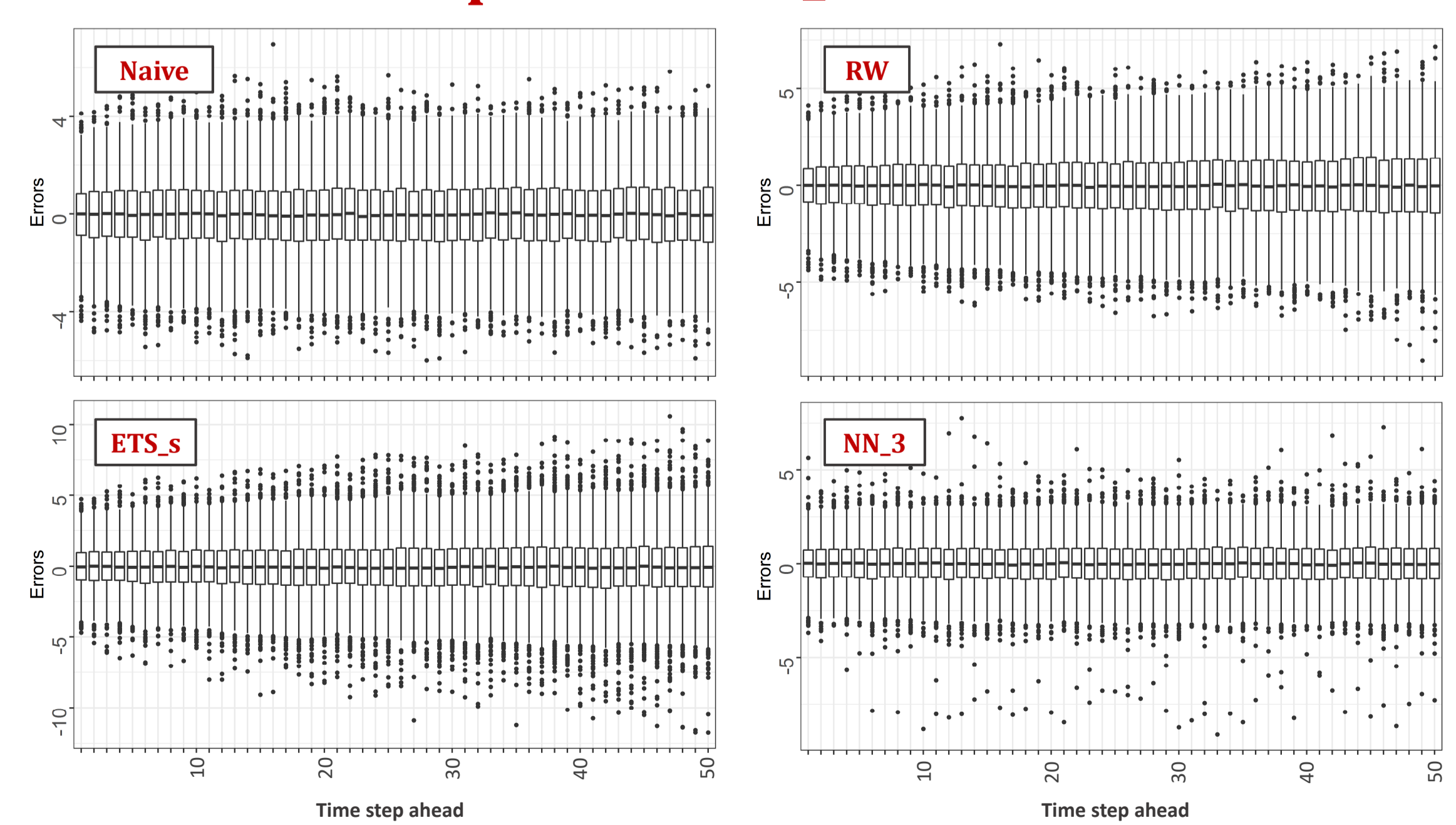
5. Forecasting methods

- We use the following forecasting methods originating from the implementation of several popular forecasting algorithms:

