



2017

# An R function for the estimation of trend significance under the scaling hypothesis- application in PET parametric annual time series

Aristoteles Tegos

*National Technical University of Athens*, tegosaris@yahoo.gr

Hristos Tyrallis

montchrister@gmail.com

Demetris Koutsoyiannis

*National Technical University of Athens*, dk@itia.ntua.gr

Khaled Hamed

hamedkhaled@hotmail.com

Follow this and additional works at: <http://scholarsarchive.byu.edu/openwater>

## BYU ScholarsArchive Citation

Tegos, Aristoteles; Tyrallis, Hristos; Koutsoyiannis, Demetris; and Hamed, Khaled (2017) "An R function for the estimation of trend significance under the scaling hypothesis- application in PET parametric annual time series," *Open Water Journal*: Vol. 4 : Iss. 1 , Article 6.

Available at: <http://scholarsarchive.byu.edu/openwater/vol4/iss1/6>

This Article is brought to you for free and open access by the All Journals at BYU ScholarsArchive. It has been accepted for inclusion in Open Water Journal by an authorized editor of BYU ScholarsArchive. For more information, please contact [scholarsarchive@byu.edu](mailto:scholarsarchive@byu.edu).



# An R function for the estimation of trend significance under the scaling hypothesis- application in PET parametric annual time series

## *Software Introduction*

Aristoteles Tegos<sup>1\*</sup>, Hristos Tyrallis<sup>2</sup>, Demetris Koutsoyiannis<sup>3</sup>, Khaled H. Hamed<sup>4</sup>

<sup>1,2,3</sup>*Department of Water Resources and Environmental Engineering, School of Civil Engineering, National Technical University of Athens, Iroon Polytechniou 5, 15780 Zografou, Greece*

<sup>4</sup>*Department of Irrigation and Hydraulics, Faculty of Engineering, Cairo University*

\*Corresponding Author: tegosaris@yahoo.gr

---

## ABSTRACT

We present an R function for testing the significant trend of time series. The function calculates trend significance using a modified Mann-Kendall test, which takes into account the well-known physical behavior of the Hurst-Kolmogorov dynamics. The function is tested at 10 stations in Greece, with approximately 50 years of PET data with the use of a recent parametric approach. A significant downward trend was detected at two stations. The R software is now suitable for extensive use in several fields of the scientific community, allowing a physical consistent of a trend analysis.

### *Keywords*

Hurst; Potential evapotranspiration; Parametric model; R software; Trend analysis

---

## 1.0 Introduction

Trend estimation in hydro-climatic time series has focused the attention of the scientific community (Sen, 2013). Many studies have examined the trend of precipitation, streamflow, groundwater regime, temperature, potential evapotranspiration both at annual and seasonal scales (Markonis et al. 2016, Stevens et al. 2016, Panda et al. 2012, Arora et al. 2005, Kumar et al. 2010). Specifically, trend estimation in potential or actual evapotranspiration pay the attention of the researchers (Gocic and Trajkovic, 2014, Mo et al. 2015, Tabari et al., 2011). Generally, the

trend results are mixed across different climatic regions, as Tabari et al (2011) found a positive trend for 70% of 20 Iranian meteorological stations during the period 1996-2005, but Gocic and Trajkovic (2014) calculated a significant increasing downward trend in 70% of 12 Serbian meteorological stations (study period 1980-2010). Finally, Mo et al. 2015, by investigating the areal evapotranspiration in China for the period 1981-2010 with remote sensing data, observed an increasing trend from the 1980s to the mid-1990s, followed by a decreasing trend. For the examination of physical variability, the Mann-Kendall under the independence assumption has been pro-

posed as a standard statistical measure for the valuation and quantification of trends (Ahn and Palmer, 2015).

Furthermore, different Mann–Kendall statistical methodologies have been developed and proposed, namely the Mann–Kendall under the Markovian behavior assumption after trend-free pre-whitening, the Mann–Kendall with complete autocorrelation structure and the Mann–Kendall under the long-term persistence assumption (Kumar et al. 2009). The latter test, proposed by Hamed (2008), offers a consistent framework to consider the Hurst phenomenon, which is observed in many climatological and hydrological processes, resulting in the increase of physical variability (Koutsoyiannis 2003; Koutsoyiannis and Montanari 2007). Hurst coefficient was first introduced by engineer Harold Hurst during the design of the Aswan reservoir (Sutcliffe et al. 2016) and plays a significant role in the hydrological variability (O’Connell et al, 2016). Its presence in large measured hydrometeorological samples is ubiquitous (Iliopoulou et al. 2016) Comparative analysis of different trend model shows significant differences in the totally results (Hamed 2008, Kumar et al. 2009) and thus a physical consistent framework is needed.

