



Abstract

Clustering of extremes is a statistical behavior often observed in geophysical timeseries. However, it is usually studied independently of the theoretical framework of Long-Range Dependence, or the Hurst-Kolmogorov behavior, which provides consistent theoretical and practical tools for identifying it and understanding it. Herein, a dataset of daily rainfall records spanning more than 150 years is studied in order to investigate the dependence properties of extreme rainfall at the annual and seasonal timescale. The same investigation is carried out for mean rainfall at the annual scale. The research question is focused on investigating the link between the Hurst behavior in the mean rainfall, which is already acknowledged in literature, and the Hurst behavior in extreme rainfall timeseries, which is also to be testified.

1. 150 years of rainfall extremes daily data

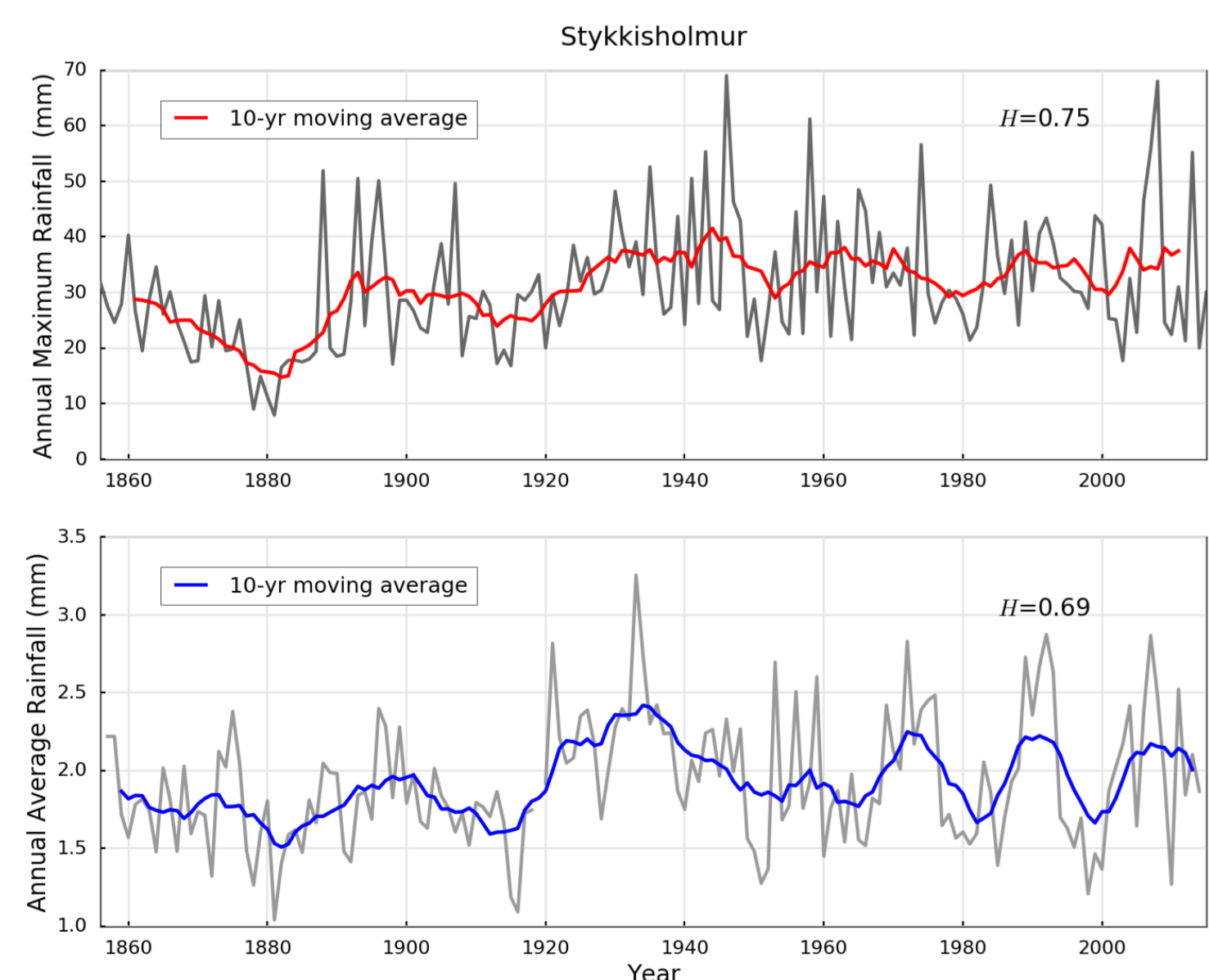
- 27 rainfall records with more than 150 years of daily data
- Collected from global databases (NOAA, ECA) and via personal contact
- Record length is paramount both in the study of extremes and in the study of long-range dependence.

