Characterizing and modeling seasonality in extreme rainfall

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Key points

- The impact of seasonality on the tail index of the probability distribution of extreme rainfall is minimal.
- Estimation uncertainty of extreme rainfall is relevant even when long historical records are available.
- Estimating seasonality in extreme rainfall by dividing the year in 4 seasons leads to overfitting and increased uncertainty.

Plain language summary

Climate studies and hydrological applications, e.g. flood risk mitigation, scheduling of season-specific engineering works and reservoir management, often require information on the time of occurrence and magnitude of extreme rainfall throughout the year. The seasonal variation of rainfall extremes is investigated in a dataset of 27 remarkably long records including at least 150 years of daily measurements. An innovative method is proposed to perform identification of seasons and simulation of extreme rainfall values for each season. Identification of the number of

seasons is achieved by evaluating alternative seasonal models according to model selection criteria. Seasonal partitioning is performed by grouping consecutive months with similarly behaving rainfall extremes. To generate synthetic data, extreme value probability distributions are employed. The results indicate that seasonal properties of rainfall extremes mainly affect the average values of seasonal maxima, while the shape of their probability distribution and its tail do not substantially vary from season to season. Uncertainty in the estimated probability distributions is important when comparing results from three different estimation methods, even for long records, where one would expect that uncertainty is limited. The overall effectiveness of the methodology is highlighted when contrasted to the conventional approach of using fixed climatological seasons.

Abstract

A comprehensive understanding of seasonality in extreme rainfall is essential for climate studies, flood prediction and various hydrological applications such as scheduling season-specific engineering works, intra-annual management of reservoirs, seasonal flood risk mitigation and stormwater management. To identify seasonality in extreme rainfall and quantify its impact in a theoretically consistent yet practically appealing manner, we investigate a dataset of 27 daily rainfall records spanning at least 150 years. We aim to objectively identify periods within the year with distinct seasonal properties of extreme rainfall by employing the Akaike Information Criterion (AIC). Optimal partitioning of seasons is identified by minimizing the within-season variability of extremes. The statistics of annual and seasonal extremes are evaluated by fitting a generalized extreme value (GEV) distribution to the annual and seasonal block maxima series. The results indicate that seasonal properties of rainfall extremes mainly affect the average values of seasonal maxima and their variability, while the shape of their probability distribution and its tail do not substantially vary from season to season. Uncertainty in the estimation of the GEV parameters is quantified by employing three different estimation methods (Maximum Likelihood, Method of Moments and Least Squares) and the opportunity for joint parameter estimation of seasonal and annual probability distributions of extremes is discussed. The effectiveness of the proposed scheme for seasonal characterization and modeling is highlighted when contrasted to results obtained from the conventional approach of using fixed climatological seasons.

Keywords: seasonality, extreme daily rainfall, AIC, seasonal clustering, Generalized Extreme Value Distribution, long rainfall records

1. Introduction

Seasonality is a dominant feature of most hydrological processes including extreme rainfall (Hirschboeck, 1988). It implies intra-annual periodic variability which pertains to both timing and magnitude of extreme rainfall. An accurate and effective characterization of seasonality is critical to a wide variety of hydrological applications. For instance, it is useful in the scheduling of various flood preparedness measures, including management of stormwater infrastructures (Dhakal et al., 2015) and reservoir operation (Chiew et al., 2003; Fang et al., 2007; Chen et al., 2010a). Similarly, seasonality characterization is exploited in advanced schemes of flood-frequency analysis incorporating causative mechanisms (e.g. Sivapalan et al., 2005; Li et al., 2016) and may be useful for medium-range flood prediction (e.g. Koutsoyiannis et al., 2008; Wang et al., 2009; Aguilar et al., 2017), for which inclusion of seasonal extreme rainfall may increase prediction skill. Modeling of seasonal rainfall extremes – which typically implies some sort of frequency analysis – may also inform the selection of design values for related infrastructure. Additionally, the latter provides support to within-year operation of water resources systems, design rainfall estimation (Golian et al., 2010; Efstratiadis et al., 2014) and probabilistic assessment of extreme events occurring in a given season. Nowadays, extreme rainfall seasonality also prompts renewed scientific interest as a field of trend analyses (Ntegeka and Willems, 2008; Dhakal et al., 2015; Tye et al., 2016; Wu and Qian, 2017).

Characterization of extreme rainfall seasonality is scarcely dealt with by the relevant literature. Most of the established methods are devised to identify the temporal span of a wet season and assess its significance, typically by a priori identifying a single wet season. For example, directional statistics are typically applied to identify the high flow season (Cunderlik et al., 2004a; Baratti et al., 2012; Chen et al., 2013) and have also been applied to characterize the timing of seasonal rainfall (Parajka et al., 2009, 2010; Lee et al., 2012). However, directional statistics are inefficient when extremes occur over multiple seasons, which is very likely in the case of rainfall (Cunderlik and Burn, 2002). Recently, Dhakal et al. (2015) provided an improvement to the traditional method of directional statistics by adopting a non-parametric approach to capture multiple modes in the timing of annual rainfall maxima. Yet they noted that the proposed method is sensitive to the subjective selection of threshold values to assess significance of circular density estimates. Multimodality of the seasonal regime is also dealt with by analyzing the monthly relative frequencies of extreme occurrences (Cunderlik and Burn, 2002; Cunderlik et al., 2004b). This approach, however, relies on the subjective identification of the monthly time step to characterize seasonality. The latter along with the four climatological seasons are often used when large-scale or global analyses are performed (e.g. Rust et al., 2009; Villarini, 2012; Serinaldi and Kilsby, 2014; Papalexiou and Koutsoyiannis, 2016) but lead to disregarding the large spatial variability of atmospheric patterns and may not align well with local behaviors (Pryor and Schoof, 2008; Dhakal et al., 2015). Moreover, the fixed partitions do not resolve the crucial question of the identification of the optimal number of seasons, therefore resulting in over-parameterization of the seasonal model of extremes due to the large number of seasons that is adopted, particularly in the 12 month model.

A sub-optimal characterization of seasonality could be a reasonable compromise when one is interested in characterizing the timing of the most extreme events only. However, technical applications often require the modeling of the frequency of extremes during the whole course of the year. In this regard, several previous studies have either considered climatological information or employed statistical methods along with some degree of subjective judgement to estimate the optimal number of seasons and their displacement in time (e.g. Durrans et al., 2003;

Chen et al., 2010a; Baratti et al., 2012; Bowers et al., 2012). Coles et al. (2003) adopted a different approach by treating seasonal temporal limits as unknown parameters to be identified within a Bayesian framework. Yet, they also identified the number of seasons a priori through subjective inference.

The above literature review highlights a methodological gap in the objective identification of the optimal number of extreme rainfall seasons and their duration. To the best of the authors' knowledge, existing methods are not suitable for directly inferring multimodality from the seasonal regime and concurrently identifying segmentation points between seasons in an objective manner.

The research herein presented proposes a two-purpose framework for (a) objective seasonality identification and (b) modelling of rainfall extremes in order to effectively estimate the seasonal probability of extreme events. To this end, we introduce two alternative methods for season identification, which are characterized by different levels of parsimony in terms of data requirements, therefore providing two options for practical applications. Our approach employs an information-theoretic framework (Akaike Information Criterion, AIC) to estimate the optimal number of seasons. In order to describe the frequency of extremes in each identified season we use the GEV probability distribution. We discuss the consistency of the model at different time scales. Finally, in order to demonstrate the efficiency of our framework we present a comparison with the traditional 4-season approach.

An extended dataset of long daily rainfall records is herein investigated, as detailed in the next section. The length of the records, the shortest one covering an observation period of 150 years, allows us to inspect the impact of uncertainty, which may be relevant for seasonal extreme value analyses (Cunderlik et al., 2004c). To reduce uncertainty we propose a robust parameterization approach of seasonal-annual distributions which is supported by empirical evidence.

2. Dataset

Our dataset includes 27 daily rainfall records each one spanning over 150 years. Eighteen of them are collected from global databases, namely, the Global Historical Climatology Network Daily database (Menne et al., 2012) and the European Climate Assessment & Dataset (Klein Tank et al., 2002). Figure 1 shows the geographical location of the stations, while Table 1 reports the coordinates of each station, the observation period, as well as the number of years that are fully covered by observations after quality control and screening of missing values. For the extraction of the annual maxima we employ a methodology proposed by Papalexiou and Koutsoyiannis (2013); accordingly, an annual maximum is not accepted if (a) it belongs to the lowest 40% of the annual maxima values and (b) 30% or more of the observations for that year are missing. For seasonal and monthly maxima we compute statistics only if number of missing values is less than 10% of the total sample (season or month). The longest series is that of Padua, that is, the longest rainfall record existing worldwide (Marani and Zanetti, 2015).

