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Coupling stochastic models of different timescales

Demetris Koutsoyiannis

Department of Water Resources, Faculty of Civil Engineering, National Technical University, Athens

Abstract. A methodology is proposed for coupling stochastic models of hydrologic processes applying to different timescales so that time series generated by the different models be consistent. Given two multivariate time series, generated by two separate (unrelated) stochastic models of the same hydrologic process, each applying to a different timescale, a transformation is developed (referred to as a coupling transformation) that appropriately modifies the time series of the lower-level (finer) timescale so that this series becomes consistent with the time series of the higher-level (coarser) timescale without affecting the second-order stochastic structure of the former and also establishes appropriate correlations between the two time series. The coupling transformation is based on a developed generalized mathematical proposition, which ensures preservation of marginal and joint second-order statistics and of linear relationships between lower- and higher-level processes. Several specific forms of the coupling transformation are studied, from the simplest single variate to the full multivariate. In addition, techniques for evaluating parameters of the coupling transformation based on second-order moments of the lower-level process are studied. Furthermore, two methods are proposed to enable preservation of the skewness of the processes in addition to that of second-order statistics. The overall methodology can be applied to problems involving disaggregation of annual to seasonal and seasonal to subseasonal timescales, as well as problems involving finer timescales (e.g., daily to hourly), with the only requirement that a specific stochastic model is available for each involved timescale. The performance of the methodology is demonstrated by means of a detailed numerical example.

1. Introduction

Very often a hydrologic stochastic process must be studied in different timescales. Therefore the problem arises of how to generate consistent time series both in a coarser, or higher-level, timescale and a finer, or lower-level, timescale. A trivial solution of this problem is to model the process in the lower-level timescale only and then aggregate to derive the process in the higher-level timescale. However, there are reasons to avoid this solution and model the process in both timescales separately, each time focusing on different important statistical properties of the process [*Salas*, 1993, p. 19.32]. For instance, if the higher- and lower-level scales are annual and seasonal, respectively, the lower-level model may focus on the periodicity and short-term memory of the process, whereas the higher-level model may focus on the long-term memory properties of the process. In other cases the higher-level process may be the output of a specialized model (e.g., a meteorological rainfall prediction model) or known from measurements (e.g., daily rainfall measurements); apparently, in such cases the aggregation approach cannot work, but rather disaggregation is needed.

Traditionally, this kind of problem is tackled by disaggregation models [*Valencia and Schaake*, 1972, 1973; *Mejia and Rousselle*, 1976; *Tao and Delleur*, 1976; *Hoshi and Burges*, 1979; *Lane*, 1979, 1982; *Salas et al*., 1980; *Todini*, 1980; *Stedinger and Vogel*, 1984; *Pereira et al*., 1984; *Stedinger et al*., 1985; *Oliveira et al*., 1988; *Grygier and Stedinger*, 1988, 1990; *Lane and Frevert*, 1990; *Santos and Salas*, 1992; *Koutsoyiannis*, 1992; *Salas*, 1993, p. 19.34; *Tarboton et al*., 1998]. These are purposely designed models to generate a process in the lower-level timescale given that in the higher-level. Specifically, they do not model the process of interest in the lower-level timescale itself, but rather they are hybrid schemes using both timescales simultaneously. Sometimes (owing to nonlinear transformations of variables) these models are not able to ensure consistency with the higher-level process. Then, adjusting procedures are necessary to restore consistency [*Grygier and Stedinger*, 1988, 1990; *Lane and Frevert*, 1990, p. V-22; *Koutsoyiannis and Manetas*, 1996].

However, there is the possibility of not designing and implementing a special model for disaggregation as a hybrid scheme incorporating both timescales. On the contrary, there may be available a model of the lower-level timescale with no reference to the higher-level timescale. The problem is then how a time series generated by the lower-level model can be modified so as to be consistent with a given higher-level time series, without affecting the stochastic structure implied by the lower-level model. (Practically, this is equivalent to the use of adjusting procedures mentioned before.) In a recent study, *Koutsoyiannis and Manetas* [1996] showed that this is possible without using any kind of disaggregation model but only using adjusting procedures on top of the separate lower-level model. Their adjusting procedures are accurate in the sense that they do not modify certain statistics of the lower-level process. In that study a contemporaneous seasonal autoregressive (PAR(1)) model was used as the lower-level model.

The present study is a generalization of that by *Koutsoyiannis and Manetas* [1996] in several senses. On the basis of a generalized mathematical proposition a wider transformation for modifying the lower-level time series, so as to be consistent with the higher-level time series, is introduced. Several forms

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of this transformation (referred to as coupling transformation) are studied. Apart from ensuring consistency with higher-level time series and reproducing second-order statistics of the lower-level variables within a certain period (higher-level time step), the transformation preserves lagged covariances of lower-level variables with lower- and higher-level variables of previous and next periods as well. Thus a well-known defect of disaggregation models, i.e., their inconsistency in preserving accurately lagged covariances among lower- and higher-level variables [*Lane*, 1982; *Stedinger and Vogel*, 1984], is remedied. In addition, the most general form of the proposed coupling transformation is true multivariate; that is, it is applied simultaneously to all the variables of all locations involved in the problem examined, rather than adjusting the variables of each location separately. Furthermore, the methodology proposed can be applied not only to the simple PAR(1) model but to any type of stationary or seasonal stochastic model for any timescale, with the only requirement that a specific stochastic model is available for each involved timescale.

The theoretical background of the methodology proposed is presented in section 2. The specific forms of the coupling transformation are studied in section 3, while the methods for evaluating their parameters are given in section 4. The problem of preservation of the coefficients of skewness of the variables involved is examined separately in section 5. A numerical example that demonstrates the performance of the methodology is given in section 6, and conclusions are drawn in section 7. To increase readability, several mathematical derivations are excluded from the paper.[1]

2. Theoretical Background

Let a hydrologic process, such as rainfall, runoff, etc., defined at n locations and studied in discrete time using two different timescales, the higher-level timescale with time step δ_H and the lower-level timescale with time step δ_L such that $k := \delta_H/\delta_L$ be an integer. We denote the higher- and lower-level discrete time processes by $\mathbf{Z}_p = [Z_p^1, \ldots, Z_p^n]^T$ and $\mathbf{X}_s = [X_s^1, \ldots, X_s^n]^T$, respectively, where superscript T denotes the transpose of a vector or matrix and subscripts p and s are integer time indices that stand for period and subperiod, respectively, with common origin (i.e., at the time origin $p = 0$ and $s = 0$). Generally, in this paper we use upper case letters for random variables and lower case letters for values, parameters, or constants. Furthermore, we use bold letters for arrays or vectors and italic letters for their elements. Higher- and lower-level processes are related by

$$\sum_{s=(p-1)k+1}^{pk} \mathbf{X}_s = \mathbf{Z}_p. \tag{1}$$

We assume that two separate stochastic models have been built, one for the higher-level process $\mathbf{Z}_p$ and one for the lower-level process $\mathbf{X}_s$, without link or reference between them. To increase readability, we can refer to the simple example where $\mathbf{Z}_p$ and $\mathbf{X}_s$ represent the annual and monthly flows at n locations, modeled as an AR(1) (autoregressive process of order 1) and a PAR(1) (periodic or seasonal autoregressive process of order 1), respectively. These models are expressed by

$$\mathbf{Z}_p = \mathbf{a}'\mathbf{Z}_{p-1} + \mathbf{b}'\mathbf{V}'_p, \tag{2}$$

$$\mathbf{X}_s = \mathbf{a}_s\mathbf{X}_{s-1} + \mathbf{b}_s\mathbf{V}_s, \tag{3}$$

where all $\mathbf{a}'$, $\mathbf{b}'$, $\mathbf{a}_s$, and $\mathbf{b}_s$ are $(n \times n)$ matrices of parameters and $\mathbf{V}'_p$ and $\mathbf{V}_s$ $(s, p = \ldots, 0, 1, 2, \ldots)$ are vectors of innovations (independent, both in time and location, random variables) with size n. The time indices s, p can take any integer value but in our example the parameters $\mathbf{a}_s$ and $\mathbf{b}_s$ are periodic functions of s with period k, whereas $\mathbf{a}'$ and $\mathbf{b}'$ do not vary with p.

We emphasize that models (2) and (3) and the relevant assumptions are given here just as simple model examples of a generalized methodology that can be combined with any type of multivariate stochastic models with any distribution functions and that can perform in any timescale. In fact, as no assumption is implied about the involved models, the methodology can be combined with linear and nonlinear stochastic models also including generalized autocovariance models [*Koutsoyiannis*, 2000], nonparametric models [*Lall and Sharma*, 1996; *Sharma et al.*, 1997], and hybrid models [*Srinivas and Srinivasan*, 2000]. Moreover, the modeling timescales need not be annual and monthly as in the examples used herein, but they can be much finer such as daily and hourly, even though distribution functions at those timescales are much more asymmetric.

We also notice that higher- and lower-level models need not be fully compatible. For example, models (2) and (3) are not fully compatible, as the covariance structure implied by (3) for the higher-level process $\mathbf{Z}_p$ (determined by invoking (1)) is not identical with that implied by (2) (it can be easily verified that the sum of AR(1) or PAR(1) processes cannot be an AR(1) process). If the models were fully compatible, the problem examined would be trivial, as the lower-level model would actually incorporate the higher-level model. Thus the problem acquires its interest in the case of (partially) incompatible models each focusing on different important statistical properties of the stochastic processes examined. For instance, the lower-level model may focus on the periodicity and the short-term memory of the process, whereas the higher-level model may focus on the long-term memory properties of the process. Apparently, (2) is not adequate for the latter case, and more complex models such as that proposed by *Koutsoyiannis* [2000] must be used instead.

