European Geosciences Union

National Technical University of Athens
School of Civil Engineering

Stochastic simulation of hydrological timeseries for data scarce regions using Hurst-Kolmogorov dynamics

Case study: Municipality of Western Mani

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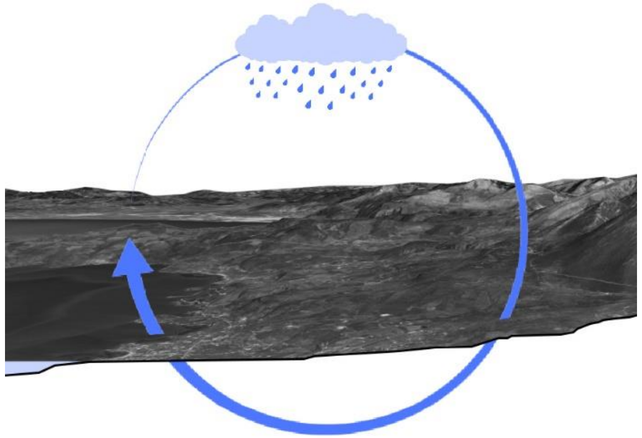
Introduction

- The Municipality of Western Mani (WM) is located in the southern part of Greece in Peloponnese. The region has a high rate of rainfalls mainly in the mountainous areas.
- Rainfall is mainly observed during the autumn and winter months, from October to March, while there is a significant decrease in the summer.[1]
- The problem that arises, is mainly the quantitative aspect of water.[2-5] The geological background, is extremely permeable, as it consists notably of karstic limestone. Therefore, there are limited surface water resources with limited supply[6].

European map which depicts Greece
Source : https://vemaps.com/europe-continent/eu-c-04#google_vignette

Greekm map which depicts Western Mani
Source: https://www.vecteezy.com/vector-art/2292777-greece-detailed-map-with-states



The water cycle

Aim of the study

The problem

- Water quantity

Challenges

- Data scarce region

Aim of the study

- Estimate the surface water availability

The method

- Stochastic simulation
- Generation of synthetic rainfall time series using data from an adjacent basin , reproducing seasonality and persistence.

Karstified limestone at Lagada’s region

Precipitation Data

Meteorological station

- Kalamata's station [7] (18 km from WM),

1951-2018 (68 yrs)

Sufficient length

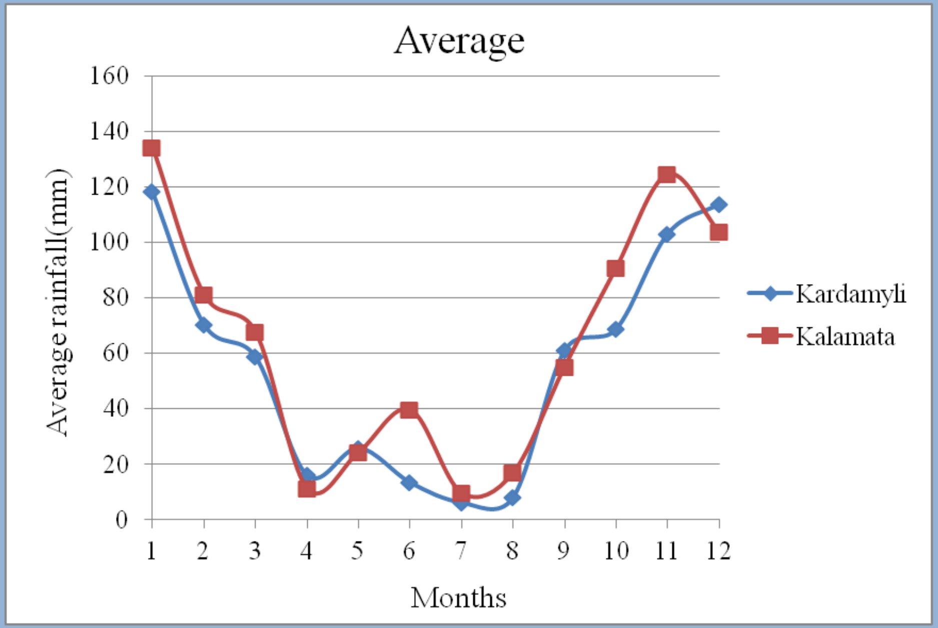
- Kadamylis' station [8] (in WM),
- 2014-2021 (8 yrs)

The length of the timeseries is insufficient for the estimation of stochastic properties.

- Both stations are in the same altitude (13 m).

We use Kalamatas' rain data as:

1. Kalamatas' is the closest meteorological station to the case study with long time series.
2. The rainfall regime in the two stations is similar.