3. Methodology

3.1 Season identification

The methodology that we propose to identify seasons is inspired by cluster analysis and model selection techniques. Seasons are regarded as groups (clusters) of consecutive months with similar behavior of extremes. The question of selecting the number of seasons that best describe the dataset is addressed here via a model selection process under the assumption that different numbers of clusters (seasons) represent alternative plausible models for the dataset. Two alternative methods for season identification characterized by different level of parsimony are considered here and described below.

In what follows, we denote random variables and their realizations by upper and lower case symbols, respectively. We also use bold characters for vectors. We denote season, month and year with the indexes $i = 1,..n$, $j = 1,...,12$, and $k = 1,..k_{\max}$, respectively, where n is the number of seasons and $k_{\max}$ is the record length in years. We assume that n is fixed a priori and denote with $\boldsymbol{c}_i$ the vector containing the j values of contiguous months belonging to the same season i, and with s_i its size. Accordingly, we define the following random variables:

- $R_{i,j,k}$ is the maximum daily rainfall amount of season i, month j and year k;
- $R_{i,j}$ is the temporal average of maximum daily rainfall of month j of season i along the record, namely, $R_{i,j} \coloneqq \frac{1}{k_{max}} \sum_{k=1}^{k_{max}} R_{i,j,k}$;
- R_i is the temporal average of the $R_{i,j}$ values along the season i, $R_i \coloneqq \frac{1}{s_i} \sum_{j \in \boldsymbol{c}_i} R_{i,j}$.

For instance, $r_{2,5,12}$ for season i=2 defined by $\boldsymbol{c}_2$ =[5,6,7] denotes the maximum daily rainfall observed in May of the 12th year of a given record and belonging to the 2nd identified season of the year, which also includes months June and July; likewise, $r_{2,5}$ is the sample average of maximum rainfall observed in all May days of the record, while, r_2 is the sample average of all monthly averages belonging to season 2, in this case of May, June and July.

We call the first method for season identification the SSD algorithm. It is based on the computation of Sum of Squared Deviations (SSD) of the $R_{i,j}$ values from their seasonal average, R_i for all seasons according to the equation:

$$\mathrm{SSD} = \sum_{i=1}^{n} \sum_{j \in \boldsymbol{c}_i} (R_{i,j} - R_i)^2 \qquad (1)$$

This metric is evaluated for each possible clustering combination c_i of consecutive months for the given number of seasons, thus enabling the identification of the lower value of SSD, which identifies the optimal partition of the year into n seasons. We require a season to span at least two months and allow the algorithm to group months across different calendar years. The requirement for a season to span at least two months implies that the maximum number of seasons is 6, but preliminary investigations showed that more than three seasons are rarely present in extreme rainfall. Therefore, we limit our attention to n values ranging in the interval [1-4].

Essentially, the SSD algorithm minimizes the within-cluster variance of the average value over the years of the monthly rainfall maxima and can be considered as a simplification of the well-known *k*-means algorithm (MacQueen, 1967). Since seasons may include contiguous months only, and the algorithm deals with only 12 data points to cluster - the average over the years of daily maximum rainfall values for each month - the number of possible combinations is relatively low and the method is parsimonious.

In order to identify the optimal number of seasons we define alternative probabilistic models, with different level of parsimony, to describe the frequency of occurrence of extreme events in each season and assess their ability to optimally fit the observed record. Accordingly, we first select a trial value for the number n of seasons in the range [1-4] and partition them by applying the above SSD algorithm. To describe the probability distribution of rainfall in each season and the whole year we form a mixture model with n seasonal components, each described by its own probability distribution. Hence, according to the law of total probability, the probability distribution of the seasonal model for a generic seasonal random variable U takes the form:

$$f_U(u; \boldsymbol{a}_1, \dots, \boldsymbol{a}_n) = \sum_{i=1}^{n} w_i f_{U_i}(u_i; \boldsymbol{a}_i) \tag{2}$$

where w_i are weights adding up to 1. They are obtained as the ratio of the season's length in months, s_i, versus the whole twelve-month period, i.e. $w_i = s_i/12$; and $\boldsymbol{a}_i$ is a seasonal parameter vector. Here f_{U_i} is a seasonal probability distribution for U describing realizations u_i in season i. Note that by applying the law of total probability instead of deriving the annual probability distribution as the product of the seasonal ones, we avoid relying on the assumption of independence of the random variables U_i, which was adopted in other studies (Durrans et al., 2003). Therefore, this is a more general approach also appropriate for the rarer cases of rainfall maxima being correlated among seasons.

The above step requires identifying and fitting a candidate model for the f_{U_i} probability distribution. We propose two alternative models for the seasonal probability distribution $f_{U_i}(u_i, \boldsymbol{a}_i)$ which are characterized by different level of complexity.

The first option, which we call Average Based (AB) method, identifies the random variable U_i as the monthly temporal average $R_{i,j}$. Then, we assume that $f_{R_{i,j}}(r_{i,j}, \boldsymbol{a}_i)$ is a uniform distribution given by:

$$f_{R_{i,j}}\left(r_{i,j}, \boldsymbol{a}_i\right) = \frac{1}{b_i} \tag{3}$$

where in this case $\boldsymbol{a}_i$ contains only one parameter, namely, $b_i = \max_{j \epsilon \boldsymbol{c}_i} r_{i,j}$. Preliminary analyses showed that the uniform distribution provides an efficient representation of the frequency of the $r_{i,j}$ realizations, by minimizing the number of involved parameters. The above approach imposes an upper limit to the average value of the monthly maximum rainfall depth and sets the lower limit to zero.

The second option, which we call Complete Data (CD) method, identifies the random variable U_i as the maximum daily rainfall in each month j of the season i for the year k, which has been previously introduced as $R_{i,j,k}$. Then, we assume that $f_{R_{i,j,k}}(r_{i,j,k}, \boldsymbol{a}_i)$ is described by two alternative probability distributions with a different tail behavior, i.e. one characterized by a lighter and one by a heavier right tail, in order to allow flexibility in fitting the observed rainfall maxima. The first is the two-parameter Gamma distribution, given by:

$$f_{R_{i,j,k}}\left(r_{i,j,k}, \boldsymbol{a}_i\right) = \frac{r_{i,j,k}^{\xi_i - 1} e^{-\frac{r_{i,j,k}}{\theta_i}}}{\theta_i^{\xi_i}\,\Gamma(\xi_i)} \tag{4}$$

where $\boldsymbol{a}_i = (\xi_i, \theta_i)$ is the parameter vector with ξ and θ being shape and scale parameters, respectively. The second is the two-parameter Weibull distribution:

$$f_{R_{i,j,k}}\left(r_{i,j,k}, \boldsymbol{a}_i\right) = \frac{\mu_i}{\lambda_i}\left(\frac{r_{i,j,k}}{\lambda_i}\right)^{\mu_i - 1} e^{-(r_{i,j,k}/\lambda_i)^{\mu_i}} \tag{5}$$

where $\boldsymbol{a}_i = (\mu_i, \lambda_i)$ is the parameter vector with μ and λ being shape and scale parameters, respectively. By working on the monthly maximum rainfall instead of their averages along the season, the CD method allows one to base the estimation of the probability model on a more extended dataset.

The above methodology allows several modeling options, which differ for the number of seasons, the application of either AB or CD method and the selection of either the Gamma or the Weibull distribution in the CD method. The best modeling option and the related optimal number of seasons is identified by applying the Akaike Information Criterion (AIC, Akaike, 1973, 1974). The criterion statistic for the pth candidate model, AIC_p, is given by:

$$\mathrm{AIC}_p = 2m_p - 2\ln L_p \tag{6}$$

where m_p is the number of parameters and L_p is the likelihood of the pth candidate model. The application of the criterion is straightforward as it only requires estimation of the likelihood function for the candidate probability models defined by eq. (2). The minimum AIC value identifies the best candidate model by evaluating the bias versus variance trade off; i.e., the condition in which as the model parameters increase the bias of the model estimates decreases, yet their variance increases (Burnham and Anderson, 2002). Hence, AIC provides an implicit interpretation of the principle of parsimony which is pivotal in model selection (Box and Jenkins, 1970). Although AIC has a solid foundation in information theory both in mathematical terms and also from a philosophical point of view, its use is not still widely established in hydrological applications (Laio et al., 2009). For an insightful review of AIC's properties, the reader is referred to Burnham and Anderson (2002).