Let us assume that a time series $\mathbf{z}_p$ of the process $\mathbf{Z}_p$ has been generated using model (2) (or any other linear or nonlinear, parametric or nonparametric model or even that it has been acquired from measurements) and that another time series $\tilde{\mathbf{x}}_s$ of the process $\mathbf{X}_s$ has been generated using (3) (or another appropriate stochastic model). The latter time series has been generated independently of the former, and therefore $\tilde{\mathbf{x}}_s$ do not add up to $\mathbf{z}_p$, as demanded by the additive property (1), but to some other quantities, which we will denote $\tilde{\mathbf{z}}_p$. We wish to modify the series $\tilde{\mathbf{x}}_s$ thus producing a series $\mathbf{x}_s$ consistent with $\mathbf{z}_p$, in the sense that $\mathbf{x}_s$ and $\mathbf{z}_p$ obey (1), without affecting the stochastic structure of the lower-level time series. For convenience we will assume that $\tilde{\mathbf{x}}_s$ is a realization of a stochastic process $\tilde{\mathbf{X}}_s$, identical to $\mathbf{X}_s$ (e.g., following (3)) and that the series $\tilde{\mathbf{z}}_p$ is a realization of a process $\tilde{\mathbf{Z}}_p$ defined as the sum of $\tilde{\mathbf{X}}_s$. In the ideal case that the processes $\mathbf{X}_s$ and $\mathbf{Z}_p$ are fully compatible, $\tilde{\mathbf{Z}}_p$ will be identical to $\mathbf{Z}_p$, but, as discussed

[1] Supporting appendices are available with entire article on microfiche. Order by mail from American Geophysical Union, 2000 Florida Avenue, N.W., Washington, DC 20009 or by phone at 800-966-2481; $2.50. Document 2000WR900200M. Payment must accompany order.

above, this is not the case in general (note that $\tilde{\mathbf{Z}}_p$ is derived as a summation of the lower-level process, whereas $\mathbf{Z}_p$ corresponds to the higher-level model regardless of the lower-level model). We seek for a transformation $\mathbf{f}(\tilde{\mathbf{X}}_s, \tilde{\mathbf{Z}}_p, \mathbf{Z}_p)$ whose outcome is a process identical to $\mathbf{X}_s$ and consistent to $\mathbf{Z}_p$ (it satisfies (1)). We will use the symbol $\mathbf{X}_s$ for the outcome of this transformation (i.e., $\mathbf{X}_s = \mathbf{f}(\tilde{\mathbf{X}}_s, \tilde{\mathbf{Z}}_p, \mathbf{Z}_p)$), and we will call this transformation a coupling transformation. With the followed notation we have two couples of processes, the auxiliary processes $(\tilde{\mathbf{X}}_s, \tilde{\mathbf{Z}}_p)$ and the "actual" processes $(\mathbf{X}_s, \mathbf{Z}_p)$; in each couple, lower- and higher-level processes are consistent (i.e., they satisfy (1)), but members of different couples are inconsistent. A schematic representation of the four processes involved, their links, and the steps followed to construct the "actual" lower-level process $\mathbf{X}_s$, consistent with $\mathbf{Z}_p$, is shown in Figure 1.

We can determine an appropriate linear form of the coupling transformation based on the following general proposition, specific forms of which we will extract and utilize in sections 3–6. For the generalized presentation of the proposition given below, the reader may have in mind that the vectors $\tilde{\mathbf{X}}$ and $\mathbf{X}$ contain numerous items of the auxiliary and actual lower-level processes, respectively, and the vectors $\tilde{\mathbf{Y}}$ and $\mathbf{Y}$ contain items of the higher-level processes and other variables that will be specified later. The additive property (1) is represented here by a more generalized linear relationship of the form of (6) below.

The proposition is as follows: let $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{Y}}$ be vectors of random variables with means $E[\tilde{\mathbf{X}}]$ and $E[\tilde{\mathbf{Y}}]$, variance-covariance matrices Cov $[\tilde{\mathbf{X}}, \tilde{\mathbf{X}}]$ and Cov $[\tilde{\mathbf{Y}}, \tilde{\mathbf{Y}}]$, respectively, and joint covariance matrix Cov $[\tilde{\mathbf{X}}, \tilde{\mathbf{Y}}]$. Also, let $\mathbf{Y}$ be a vector of stochastic variables independent of $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{Y}}$ with means and variance-covariance matrix identical to that of $\tilde{\mathbf{Y}}$. Define

$$\mathbf{X} := \tilde{\mathbf{X}} + \mathbf{h}(\mathbf{Y} - \tilde{\mathbf{Y}}), \tag{4}$$

where $\mathbf{h}$ is a matrix of parameters given by

$$\mathbf{h} := \mathrm{Cov}\,[\tilde{\mathbf{X}}, \tilde{\mathbf{Y}}]\{\mathrm{Cov}\,[\tilde{\mathbf{Y}}, \tilde{\mathbf{Y}}]\}^{-1}. \tag{5}$$

Then, (1) $\mathbf{X}$ has mean and variance-covariance matrix identical to those of $\tilde{\mathbf{X}}$ and joint covariance matrix with $\mathbf{Y}$ identical to that of $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{Y}}$. (2) Any linear relationships that hold among $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{Y}}$, which can be written in the form

$$\mathbf{g}_\mathbf{X}^T \tilde{\mathbf{X}} = \mathbf{g}_\mathbf{Y}^T \tilde{\mathbf{Y}}, \tag{6}$$

where $\mathbf{g}_\mathbf{X}$ and $\mathbf{g}_\mathbf{Y}$ are matrices (or vectors in case of a single linear relationship) of coefficients, hold also among $\mathbf{X}$ and $\mathbf{Y}$, that is,

$$\mathbf{g}_\mathbf{X}^T \mathbf{X} = \mathbf{g}_\mathbf{Y}^T \mathbf{Y}. \tag{7}$$

(3) The conditional variance of any element X_i of the vector $\mathbf{X}$, given $\mathbf{Y} = \mathbf{y}$, is

$$\mathrm{Var}\,[X_i | \mathbf{Y} = \mathbf{y}] = \mathrm{Var}\,[\tilde{X}_i] - \mathrm{Cov}\,[\tilde{X}_i, \tilde{\mathbf{Y}}] \cdot \{\mathrm{Cov}\,[\tilde{\mathbf{Y}}, \tilde{\mathbf{Y}}]\}^{-1}\,\mathrm{Cov}\,[\tilde{\mathbf{Y}}, \tilde{X}_i] \tag{8}$$

and is identical to the least mean square prediction error of X_i from $\mathbf{Y}$.

The proof of the proposition is given in Appendix A1 of the microfiche supplement. We mention here an interesting intermediate result regarding the proof of item 2 of the proposition: If (6) holds, $\mathbf{g}_\mathbf{X}$ and $\mathbf{g}_\mathbf{Y}$ affect the covariance matrices Cov $[\tilde{\mathbf{Y}}, \tilde{\mathbf{Y}}]$ and Cov $[\tilde{\mathbf{X}}, \tilde{\mathbf{Y}}]$ and consequently $\mathbf{h}$, so that finally

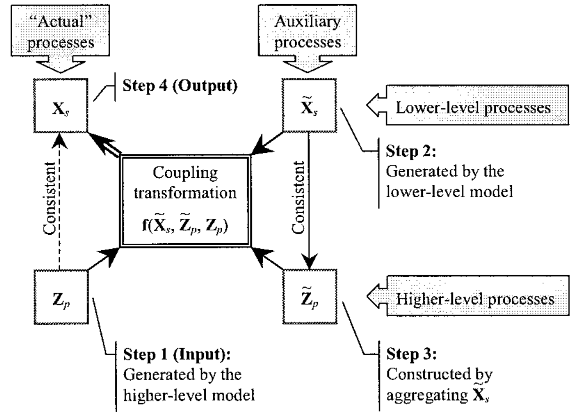

Figure 1. Schematic representation of actual and auxiliary processes, their links, and the steps followed to construct the actual lower-level process from the actual higher-level process.

$$\mathbf{g}_\mathbf{Y}^T = \mathbf{g}_\mathbf{X}^T \mathbf{h}. \tag{9}$$

We also notice that given the equality of covariances between the couples $(\tilde{\mathbf{X}}, \tilde{\mathbf{Y}})$ and $(\mathbf{X}, \mathbf{Y})$, we can substitute any covariance matrix of the first couple with the corresponding of the second couple; for example, we can write (5) as $\mathbf{h} := \mathrm{Cov}\,[\mathbf{X}, \mathbf{Y}]\{\mathrm{Cov}\,[\mathbf{Y}, \mathbf{Y}]\}^{-1}$.

Equations (4) and (5) offer the basis to develop the coupling transformation, as we will see in section 3. Item 3 of the proposition, although not used directly in developing the coupling transformation, ensures that (4) provides the best possible conditional estimate of $\mathbf{X}$ given $\mathbf{Y}$ in a least squares sense.

3. Specific Forms of Coupling Transformation

In this section we will develop several forms of the coupling transformation, starting from the simplest case of a single variate model and proceeding toward more complex cases. For mathematical convenience the transformation will extend to the lower-level variables of one period only (rather than extending to all simulated periods simultaneously) yet considering the necessary links to previous and next periods. For notational convenience we will assume that the time origin coincides with the origin of the examined period so that $p = 1$. Thus we will write (1) as

$$\sum_{s=1}^{k} \mathbf{X}_s = \mathbf{Z}_1. \tag{10}$$

We introduce the following notational conventions of covariances among lower- and/or higher-level variables:

$$\begin{aligned} \boldsymbol{\sigma}_{sr} &:= \mathrm{Cov}\,[\mathbf{X}_s, \mathbf{X}_r] \equiv \boldsymbol{\sigma}_{rs}^T, \qquad \boldsymbol{\varphi}_{pr} := \mathrm{Cov}\,[\mathbf{Z}_p, \mathbf{Z}_r] \equiv \boldsymbol{\varphi}_{rp}^T, \\ \boldsymbol{\tau}_{sp} &:= \mathrm{Cov}\,[\mathbf{X}_s, \mathbf{Z}_p], \qquad \boldsymbol{\tau}'_{ps} := \mathrm{Cov}\,[\mathbf{Z}_p, \mathbf{X}_s] \equiv \boldsymbol{\tau}_{sp}^T. \end{aligned} \tag{11}$$

The evaluation of these parameters will be discussed in section 4.