The Hurst-Kolmogorov (HK) behavior

- The Hurst-Kolmogorov (HK) behavior, is also known as Long Term Persistence (LTP).

It seems that in natural systems, randomness and predictability are intrinsic and can be deterministic and random at the same time, depending on the prediction horizon and the time scale[9]. Particularly, all these processes are characterized by high unpredictability due to clustering of events.

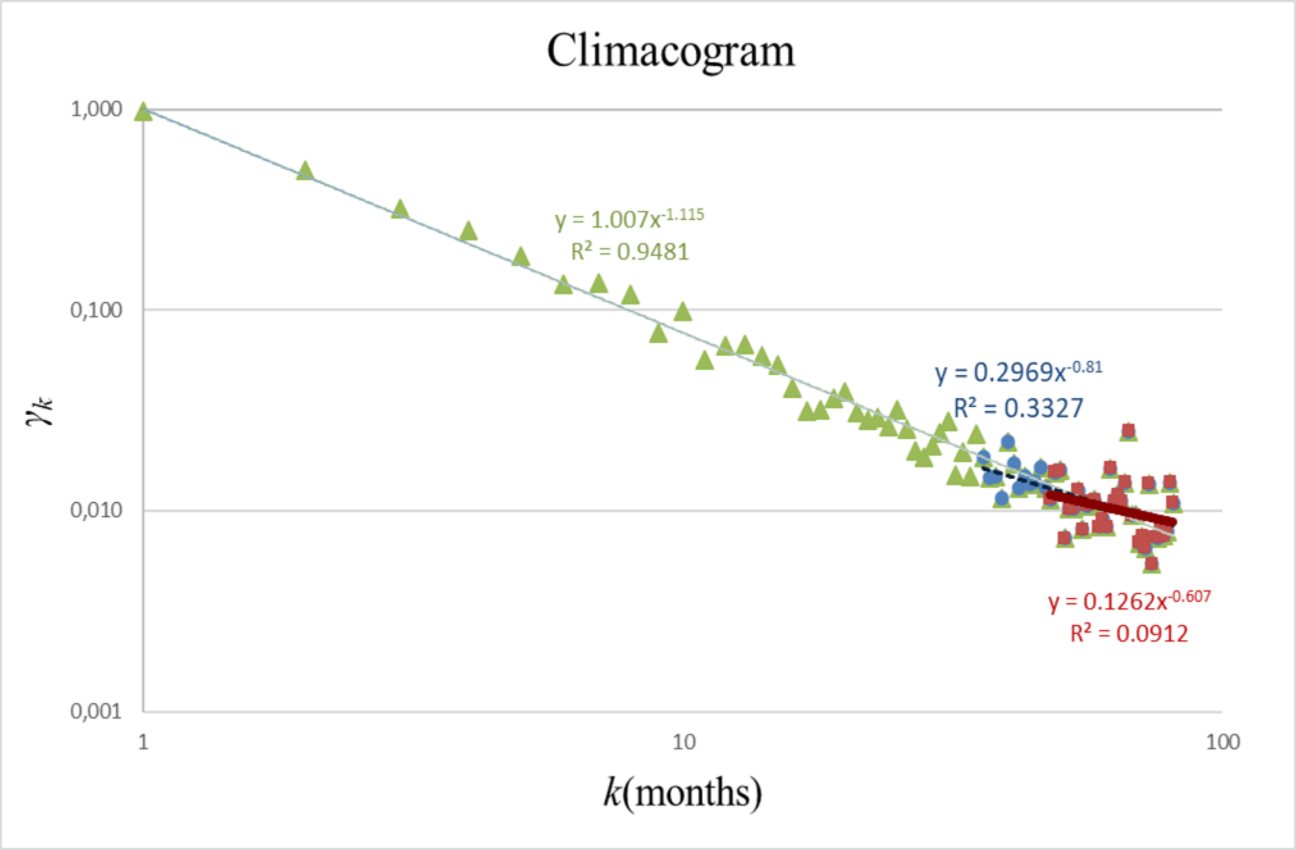
- Although, in HK dynamics the marginal distribution of the process maybe arbitary, the most commonly used distribution is the Gaussian one, which results in the well-known fractional-Gaussian-noise, described by Mandelbrot and van Ness [10], i.e. the power-law decay of variance as a function of scale γ(k), is defined for a process [11]. The HK behavior is quantified by the Hurst parameter(0<H<1). In the simplest case, the variance γ(k) of the process at timescale k, known as climacogram, is given by:

$$\gamma(k) = \frac{\lambda}{(k/\Delta)^{2(1-H)}}$$

Where Δ, is a characteristic timescale and λ is the variance at this time scale. For 0<H<0.5, the HK process exhibits an anti-persistent behavior, H=0.5 corresponds to the white noise process, and for 0.5<H<1 the process exhibits LTP(clustering).