- Croatia (1)
- Czech Republic (1)
- Greece (1)
- Finland (1)
- Italy (7)
- Iceland (1)
- Germany (2)
- UK (2)
- Netherlands (3)
- Portugal (1)
- Sweden (1)



2. Identifying LRD in extremes

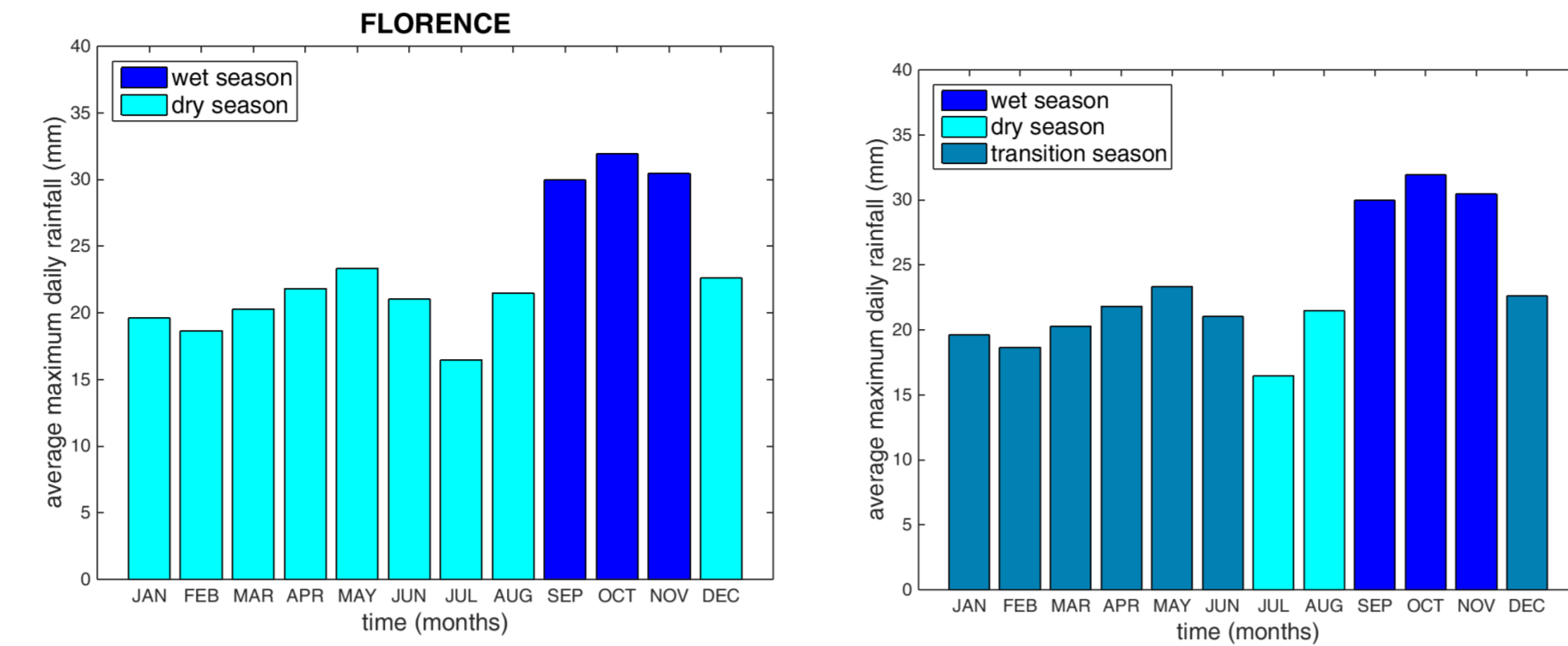
- Clustering of extremes in geophysical processes has been recursively studied in literature (see Serinaldi and Kilsby, 2016 for a brief review).
- Here we evaluated the LRD properties of rainfall extremes derived from long rainfall records, by estimating the Hurst exponent, already shown to be present in annual rainfall (e.g. Iliopoulou et al. 2016)



Annual average and Annual maxima timeseries exhibiting LRD

- For estimating the Hurst exponent H we employ the mean aggregated variance method: $\sigma^{(k)} = \frac{\sigma}{k^{1-H}}$ where $\sigma^{(k)}$ the variance of the process aggregated and averaged at scale k
- Although the variance estimator is biased (Koutsoyiannis, 2003), in cases of weak presence of LRD the bias, such as in annual rainfall, the bias is negligible.