3.1. Preserving the Additive Property

In the simplest case we assume a single-site model with lower-level variables $X_1, \ldots, X_k$ adding up to the higher-level variable Z_1. We apply the proposition of section 2 setting $\mathbf{X} = [X_1, \ldots, X_k]^T$ and $\mathbf{Y} = [Z_1]$. In the single-site case examined, we have σ_{sr} = Cov $[X_s, X_r] \equiv \sigma_{rs}$, τ_{sp} = Cov $[X_s, Z_p] \equiv \tau'_{ps}$, and φ_{pr} = Cov $[Z_p, Z_r] \equiv \varphi_{rp}$. The additive property (10) can be written in the form (6) with $\mathbf{g}_{\mathbf{X}}^T = [1, 1, \ldots, 1]$ and $\mathbf{g}_{\mathbf{Y}}^T = [1]$. The parameter matrix $\mathbf{h}$ is (from (5))

$$\mathbf{h} = \frac{1}{\varphi_{11}} [\tau_{11}, \ldots, \tau_{k1}]^T, \tag{12}$$

and thus the coupling transformation (4) can be written for each subperiod s as

$$X_s := \tilde{X}_s + \frac{\tau_{s1}}{\varphi_{11}} (Z_1 - \tilde{Z}_1). \tag{13}$$

This is the simple adjusting procedure developed by *Koutsoyiannis and Manetas* [1996]. Note that each of the coefficients τ_{s1}/φ_{11} for a specific s represents the ratio of the covariance of each lower-level variable X_s with the higher-level variable Z_1 (τ_{s1}) to the variance of the higher-level variable Z_1 (φ_{11}). Thus (13) distributes the departure $(Z_1 - \tilde{Z}_1)$ of the additive property to each lower-level variable proportionally to the covariance of this lower-level variable with the higher-level variable. Note also that the covariances τ_{s1} for all s add up to the variance φ_{11} (see also section 4), and thus the coefficients τ_{s1}/φ_{11} for all s add up to 1, as they should. Thus the sum of all X_s will equal Z_1 regardless of the values of $\tilde{X}_s$; that is, the preservation of the additive property is ever assured. The special case where the lower-level variables are independent (the process X_s is white noise), although unusual, provides better understanding of the rationale of (13). In this case, τ_{s1} equals the variance of the lower-level variable X_s, so that the distribution of the departure $(Z_1 - \tilde{Z}_1)$ to each lower-level variable becomes proportional to the variance of the variable. Interestingly, *Grygier and Stedinger* [1988] and *Lane and Frevert* [1990, p. V-22] had proposed a similar empirical adjusting procedure but using the standard deviation in place of the variance of each of the lower-level variables. We must emphasize, however, that the exact transformation that assures preservation of the additive property, means, and second-order moments of the process in the general case of dependent variables is expressed as in (13) in terms of the covariances τ_{s1} rather than variances or standard deviations of the different lower-level variables.

3.2. Linking With Lower-Level Variables of the Previous Period

The simple transformation in section 3.1 preserves the additive property and correlations of lower-level variables within the examined period. However, it does not preserve explicitly the correlations of lower-level variables with subperiods of previous periods. For example, it does not preserve explicitly the correlation of the first lower-level variable X_1 with the last lower-level variable of the previous period X_0. This can be easily remedied by setting $\mathbf{X} = [X_1, \ldots, X_k]^T$ and $\mathbf{Y} = [X_0, Z_1]^T$. In this case, $\mathbf{g}_{\mathbf{X}}^T = [1, 1, \ldots, 1]$ and $\mathbf{g}_{\mathbf{Y}}^T = [0, 1]$ so that $\mathbf{g}_{\mathbf{X}}^T \mathbf{X}$ is the sum of all lower-level variables and $\mathbf{g}_{\mathbf{Y}}^T \mathbf{Y}$ is the higher-level variable Z_1. The parameter matrix $\mathbf{h}$ is

$$\mathbf{h} = \begin{bmatrix} \sigma_{10} & \tau_{11} \\ \vdots & \vdots \\ \sigma_{k0} & \tau_{k1} \end{bmatrix} \begin{bmatrix} \sigma_{00} & \tau_{01} \\ \tau_{01} & \varphi_{11} \end{bmatrix}^{-1}. \tag{14}$$

Thus the coupling transformation (4) can be written for each subperiod s as

$$X_s := \tilde{X}_s + \frac{1}{\sigma_{00}\varphi_{11} - \tau_{01}^2} [(\varphi_{11}\sigma_{s0} - \tau_{01}\tau_{s1})(X_0 - \tilde{X}_0) + (\sigma_{00}\tau_{s1} - \tau_{01}\sigma_{s0})(Z_1 - \tilde{Z}_1)]. \tag{15}$$

This notion can be extended to include a greater number of previous lower-level variables, as we will see in section 3.3. Here an explanation of the rationale of the different terms of (15) is no longer simple as it was in (13) (this in even more the case for the equations of sections 3.3 and 3.4). We can only say that (15) performs two kinds of adjustment to the auxiliary lower-level variables $\tilde{X}_s$: First, it distributes the departure $(Z_1 - \tilde{Z}_1)$ among the different lower-level variables so as to restore the additive property. Second, it modifies $\tilde{X}_s$ in proportion to the departure $(X_0 - \tilde{X}_0)$ so as to restore the dependence with lower-level variables of the previous period. It may be easily verified that the coefficients used to distribute $(Z_1 - \tilde{Z}_1)$ add up to 1, as they should, whereas the coefficients used for $(X_0 - \tilde{X}_0)$ add up to 0, as they should, too.

3.3. Linking With Next Higher-Level Variables

Linking the lower-level variables of the current period with those of the previous period, in the way discussed in section 3.2, may be regarded as linking with lower-level variables of the next subperiod as well. Specifically, at the stage of generating the lower-level variables of the next period the correlation with the lower-level variables of the current period will be preserved in the manner discussed in section 3.2.

However, this is not absolutely correct, because in this manner the lagged correlations between lower- and higher-level variables are not considered explicitly. As shown by *Stedinger and Vogel* [1984], the departures in preserving such lagged correlations are responsible for inconsistencies in preserving correlations between lower-level variables of different periods; this problem was first reported by *Lane* [1982], and contributions to overcome it were made by *Stedinger and Vogel* [1984], *Lin* [1990], *Koutsoyiannis* [1992], and *Koutsoyiannis and Manetas* [1996].

The developed general proposition allows for an effective tackling of this problem. In addition to correlations with the previous lower-level variables, discussed in section 3.2, we will also consider the preservation of correlations between the lower-level variables of the current period and the higher-level variable of the next period. We note that the correlation of the former with the higher-level variables of the previous periods has been already considered indirectly (through correlations with the corresponding lower-level variables), whereas the correlation with the higher-level variable of the current period has been incorporated explicitly (through the coupling transformation).

We will distinguish between two cases regarding the succession of generation steps of higher- and lower-level variables. In the first case all higher-level variables of all periods are generated before the generation of lower-level variables. In the second case the generation of lower-level variables of one period follows the generation of the higher-level variable of that period and precedes that of the next period.

In the first case, at the step of generating the lower-level variables of the current period, the higher-level variable of the next period (Z_2) is already known, and the correlation with it must be considered and preserved. This may be essential especially when this correlation is high (e.g., for fine timescales). To this aim we must append Z_2 to the vector $\mathbf{Y}$ again setting

$\mathbf{X} = [X_1, \ldots, X_k]^T$. In addition, to acquire a more generalized solution than that of section 3.2, which was appropriate for the specific model (3), we append to $\mathbf{Y}$ a number q (depending on the lower-level model used) of lower-level variables prior to X_0 so that finally $\mathbf{Y} = [X_{-q}, \ldots, X_0, Z_1, Z_2]^T$. The vectors $\mathbf{g_X}$ and $\mathbf{g_Y}$ become $\mathbf{g_X}^T = [1, 1, \ldots, 1]$ and $\mathbf{g_Y}^T = [0, \ldots, 0, 1, 0]$. The parameter matrix $\mathbf{h}$ is

$$\mathbf{h} = \begin{bmatrix} \sigma_{1,-q} & \cdots & \sigma_{10} & \tau_{11} & \tau_{12} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \sigma_{k,-q} & \cdots & \sigma_{k,0} & \tau_{k,1} & \tau_{k,2} \end{bmatrix} \cdot \begin{bmatrix} \sigma_{-q,-q} & \cdots & \sigma_{-q,0} & \tau_{-q,1} & \tau_{-q,2} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \sigma_{0,-q} & \cdots & \sigma_{00} & \tau_{01} & \tau_{02} \\ \tau'_{1,-q} & \cdots & \tau'_{10} & \varphi_{11} & \varphi_{12} \\ \tau'_{2,-q} & \cdots & \tau'_{20} & \varphi_{21} & \varphi_{22} \end{bmatrix}^{-1}. \quad (16)$$

The coupling transformation (4) can be written for each subperiod s as

$$X_s := \tilde{X}_s + \mathbf{h}_s[(X_{-q} - \tilde{X}_{-q}), \ldots, (X_0 - \tilde{X}_0), (Z_1 - \tilde{Z}_1), (Z_2 - \tilde{Z}_2)]^T, \quad (17)$$

where $\mathbf{h}_s$ is the sth row of $\mathbf{h}$.