Therefore, the workflow for season identification is as follows:

1. A trial value is adopted for the number n of seasons in the range [1-4];
2. The n seasons are partitioned by applying the SSD algorithm therefore identifying the vectors $\boldsymbol{c}_i$, $i = 1,\ldots, n$, of the indexes of the months that are included in each season;
3. AB and CD methods are applied to estimate the probability distribution of $R_{i,j}$ and $R_{i,j,k}$, respectively, in each season;
4. AIC is computed for candidate models;
5. The procedure is repeated for the other values of n in the range [1-4];
6. The resulting AIC values are compared therefore identifying the optimal number of seasons, and their partition, for AB and CD methods.
7. If n values resulting from AB and CD methods are the same, then the procedure is terminated and the optimal partition of seasons is uniquely identified;

8. If the estimated n values differ, then the user is allowed to select the preferred partition of seasons based on the suitability of $R_{i,j}$ instead of $R_{i,j,k}$, for the considered design problem.

3.2 Extreme value analysis

3.2.1 Fitting the GEV distribution

Once the optimal number of seasons and their partition have been identified, to estimate seasonal extremes one needs to fit a suitable probabilistic model for the seasonal block maxima series. The latter is formed by extracting from each identified season the maximum daily rainfall observed in each year. It is worth noting that distributions that were previously considered for seasonal partitioning (the Gamma and the Weibull) are not suited for fitting extreme values and therefore are not an option for the current target.

Extreme Value Theory (EVT) suggests that the distribution of the maximum of v independent and identically distributed (i.i.d.) random variables asymptotically converges to three limiting laws (Fisher and Tippett, 1928), which are the Gumbel distribution (Type I), the Fréchet distribution (Type II) and the reversed Weibull (Type III), that can be unified under the following single analytical form provided independently by von Mises (1936) and Jenkinson (1955) and known as Generalized Extreme Value (GEV) distribution:

$$G_X(x) = \exp\left(-\left(1+\kappa\frac{x-\psi}{\sigma}\right)^{-\frac{1}{\kappa}}\right), \quad 1+\kappa\frac{x-\psi}{\sigma} \geq 0 \qquad (7)$$

where X is a generic random variable and $\kappa \in \mathbb{R}$ is a shape parameter, $\sigma > 0$ is a scale parameter and $\psi \in \mathbb{R}$ is a location parameter. The Type I distribution emerges for $\kappa = 0$, the Type II for $\kappa > 0$, while the Type III emerges as the limiting distribution for $\kappa < 0$, but it is not used for modeling rainfall extremes as it is upper limited. GEV is the limiting distribution for extremes from any parent distribution of the underlying stochastic process. Therefore, it could be the limiting distribution also in the case of monthly rainfall maxima described by the Gamma and Weibull distributions as in the CD method above. Leadbetter (1974) showed that convergence to GEV is guaranteed even in the presence of short-range correlation in the underlying stochastic process. In our case, the implication is that GEV emerges as limiting distribution even if rainfall maxima are correlated. Koutsoyiannis (2004a) has shown mathematically that GEV still emerges as asymptotical distribution in the presence of different parent distributions from season to season. In practical applications, though, in which a maximum value is extracted from a small number of events, the asymptotic condition is unlikely to hold. In this respect, Koutsoyiannis (2004a) demonstrated that the convergence of the distribution of maxima to the GEV with a positive shape parameter (Type II) is good even for a small number of events and also for parent distributions belonging to the domain of attraction of the Gumbel (Type I), due to the increased flexibility of the three-parameter distribution. On the contrary, convergence rates to the Gumbel distribution are very slow even for distributions belonging to the domain of attraction of the Gumbel family (see also Papalexiou and Koutsoyiannis, 2013).

Here, we assume that the underlying stochastic process is given by the series of the monthly maxima of daily rainfall in each season. We aim to fit with the GEV distribution the seasonal samples that are obtained by extracting from each season i and each year k the maximum daily value $r^*_{i,k}$ therefore obtaining a block maxima series, which is assumed to be a realization of the random variable $R^*_{i,k}$. We also fit the series of the annual maxima r^*_k which is assumed to be a realization of the random variable R^*_k. This approach shall allow one to estimate the extremes for the seasonal periods and the total annual period and ensures that both the seasonal and annual approaches refer to the same sample size when fitting the GEV, as the block maxima sampling method is used, i.e., one extreme event is sampled on a yearly basis for both the seasonal and annual periods.

3.2.2. Investigating consistency of seasonal and annual distributions

A considerable part of related literature (e.g. Buishand and Demaré, 1990; Durrans et al., 2003; Chen et al., 2010b; Baratti et al., 2012) has focused on the estimation of seasonal and annual flood frequency distributions and their inter-relationship. Usually, it is suggested that an independent fitting of seasonal and annual distributions may lead to inconsistency among them, manifested as a "crossing over" effect. The latter means that for extremely rare events seasonal quantiles may be higher than their annual counterparts. To resolve this inconsistency, a variety of methods for the joint estimation of the seasonal and annual distributions has been proposed.

Durrans et al. (2003) attributed distributional inconsistencies in seasonal-annual frequency analysis to three possible reasons: (a) the arbitrary parameterization of seasonal and annual distributions, (b) stochastic dependence among them and (c) estimation uncertainty. In this respect, we believe that the arbitrary specification of seasonal samples is also a major reason causing distributional inconsistencies (such a case is discussed and illustrated later in section 4.5). In our case though, we argue that the above inconsistency should rather be viewed as an empirical evidence of estimation uncertainty, which is particularly relevant in extreme value studies (Coles et al., 2003; Koutsoyiannis, 2004a). This is further supported by observing that the crossing over effect is manifested in the domain of extremely rare events, where uncertainty is prominent.

To inspect the impact of estimation uncertainty, we fit the GEV probability distribution by applying three different methods, namely, maximum likelihood (ML), method of moments (MM) and a least squares estimation method (LS) for an improved fitting of the extremes

(Koutsoyiannis, 2004b). We further investigate estimation uncertainty in each of the three methods by computing 95% Monte Carlo Prediction Limits (MCPL) for the resulting GEV quantiles. MCPL are estimated by applying a Monte Carlo simulation which is structured according to the following steps: (1) we estimate the GEV parameters by each method, (2) produce 1000 synthetic GEV series for each derived parameter set, (3) re-estimate the parameters by the same method, (4) compute the resulting GEV quantiles for each of them and then (5) identify the 95% confidence region for each quantile value. The scope is to assess whether the crossing over falls within the limits of the estimation uncertainty as evaluated from applying a set of different parameter estimation methods. To further reduce fitting uncertainty, we propose a simpler alternative to joint parameterization, i.e. the joint estimation of a common shape parameter among seasonal-annual distributions – since the shape parameter is the most difficult to estimate accurately − and we discuss how this choice is supported by empirical evidence.

4. Results

4.1 Season identification for the observed records

Table 2 shows the AIC values resulting from season identification for the available stations. Following Burnham and Anderson (2004), we denote with ΔAIC the difference in the AIC value of each model with respect to the best one. Therefore, the zero ΔAIC model is the best model, while models with ΔAIC<2 and ΔAIC>10 are assumed to have good and little support, respectively. An example of seasonal partition for the case of Florence is shown in Figure 2a and Figure 2b for 2 and 3 seasons. We refer to this type of figures as *climatograms*, though the term is typically used for plots depicting both rainfall and temperature climatological regimes.

The results point out that both methods identified the one-season (annual) model as the best solution for 11 stations (with 6 stations being the same for both methods). In four stations, the one-season model was preferred by the CD method, while the two-season solution was indicated by the AB method. On further investigation, it was found that neither the Gamma, nor the Weibull provided satisfactory likelihood values for these stations. As a result, the more parsimonious one-season model was preferred by the AIC. The three-season model is identified as the best solution for five stations with the CD method, while the AB method did not select $n = 3$ for any station. This result was expected as the AB method exploits information from a limited dataset and therefore parsimonious models are likely to provide better AIC values. The Gamma distribution is selected as the best model in 21 cases and the Weibull for the remaining 6.