This study presents an R function embedded in an automatic and user-friendly environment following modern views of water resources modeling tools (Guo et al. 2016, Turner and Ganelli 2016). The package implements the modified Hamed’s (2008) framework and the procedure is tested in annual parametric PET time series from 10 sites in Greece, which cover the period 1950-2000.

## 2. Materials and methods

### 2.1 The parametric-PET model

The parametric model was first introduced by Koutsoyiannis and Xanthopoulos in 1998, mainly as a framework to fill and extend PET time series and included the calibration of the parametric model in a Penman-Monteith time series. Later, Tegos et al. (2009, 2013, 2015) implemented the model at point and regional scale in Greece territory and compared the results with Hargreaves and Oudin models.

Recently, an extended and comparative analysis in different climatic regimes were made which included the development of the model in the well-known CIMIS network (California, U.S.A) as well in European stations (Tegos et al. 2015). The results of the implementation were quite satisfactory and the framework allowed consistent monthly and annual PET estimation at point and especially in regional scale. Another key conclusion was the better agreement with Penman-Monteith measured samples against other world-recognized radiation-based models such as Hargreaves, Oudin, Jensen and McGuinness. The most recently application was the daily and monthly implementation of the model for the PET mapping in an irrigated plain of Greece (Malamos et al., 2015).

The mathematical expression of the parametric model for every time step is the following:

$$PET = \frac{aR_a - b}{1 - cT} \quad (1)$$

where, PET potential evapotranspiration (mm)  $R_a$  ( $\text{KJm}^{-2}$ ) is the extraterrestrial radiation,  $a$  ( $\text{KgK}^{-1}$ ),  $b$  ( $\text{Kgm}^{-2}$ ) and  $c$  ( $\text{C}^{-1}$ ) are the calibrated parameters, while  $T$  ( $^{\circ}\text{C}$ ) is the mean air temperature.

The extraterrestrial radiation  $R_a$ , for each day of the year and for different latitudes is estimated from the solar constant, the solar declination and the time of the year by the formula:

$$R_a = \frac{24(60)}{\pi} G_{sc} d_r [\omega_s \sin(\varphi) \sin(\delta) + \cos(\varphi) \cos(d) \sin(\omega_s)] \quad (2)$$

where  $R_a$  ( $\text{MJm}^{-2}\text{d}^{-1}$ ) extraterrestrial radiation,  $G_{sc}$  solar constant =  $0.0820 \text{ MJm}^{-2}\text{min}^{-1}$ ,  $d_r$  inverse relative distance Earth-Sun,  $\omega_s$  (rad) sunset hour angle,  $\varphi$  latitude (rad),  $\delta$  solar declination (rad).

### 2.2 Mann-Kendall test under the scaling hypothesis

The Mann-Kendall test under the scaling hypothesis consists of three consecutive hypothesis tests, namely O (Original MK test), H (Hurst Parameter test) and M. The mathematic background and framework are presented from Hamed (2008). Let  $H_0i$  denote the null hypothesis

of each test and let  $H1i$  denote the alternative hypothesis, where  $i = O, H, M$  denotes the step of the Mann-Kendall test under the scaling hypothesis. We define:

- $H0O$ : No trend under the independence assumption
- $H1O$ : Significant trend under the independence assumption.
- $H0H$ : No significant LTP.
- $H1H$ : Significant LTP.
- $H0M$ : No trend under LTP assumption.
- $H1M$ : Significant trend exists under LTP assumption.
- Then the three steps of the test are summarized by the following sequences
  - $\{H0O\}$ : No trend.
  - $\{H1O\}$ : Possible significant trend. Proceed to step H.
  - $\{H1O, H0H\}$ : Significant trend exists.
  - $\{H1O, H1H\}$ : Possible LTP effect. Proceed to step M.
  - $\{H1O, H1H, H0M\}$ : No trend.
  - $\{H1O, H1H, H1M\}$ : Significant trend exists.

Hurst coefficient can be defined by a simple power-law relationship of its standard deviation:

$$\sigma^\kappa = \kappa^{H-1} \sigma$$

where  $\sigma = \sigma(1)$  and  $H$  is the entropy production in logarithmic time (Koutsoyiannis 2011), and the parameter ranges between 0 and 1. For values  $H > 0.5$ , the process exhibits long-term persistence, while for  $H < 0.5$  the process is anti-persistent.