In the second case mentioned above (which is met rather rarely), at the step of generation of the lower-level variables of the current period, the higher-level variable of the next period Z_2 is not known, and thus the analysis of section 3.2 suffices. Just before that step, the higher-level variable Z_1 of the current period is to be generated. At this time the lower-level variables of the previous periods ($X_0, X_{-1}, \ldots$) are already known, and correlation with them must be preserved. However, this is not done automatically by the higher-level model itself (e.g., by (2)). Using the general proposition, we can remedy this problem as well if we set $\mathbf{X} = [Z_1]$ and $\mathbf{Y} = [X_{-q}, \ldots, X_0]^T$, i.e., the vector of $q + 1$ lower-level variables of the previous periods. In this case, $q + 1$ can be chosen equal to k, the number of lower-level variables of one period, but it can be lower or greater than this value as well, depending on how large the correlation of higher-level to lagged lower-level variables is. The parameter matrix now is

$$\mathbf{h}_Z = [\tau'_{1,-q} \ldots \tau'_{10}] \begin{bmatrix} \sigma_{-q,-q} & \cdots & \sigma_{-q,0} \\ \vdots & \ddots & \vdots \\ \sigma_{0,-q} & \cdots & \sigma_{00} \end{bmatrix}^{-1}, \quad (18)$$

and the coupling transformation is

$$Z_1 := \tilde{Z}_1 + \mathbf{h}_Z[(X_{-q} - \tilde{X}_{-q}), \ldots, (X_0 - \tilde{X}_0)]^T. \quad (19)$$

The generation of the lower-level variables of the current period follows that of the higher-level variable. This is done by (14) and (15) if only one previous lower-level variable is considered or otherwise by the more general relationship

$$X_s := \tilde{X}_s + \mathbf{h}_s[(X_{-q} - \tilde{X}_{-q}), \ldots, (X_0 - \tilde{X}_0), (Z_1 - \tilde{Z}_1)]^T, \quad (20)$$

where $\mathbf{h}_s$ is the sth row of the matrix $\mathbf{h}$ that is now given by

$$\mathbf{h} = \begin{bmatrix} \sigma_{1,-q} & \cdots & \sigma_{10} & \tau_{11} \\ \vdots & \ddots & \vdots & \vdots \\ \sigma_{k,-q} & \cdots & \sigma_{k,0} & \tau_{k,1} \end{bmatrix} \begin{bmatrix} \sigma_{-q,-q} & \cdots & \sigma_{-q,0} & \tau_{-q,1} \\ \vdots & \ddots & \vdots & \vdots \\ \sigma_{0,-q} & \cdots & \sigma_{00} & \tau_{01} \\ \tau'_{1,-q} & \cdots & \tau'_{10} & \varphi_{11} \end{bmatrix}^{-1}. \quad (21)$$

Here (20) and (21) have been derived from (17) and (16), respectively, by omitting all elements referring to Z_2.

3.4. Multivariate Case

The above forms of the coupling transformation can be applied location by location in the case of a multivariate process. However, in this manner, the cross correlations of the lower-level variables will be altered by the single-location coupling transformations of different locations. The same coupling transformations can be formulated in a true multivariate form, so that cross correlations are explicitly preserved. This is a very simple task, as it suffices to write the same relationships in multivariate form. We will give here the multivariate version of the most general case of section 3.3; the other cases are remedied in a similar manner.

The vector $\mathbf{X}$ is formed by appending all vectors of lower-level variables of the current period, and the vector $\mathbf{Y}$ is constructed in a similar manner, i.e.,

$$\mathbf{X} = \begin{bmatrix} \mathbf{X}_1 \\ \vdots \\ \mathbf{X}_k \end{bmatrix}, \qquad \mathbf{Y} = \begin{bmatrix} \mathbf{X}_{-q} \\ \vdots \\ \mathbf{X}_0 \\ \mathbf{Z}_1 \\ \mathbf{Z}_2 \end{bmatrix}. \quad (22)$$

Thus $\mathbf{X}$ and $\mathbf{Y}$ have kn and $(q + 3)n$ elements, respectively. The matrices $\mathbf{g_X}$ and $\mathbf{g_Y}$, needed to express the additive property in the multivariate form (10), are constructed as in section 3.3 but replacing 1 with the $n \times n$ identity matrix $\mathbf{I}$ and 0 with the $n \times n$ zero matrix $\mathbf{O}$, i.e.,

$$\mathbf{g_X}^T = [\mathbf{I}, \mathbf{I}, \ldots, \mathbf{I}] \qquad \mathbf{g_Y}^T = [\mathbf{O}, \ldots, \mathbf{O}, \mathbf{I}, \mathbf{O}]. \quad (23)$$

For example, in a problem with $k = 3$ lower-level variables, $n = 2$ locations, and $q = 0$, the relevant vectors and matrices become

$$\mathbf{X} = \begin{bmatrix} X_1^1 \\ X_1^2 \\ X_2^1 \\ X_2^2 \\ X_3^1 \\ X_3^2 \end{bmatrix}, \qquad \mathbf{Y} = \begin{bmatrix} X_0^1 \\ X_0^2 \\ Z_1^1 \\ Z_1^2 \\ Z_2^1 \\ Z_2^2 \end{bmatrix},$$
$$\mathbf{g_X}^T = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}, \quad (24)$$
$$\mathbf{g_Y}^T = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

It can be directly verified from this example that the relationship (7) (i.e., $\mathbf{g_X}^T \mathbf{X} = \mathbf{g_Y}^T \mathbf{Y}$) is identical to the additive property (10).

The parameter matrix $\mathbf{h}$ is constructed as in section 3.3 but replacing each scalar covariance σ, τ, τ', or φ with its corresponding $n \times n$ matrix $\boldsymbol{\sigma}$, $\boldsymbol{\tau}$, $\boldsymbol{\tau}'$, or $\boldsymbol{\varphi}$, respectively. Thus

$$\mathbf{h} = \begin{bmatrix} \boldsymbol{\sigma}_{1,-q} & \cdots & \boldsymbol{\sigma}_{10} & \boldsymbol{\tau}_{11} & \boldsymbol{\tau}_{12} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \boldsymbol{\sigma}_{k,-q} & \cdots & \boldsymbol{\sigma}_{k,0} & \boldsymbol{\tau}_{k,1} & \boldsymbol{\tau}_{k,2} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{\sigma}_{-q,-q} & \cdots & \boldsymbol{\sigma}_{-q,0} & \boldsymbol{\tau}_{-q,1} & \boldsymbol{\tau}_{-q,2} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \boldsymbol{\sigma}_{0,-q} & \cdots & \boldsymbol{\sigma}_{00} & \boldsymbol{\tau}_{01} & \boldsymbol{\tau}_{02} \\ \boldsymbol{\tau}'_{1,-q} & \cdots & \boldsymbol{\tau}'_{10} & \boldsymbol{\varphi}_{11} & \boldsymbol{\varphi}_{12} \\ \boldsymbol{\tau}'_{2,-q} & \cdots & \boldsymbol{\tau}'_{20} & \boldsymbol{\varphi}_{21} & \boldsymbol{\varphi}_{22} \end{bmatrix}^{-1}. \tag{25}$$

The coupling transformation (4) is

$$\mathbf{X} := \tilde{\mathbf{X}} + \mathbf{h}[(\mathbf{X}_{-q}^T - \tilde{\mathbf{X}}_{-q}^T), \ldots, (\mathbf{X}_0^T - \tilde{\mathbf{X}}_0^T), (\mathbf{Z}_1^T - \tilde{\mathbf{Z}}_1^T), (\mathbf{Z}_2^T - \tilde{\mathbf{Z}}_2^T)]^T. \tag{26}$$

In the second case mentioned in section 3.3, again we have two steps. At the first step, concerning the generation of the higher-level variables, the corresponding multivariate variables are $\mathbf{X} = \mathbf{Z}_1$ and $\mathbf{Y} = [\mathbf{X}_{-q}^T, \ldots, \mathbf{X}_0^T]^T$. The parameter matrix $\mathbf{h}_\mathbf{Z}$ now is

$$\mathbf{h}_\mathbf{Z} = [\boldsymbol{\tau}'_{1,-q} \ldots \boldsymbol{\tau}'_{10}] \begin{bmatrix} \boldsymbol{\sigma}_{-q,-q} & \cdots & \boldsymbol{\sigma}_{-q,0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{\sigma}_{0,-q} & \cdots & \boldsymbol{\sigma}_{00} \end{bmatrix}^{-1}, \tag{27}$$

and the coupling transformation is

$$\mathbf{Z}_1 := \tilde{\mathbf{Z}}_1 + \mathbf{h}_\mathbf{Z}[(\mathbf{X}_{-q}^T - \tilde{\mathbf{X}}_{-q}^T), \ldots, (\mathbf{X}_0^T - \tilde{\mathbf{X}}_0^T)]^T. \tag{28}$$

At the second step, concerning the generation of the lower-level variables, the vectors of variables are $\mathbf{X} = [\mathbf{X}_1^T, \ldots, \mathbf{X}_k^T]^T$ and $\mathbf{Y} = [\mathbf{X}_{-q}^T, \ldots, \mathbf{X}_0^T, \mathbf{Z}_1^T]^T$, the parameter matrix is

$$\mathbf{h} = \begin{bmatrix} \boldsymbol{\sigma}_{1,-q} & \cdots & \boldsymbol{\sigma}_{10} & \boldsymbol{\tau}_{11} \\ \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{\sigma}_{k,-q} & \cdots & \boldsymbol{\sigma}_{k,0} & \boldsymbol{\tau}_{k,1} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{\sigma}_{-q,-q} & \cdots & \boldsymbol{\sigma}_{-q,0} & \boldsymbol{\tau}_{-q,1} \\ \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{\sigma}_{0,-q} & \cdots & \boldsymbol{\sigma}_{00} & \boldsymbol{\tau}_{01} \\ \boldsymbol{\tau}'_{1,-q} & \cdots & \boldsymbol{\tau}'_{10} & \boldsymbol{\varphi}_{11} \end{bmatrix}^{-1}, \tag{29}$$

and the coupling transformation is

$$\mathbf{X} := \tilde{\mathbf{X}} + \mathbf{h}[(\mathbf{X}_{-q}^T - \tilde{\mathbf{X}}_{-q}^T), \ldots, (\mathbf{X}_0^T - \tilde{\mathbf{X}}_0^T), (\mathbf{Z}_1^T - \tilde{\mathbf{Z}}_1^T)]^T. \tag{30}$$

4. Evaluation of Parameters of Coupling Transformation

We have seen in section 3 that all forms of the coupling transformation involve three categories of parameters, defined in (11). These are (1) covariances between lower-level variables, denoted by σ; (2) covariances between higher-level variables, denoted by φ; and (3) covariances between lower- and higher-level variables, denoted by τ or τ'.

A first option to evaluate these parameters would be to refer to the historical data. This, however, must be avoided for several reasons: (1) because in this way we would introduce a vast number of parameters dependent on, and estimated from, the data, (2) because usually historical data records are limited and inadequate to estimate such a large parameter set, and (3) because such a large parameter set, if estimated from historical data, may not be consistent with the higher- or lower-level models, which are usually expressed in terms of a parameter set as parsimonious as possible. The alternative is to let models determine the parameters (more specifically, the lower-level model, as will be explained in the following paragraphs). There are two options to do this, one numerical and one analytical.