Naive <i>simple</i>	RW <i>simple</i>	auto_ARFIMA <i>ARFIMA</i>	BATS <i>state space</i>
ETS_s <i>state space</i>	SES <i>exponential smoothing</i>	Theta <i>exponential smoothing</i>	NN_1 <i>neural networks</i>
NN_2 <i>neural networks</i>	NN_3 <i>neural networks</i>	RF_1 <i>random forests</i>	RF_2 <i>random forests</i>
RF_3 <i>random forests</i>	SVM_1 <i>support vector machines</i>	SVM_2 <i>support vector machines</i>	SVM_3 <i>support vector machines</i>

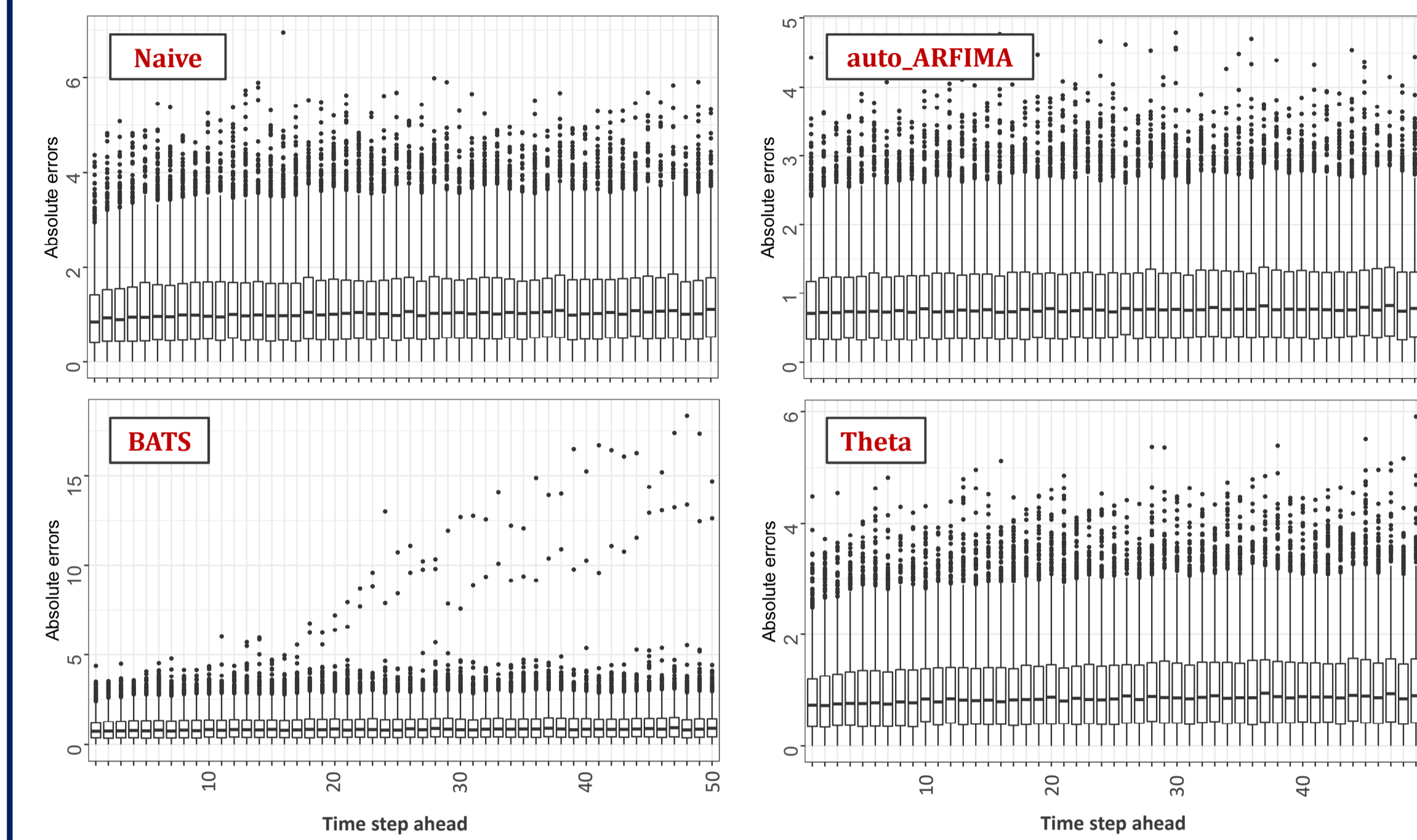
- We apply the simple, auto_ARFIMA, state space, exponential smoothing and NN_3 methods using the R package forecast (Hyndman and Khandakar 2008, Hyndman et al. 2017) and the remaining forecasting methods using the R package rminer (Cortez 2010, 2016), as also several built in R algorithms (R Core Team 2017).
- The R package rminer uses the nnet algorithm of the nnet R package (Venables and Ripley 2002), the randomForest algorithm of the randomForest R package (Liaw and Wiener 2002) and the ksvm algorithm of the kernlab R package (Karatzoglou et al. 2004) for the application of the neural networks, random forests and support vector machines respectively.
- The source code for the implementation of the forecasting methods, as well as generalized information about their performance when applied to linear stochastic processes, can be found in Papacharalampous et al. (2017a).