To inspect the spatial coherence of the results, we present maps of the two regions of the dataset having neighboring stations, i.e. Europe and Australia (Figure 3). We group the stations in six clusters of similarity in their seasonal patterns and we also mark single stations for which similarity falls below the accepted threshold. As similarity index we define the ratio of the number of the wet season months that the stations in the cluster have in common versus the span of each wet season and we require it to be at least 60% for each station in the cluster. More specifically, Clusters 1 and 2 have 67% and 80% similarity, respectively, for both methods, while, Clusters 3, 4 and 5 exhibit 100%-75%, 60%-75% and 80%-67% for the AB and CD methods, respectively. On top of the maps, we also plot Köppen maps of climate classification by Chen and Chen (2013) covering the period 1901-2010, in order to allow a direct comparison of the observed spatial patterns to the climatological ones. Some interesting insights can be derived. First, spatial coherence does not fully coincide with climatological coherence and vice versa, and this is especially true in regions with complex topography/climatology. For example, in the wider Alpine region, where climate shows great diversity, the stations are less spatially consistent than in Central Europe. On the contrary, stations belonging to a Mediterranean climate (Cluster 1) show consistent patterns. In general, we notice that patterns are coherent on both levels: neighboring stations show very high similarity (e.g. Cluster 3) and far apart stations belonging to a climatically homogenous region show medium to high similarity (see, e.g., Cluster 2).

4.2 Assessing temporal change in observed seasonality

To demonstrate the applicability of the proposed season identification method in the inspection of temporal changes in seasonality, we analyze the four longest records of the dataset, i.e. the stations of Padua (275 years), Prague (211 years), Bologna (195 years) and Radcliffe (188 years). We split the observation period into equally sized sub-periods and apply the methodology independently to each period. We employ four sub-periods for the significantly longer station of Padua and three for the other records.

Results are shown in Table 3. It can be seen that changes, both in the number and duration of seasons, are likely to emerge within each sub-period. For example, seasonality in Prague during the 2nd period changed in terms of the span of the wet season, but a two-season regime was selected for all sub-periods. Results for 3rd and 1st window coincide. These characteristics of the methodology make it useful for climate change analysis.

4.3 Fitting the GEV distribution

Subsequently to the identification of seasons, we fitted the GEV distribution via maximum likelihood (ML) estimation to each of the seasonal sets (or the annual set if one season was identified). Table 4 contains summary statistics of the GEV fitting for wet and dry seasons, as well as for the whole year, for the cases where the two- and three-season model were found prevalent under AB and/or CD methods. Summary statistics for the transition season (placed between the wet and dry season) in the three season model are omitted since the sample is small (5 stations). The main differences in the seasonal distributions lie in the values of the scale and location parameters, which are in their vast majority (93.8% and 100%, respectively, under AB method and 100% and 100%, respectively, under CD method) higher for the wet season compared to the dry. What might be less anticipated is that there is limited seasonal variation in the value of the shape parameter κ, which is related to the shape of the tail of the seasonal maxima distribution. Hence, it is justifiable to represent the two seasons and the whole year by a common value for the shape parameter, therefore increasing robustness of the method, which is a desirable feature. Additionally, for the majority of the stations, the shape parameter takes positive values indicating the appropriateness of heavier-tailed distributions for modeling of extremes. It is also clear that the wet extreme properties are quite close to the annual maxima ones, which indicates that the annual maxima distribution is dominated by the wet season.

The singular cases of the stations of Prague from Czech Republic and Florence from Italy are plotted in Figures 4a and 4b. In the second case, there is small deviation between the wet season and the annual period, while in the first case the two lines are almost identical. In both cases, the dry-season probability line lies considerably lower. In the second case, in which the three-season model is preferred by the CD method (while two seasons were preferred by the AB method), the probability line of the transition season lies in the space between the wet and dry seasons' probability lines, as expected.

4.4 Assessing estimation uncertainty in seasonal-annual GEV parameterization

The crossing over effect mentioned in Section 3.2.2 is observed in five cases (Eelde, Genoa, Hohenpeissenberg, Milan and Zagreb), where we found that the wet-season probability distribution lies higher than the annual one in the area of extremely rare events. We focus on the station in Genoa where the effect is more pronounced. As mentioned in Section 3.3, we performed additional parameter estimation by applying the method of moments (MM) and the least squares algorithm (LS). Figure 5a shows results from the application of the three estimation methods for the annual maximum series along with uncertainty bounds computed within each method by means of Monte Carlo analysis. Uncertainty bounds in the area of extremely rare events, where the crossing over effect is also observed, are large. The larger annual maxima fall within the 95% limits of the annual maxima GEV distribution only for the LS method. This is due to the better fitting capability of the LS algorithm for extremely rare events (Koutsoyiannis, 2004b). To further improve the fitting we also estimate via LS a common shape parameter for the three distributions (two seasonal GEV and the annual one). In these cases as well, the choice of a common shape parameter is supported by empirical evidence from the previous independent fitting. The crossing over effect is significantly mitigated (Figure 5b), with a remaining positive difference between the quantiles of the wet season and annual distribution of 10 mm for the 0.5% annual exceedance probability, which is considered not significant in view of the large uncertainty in the high-quantile domain. The results for the other cases also showed that the crossing over effect was resolved.

The importance of taking estimation uncertainty into consideration is additionally showcased by applying ML, MM and LS estimation methods to the entire set of stations, as shown in Table 5. One notices that uncertainty is higher in the estimation of the shape parameter, as already discussed in literature (Koutsoyiannis, 2004b; Papalexiou and Koutsoyiannis, 2013). The fact that this result is empirically confirmed for the long rainfall records considered here is a further confirmation that for practical applications uncertainty in the estimation of extremes is unavoidable even when dealing with long records.

4.5 A comparison to traditional methods of seasonal clustering

We compare our method to the climatological 4-season approach, which divides the annual period in Winter, Spring, Summer and Fall seasons. First, to highlight that site-specific season identification is important, we compare the monthly maxima plots for two stations in Europe for our method and the fixed seasonal partition (Figure 6). It is clear that climatological seasons are an inefficient partition for analyzing the extreme rainfall properties, and may also be a rather crude method for delineating the extreme's properties in multi-site analyses where seasonal differences in climate may be very pronounced among stations. As an example, seasonality of maximum rainfall in Jena (Germany) is completely out of phase with respect to Athens (Greece). The same could be argued for trend studies employing fixed characterizations of seasonality. For instance, the question of whether winter rainfall has increased is potentially ill-conceived, as it mostly pertains to a subjective interpretation of seasonality. A more relevant question is whether rainfall in the major rainy season has significantly changed, but such a change is unlikely to be identified by considering an arbitrary partition in seasons.

To demonstrate the effect that a fixed 4-season partition could have on the estimation of extreme value properties, we focus on the rainfall record of Athens. By applying the 4-season partition one obtains an apparent overfitting, as the seasonal lines are not clearly separated and even cross each other at several points (Figure 7b). It is evident that an inappropriate characterization of seasonality provides no valuable and practical information for seasonal planning and decision-making while, in fact, it obscures the presence of the existing seasonal regime (Figure 7a). Additionally, in the presence of parameter uncertainty and given the short record lengths that are usually available, adopting subjective characterizations of seasonality for the study of extreme values entails the risk of disproportionately increasing estimation uncertainty. The consequences of overfitting are even more obvious in stations with very low or no seasonality.

5. Discussion and Conclusions

An objective methodology is proposed to allow season identification in extreme daily rainfall and the study of the resulting extreme properties in each season. The methodology is evaluated on an extended dataset comprising 27 rainfall stations covering a period of more than 150 years of daily observations. In the following, we discuss methodological and modeling issues, the results of the extreme value analysis and their comparison to the no-seasonality approach, as well as relative strengths and potential limitations of our method.

The season identification methodology herein proposed is based on the SSD algorithm, a simplified version of the *k*-means clustering algorithm, whose results are evaluated by exploiting the model selection properties of the Akaike Information Criterion (AIC). The method is able to identify the optimal modeling option for the seasonal extreme rainfall for a given dataset, discerning among the existence of 1 (no dominant season) to 4 seasons in the extreme rainfall properties and identifying their temporal span. Since AIC is a measure of relative performance of models, this task should be performed after thorough consideration of the appropriateness of the candidate seasonal distributions to be assessed. In that respect, our methodology provides additional flexibility as multiple probabilistic models may be simultaneously assessed. Overall, the methodology shows good spatial coherence, which makes it potentially appropriate for regionalization studies, and its flexibility allows one to inspect temporal changes in a range of ways, which is also a desirable feature concerning climatic variability and trend studies.