The numerical option is based on stochastic simulation and is fully generalized, as it can perform with any type of lower-level model: We can generate a synthetic data record of lower-level variables $\tilde{\mathbf{X}}$ with an appropriate length and aggregate it to obtain the higher-level variables $\tilde{\mathbf{Z}}$. As covariances between $\tilde{\mathbf{X}}$ and/or $\tilde{\mathbf{Z}}$ equal those of $\mathbf{X}$ and/or $\mathbf{Z}$, we can use these synthetic data records to directly estimate the parameters. This option has the advantage of being simple and independent of the type of model. However, it has the disadvantages of the approximate character of estimations and the computational effort needed.

The analytical option is case-specific and uses the properties of the lower-level model chosen to determine the needed parameters theoretically. Owing to its exact character and the fast evaluation of parameters this option is the most preferable whenever analytical equations can be established for the model chosen. Below, we will give the equations that are necessary to evaluate the needed parameters for a list of very common lower-level models of the literature [e.g., *Salas et al.*, 1980; *Bras and Rodriguez-Iturbe*, 1985; *Lane and Frevert*, 1990; *Grygier and Stedinger*, 1990; *Salas*, 1993]. The derivations of equations are given in Appendix A3 of the supplement and may serve as a basis for extending the list given here with more models.

For the simple PAR(1) example defined by (3), the covariance of lower-level variables for any lag $(s - r)$ is given by

$$\boldsymbol{\sigma}_{sr} := \mathrm{Cov}\,[\mathbf{X}_s, \mathbf{X}_r] = \mathbf{a}_s\mathbf{a}_{s-1} \ldots \mathbf{a}_{r+1}\boldsymbol{\sigma}_{rr}, \qquad s > r \tag{31}$$

so that all lagged covariances among lower-level variables $\boldsymbol{\sigma}_{sr}$ are determined in terms of lag-zero cross covariances $\boldsymbol{\sigma}_{ss}$ and the model parameters $\mathbf{a}_s$. Similar (although somehow more complex) is the situation with other common hydrologic stochastic models. Thus the PAR(2) model, expressed by

$$\mathbf{X}_s = \mathbf{a}_s\mathbf{X}_{s-1} + \mathbf{e}_s\mathbf{X}_{s-2} + \mathbf{b}_s\mathbf{V}_s, \tag{32}$$

where all $\mathbf{a}_s$, $\mathbf{e}_s$, and $\mathbf{b}_s$ are $(n \times n)$ matrices of parameters, results in

$$\boldsymbol{\sigma}_{sr} = \mathbf{a}_s\boldsymbol{\sigma}_{s-1,r} + \mathbf{e}_s\boldsymbol{\sigma}_{s-2,r}, \qquad s > r. \tag{33}$$

By applying this relationship recursively, for $s = r + 1, r + 2, \ldots$, we can find any lagged covariance of lower-level variables in terms of lag-zero and lag-one covariance matrices ($\boldsymbol{\sigma}_{ss}$ and $\boldsymbol{\sigma}_{s-1,s}$) and the model parameters $\mathbf{a}_s$ and $\mathbf{e}_s$.

Similarly, the PARMA(1, 1) model, expressed by

$$\mathbf{X}_s = \mathbf{a}_s\mathbf{X}_{s-1} + \mathbf{b}_s\mathbf{V}_s + \mathbf{e}_s\mathbf{V}_{s-1}, \tag{34}$$

where $\mathbf{a}_s$, $\mathbf{b}_s$, and $\mathbf{e}_s$ are $(n \times n)$ matrices of parameters, results in

$$\begin{aligned} \boldsymbol{\sigma}_{sr} &= \mathbf{a}_s\boldsymbol{\sigma}_{rr} + \mathbf{e}_s\mathbf{b}_r^T, \qquad s = r + 1 \\ \boldsymbol{\sigma}_{sr} &= \mathbf{a}_s\boldsymbol{\sigma}_{s-1,r}, \qquad s > r + 1. \end{aligned} \tag{35}$$

By applying this relationship recursively, for $s = r + 1, r + 2, \ldots$, we can find any lagged covariance of lower-level variables in terms of the lag-zero covariance matrix $\boldsymbol{\sigma}_{ss}$ and the model parameters $\mathbf{a}_s$, $\mathbf{b}_s$, and $\mathbf{e}_s$.

Similar relationships are extracted for PAR or PARMA models of higher order. If the processes are not periodic (seasonal) but are stationary, the same equations apply but in a simplified form as all parameter matrices do not depend on subperiod.

It is very common in stochastic hydrology the case that the lower-level model is expressed in terms of the logarithmic transformation of the lower-level variables, for example, in terms of

$$\mathbf{X}_s^* := \ln(\mathbf{X}_s - \mathbf{c}_s), \tag{36}$$

where $\mathbf{c}_s$ is a vector of parameters estimated in such a manner that $\mathbf{X}_s^*$ is (approximately) normally distributed. In this case, relationships (31)–(35) express the covariances of the logarithmic transformations of variables. It is easy then to derive the covariances of the untransformed variables (which will be used then in the transformation) through the relation

$$\sigma_{sr}^{lj} = (\mu_s^l - c_s^l)(\mu_r^j - c_r^j)[\exp(\sigma_{sr}^{lj*}) - 1], \tag{37}$$

valid for any s, r, l, and j, where $\sigma_{sr}^{lj} = \operatorname{Cov}[X_s^l, X_r^j]$, $\sigma_{sr}^{lj*} = \operatorname{Cov}[X_s^{l*}, X_r^{j*}]$, and

$$\mu_s^l = E[X_s^l] = c_s^l + \exp(\mu_s^{l*} + \sigma_{ss}^{ll*}/2), \tag{38}$$

with $\mu_s^{l*} = E[X_s^{l*}]$.

In conclusion, any lagged covariance matrix of lower-level variables ($\boldsymbol{\sigma}_{sr}$) can be determined in terms of the lower-level model parameters by either of the two methods (options) proposed. The next step is to determine covariances between lower-level and higher-level variables ($\boldsymbol{\tau}_{sp}$). This is rather easy, because (1) implies that

$$\boldsymbol{\tau}_{sp} = \operatorname{Cov}[\mathbf{X}_s, \mathbf{Z}_p] = \sum_{r=(p-1)k+1}^{pk} \operatorname{Cov}[\mathbf{X}_s, \mathbf{X}_r] \tag{39}$$

or

$$\boldsymbol{\tau}_{sp} = \sum_{r=(p-1)k+1}^{pk} \boldsymbol{\sigma}_{sr}. \tag{40}$$

What remains is the determination of lagged covariances between higher-level variables ($\boldsymbol{\varphi}_{sp}$). Again, using (1) we get

$$\boldsymbol{\varphi}_{pr} = \operatorname{Cov}[\mathbf{Z}_p, \mathbf{Z}_r] = \sum_{s=(p-1)k+1}^{pk} \operatorname{Cov}[\mathbf{X}_s, \mathbf{Z}_r] \tag{41}$$

or

$$\boldsymbol{\varphi}_{pr} = \sum_{r=(p-1)k+1}^{pk} \boldsymbol{\tau}_{sr}. \tag{42}$$

We emphasize that the above estimation of $\boldsymbol{\varphi}_{pr}$ has been based on the lower-level model, although it could also be based on the higher-level model. However, in the latter case, possible incompatibilities of the two models would have negative consequences in preservation of the additive property. This is easily demonstrated through the simplest transformation (13): If φ_{11} is estimated from the lower-level model (equation (42)), that is, as the sum of τ_{s1} for all s, then the coefficients τ_{s1}/φ_{11} add up to 1 and (13) preserves the additive property. On the contrary, if φ_{11} were estimated directly from the higher-level model, possibly it would have some departure from the sum of τ_{s1} (because of incompatibilities of models), which would result in violation of the additive property. This situation, that is, the estimation of higher-level covariances using the lower-level model, may seem strange at first glance. However, a more careful consideration of the context where these estimations of covariances are used shows that it is absolutely justified. Specifically, these estimations are not used in the higher-level model at all. On the contrary, this is an independent model that is fitted in a different manner (the appropriate one for the specific model chosen, which is beyond the scope of this paper). Moreover, the higher-level model is run in an initial modeling phase, previous to that of the lower-level model. In turn, the lower-level model is fitted with a procedure that is appropriate for this specific model, which again is beyond the scope of this paper. Thus the estimations of covariances described in the present section are used only in the coupling transformation, which applies to the values generated by the lower-level model. Therefore it is natural to infer these parameters using the lower-level model only.

5. Preservation of Skewness

The preservation of skewness is often of great importance, as hydrologic processes, particularly in small timescales, exhibit nonsymmetric distributions. In all analyses of the previous sections the skewness of the processes, either higher- or lower-level, was not considered. On the contrary, the analyses focused on second, marginal or joint moments of the processes, the preservation of which was proved theoretically. Unfortunately, the preservation of third moments is hard to be handled in an analytical manner.

From the general relation (4), used to develop the various forms of the coupling transformation, we may conclude that the marginal third moments of $\mathbf{X}$ do not necessarily equal those of $\tilde{\mathbf{X}}$. Specifically, we may assume that the marginal third moment of the term $\mathbf{h}(\mathbf{Y} - \tilde{\mathbf{Y}})$ is zero, because of symmetry, and thus it is not responsible for differences between the coefficients of skewness of $\tilde{\mathbf{X}}$ and $\mathbf{X}$. However, apart from that marginal moment, joint third moments of $\tilde{\mathbf{X}}$ and $\mathbf{h}\,\tilde{\mathbf{Y}}$ may create such differences. These joint third moments are difficult to determine analytically. Generally, because $\mathbf{X}$ in (4) is expressed as a linear combination of $\tilde{\mathbf{X}}$ and other variables, we expect that the coefficient of skewness of $\mathbf{X}$ will be lower than that of $\tilde{\mathbf{X}}$ (from the central limit theorem we know that under certain conditions, linear combinations of variables tend to have symmetric distributions). Indeed, numerical investigations confirm this observation. Since an analytical solution is too complicated (if not intractable), we seek for approximate numerical methods. We will discuss two such methods.