For the test implementation, we used the R function `MannKendallLTP` from the `HKprocess` R package (Tyralis, 2015). The R function computes the p-value in each step of the test. If the p-value is higher than a predefined significance level  $\alpha$  (e.g.  $\alpha = 0.05$ ), then we cannot reject  $H0$ . A p-value less than or equal to  $\alpha$  gives evidence that  $H1$  is true.

### 2.3 Study area and procedures

Ten meteorological stations (National Meteorological Services of Greece) well-distributed over Greece were used. Table 1 presents the list of the meteorological stations used in our study.

Stations	$\varphi(o)$	$z$ (m)
Heraklion	35.20	39
Ioannina	39.42	484
Kavala	40.54	63
Kerkyra	39.37	2
Kozani	40.18	626
Larissa	39.39	74
Lemnos	39.54	17
Methoni	36.50	34
Skyros	38.54	5
Tripoli	37.32	663

**Table 1.** Meteorological stations with their latitude ( $\varphi^\circ$ ) and elevation ( $z$ ).

Based on our previous study (Tegos et al. 2013) the parametric model was calibrated and tested in monthly time step for the period 1968-1989. For the purposes of this study, monthly air temperature data for the period 1950-2000 were collected and the parametric model was applied to the total length. Finally, every monthly time series was aggregated into annual step with the use of the `HYDROGNOMON` software (Kozanis et al. 2010).

### 3. Results

Table 2 presents the results of our analysis. In seven out of the ten stations tested, no trends were found under the independence assumption. The estimate of the Hurst parameter for annual PET time series varies in the range from 0.43 to 0.76. Out of the three stations that had significant trends under the independence assumption, only two stations (Ioannina, Limnos) showed a significant downward trend.

In Figure 1, we present the PET at Ioannina. In Table 2, we observe a significant trend under the independence assumption. This assumption is valid. At Kerkyra (see Figure 2) we do not observe any significant trend. At Larissa (Figure 3), we find a significant trend under the independence assumption; however, this trend is not significant under the long-term dependence assumption. Finally, we observe a significant trend under a valid independence assumption at Limnos (see Figure 4).

#### 4. Discussions and conclusions

We present an R function that implements the Mann-Kendall test under the long-term persistence hypothesis. The test applied and tested in annual time series of PET estimated from a recent parametric approach. The parametric model estimation allows the consistent estimation of the PET with minimal data requirements and it's useful for climatic studies when crucial hydrometeorological data are missing (wind velocity, relative humidity, extraterrestrial radiation). The results of our preliminary case study analysis show that in seven cases, no significant trend was detected under the independence assumption. In one case, no significant trend was detected under the long-term persistence assumption, while the trend was significant under the independence assumption. In the remaining two cases, we found a significant downward trend under both the independence and the long-term persistence assumptions. In summary, an R function is ready and user-friendly for use in other field of water resources studies.

#### Acknowledgement

The authors wish to kindly acknowledge one anonymous reviewer for his/her constructive suggestions which improved earlier version of this manuscript.

#### Appendix A Supplementary material

Supplementary data and code for reproducing the analysis of this paper as well as additional Figures, associated with the present study but not included here for brevity, are available as supporting material in Appendix A.

#### Software Availability

Name of software: MannKendallLTP R function in the HKprocess R package  
 Developers: Hristos Tyralis  
 Contact: Hristos Tyralis, Athens 10443, contact: montchrister@gmail.com  
 Year first available: 2016  
 Required software: R ( $\geq 3.2.3$ )  
 Cost: Free. The function is public available in the R package at <https://CRAN.R-project.org/package=HKprocess>

Stations	Hurst parameter estimate	Mann-Kendall 2-sided $p$ -value (Step <b>O</b> )	Significance of $H$ , 2-sided $p$ -value (Step <b>H</b> )	Mann-Kendall LTP 2-sided $p$ -value (Step <b>M</b> )	Trend identification
Heraklion	0.67	0.31			{H00}, no trend
Ioannina	0.58	<b>0.05</b>	0.27		{H1O,H0H}, trend exists
Kavala	0.76	0.63			{H00}, no trend
Kerkyra	0.71	0.90			{H00}, no trend
Kozani	0.63	0.31			{H00}, no trend
Larissa	0.76	<b>0.04</b>	<b>0.00</b>	0.42	{H1O,H1H,HOM} no trend
Lemnos	0.74	<b>0.00</b>	0.26		{H1O,H0H}, trend exists
Methoni	0.69	0.06			{H00}, no trend
Skyros	0.46	0.40			{H00}, no trend
Tripoli	0.43	0.46			{H00}, no trend

**Table 2.** Summary results of the application of the Mann-Kendall modified test to the PET data. The Hurst parameter was estimated using the maximum likelihood estimator (Tyralis and Koutsoyiannis 2011). The trend identification is performed for a predefined level  $\alpha = 0.05$  in each step.