3. LRD in Seasonal and Annual Maxima



Algorithmic Partition in seasons

Hurst exponent analysis

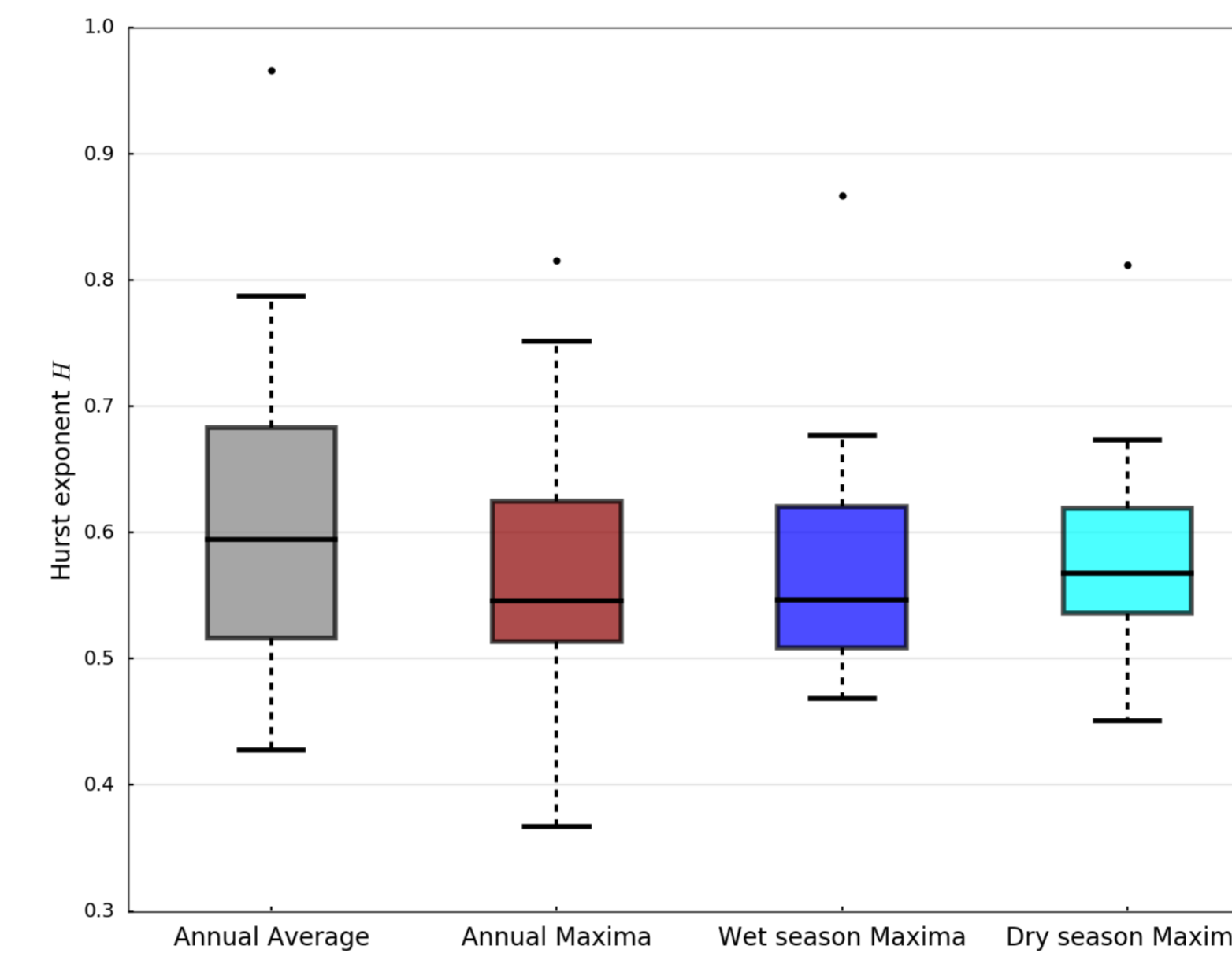
Statistic	$H_{\text{annual_average}}$	$H_{\text{annual_maxima}}$	$H_{\text{wet_season}}$	$H_{\text{dry_season}}$
Number of stations	27	27	16	16
Mean	0.608	0.568	0.576	0.581
Standard deviation	0.126	0.094	0.1	0.088

Seasonal identification is achieved following Iliopoulou et. al (2016):