Let $\zeta_s^l := E[(X_s^l - \mu_s^l)^3]$ and $\tilde{\zeta}_s^l := E[(\tilde{X}_s^l - \mu_s^l)^3]$ be the third central moments of X_s^l and $\tilde{X}_s^l$, respectively, where $\mu_s^l = E[X_s^l] = E[\tilde{X}_s^l]$. From the properties of the lower-level model we know ζ_s^l. In the first method we assume that $\tilde{\zeta}_s^l$ shall be different from (generally, higher than) ζ_s^l, and we seek for the value of $\tilde{\zeta}_s^l$ that results in the correct value of ζ_s^l. This can be determined by iterative stochastic (Monte Carlo) simulation. At the ith iteration we assume a trial value $(\tilde{\zeta}_s^l)_i$, starting with an initial value $(\tilde{\zeta}_s^l)_0 = \zeta_s^l$. We run the lower-level model to obtain a synthetic time series $\tilde{\mathbf{x}}_s$ with a sufficient length and the coupling transformation to derive the series $\mathbf{x}_s$. From the latter we estimate the sample third moments which we denote $(\hat{\zeta}_s^l)_i$ (for location l, subperiod s, and iteration i). We then modify $\tilde{\zeta}_s^l$ according to the rule

$$(\tilde{\zeta}_s^l)_{i+1} = (\tilde{\zeta}_s^l)_i + [\zeta_s^l - (\hat{\zeta}_s^l)_i]/c \tag{43}$$

and proceed to the next iteration. The denominator c in (43) is a number greater than 1 (e.g., $c = 2$) that enhances numerical stability in the route to the final solution. Normally, this procedure will stop when the attained sample third moments $(\hat{\zeta}_s^l)$ match the theoretical ones (ζ_s^l) for all l and s. However, given the Monte Carlo character of the method, we must relax the convergence criterion and accept the solution of iteration i if for all l and s

$$|(\hat{\zeta}_s^l)_i - \zeta_s^l| \le \max_i \{|(\hat{\tilde{\zeta}}_s^l)_i - (\tilde{\zeta}_s^l)_i|\}, \tag{44}$$

where $\hat{\tilde{\zeta}}_s^l$ denotes the sample third moment of the synthetic time series $\tilde{\mathbf{x}}_s$. Practically, this means that a deviation of the sample skewness, after performing the coupling transformation, from its theoretical value can be acceptable if it is lower than or equal to the corresponding deviation without applying the coupling transformation.

The second method is based on conditional sampling in a manner much the same as that proposed by *Koutsoyiannis and Manetas* [1996]. Here we demand that the departure of $\tilde{\mathbf{Y}}$ and $\mathbf{Y}$ in the coupling transformation be small enough so that the addition of the term $\mathbf{h}\,(\mathbf{Y} - \tilde{\mathbf{Y}})$ to $\tilde{\mathbf{X}}$ does not affect the statistics of the latter. Therefore we can assume that $\tilde{\zeta}_s^l = \zeta_s^l$. To achieve a vector $\tilde{\mathbf{Y}}$ close to the known $\mathbf{Y}$, we must keep repeating the generation process for the variables of each period (rather than performing a single generation only) until the distance of $\tilde{\mathbf{Y}}$ from $\mathbf{Y}$ is lower that an accepted limit. This distance can be defined as

$$\Delta = (1/m)\|\mathbf{Y}' - \tilde{\mathbf{Y}}'\|, \tag{45}$$

where $\mathbf{Y}'$ and $\tilde{\mathbf{Y}}'$ are $\mathbf{Y}$ and $\tilde{\mathbf{Y}}$ standardized by standard deviation (i.e., $Y_s'^l = Y_s^l/\{\mathrm{Var}\,[Y_s^l]\}^{1/2}$, $\tilde{Y}_s'^l = \tilde{Y}_s^l/\{\mathrm{Var}\,[Y_s^l]\}^{1/2}$), m is the common size of $\mathbf{Y}$ and $\tilde{\mathbf{Y}}$, and $\|\ \|$ denotes the Euclidian norm (other norms such as the maximum norm were found to behave worse).

Because in the initial generation scheme proposed, the variables $\tilde{\mathbf{X}}$ are generated independently of the higher-level variables, it was assumed in the proposition of section 2 that $\mathbf{Y}$ is independent of $\tilde{\mathbf{X}}$. However, repetition apparently introduces dependence of $\tilde{\mathbf{X}}$ on $\mathbf{Y}$. Hence the question arises whether the conclusions of the proposition are still valid for $\mathbf{Y}$ dependent on $\tilde{\mathbf{X}}$ (and $\tilde{\mathbf{Y}}$) or not. For an intuitive answer to that question we observe that the case of independent $\mathbf{Y}$ and $\tilde{\mathbf{X}}$ is the worst to manage. If the covariance Cov $[\tilde{\mathbf{X}}, \mathbf{Y}]$ approaches (or matches) the true covariance Cov $[\mathbf{X}, \mathbf{Y}]$, instead of being zero, then it is easier for the coupling transformation to preserve the statistical properties of interest. Furthermore, the applications given by *Koutsoyiannis and Manetas* [1996] for the similar case of using their adjustment procedure, which is equivalent to the simplified transformation of section 3.1, and those given here in section 6 verify empirically a positive answer to the above question.

That the answer is positive, under certain conditions, can be proved theoretically. Specifically, it is shown in Appendix A2 of the supplement that under the assumption that $\mathbf{Y}$ is no more independent from $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{Y}}$ but correlated to both, such that

$$\mathrm{Cov}\,[\tilde{\mathbf{X}}, \mathbf{Y}] = \mathbf{h}\ \mathrm{Cov}\,[\tilde{\mathbf{Y}}, \mathbf{Y}], \tag{46}$$

where $\mathbf{h}$ is given from (5), the proposition of section 2 remains valid in all its items. We note that the condition (46) holds in the case of $\mathbf{Y}$ independent from $\tilde{\mathbf{X}}$ and $\tilde{\mathbf{Y}}$, as both its sides are zero. Also, it holds in the other extreme case where both Cov $[\tilde{\mathbf{X}}, \mathbf{Y}]$ and Cov $[\tilde{\mathbf{Y}}, \mathbf{Y}]$ match the true covariances Cov $[\mathbf{X}, \mathbf{Y}]$ and Cov $[\mathbf{Y}, \mathbf{Y}]$, respectively. This can be verified using (5). Numerical investigation with the proposed repetition scheme shows that the condition holds in intermediate cases as well.

Both the above methods can lead to sufficient preservation of skewness (see section 6), although none of them is ideal. Their common disadvantages are the approximate character and the repetitive application, which increases computer time (although this time is not an obstacle as it ranges from less than a minute to some minutes for typical hydrological problems run on a modern PC to generate some thousands of synthetic data). Their common advantages are their simplicity and independence of the type of models. The Monte Carlo simulation method seems to need fewer repetitions than the conditional sampling method, and, once its additional parameters $(\tilde{\zeta}_s^l)$ are evaluated, it does not need further repetitions in subsequent applications of the model. However, to get adequate estimates of these parameters a large length of simulation record (e.g., 10,000 years or more) is needed. Another weak point of the Monte Carlo method is the fact that it results in coefficients of skewness higher than those of the original lower-level model, and this may be a problem if the latter are already large. Generally, we can consider the Monte Carlo method preferable if a model must be set up once and thereafter run several times. Conversely, if the model is to run only once, the conditional sampling method may be preferable.

Apart from the computational cost, i.e., the increase of computer time due to repetition, no additional cost is implied by either of the two approaches for preservation of skewness. Specifically, there is no negative effect in preserving other properties of the lower-level processes such as the second-order moments. On the contrary, the conditional sampling method may have positive effects in preserving second-order moments by simplified forms of the coupling transformation, as it is demonstrated in section 6.

6. Performance Demonstration

The entire modeling framework for both timescales can be summarized in the following steps composing two groups. The steps of the first group correspond to model choice and fitting: (1) Choose a model for the higher-level (coarse) timescale and fit it using the appropriate method for that model. (2) Choose a model for the lower-level (fine) timescale and fit it using the appropriate method for that model. (3) Decide the composition of the vector $\mathbf{Y}$ according to the specific needs of the problem as discussed in section 3. In the most general multivariate case, use the composition defined in (22). (4) Define the appropriate matrix $\mathbf{h}$ of the coupling transformation using the corresponding equations of section 3. In the most general multivariate case, use (25). (5) Evaluate the items of matrix $\mathbf{h}$ using either of the methods of section 4.

The steps of the second group perform the generation: (6) Use the higher-level model to produce a series of $\mathbf{Z}$ with the desired length. (7) Use the lower-level model to produce a series of $\tilde{\mathbf{X}}$ with the same number of periods, without reference to the higher-level series. (8) At each period, evaluate the

Table 1. Model Parameters for the Test Application

Parameter Type	Parameter Values for Lower-Level Process			
	Subperiod $s = 1$		Subperiod $s = 2$	
	Location $l = 1$, X_1^1	Location $l = 2$, X_1^2	Location $l = 1$, X_2^1	Location $l = 2$, X_2^2
Mean μ_s^l	1.000	2.000	3.000	4.000
Covariance matrices				
$\boldsymbol{\sigma}_{ss}$				
$l = 1$	0.250	0.210	0.810	0.432
$l = 2$	0.210	0.490	0.432	2.560
$\boldsymbol{\sigma}_{s,s-1}$				
$l = 1$	0.225	0.120	0.090	0.076
$l = 2$	0.113	0.672	0.432	1.008
Third central moment ζ_s^l	0.125	0.240	0.437	6.550

Parameter Type	Parameter Values for Higher-Level Process	
	Location $l = 1$, Z_1^1	Location $l = 2$, Z_1^2
Mean	4.000	6.000
Covariance matrices		
$\boldsymbol{\varphi}_{11}$		
$l = 1$	1.240	1.150
$l = 2$	1.150	5.066
$\boldsymbol{\varphi}_{12}$		
$l = 1$	0.340	0.693
$l = 2$	0.192	2.863
Third central moment	0.708	10.704

vectors $\mathbf{Y}$ and $\tilde{\mathbf{Y}}$ using the values of $\mathbf{Z}$, $\tilde{\mathbf{X}}$ of the current and (if applicable) next period, and (if applicable) $\mathbf{X}$ of the previous period. (9) Apply the coupling transformation to derive $\mathbf{X}$ of the current period. (10) Repeat steps 8 and 9 for all periods.