6. Simulation experiment SE_1a: errors



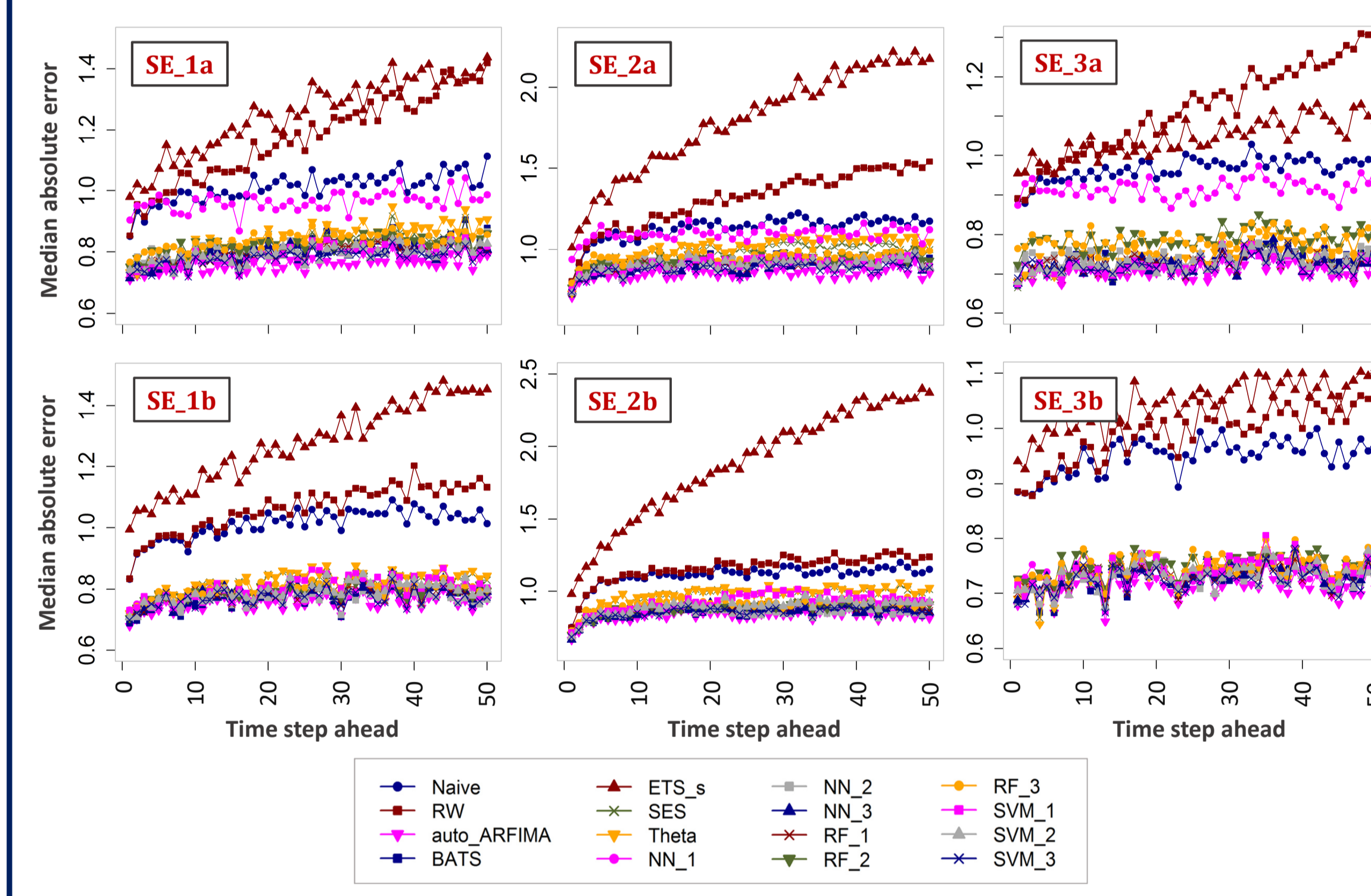
- The error evolution can differ to a great extent from the one forecasting method to the other. However, all the error distributions (see above figures) tend to be approximately symmetric around zero.
- At the first few time steps ahead we observe an apparent increase of the median and iqr values. This increase is followed by a stabilization of the error distributions for most of the forecasting methods (e.g. Naive and NN_3). On the contrary, when using the RW and ETS_s forecasting methods the errors seem to keep increasing until the last time step of the forecast horizon.
- The outliers are more frequent and lay farther from the median values when using specific forecasting methods (e.g. NN_3).