- Therefore, to account for the effect of the non-white-noise behavior of the spring outflow in our toy model, we simulate the spring outflow based on the HK model (with, for example, H = 0.8), and by following a Gaussian distribution (described earlier), through a stochastic synthesis algorithm, and particularly, through the Symmetric-Moving-Average (SMA) scheme [12, 13].

Estimation of Hurst-Kolmogorov behavior - Historic time series



We estimate the H parameter through the climacogram.

Hurst Kolmogorov behavior is apparent at the larges scales.

- From 3 years until the last scales (k=36-81) , H=0.6
- From 4 years until the last scales (k=48-81), H=0.7
- We estimate H=0.65

H=0.65 is a value in accordance with global analylis[14]

Moving Average(SMA) method

SMA scheme[15] was introduced by Koutsoyiannis(2000) and transforms a stochastic process of white noise vi into a realization of stochastic process xi, according with the equation:

$$\underline{x}_\tau=\sum_{j=-J}^{J} \alpha_{|j|}\underline{v}_{\tau+j} = \alpha_j\underline{v}_{\tau-J}+...+\alpha_1\underline{v}_{\tau-1}+\alpha_o v\underline{v}_\tau+\alpha_1 v\underline{v}_{\tau+1}+...+\alpha_j\underline{v}_{\tau+J}$$

Where a_j are weight coefficients and their number J is theoretically infinite but in practice, it takes a finite value. The method is suitable for any random autocorrelation. Function. In case of simple assimilation model, it is proved that weight coefficients are given by :

$$\alpha_\eta \approx \frac{\sqrt{(2-2H)\gamma_1}}{3-2H}\left(|\eta+1|^{H+0.5}+|\eta-1|^{H+0.5}-2|\eta|^{H+0.5}\right)$$

And the necessary number of weights is:

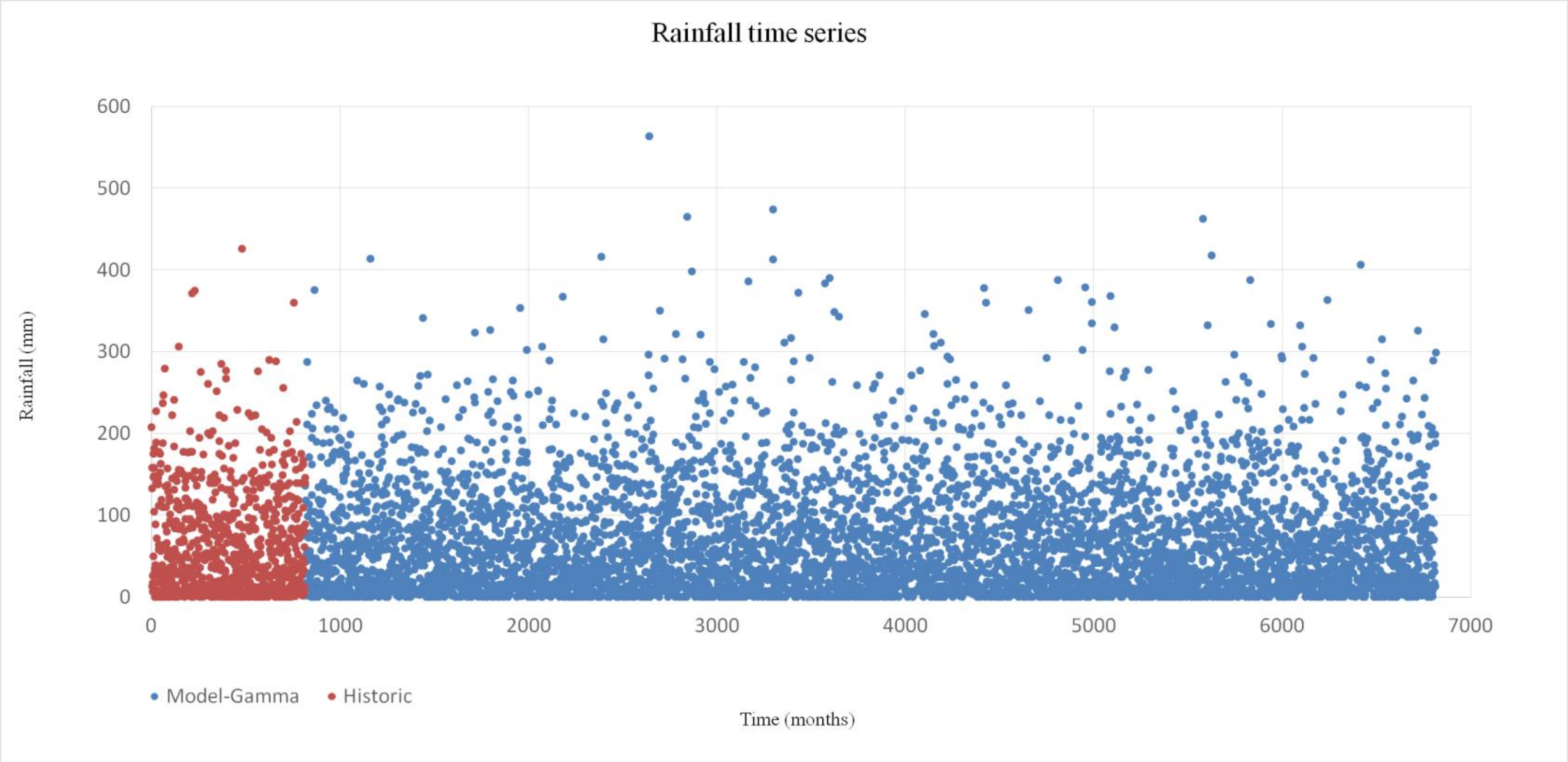
$$J \geq \max\left[m, \left(\frac{2\beta}{H^2-0.25}\right)^{1/(H-1.5)}\right]$$

