ON THE PARAMETRIC APPROACH TO UNIT HYDROGRAPH IDENTIFICATION

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ABSTRACT

Unit hydrograph identification by the parametric approach is based on the assumption of a proper analytical form for its shape, with a limited number of parameters. This paper presents various suitable analytical forms for the instantaneous unit hydrograph, originated from known probability density functions or transformations of them. Analytical expressions for the moments of area of these forms, versus their definition parameters are theoretically derived. The relation between moments and specific shape characteristics are also examined. Two different methods of parameter estimation are studied, the first being the well-known method of moments, while the second is based on the minimization of the integral error between derived and recorded flood hydrographs. The above tasks are illustrated with application examples originated from case studies in catchments of Greece.

Key words : Unit hydrograph, instantaneous unit hydrograph, identification, probability density function, probability distribution function, method of moments, optimization.

1. INTRODUCTION

Common mathematical approaches to model synthesis, include: (1) discretization techniques, i.e. determination of the model in a finite number of discrete points, and (2) parametric techniques, i.e. assumption of a proper analytical form for the model, with a limited number of parameters, and estimation of these parameters by means of known restrictions and/or the optimization of an objective function.

Both the above approaches have been used for the unit hydrograph (UH) synthesis. The first has become the most common method and is based on linear analysis (matrix inversion technique - *O'Donnell* [1986]). The second was founded by *Nash* [1959], who showed that the moments of area of the instantaneous unit hydrograph (IUH) can be easily derived from recorded hydrographs and simultaneous rainfall records. Nash also studied several suitable two-parameter analytical forms to represent the shape of IUH, the most common being the gamma-PDF form. The parametric approach has also been used in synthetic unit hydrograph derivations, with most common analytical forms the triangular and the gamma-PDF.

The first approach is generally considered as more accurate, because of the considerable number of points defining UH, while the second uses a very limited number of parameters (2 or 3) for the UH shape representation. In fact the inaccuracies due to the limited number of parameters, in the parametric approach, are minor, when compared with the uncertainties of the whole process of UH derivation from recorded data, which are met in the areal rainfall estimation, the establishment of the stage-discharge relationship, the baseflow separation, and, finally, the separation and the time distribution of the rainfall losses.

Moreover, it is often very difficult or impossible to find recorded flood events originated from entirely uniform, over the catchment, rainfall; finally, the assumption of the catchment linearity is not strictly valid, and the catchment response is not unique. Because of the above uncertainties it is very common practice in the formation of design flood hydrographs, to consider a unit hydrograph more severe than the derived from flow records (e.g. by reduction of the time to peak by 2/3, see *Sutcliffe* [1978]).

The above discussion indicates that the parametric method, though seems less accurate than the standard linear method, can be a good approximation to the unit hydrograph identification on real world catchments, since the inaccuracies introduced by the use of limited number of parameters are minor. Moreover the parametric method has some advantages, to be discussed later.

Problems associated with the application of the parametric approach are the selection of the proper analytical form for the UH representation, and, mainly, the method for the parameter estimation. These problems are systematically examined throughout this study.

2. DEFINITIONS AND GENERAL RELATIONS

Let $U_D(t)$ be the unit hydrograph for a net rainfall of duration D (DUH). The instantaneous unit hydrograph $U_0(t)$ corresponds to the case where $D = 0$. We denote by V_0 the surface runoff volume, corresponding to the unit rainfall with depth $H_0 = 10$ mm, that is

$$V_O = \int_0^{T_D} U_D(t)\ dt = \int_0^{T_O} U_O(t)\ dt = H_O \cdot A$$

where T_D and T_O are large enough time intervals, referred to as flood durations (theoretically are right bounds for the functions $U_D(t)$ and $U_O(t)$, respectively, and can be equal to ∞), and A the catchment area. Now we define the function

$$u(t) = U_O(t) \ / \ V_O \qquad (1)$$

which will be referred to as **standardized instantaneous unit hydrograph** (SIUH); it is a positive function, with dimension $(\text{time})^{-1}$, and has the property

$$\int_0^{T_O} u(t)\ dt = 1 \qquad (2)$$

In general u(t) is a single peaked function. The time to peak, t_p, and the peak value, $u_p \equiv u(t_p)$, are main characteristics of SIUH.