In terms of generated results, the adopted scheme proved to be successful for the long rainfall records considered here, by both visual evaluation of the plots of the monthly maximum rainfall values (climatograms) and assessment of the resulting extreme seasonal distributional properties. For the cases where two or three seasons are identified, the differences in the distributional properties are reflected mainly in the value of the scale and location parameters of the GEV which are significantly higher for the wet season. The shape parameter shows limited seasonal variability, which implies that the seasonal distributional properties do not differ substantially in the shape of the distribution tail. Our results also confirm other studies regarding the prevalence of heavy-tailed distributions for daily rainfall extremes (Koutsoyiannis, 2004b; Villarini, 2012; Papalexiou and Koutsoyiannis, 2013; Serinaldi and Kilsby, 2014; Mascaro, 2018). Some of these studies have also argued that a positive shape parameter emerges for extremes caused by multiple types of synoptic patterns, whereas a zero exponent (i.e. an exponential tail) may occur for a single-type of events. Apart from pronounced intra-annual variability, a positive shape parameter may be also portraying increased inter-annual variability in the extremes which has been linked to the presence of large-scale circulation patterns, i.e. the NAO, for certain stations of our dataset (Kutiel and Trigo, 2014; Marani and Zanetti, 2015). In principle, we believe that our findings are in agreement with previous research and strengthen the assumption that a heavier-tail behaviour better captures conditions of enhanced natural variability and complex atmospheric forcing, as revealed by the inspection of our long and spatially sparse dataset.

In comparison to the no-seasonality approach, in some cases the annual maxima series are found to be dominated by extreme events occurring in the wet season. This result is pointed out by the closeness in the estimated GEV parameter values between the annual and the wet season's probability distribution of extreme events. It also indicates that annual frequency analyses may suffice for studying the annual maxima (AM). Actually, studying the AM series is more in favor of a conservative design approach, since the former takes into account the rare cases of extreme events of significant magnitude happening in the dry season. Furthermore, since the majority of AM in records with pronounced seasonality still stems from the wet season, strong seasonality is not significantly violating the i.i.d. assumption in the GEV approach. A similar remark was also made by Allamano et al. (2011). However, for intra-annual hydrological design and management, it is crucial to take seasonal variability into account. The wet season maxima series contain valuable information on the timing of occurrence of the most extreme events, although it is likely that in some cases, their magnitude will be close to the AM estimated one. Yet when dry periods are of interest, using the AM series instead, i.e. adopting a no-seasonality approach, is likely to lead to costly overestimation of design values and floodwater waste.

A few key strengths of our methodology should be underlined. In general, estimation uncertainty in extreme studies is a known issue especially for the estimation of the shape parameter of the GEV distribution. Here, we show how an alternative choice of estimation methods, improving the model performance in the domain of extreme events, may resolve inconsistencies deriving from an independent seasonal and annual fitting. Given the latter, we consider the need for the laborious joint estimation of seasonal-annual distributions to be questionable and we propose a simpler procedure based on the estimation of a common shape parameter for the seasonal-annual parameterization, which is shown to increase robustness of the statistical model. On the whole, the entire methodology is compared to a conventional partition in fixed seasons and its advantageous features are highlighted both in that it enables consistent identification of seasonal regimes at single-site and multi-site levels, as opposed to arbitrary partitions, and that it consequently allows a more informed and parsimonious fitting of the GEV distribution to seasonal extremes.

A few limitations should be taken into account. We note that in case where the Average Based (AB) and the Complete Data (CD) methods diverge, there is some remaining degree of subjectivity in the choice for the most appropriate scheme. This constitutes a potential limitation of our method as results may not be fully conclusive. Yet this may be resolved if an equifinality framework is adopted and both options are considered. Additionally, it should be noted that the performance of AIC largely depends on the quality of the considered candidate models. Although the chosen distributions are representative of a variety of statistical behaviors, it is possible that there may be exceptions for which they do not perform well. Increasing the set of candidate distributions is another option to achieve a greater degree of confidence within a multi-model approach.

Despite these limitations, we believe that our findings have direct applications both in the theoretical conceptualization of seasonality in extreme rainfall and in engineering applications. On a methodological level, they contribute to a wider establishment of model selection techniques, in this case AIC, in hydrological studies and pave the way for the objective identification of seasonality via automated schemes which are required for global-scale hydrology.

Acknowledgments

We greatly thank the Radcliffe Meteorological Station (Burt and Howden, 2011), the Icelandic Meteorological Office (Trausti Jónsson), the Czech Hydrometeorological Institute, the Finnish Meteorological Institute, the National Observatory of Athens, the Department of Earth Sciences of the Uppsala University and the Regional Hydrologic Service of the Tuscany Region (servizio.idrologico@regione.toscana.it) for providing the required data for each region respectively. We are also grateful to Professor Ricardo Machado Trigo (University of Lisbon) for providing the Lisbon timeseries and to Professor Marco Marani (University of Padua) for providing the Padua timeseries. All the above data were freely provided after contacting the acknowledged sources. The Eelde and Den Helder timeseries are publicly available by the data providers in the ECA&D project (http://www.ecad.eu), while the remaining 16 timeseries can be freely accessed at the GHCN-Daily database (https://data.noaa.gov/dataset/global-historical-climatology-network-daily-ghcn-daily-version-3). We greatly thank Professor Marco Marani and Professor Attilio Castellarin (University of Bologna) for their helpful discussions during an early stage of this research. We are also grateful to the anonymous reviewers and the Associate Editor for providing very constructive remarks and suggestions.

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Tables

Table 1 Characteristics of the 27 daily rainfall stations used in the analysis. The last column shows the final number of years that were accepted for the analysis after quality control.

Stations	Country	Latitude	Longitude	Start year	End year	Years accepted
Bologna	Italy	44.5	11.346	1813	2007	195
Palermo	Italy	38.11	13.351	1797	2008	175
Mantova	Italy	45.158	10.797	1840	2008	160
Milan	Italy	45.472	9.1892	1858	2008	151
Genoa	Italy	44.414	8.9264	1833	2008	176
Florence	Italy	43.8	11.2	1822	1979	155
Padua	Italy	45.866	11.526	1725	2013	275
Newcastle	Australia	-32.919	151.8	1862	2015	151
Deniliquin	Australia	-35.527	144.95	1858	2014	154
Melbourne	Australia	-37.807	144.97	1855	2015	160
Robe	Australia	-37.163	139.76	1860	2015	152
Sydney	Australia	-33.861	151.21	1858	2015	157
Jena Sternwarte	Germany	50.927	11.584	1826	2015	179
Hohenpeissenberg	Germany	47.802	11.012	1781	2015	182
Armagh	UK	54.35	-6.65	1838	2001	164
Radcliffe	UK	51.761	-1.2639	1827	2014	188
Zagreb	Croatia	45.817	15.978	1860	2015	154
Vlissingen	Netherlands	51.441	3.5956	1854	2015	158
Eelde	Netherlands	53.124	6.5847	1846	2015	169
Den Helder	Netherlands	52.933	4.75	1850	2015	165
Helsinki	Finland	60.167	24.933	1845	2015	171
Lisbon	Portugal	39.2	-9.25	1863	2013	150
Prague	Czech republic	50.051	14.246	1804	2014	211
Uppsala	Sweden	59.86	17.63	1836	2014	179
Stykkisholmur	Iceland	65.083	-22.733	1856	2015	160
Athens	Greece	37.973	23.72	1863	2014	152

Toronto	Canada	43.667	-79.4	1840	2015	162

Table 2 ΔAIC differences among the seasonal models (one, two or three seasons) under Average Based (AB) and Complete Data (CD) methods. A zero ΔAIC value indicates the model with the smallest AIC value which stands for the best model.