Where m is the number of autocorrelations that have to be maintained and β the accuracy coefficient(β=0.001). The method can maintain the asymmetry coefficient too $C_s(x)$ of xi process, if the white noise has asymmetry coefficient $C_s(v)$ is given by:

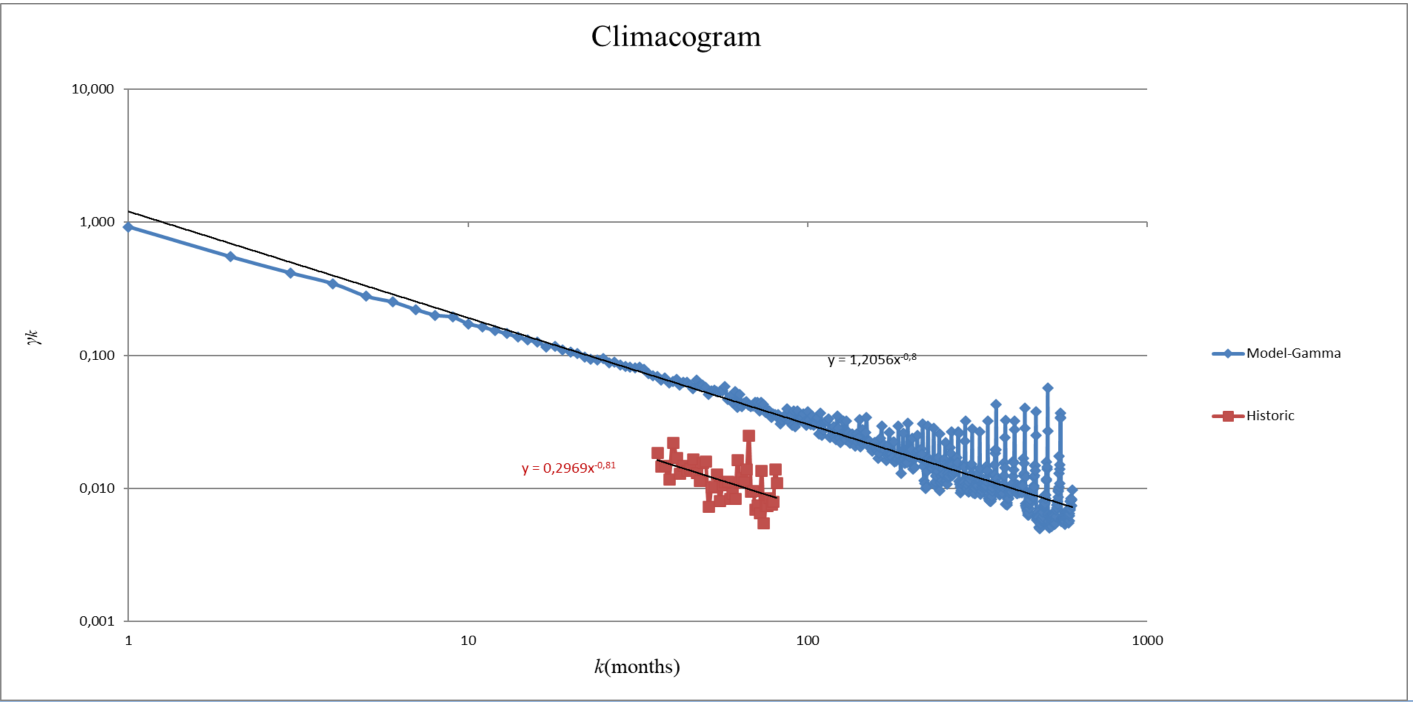
$$(\alpha_o{}^3+2\sum_{j=1}^{q} \alpha_j{}^3)C_s{}^{(v)}=C_s{}^{(x)}\gamma_1^{\frac{3}{2}}$$