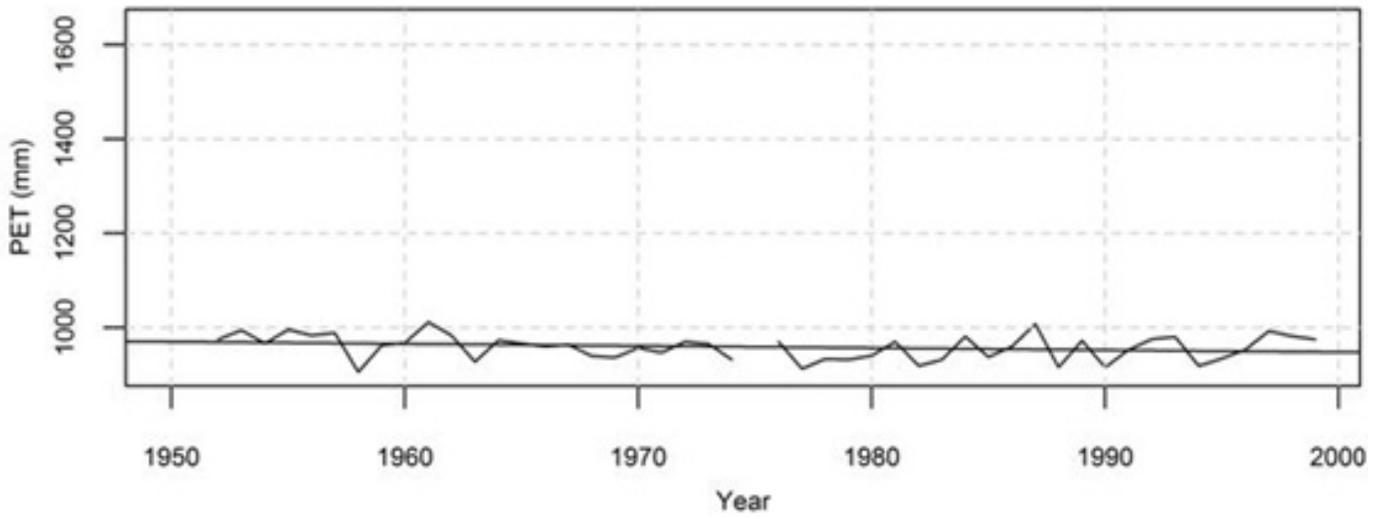


Figure 1. Annual PET at Ioannina

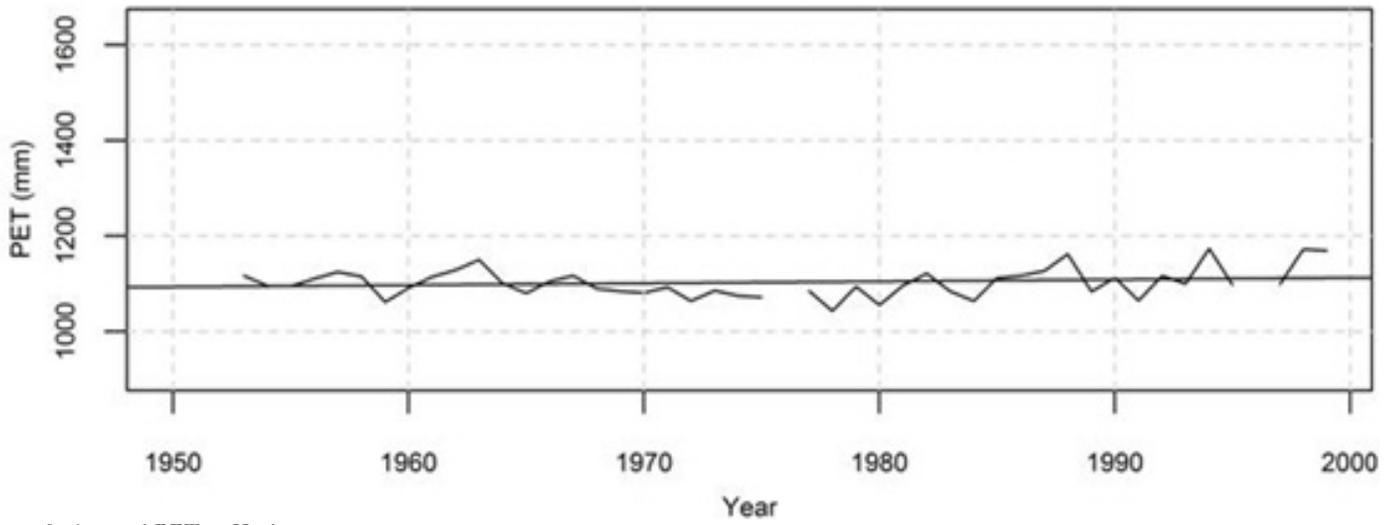


Figure 2. Annual PET at Kerkyra

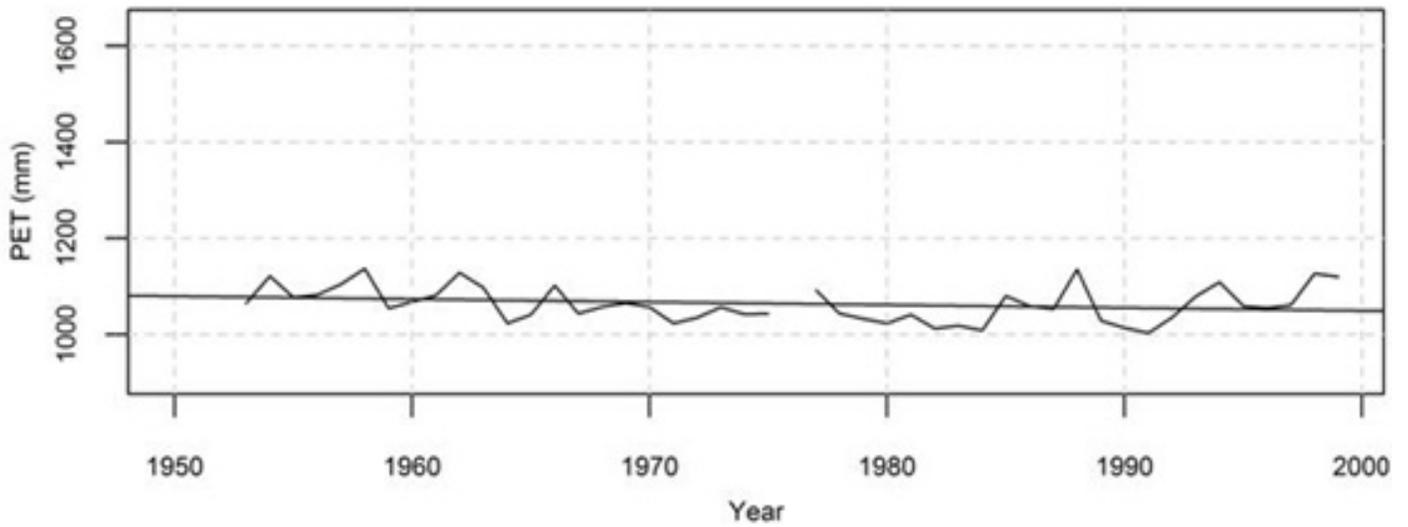


Figure 3. Annual PET at Larissa

## References

- Ahn, K. H., & Palmer, R. N. (2015). Trend and Variability in Observed Hydrological Extremes in the United States. *Journal of Hydrologic Engineering*, 21(2), 04015061.
- Arora, Manohar, N. K. Goel, and Pratap Singh. "Evaluation of temperature trends over India." *Hydrological sciences journal* 50.1 (2005).
- Gocic, Milan, and Slavisa Trajkovic. "Analysis of trends in reference evapotranspiration data in a humid climate." *Hydrological Sciences Journal* 59.1 (2014): 165-180.
- Guo, D., Westra, S., & Maier, H. R. (2016). An R package for modelling actual, potential and reference evapotranspiration. *Environmental Modelling & Software*, 78, 216-224
- Hamed KH (2008) Trend detection in hydrologic data: The Mann-Kendall trend test under the scaling hypothesis. *Journal of Hydrology* 349(3-4):350-363.
- Iliopoulou, T., Papalexiou, S. M., Markonis, Y., & Koutsoyiannis, D. (2016). Revisiting long-range dependence in annual precipitation. *Journal of Hydrology*.
- Kozanis, S., Christofides, A., Mamassis, N., Efstathiadis, A. and Koutsoyiannis, D., 2010, May. Hydrognomon—open source software for the analysis of hydrological data. In *EGU General Assembly Conference Abstracts* (Vol. 12, p. 12419).
- Koutsoyiannis, D., 2003. Climate change, the Hurst phenomenon, and hydrological statistics. *Hydrological Sciences-Journal-des Sciences Hydrologiques* 48 (1).