Stations	AB method			CD method					
	Uniform distribution			Weibull distribution			Gamma distribution		
	Number of seasons			*Number of seasons*					
	1	2	3	1	2	3	1	2	3
Bologna	0	0.047	2.9	27.55	27.51	35.12	0	4.91	9.04
Palermo	5.29	0	1.72	0	50.32	41.37	0.372	9.6	5.15
Mantova	0	0.414	3.92	28.22	25.76	13.43	0	8.58	7.12
Milan	0	0.639	3.86	8.633	10.75	0	30.72	33.5	34.7
Genoa	3.38	0	0.67	9.215	0	6.641	5.797	10.5	18
Florence	1.85	0	3.62	15.52	8.421	0	48.55	54.8	9.9
Padua	0	1.058	4.914	0	9.72	11.12	100.19	56.43	65.84
Newcastle	0.75	0	3.88	60.23	34.25	40.68	8.414	0	15.7
Deniliquin	0	2.894	6.41	18.11	16.3	20.18	0	3.68	9.11
Melbourne	0	0.903	4.58	139.1	76.04	78.35	37.31	0	5.27
Robe	1.45	0	5.09	3.475	36.71	40.45	0	1.7	7.29
Sydney	0	0.556	2.88	37.92	40.93	41.35	0	0.02	2.24
Jena Sternwarte	4.6	0	3.69	208.1	123.2	131.6	46.77	0	1.45
Hohenpeissenberg	5.5	0	3.25	85.83	63.77	67.7	0	8.93	6.95
Armagh	0	0.78	3.71	161.3	103.2	106.9	3.412	0	3.04
Radcliffe	0	0.682	3.68	129.6	70.5	77.79	0	0.74	0.75
Zagreb	0.5	0	3.68	42.95	17.99	23.94	0	19.9	25.87
Vlissingen	0.66	0	3.34	78.03	36.79	36.18	0	4.02	9.84
Eelde	1.79	0	3.55	135.8	61.08	67.77	3.338	0	6.24
Den Helder	0.36	0	3.7	201.4	137.5	118.7	27.93	3.58	0
Helsinki	1.9	0	3.69	108.9	48.3	35.58	1.161	0	6.95
Lisbon	8.18	0	7.07	58.26	0	3.646	72.71	7.33	3.04
Prague	2.7	0	1.44	133.3	64.84	58.67	36.75	0.06	0
Uppsala	4.73	0	3.17	187	72.35	58.84	27.02	0.38	0
Stykkisholmur	0	0.657	4.39	103.1	75.99	82.57	2.13	0	6.42
Athens	8.11	0	2.81	104	19.73	23.16	65.54	0	1.45
Toronto	0	1.704	5.14	183.5	121.3	113.1	18.24	0	2.2

Table 3 Temporal changes in seasonality identified by application of Average Based (AB) and Complete Data (CD) methods for non-overlapping sub-periods for the four longest stations of the dataset. For the longer station of Padua, an additional sub-period is investigated (4th window).

Station	Record length	Number of Seasons							
		1st window		2nd window		3rd window		4th window	
		Method		Method		Method		Method	
		AB	CD	AB	CD	AB	CD	AB	CD
Padua	1725-2013	1	1	1	1	1	1	1	1
Bologna	1813-2007	1	1	2	1	1	1	–	–
Radcliffe	1827-2014	1	1	1	1	1	1	–	–
Prague	1804-2014	2	1	2*	2*	2	2	–	–
		Span of wet season in months for Prague							
		5-8	–	5-9	5-9	5-8	5-8	–	–

Table 4 Comparative statistics of the GEV annual and seasonal parameters, i.e., shape parameter κ, scale parameter σ and location parameter ψ, as estimated via Maximum Likelihood method for the stations in which two or three seasons are identified by Average Based (AB) and Complete Data (CD) methods. The last column of each table shows the percentage (%) of stations in which the parameter value for the wet season is higher than the corresponding value for the dry season.

AB method (16 stations)						CD method (16 stations)			
Parameter		Annual	Wet season	Dry season	Parameter Variation: (wet>dry)%	Annual	Wet season	Dry season	Parameter Variation: (wet>dry)%
κ	Mean	0.1121	0.091	0.097	62.5	0.115	0.106	0.104	45
	Percent Positive	93.8	93.8	87.5	-	93.8	93.8	75	-
σ	Mean	12.207	12.706	8.747	93.8	12.187	13.238	9.2872	100
ψ	Mean	39.265	35.602	23.772	100	39.652	40.998	34.333	100

Table 5 Statistics of the GEV parameters, i.e., shape parameter κ, scale parameter σ and location parameter ψ, as estimated for the Annual Maxima series for all stations (27) via Maximum Likelihood (ML), method of moments (MM) and Least Squares method (LS).

Parameter of the annual model		ML	MM	LS
κ	Mean	0.099	0.062	0.120
	Percent Positive	92.6	88.9	96.3
σ	Mean	12.638	10.500	12.732
ψ	Mean	40.510	42.246	40.295

Figures

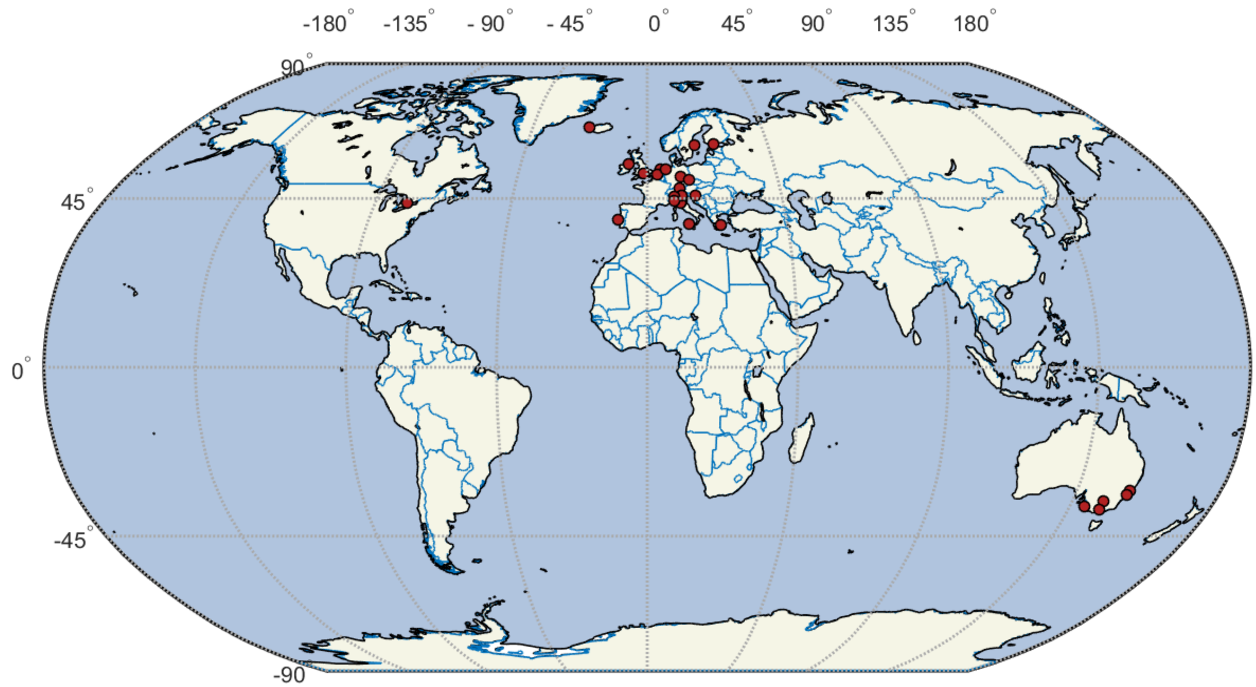

Figure 1. Map of the 27 analyzed stations with daily rainfall records spanning over 150 years.