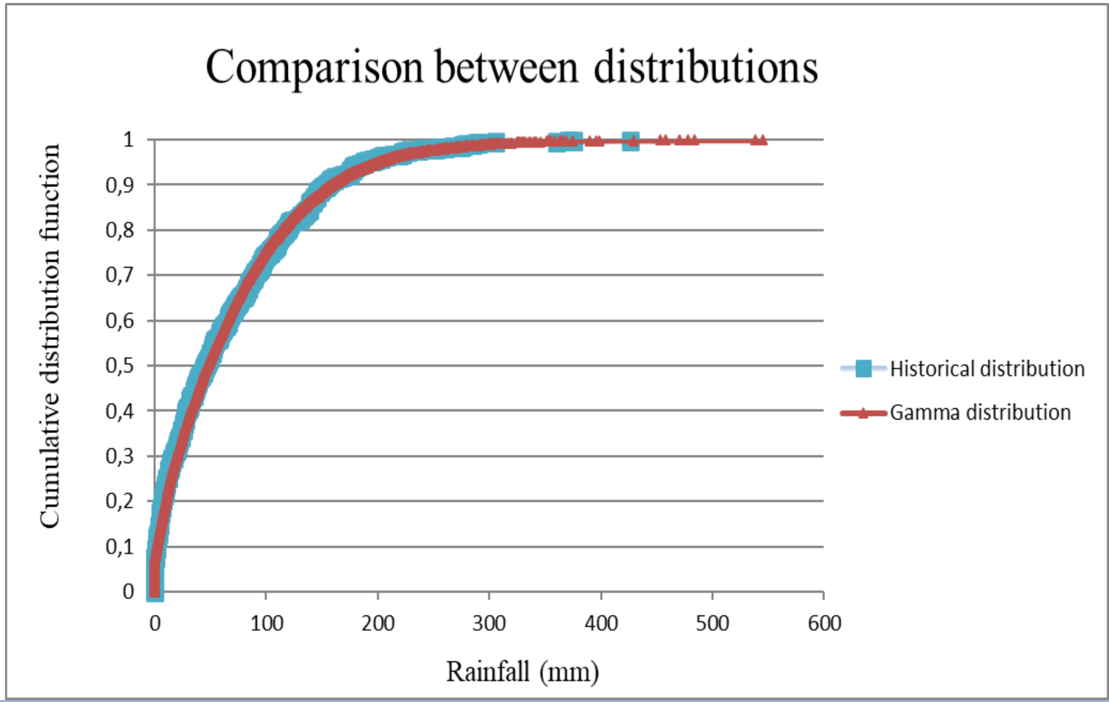
SMA method-Gamma distribution for one realization

- We use standardization to maintain:
    1. The average of each month
    2. The standard deviation of each month
- We use Gamma distribution to reproduce the skewness of the monthly rainfall series the synthetic time series. The asymmetry of total monthly time series.
- We use the SMA algorithm to reproduce the Hurst-Kolmogorov (HK) behavior



Validation of synthetic time series





From the historical climagogram we received a value of H=0.65, the same as the synthetic one.
Also, the inclination of historical and modelled climacogram , at big scales is the same.

- It is proved that SMA model maintain HK behavior.
- Gamma distribution fits very well at historical distribution.

Conclusions

- Synthetic time series inspect the variability of the rainfall.
- Stochastic simulation of hydrological time series with the SMA method which includes HK dynamics is an excellent tool to simulate natural processes as rainfall.
- The synthetic time series which we have created (500 years) could be used as the rainfall input to the design of the hydraulic infrastructures of Western Mani, and to address the increasing water needs of the region with proper infrastructures.
- This project highlights the power and the flexibility of the stochastic calculus to study and unify the study of issues which are not easy to approach and characterize in an objective and interpretable manner, such as the rainfall.
- However, as the case study of Wastern Mani is a multi parametric problem, further studies will analyze this issue in the same section[1, 16,17]

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Thank you for your attention!