

**Renewable Energy & Hydroelectric Works**  
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# **Hydrological design of small hydropower plants: Supplementary notes**

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# Problem setting

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- Given data for small hydroelectric plants (SHPs) :
  - Streamflow time series upstream of the intake,  $q$ ;
  - Gross head,  $h$  (practically constant);
  - Power plant efficiency,  $\eta$ , expressed as function of discharge;
  - Maximum discharge that can pass from the turbines (nominal flow),  $q_{max}$
  - Minimum discharge for energy production,  $q_{min}$  (typically, 10-30% of  $q_{max}$ )
- Flow passing through the turbines:

$$q_T = \min(q, q_{max})$$

- Power is produced for  $q_T > q_{min}$ :

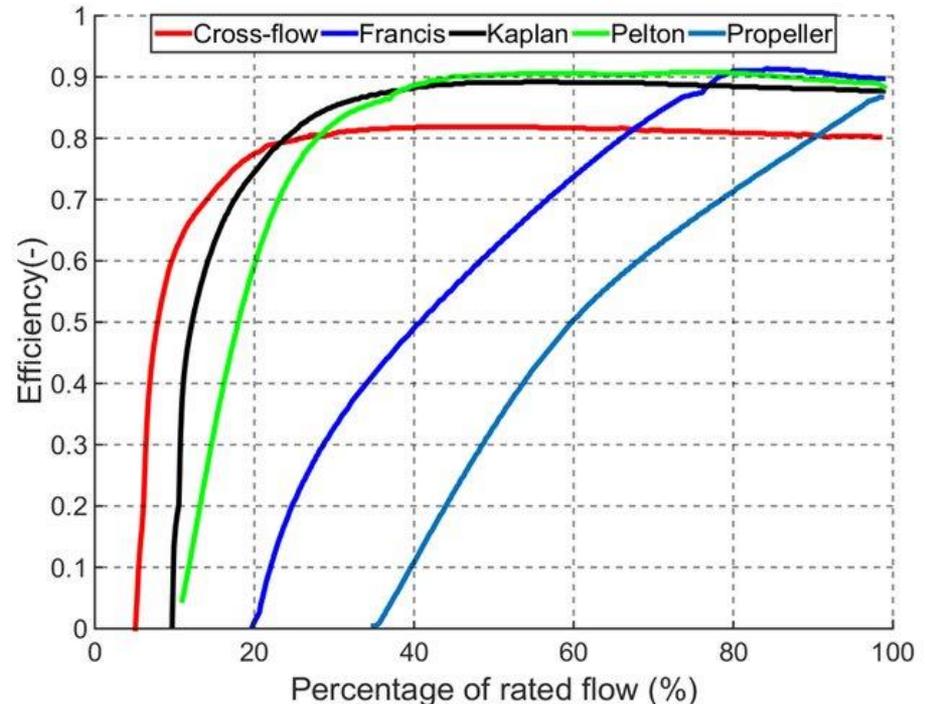
$$P = \gamma \eta q_T h_n$$

where  $\gamma$  is the specific weight of water (9.81 KN/m<sup>3</sup>) and  $h_n$  is the net head, i.e. the gross head,  $h$ , after subtracting hydraulic losses,  $h_L$ .

- Hydraulic losses include friction and local ones, which are function of discharge and the penstock properties (roughness, length, diameter, geometrical transitions).
- Large hydroelectric reservoirs allow for controlling outflows, thus their turbines are normally working with the nominal flow (which maximizes  $\eta$ ). In contrast, SHPs are operating with any flow conditions, thus  $\eta$  is strongly varying across the feasible flow range ( $q_{min}, q_{max}$ ).

# Some remarks on turbine efficiency

- The efficiency curve for specific turbine dimensions (e.g., diameter runner) is usually expressed by means of **nomographs**, as percentage of rated flow,  $q_T/q_{max}$ .
- Nomographs are provided by the turbine manufacturer and they are obtained by data extrapolation from a **reduced scale model**. Since it is not possible to exactly preserve dynamical, geometrical, and kinematical similarity between the model and the prototype, it is also not possible to precisely estimate the efficiency.
- Although empirical corrections are employed to better reflect the **prototype performance**, actual efficiency is unknown, since it also depends on constructive and operational characteristics of the power plant, as well as changes due to deterioration, damage and aging of the equipment over time.
- In general, **efficiency increases with scale**, i.e. discharge and turbine size.
- Pelton, Crossflow and Kaplan machines retain high efficiency even when running below their design flow; in contrast the efficiency of Francis turbines falls away sharply if run at below half its normal flow.



# Simulation-optimization context (single turbine)

- Design variable: power capacity,  $P$ .
- Design objectives (conflicting):
  - Minimization of capital costs, expressed as function of  $P$  (preliminary analysis);
  - Maximization of mean energy production, which depends on  $P$  and the statistical regime of streamflows;
- Assumptions:
  - Given streamflow data,  $q$ , after subtracting environmental flow requirements;
  - Given turbine type, which known efficiency curve, expressed as  $\eta = f(q/q_{max})$ ;
  - Minimum flow for turbine operation expressed as known percentage of the nominal one, i.e.  $q_{min} = a q_{max}$ ;
  - Hydraulic calculations are omitted, thus the net head is set equal to or little smaller from the gross one (valid assumption for large elevation differences);
- The nominal flow is given by:

$$q_{max} = \frac{P}{\gamma \eta_{max} h_n}$$

- At each time step, the energy production is calculated by considering the flow passing through the turbines to produce energy, within the range  $q_{min}$  and  $q_{max}$ .
- The mean energy production is estimated on the basis of simulated energy.

# Simulation-optimization for multiple turbines

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- Design variable: power capacity of each turbine,  $P_i$ .
- Objective: estimation of mean annual energy production (= sum of individual energy data, provided through simulation).
- Assumptions:
  - Turbine types are known (thus the efficiency curve of each turbine as well as the flow limits  $q_{min}$  and  $q_{max}$  are also known);
  - Turbines operate in hierarchical order, which is a priori specified;
- Example with a SHP comprising two turbines, A (master) and B (secondary), receiving a total streamflow,  $q$ :
  - If  $q > q_{max,A} + q_{max,B}$  then the total power produced is  $P_A + P_B$ .
  - If  $q < \min(q_{min,A}, q_{min,B})$  then any power is produced;
  - For any intermediate case, the flow is by priority conducted to turbine A, while the remaining flow,  $q - q_A$  (if exists) passes from turbine B; the remaining flow produces energy only if exceeds  $q_{min,B}$ ;
- The above policy is the simplest one, but not the overall optimal, because of the nonlinearities induced by the efficiency curves of turbines. In a more rigorous optimization context, the operation of two turbines accounts for the **maximization of the combined efficiency** of the system across all feasible flows, which ensures the maximum energy production.