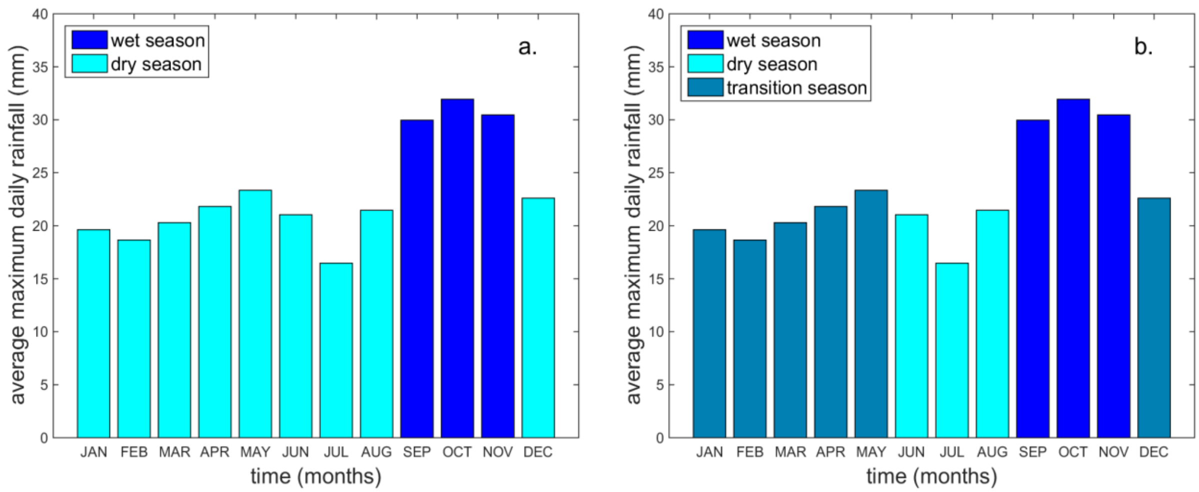

Figure 2. Climatograms showing the partition in two seasons (a) and three seasons (b) after application of the SSD clustering algorithm for the station of Florence.

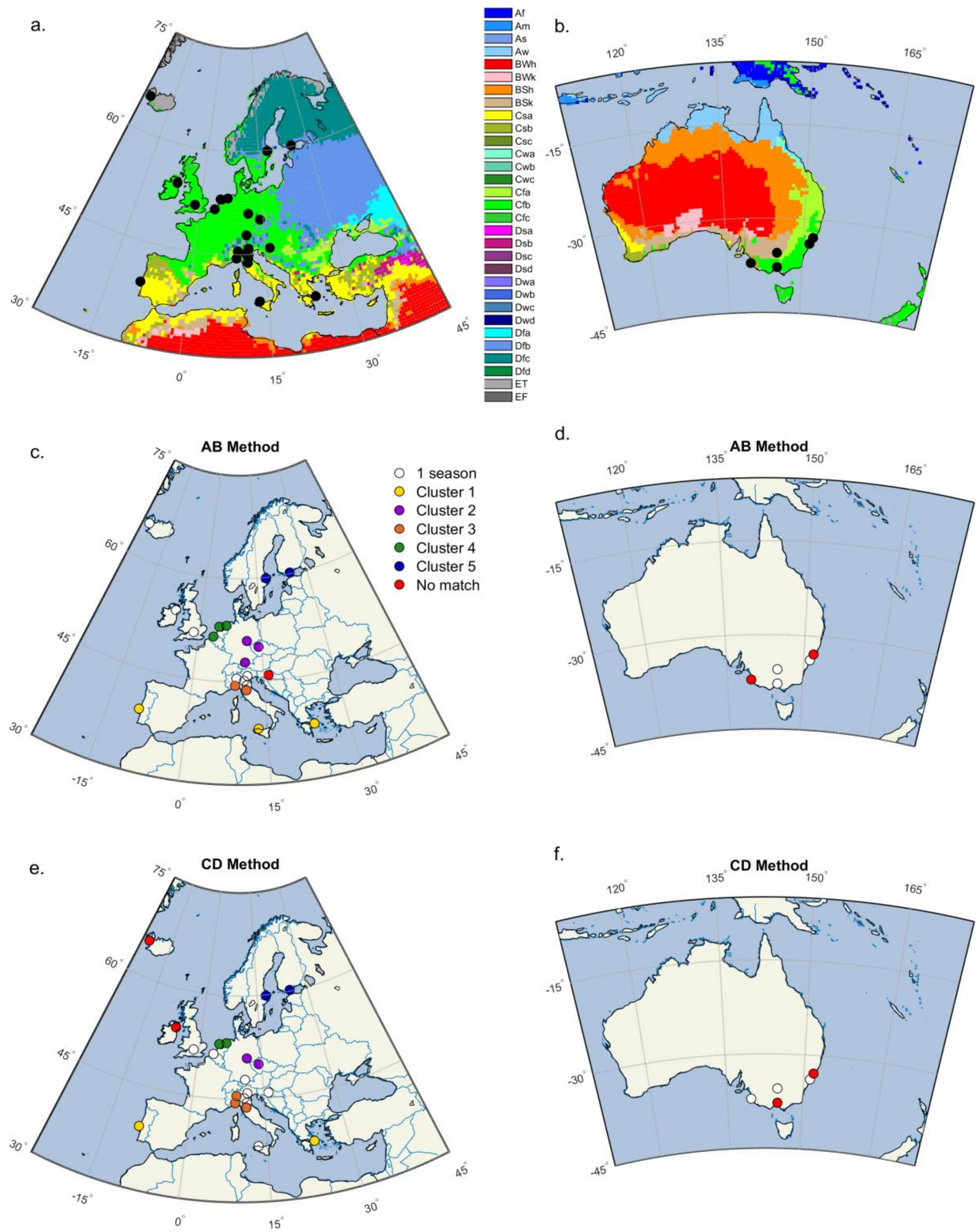

Figure 3. Spatial and climatological coherence of the identified seasons for the regions of Europe (a,c,e) and Australia (b,d,f). Figures a,b show the location of the stations on a Köppen climatological map, while the rest show the stations clustered by similarity. White dots represent stations having one season; the remaining dots denote stations having at least 60% overlap of months belonging to the wet season. Red dots denote stations with a lower percentage of similarity to their neighboring stations.

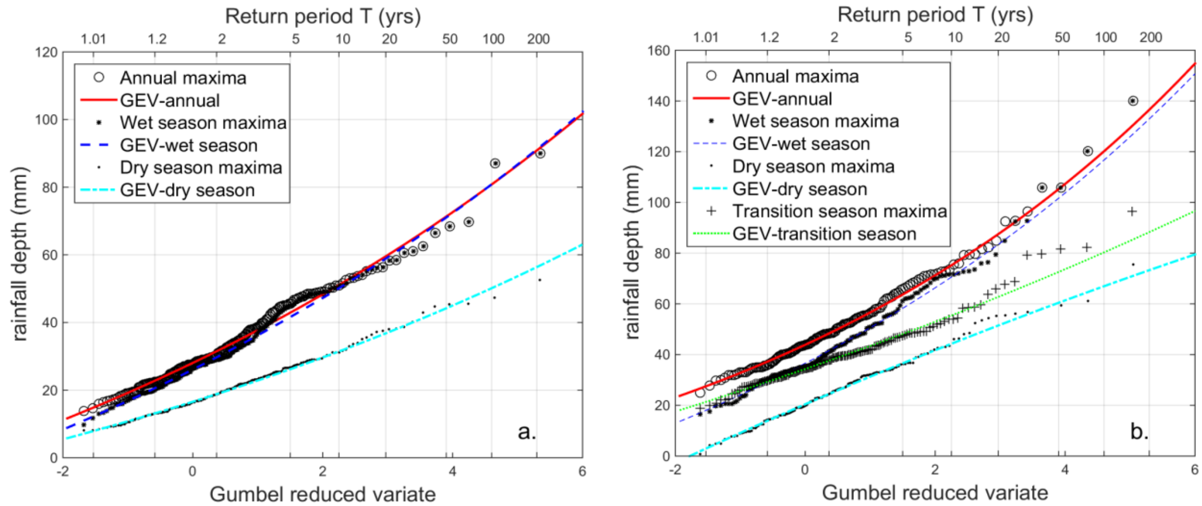

Figure 4. Gumbel probability plots of the fitting of the GEV distribution to the annual maxima (red solid line), to the wet season maxima (blue dashed line) and to the dry season maxima (cyan dash-dotted line) for the stations of Prague (a) and Florence (b). For the station of Florence (b), the fitting of the GEV distribution to the transition season maxima (green dotted line) is also shown.