For the preservation of skewness the algorithm becomes slightly more complex because of repetition as described in section 5. The different forms and methods of the proposed framework for coupling stochastic models of different timescales are demonstrated through a simple numerical example involving two locations and two lower-level variables per period. The higher- and lower-level models used are those of (2) and (3), respectively (we note that models (2) and (3) are not fully compatible with one another). Several sets of model parameters were examined; here we present the results of a representative case with the parameter set shown in Table 1.

The following forms of the coupling transformation were examined: (1) full transformation, multivariate mode, as in section 3.4 (F/M); (2) full coupling transformation, single variate mode, that is, transformation applied separately to each location, as in section 3.3 (F/S); (3) transformation without link to the higher-level variable of the next period, as in section 3.2, but in multivariate mode (N+/M); (4) transformation without link to the lower-level variables of the previous period, multivariate mode (N−/M); (5) simplified transformation, single variate mode, as in section 3.1 (S/S); and (6) modified simplified transformation, single variate mode (S1/S). The modification in item 6 of the above list (in comparison to item 5) consists of using the value of the last lower-level variable of the previous period for initialization of the PAR(1) model (3) in each period, although this is not used by the coupling transformation.

For comparison, results of the noncoupled lower-level model (NC) are also presented. In all cases the model generated synthetic series of 10,000 periods, from which the sample statistics were computed and compared to the theoretical values.

In Figure 2 we compare the marginal statistics (means, standard deviations, and coefficients of skewness) of all lower-level variables, obtained by the different transformation forms, to their theoretical values. As anticipated, all forms of coupled models preserved perfectly the means and standard deviations, but no form preserved the coefficients of skewness (apart, of course, from the noncoupled model). Figure 3 shows the temporal correlation coefficients of the lower-level variables with previous lower-level variables and current and next higher-level variables, as derived by the various forms of the coupling transformation for the test application. We observe that only the full transformation form, either in multivariate or single-variate mode (F/M or F/S), has a perfect behavior in preserving all these correlations. Transformation form N+/M fails to reproduce some of the correlations with higher-level variables of next period; also, it has a lower performance in preserving correlations with previous lower-level variables. Transformation forms N−/M and S/S exhibit a poor behavior in preserving correlations with previous lower-level variables (particularly, those of previous period); the situation is improved with model S1/M. Also, both simplified models (S/S and S1/S) fail to reproduce the correlations with next higher-level variables. Figure 4 shows the lag-zero cross-correlation coefficients attained by the various transformation forms. As anticipated, the multivariate forms (F/M, N+/M, and N−/M) performed very well, whereas single-variate models (F/S, S/S, and S1/S) failed to preserve cross correlations.

To improve the preservation of the coefficients of skewness, we applied both methods discussed in section 5. In Figure 5 we present the results of the Monte Carlo method for the full

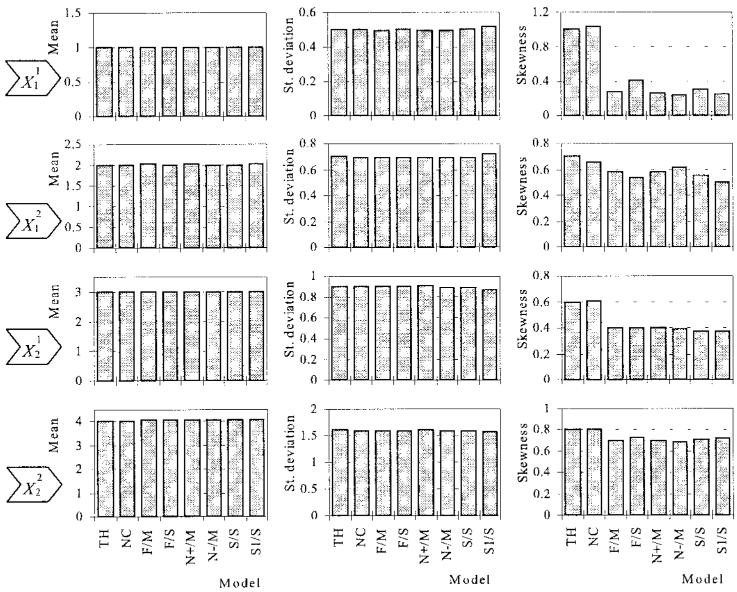

Figure 2. Comparison of marginal statistics of the lower-level variables (X_s^l, indicated by the block arrows to the left) as derived by various forms of the coupling transformation for the test application. Abbreviations are as follows: TH, theoretical values; NC, noncoupled lower-level model; F/M, full coupling transformation, multivariate mode; F/S, full transformation, single variate mode; N+/M, transformation without link to the next higher-level variable, multivariate mode; N−/M, transformation without link to the previous lower-level variable, multivariate mode; S/S, simplified transformation, single variate mode; and S1/S, modified simplified transformation (starting with the known value of the previous lower-level variable), single variate mode.

transformation form in multivariate mode (F/M). We observe that after the tenth iteration, the attained coefficients of skewness become close to the theoretical ones. The criterion of (44) becomes true for iteration 13; the fluctuation of the attained coefficients of skewness of most variables that appears beyond iteration 13 is anticipated because of the Monte Carlo character of the method. We notice that the differences of the assumed and attained (after applying the coupling transformation) coefficients of skewness, which correspond to $\tilde{\zeta}_s^l$ and $\hat{\zeta}_s^l$, respectively, may be very large (e.g., for variable X_1^1).

We also applied the method of conditional sampling using repetitions for the full (F/M), the simplified (S/S), and the

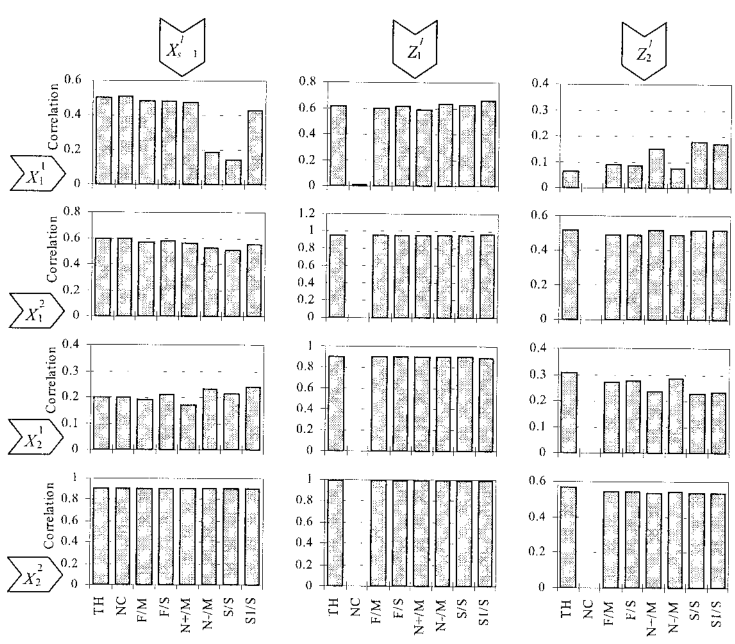

Figure 3. Comparison of temporal correlation coefficients of the lower-level variables (X_s^l, indicated by the block arrows to the left) with previous lower-level variables and current and next higher-level variables (X_{s-1}^l, Z_1^l, and Z_2^l, respectively, indicated by the block arrows at the top) as derived by various forms of the coupling transformation for the test application. Abbreviations are the same as in Figure 2.

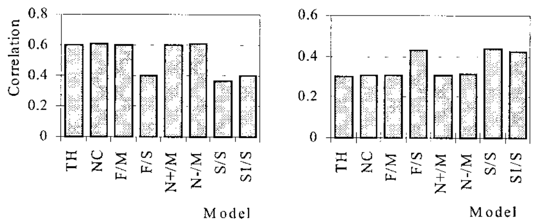

Figure 4. Comparison of cross-correlation coefficients of the lower-level variables of the (left) first and (right) second subperiod as derived by various forms of the coupling transformation for the test application. Abbreviations are the same as in Figure 2.

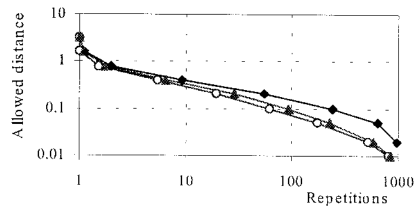

Figure 6. Average number of repetitions required to achieve the preset allowed distance between **Y** and $\tilde{\mathbf{Y}}$ using the conditional sampling method for the full (F/M, diamonds), the simplified (S/S, triangles), and the modified simplified (S1/S, circles) coupling transformations.

modified simplified (S1/S) forms of the coupling transformation. In Figure 6 we plotted the average number of repetitions required to achieve a certain distance Δ (defined in (45)). Figure 7 shows the attained coefficients of skewness using the conditional sampling method, as a function of the mean number of repetitions. We observe that all three transformation forms examined have roughly the same performance. Adequate values of sample coefficients of skewness are obtained with 50–100 repetitions. We also examined in this case the preservation of correlation coefficients of the lower-level variables with the previous lower-level variables and the next higher-level variables (Figure 8) and cross-correlation coefficients (Figure 9). We observe that both S/S and S1/S forms, which failed to preserve all these statistics if applied without repetitions (Figures 3 and 4), result in adequate preservation of cross correlations after 50–100 repetitions. In addition, the S1/S model performs well in preserving correlation coefficients of the lower-level variables with the previous lower-level variables after 50–100 repetitions. However, none of the two simplified forms could approach the theoretical correlation coefficients of the lower-level variables with the next higher-level variables, even after 1000 repetitions. In conclusion, repetition, apart from its usefulness for preserving coefficients of skewness, also improves preservation of autocorrelation and cross-correlation coefficients of lower-level variables of simplified model versions. Notably, this is done at no additional computational cost.