- Koutsoyiannis, D., Montanari, A., 2007. Statistical analysis of hydroclimatic time series: uncertainty and insights. *Water Resources Research* 43, W05429.
- Koutsoyiannis D. 2011. Hurst–Kolmogorov dynamics as a result of extremal entropy production. *Physica A: Statistical Mechanics and its Applications*, 390(8): 1424-1432
- Koutsoyiannis, D., and Th Xanthopoulos. "Engineering hydrology." *Edition 3* (1999): 418.
- Kumar, S., Merwade, V., Kam, J., & Thurner, K. (2009). Streamflow trends in Indiana: effects of long term persistence, precipitation and subsurface drains. *Journal of Hydrology*, 374(1), 171-183.
- Markonis, Y., Batelis, S. C., Dimakos, Y., Moschou, E., & Koutsoyiannis, D. (2016). Temporal and spatial variability of rainfall over Greece. *Theoretical and Applied Climatology*, 1-16.
- Mo, X., Liu, S., Lin, Z., Wang, S., & Hu, S. (2015). Trends in land surface evapotranspiration across China with remotely sensed NDVI and climatological data for 1981–2010. *Hydrological Sciences Journal*, 60(12), 2163-2177.
- O'Connell, P. E., Koutsoyiannis, D., Lins, H. F., Markonis, Y., Montanari, A., & Cohn, T. (2016). The scientific legacy of Harold Edwin Hurst (1880–1978). *Hydrological Sciences Journal*, 1-20.
- Panda, D. K., A. Mishra, and A. Kumar. "Quantification of trends in groundwater levels of Gujarat in western India." *Hydrological Sciences Journal* 57.7 (2012): 1325-1336.
- Stevens, Andrew J., Derek Clarke, and Robert J. Nicholls. "Trends in reported flooding in the UK: 1884–2013." *Hydrological Sciences Journal* 61.1 (2016): 50-63.
- Sutcliffe, J., Hurst, S., Awadallah, A. G., Brown, E., & Hamed, K. (2016). Harold Edwin Hurst: the Nile and Egypt, past and future. *Hydrological Sciences Journal*, 1-14.