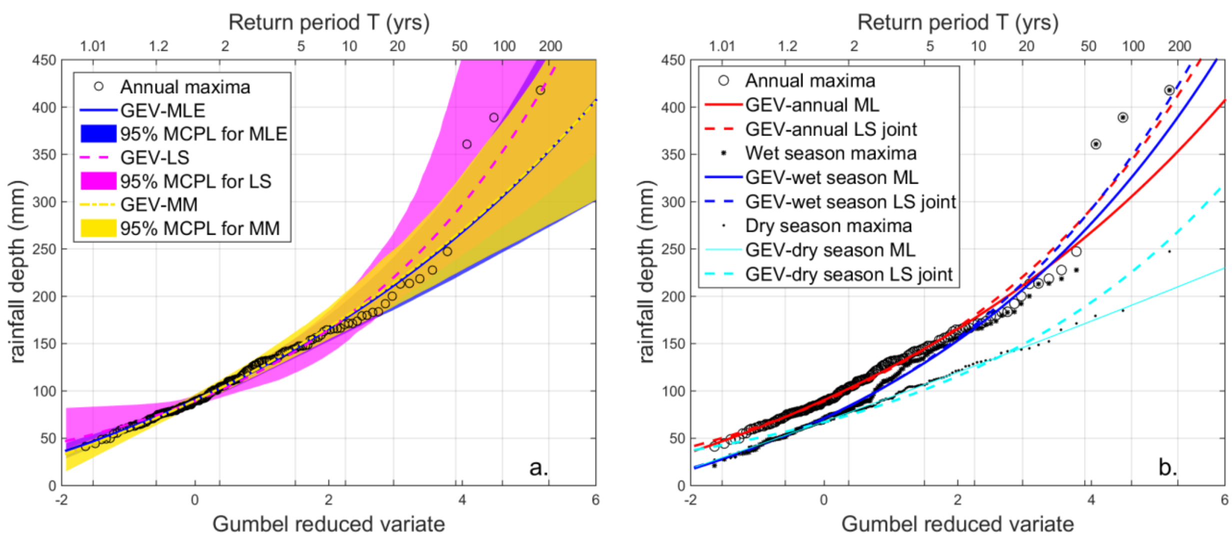

Figure 5. Gumbel probability plot of the fitting of the GEV distribution to the annual maxima by the maximum likelihood method (blue color), least-squares method (magenta color) and method of moments (yellow color) along with 95% Monte Carlo Prediction Limits (MCPL) for each method for the station of Genoa (a). The crossing over distance observed in the area of high return periods, where the wet-season probability line (blue solid line) crosses the annual probability line (red solid line), is greatly eliminated when a common shape parameter is employed via the least-squares method (b).

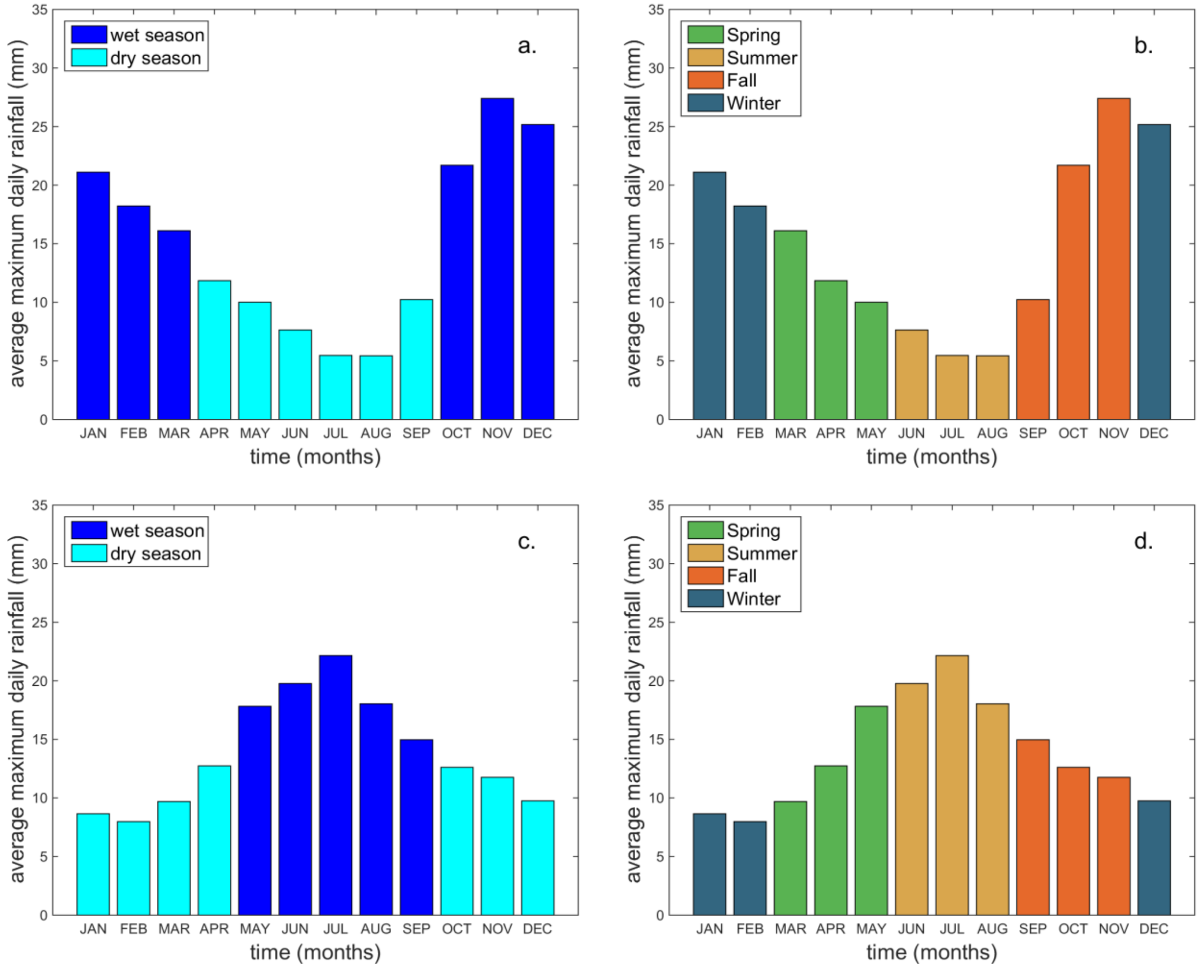

Figure 6. Partition in seasons resulting from application of the proposed season identification method versus the fixed 4-season partition for the stations of Athens (a, b respectively) and Jena (c, d respectively).

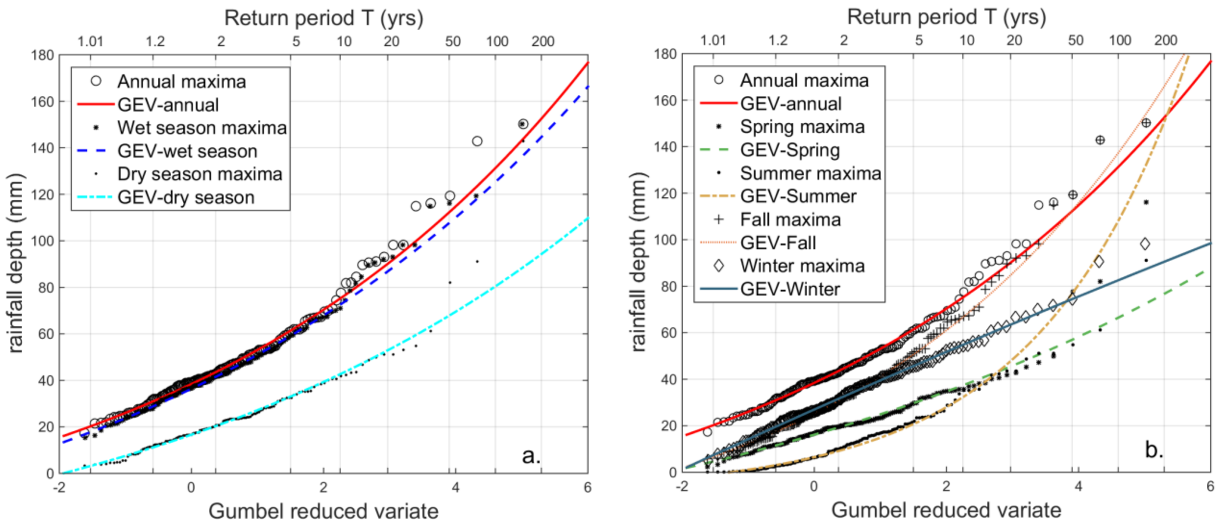

Figure 7. Gumbel probability plots of the fitting of the GEV distribution to the annual and seasonal maxima resulting from the proposed season identification method (a) vs Gumbel probability plot of the fitting of the GEV distribution to the annual and seasonal maxima according to the fixed 4-season partition (b) for the station of Athens.