7. Simulation experiment SE_1a: absolute errors



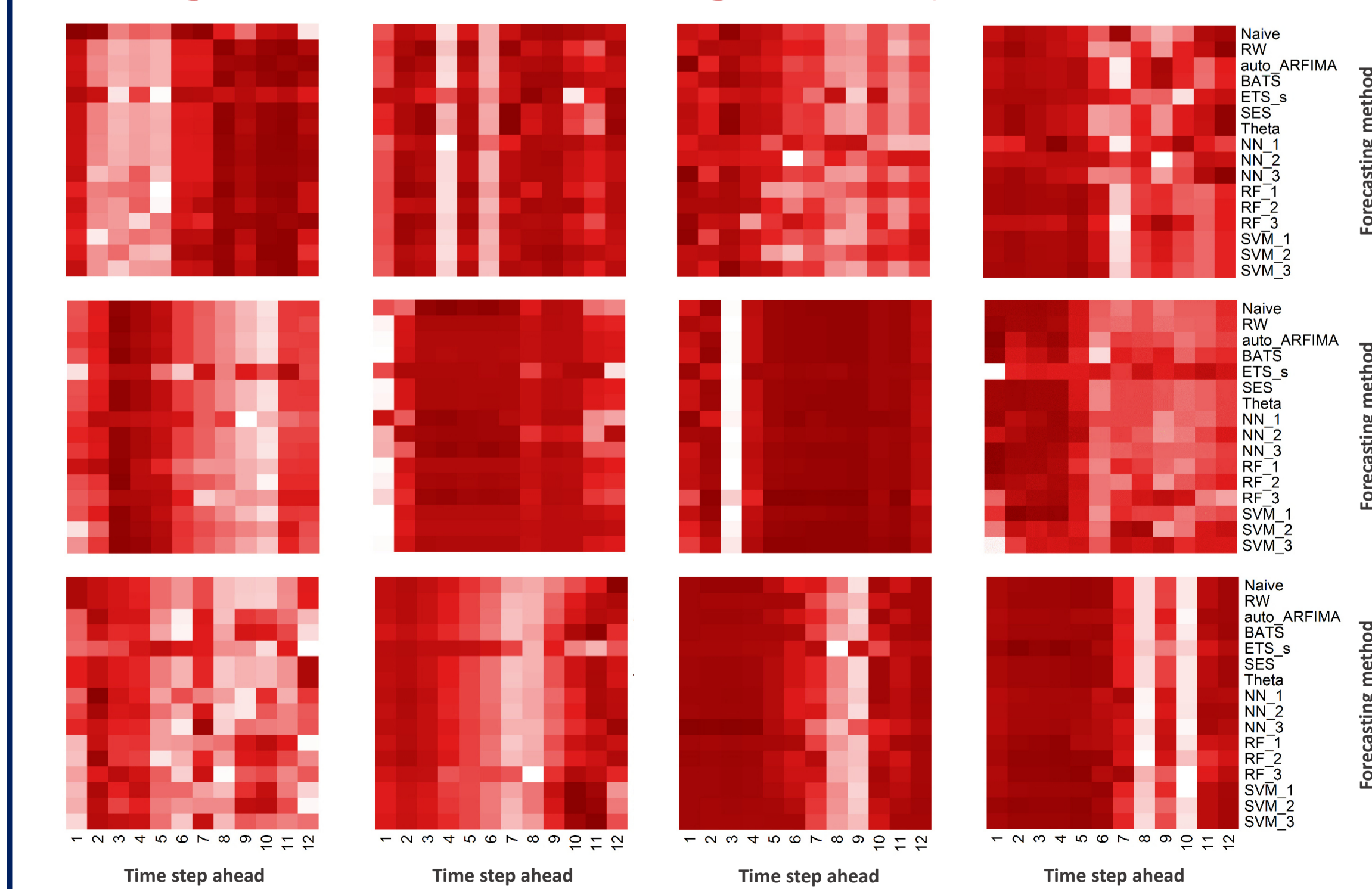
- The auto_ARFIMA and Theta forecasting methods are proven more accurate than the Naive benchmark.
- The same applies to BATS, which however produces far outliers. The latter tend to be farther from the median values, as the time step increases.
- Particularly noteworthy is the fact that forecasting methods sharing a quite similar performance within the experiments of Papacharalampous et al. (2017a) are somehow differentiated through the experiments of the present study, e.g. auto_ARFIMA and BATS, Naive and RW (see 6).