- We identify the optimal temporal partition for a given number of seasons as the one that minimizes the Sum of Squared Deviations:

$$SSD = \sum_{j=1}^k \sum_{x \in C_j} |x - \bar{x}_j|^2$$

- And select the number of season by applying AIC to the mixture seasonal probability model



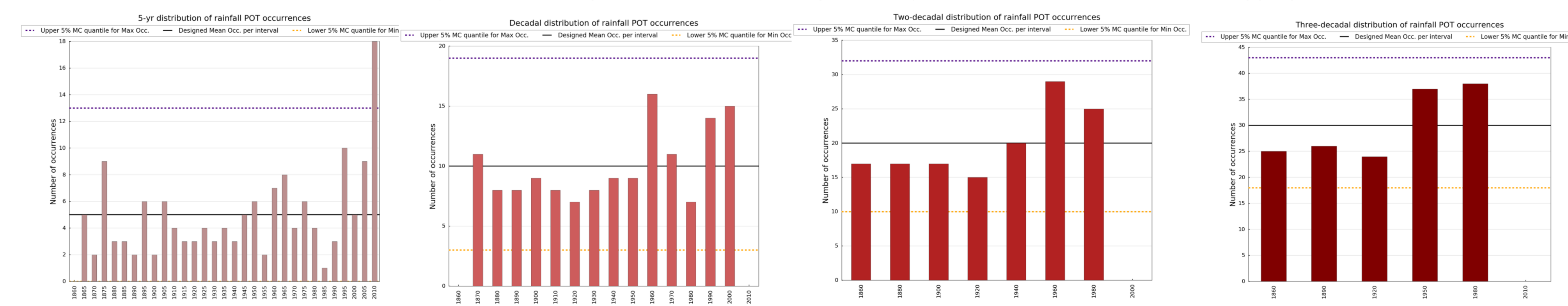
4. Are Peaks Over Threshold Poisson-distributed?

A simple Monte Carlo experiment

- We select the average number of events λ per interval equal to the number of years in the interval, e.g. 10 for decade.
- We sample the λn maximum daily rainfall values from the whole record, where n the available number of intervals. Therefore, approximately the same POT sample is partitioned in all cases, except for cases of intervals affected from missing values and excluded.
- We generate 10000 samples of Poisson distribution with the same λ and equal length and estimate the sample minimum and maximum for each.
- Below, the upper 5% of the sample maxima values is plotted along with the lower 5% of the minima, and both used to form confidence regions for Poisson distribution.

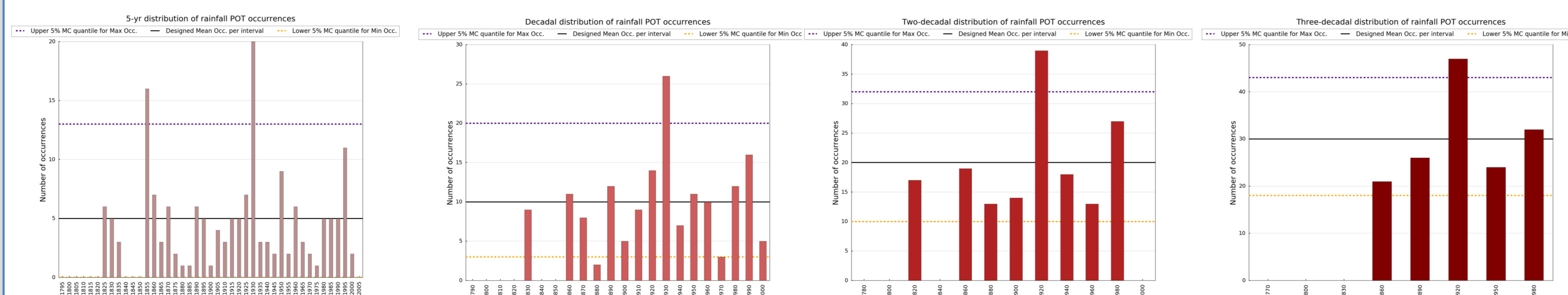
Lisbon, Portugal

- $H_{\text{Annual Maxima}} = 0.5$
- POT occurrences show small variability from the designed mean, which is significantly smoothed out as timescale of aggregation increases