- Tabari, H., Marofi, S., Aeni, A., Talaei, P. H., & Mohammadi, K. (2011). Trend analysis of reference evapotranspiration in the western half of Iran. *Agricultural and Forest Meteorology*, 151(2), 128-136.
- Tegos, A., Efstathiadis, A., & Koutsoyiannis, D. (2013). A parametric model for potential evapotranspiration estimation based on a simplified formulation of the Penman-Monteith equation. *Evapotranspiration-an overview*. InTech, Rijeka, Croatia, 143-165.
- Tegos, A., Malamos, N., & Koutsoyiannis, D. (2015). A parsimonious regional parametric evapotranspiration model based on a simplification of the Penman–Monteith formula. *Journal of Hydrology*, 524, 708-717.
- Tegos, A., Efstathiadis, A., Malamos, N., Mamassis, N., & Koutsoyiannis, D. (2015). Evaluation of a parametric approach for estimating potential evapotranspiration across different climates. *Agriculture and Agricultural Science Procedia*, 4, 2-9.
- Tegos, A., N. Mamassis, and D. Koutsoyiannis. "Estimation of potential evapotranspiration with minimal data dependence." *EGU General Assembly Conference Abstracts*. Vol. 11. 2009.
- Turner, S. W. D., & Galelli, S. (2016). Water supply sensitivity to climate change: An R package for implementing reservoir storage analysis in global and regional impact studies. *Environmental Modelling & Software*, 76, 13-19.
- Tyralis H, Koutsoyiannis D (2011) Simultaneous estimation of the parameters of the Hurst-Kolmogorov stochastic process. *Stochastic Environmental Research and Risk Assessment* 25(1):21-33.
- Tyralis H. (2016). HKprocess: Hurst-Kolmogorov process. R package version 0.0-2. <https://CRAN.R-project.org/package=HKprocess>