8. Simulation experiments: median absolute errors



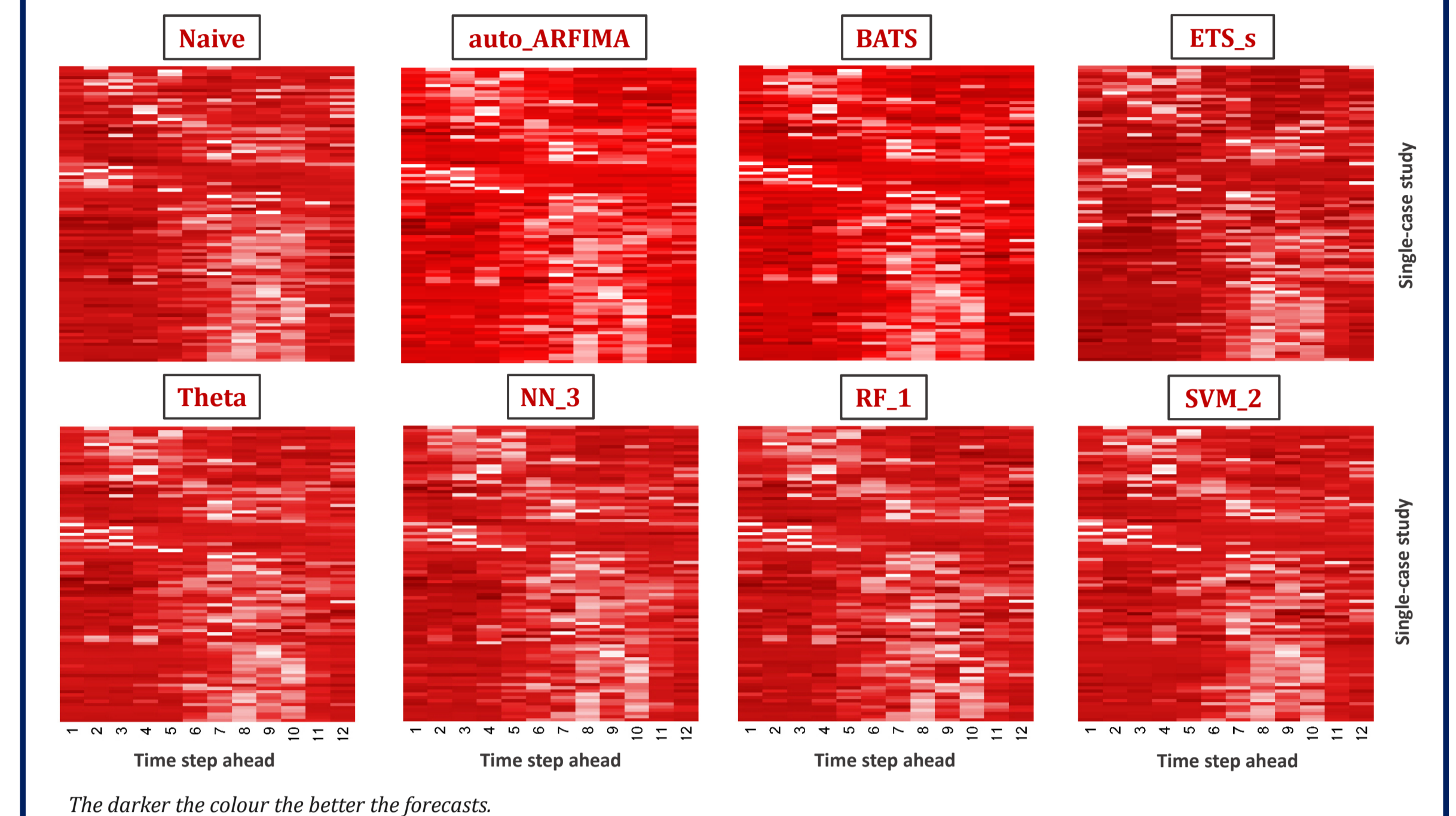
- The results vary from the one simulation experiment to the other to an extent depending on the forecasting method.

9. Single-case studies using monthly streamflow data



- The darker the colour the better the forecasts.

10. Cross-case synthesis



- The relative magnitude of the errors seems to strongly depend on the individual case examined.
- The effect of the forecasting method used or the time step of the forecasting horizon on the error evolution cannot be extracted from the figures presented in 9 and 10, neither from any other single- or multiple-case study.

11. Contribution of the present study

- We deliver generalized results on the error evolution in multi-step ahead forecasting using the recursive technique by comparing the performance of 16 forecasting methods under this specific light.
- The present study is an expansion of Papacharalampous et al. (2017a), as it provides complementary information about the forecasting methods also implemented in the latter.
- Our findings indicate that the error evolution can differ to a great extent from the one forecasting method to the other. This specific information can be used to decide on a forecasting method, since some forecasting methods have been proven more useful than others.
- However, due to the stochastic nature of forecasting, the errors computed at each time step of a forecast horizon within a specific case study strongly depend on the case examined and can be either small or large, regardless the forecasting method used and the time step of our interest.
- In fact, the limitations accompanying time series forecasting emphasized by Koutsoyiannis et al. (2008), as also by Papacharalampous et al. (2017a) and Papacharalampous et al. (2017b), are highly perceivable here as well.
- These limitations might impose the implementation of probabilistic forecasting methodologies (e.g. using Bayesian statistics, as in Tyralis and Koutsoyiannis 2014) instead of point forecasting.

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