Palermo, Italy

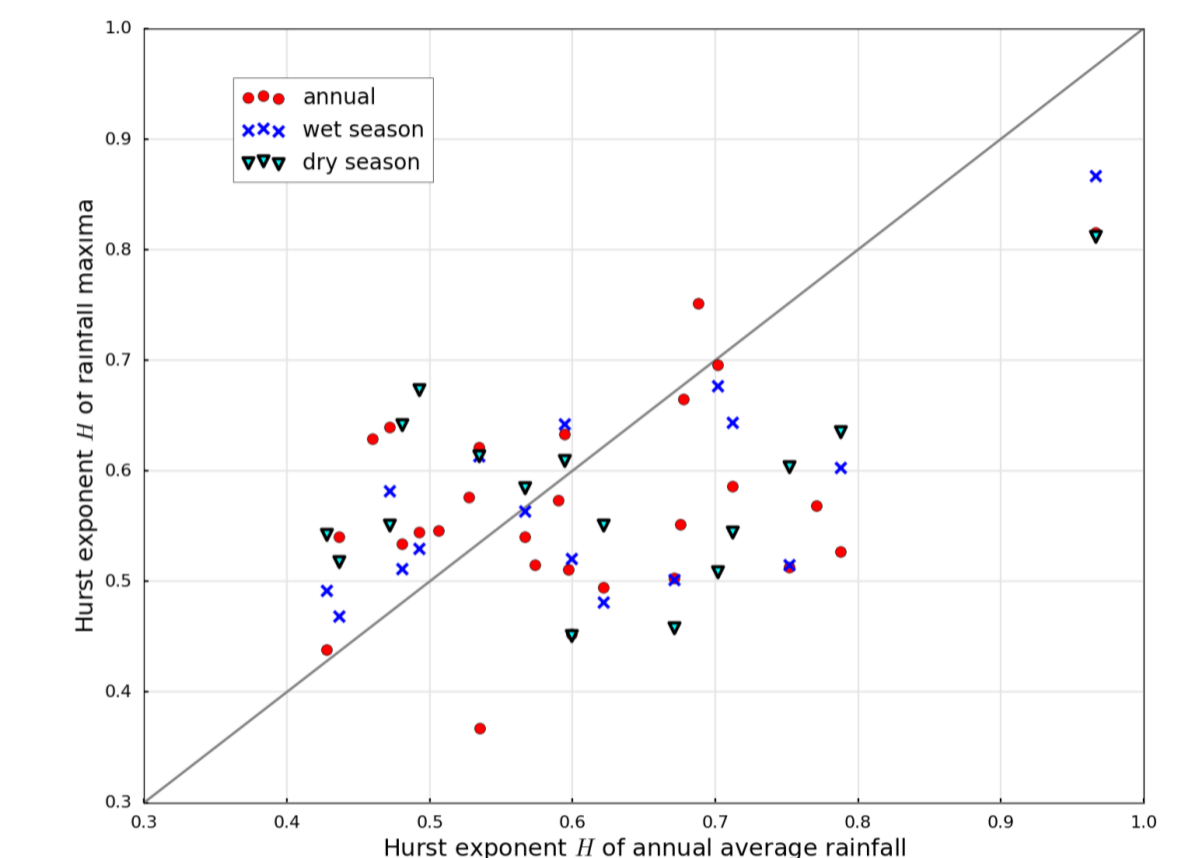
- $H_{\text{Annual Maxima}} = 0.7$
- POT occurrences show larger variability from the designed mean, which persists at larger timescales as well



5. Does LRD propagate from average behaviour to extreme behaviour?

	Number of records	$H_{\text{annual_average}}$	$H_{\text{annual_maxima}}$
Records with exceedances	12	0.65	0.619
Records of non-exceedances	15	0.575	0.527

Links between LRD in annual average and dependence of extremes



6. Implications for the statistical inference

12 records exhibit at least one exceedance of the Poisson MC limits during the examined period; among which, 9 show at least one exceedance of the upper limit.

Timescale	5yr	1 decade	2 decades	3 decades
Number of stations with exceedances	7	9	12	7
Upper limit exceedances	7	5	6	5

- It is known that the limiting distribution of maxima is unaltered even in cases of weakly dependent occurrences of extreme events (Leadbetter, 1983).
- Dependence does not alter either the classical Return Period mathematic formulation as the mean inter-arrival time between two extreme events; yet it may affect the corresponding probability of failure, which is significantly increased in strongly correlated processes, $\rho > 0.5$ (Volpi et al. 2015).

Conclusions

- Weak presence of LRD in annual and seasonal extremes.
- Evidence on links between LRD in mean and LRD in extreme behaviour.
- Record length is pivotal as it enables the exploration of clustering at larger scales
- Violations of the Poisson distribution of extremes are present at these timescales; yet they do not challenge probabilistic concepts of Extreme Value Theory.
- If existing, clustering behaviour may be exploited to condition the waiting time to the next occurrence.

References

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