7. Summary and Conclusions

A methodology is proposed for coupling stochastic models of hydrologic processes applying to different timescales so that time series generated by the different models be consistent.

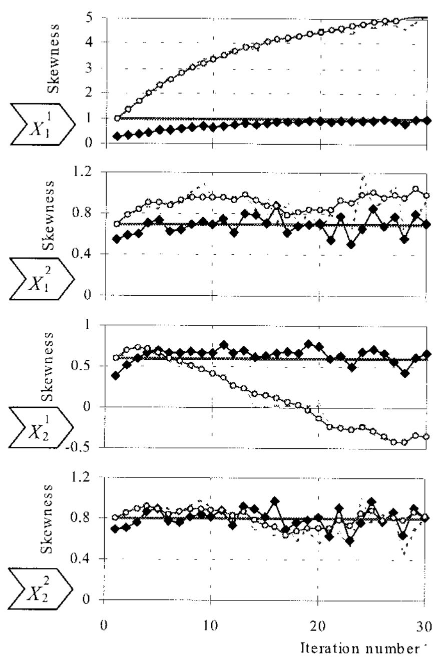

Figure 5. Hypothesized (circles, corresponding to $\tilde{\zeta}_s^l$) and attained (diamonds, corresponding to $\hat{\zeta}_s^l$) coefficients of skewness for each of the lower-level variables (shown in the block arrows to the left), as a function of the iteration number, for a test application of the full coupling transformation (F/M and Monte Carlo method). Lines without symbols and dotted lines represent the theoretical values (corresponding to ζ_s^l) and the values obtained from the noncoupled lower-level model (NC, corresponding to $\hat{\tilde{\zeta}}_s^l$), respectively.

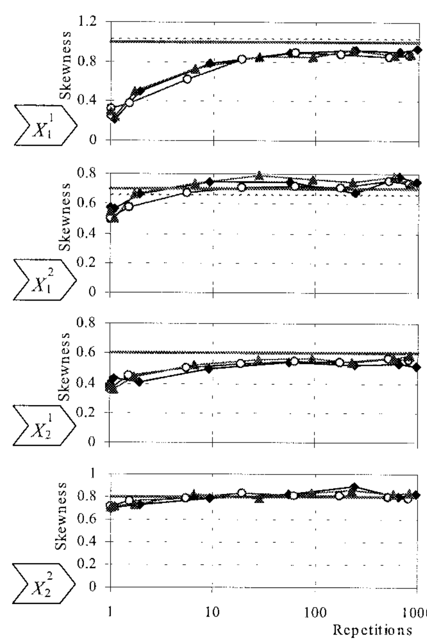

Figure 7. Attained coefficients of skewness of the lower-level variables (X_s^l, indicated by the block arrows to the left) as a function of the mean number of repetitions for the full (F/M, diamonds), the simplified (S/S, triangles), and the modified simplified (S1/S, circles) coupling transformations. Lines without symbols and dotted lines (indistinguishable when they coincide with lines without symbols here and in Figures 8 and 9) represent the theoretical values and the values obtained from the noncoupled lower-level model (NC), respectively.

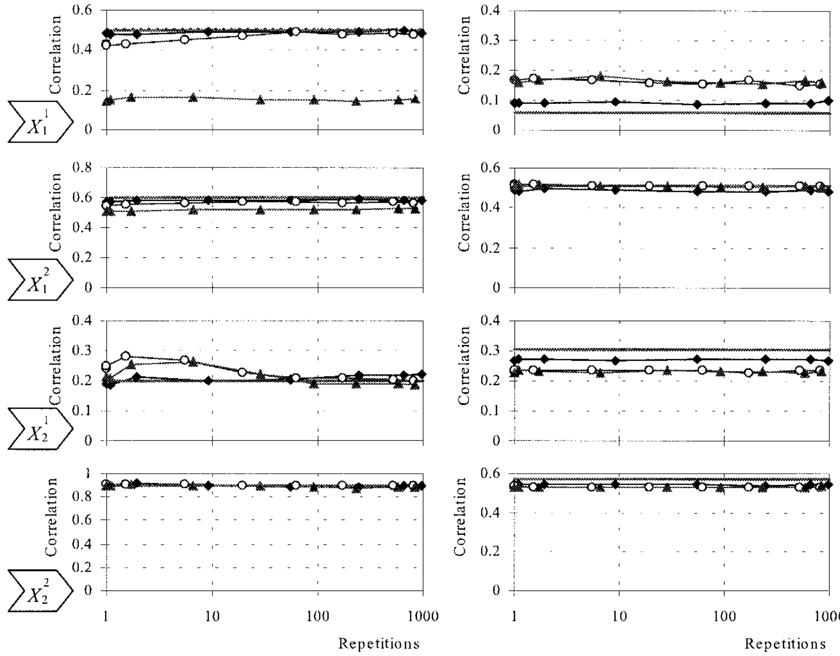

Figure 8. Attained correlation coefficients of the lower-level variables (X_s^l, indicated by the block arrows to the left) with (left) the previous lower-level variables and (right) the next higher-level variables as a function of the mean number of repetitions for the full (F/M, diamonds), the simplified (S/S, triangles), and the modified simplified (S1/S, circles) coupling transformations. Lines without symbols and dotted lines represent the theoretical values and the values obtained from the noncoupled lower-level model (NC), respectively.

Given two multivariate time series, generated by two separate (unrelated) stochastic models of the same hydrologic process, each applying to a different timescale, a transformation is developed (referred to as a coupling transformation) that appropriately modifies the time series of the lower-level timescale so that this series becomes consistent with the time series of the higher-level timescale without affecting the second-order stochastic structure of the former and also establishes appropriate correlations between the two time series. The coupling transformation is based on a developed generalized mathematical proposition, which ensures preservation of marginal and joint second-order statistics and linear relationships between lower- and higher-level processes. The methodology can be applied to problems involving disaggregation of annual to seasonal and seasonal to subseasonal timescales, as well as problems involving finer timescales, with the only requirement that a specific stochastic model is available for each involved timescale. An implementation of the methodology for disaggregation of daily rainfall into hourly rainfall at many locations (a problem much more demanding than disaggregation of annual to seasonal quantities, because of the intermittent aspect of the process and the very asymmetric marginal distributions) is under way.

Several specific forms of the coupling transformation are studied. The simplest of them, S/S and S1/S, are single variate and do not consider any link to higher- or lower-level variables of previous or next periods; the difference between the two is that S1/S uses some of the already generated variables of the previous period for its initialization, whereas S/S does not. The most detailed form, F/M, is multivariate and incorporates appropriate links to higher- and lower-level variables of previous and next periods. In addition, techniques for evaluating parameters of the coupling transformation based on second-order moments of the lower-level process are studied. Specific implementations of these techniques are given for the very common cases where the lower-level process (or its logarithmic transformation) is multivariate PAR(1), PAR(2), or PARMA(1, 1).

Although the coupling transformation can explicitly preserve means and second-order statistics of the processes in-

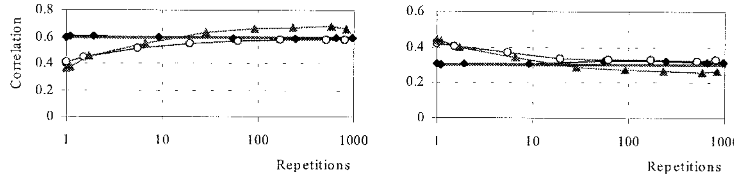

Figure 9. Attained cross-correlation coefficients of the lower-level variables of the (left) first and (right) second subperiod as a function of the mean number of repetitions for the full (F/M, diamonds), the simplified (S/S, triangles), and the modified simplified (S1/S, circles) coupling transformations. Lines without symbols and dotted lines represent the theoretical values and the values obtained from the noncoupled lower-level model (NC), respectively.

volved, it introduces bias to the coefficients of skewness and any other parameters that cannot be related to means and second-order statistics (e.g., probabilities of dry intervals in the case of the fine-scale rainfall process). Because of its linearity the coupling transformation encompasses the effects of the central limit theorem. Thus the transformed series tend to be Gaussian (their coefficients of skewness are reduced). Unlike second-order statistics, third moments and coefficients of skewness are too complicated to handle analytically. However, two approximate methods that enable preservation of skewness of the processes are studied. The first introduces negative bias to the coefficients of skewness of the lower-level processes, the magnitude of which is determined by Monte Carlo simulation, to counterbalance the bias introduced by the application of the coupling transformation. The second uses repetition as a means of conditional sampling, and, in that way, it prevents the lower-level variables from departing (in terms of their sum) significantly from the known higher-level variables, thus reducing bias to a negligible level.

A detailed numerical example of the application of the methodology demonstrates that it behaves as it should. The full multivariate (F/M) form preserves all temporal and spatial correlations of lower-level variables either with other lower-level variables or with higher-level variables, whereas simplified forms fail to preserve some of these correlations. All forms preserve first and second marginal moments but fail to preserve third moments. The latter are preserved only after application of either of the two methods developed for that purpose, Monte Carlo simulation or conditional sampling. The latter, apart from its usefulness for preservation of skewness coefficient, also improves (at no additional computational cost) preservation of autocorrelation and cross-correlation coefficients of lower-level variables for simplified forms of the coupling transformation.

Among the different forms of the coupling transformation studied, the full multivariate one (F/M) is the most preferable as it preserves explicitly the greater number of statistics. Between the two methods for preserving skewness, the Monte Carlo method may be preferable if a model must be set up once and thereafter run several times. Conversely, if the model is to run only once, the conditional sampling method may be preferable. Besides, if a simplified form of the coupling transformation is chosen, then it must be combined with the conditional sampling method to improve preservation of statistics that are not explicitly considered in the transformation.

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D. Koutsoyiannis, Department of Water Resources, Faculty of Civil Engineering, National Technical University, Athens, Heroon Polytechneiou 5, GR-157 80 Zographou, Greece. (dk@hydro.ntua.gr)

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