Furthermore let $S_D(t)$ be the S-curve (DSC) derived from the DUH, which corresponds to rainfall intensity equal to H_O/D, and infinite duration. The DSC is related to DUH by

$$U_D(t) = S_D(t) - S_D(t-D) \qquad (3)$$

and to IUH by

$$S_D(t) = \frac{1}{D} \int_0^{t} U_O(t)\ dt \qquad (4)$$

Additionally we define the function

$$s(t) = S_D(t) \cdot (D/V_0) \tag{5}$$

which is dimensionless, independent of the duration D, and has the properties

$$s(0) = 0 \ , \ s(T_0) = 1 \tag{6}$$

This function will be referred to as **standardized S-curve (SSC)**. SSC and SIUH are related by

$$s(t) = \int_0^t u(t) \, dt \tag{7}$$

$$u(t) = \frac{ds(t)}{dt} \tag{8}$$

Finally let U_n be the n-th central moment of area of the function u(t), U'_n the n-th moment about the origin, and U''_n the n-th moment about the right bound T_0 (if exists). These are defined by

$$U_n = \int_0^{T_0} (t-t_U)^n \, u(t) \, dt \tag{9}$$

$$U'_n = \int_0^{T_0} t^n \, u(t) \, dt \tag{10}$$

$$U''_n = \int_0^{T_0} (T_0-t)^n \, u(t) \, dt \tag{11}$$

where $t_U = U'_1$ is the distance of the center of area of SIUH from the origin, known as **lag time**. (Note the term (TO-t) in (11), which is opposite to the usual).

Each family of moments can theoretically determine the complete shape of SIUH, when infinite number of them is known. In reality, only a limited number of them can be estimated; nevertheless this limited number keeps substantial information about the shape, which is very helpful for the IUH identification.

Given a specific analytical form of SIUH, it is an easy matter to derive the DUH for whichever duration D. This can be done by subsequent application of equations (7), (5) and (3).

3. ANALYTICAL FORMS FOR THE SIUH REPRESENTATION

The functions u(t) and s(t) are mathematically similar to the families of probability density functions (PDFs) and distribution functions (CDFs). Those single-peaked PDFs which are left bounded by zero are proper for the representation of the SIUH. Normally they should have a right bound, too, but this is not necessarily considered as a strict theoretical requirement, since all PDFs tend to zero for large values of their argument.

Eight particular analytical forms have been systematically examined in this study. All of them are originated from known probability density functions (PDFs), or transformations of them. They have two or three parameters and are left-bounded by zero or double-bounded. These forms are described below; in their analytical expressions, generally, a denotes a scale parameter, while b and c denote shape parameters. The expressions of their main features (theoretically derived in the present study, except the ones of well known functions) are summarized in Tables 1 and 2.

1. Double triangular (DT) (double-bounded / two-parameter)

In fact this form is a single triangle consisting of two successive triangular PDFs (thus the characterization "double"), the first with a negative skewness and the second with a positive one. The SIUH is expressed by

$$u(t) = \begin{cases} \dfrac{2t/a}{ab}, & 0 \le t/a \le b \\[2ex] \dfrac{2(1-t/a)}{a(1-b)}, & b \le t/a \le 1 \end{cases} \qquad (12)$$

The double-triangular form has been widely used for the expression of synthetic unit hydrographs (for example see *Sutcliffe* [1978]), but not so much for the IUH itself.

2. Gamma (Γ) (left-bounded / two-parameter)

This form has been suggested by *Nash* [1959], and it is the most common for the IUH analytical expression, either synthetic or from recorded data. Its analytical expression is

$$u(t) = \frac{(t/a)^{b-1}\, e^{-t/a}}{a\Gamma(b)}, \qquad t \ge 0 \qquad (13)$$

3. Log-Normal (LN) (left-bounded / two-parameter)

This form was also suggested by *Nash* [1959]; its analytical expression is

$$u(t) = \frac{1}{t(\pi b)^{1/2}} e^{\frac{-\ln^2(t/a)}{b}} , \qquad t \geq 0 \tag{14}$$

4. Weibull (W) (left-bounded / two-parameter)

Originating from the Weibull distribution function we get the following form for SSC

$$s(t) = 1 - e^{-(t/a)^b} , \qquad t \geq 0 \tag{15}$$

5. Beta (B) (double-bounded / three-parameter)

Beta density function with an extra scale parameter $a = T_0$, can give a nice form for the SIUH representation, that is

$$u(t) = \frac{(t/a)^{b-1}(1-t/a)^{c-1}}{aB(b, c)} , \qquad 0 \leq t \leq a \tag{16}$$

6. Double-Power (DP) (double-bounded / three-parameter)

This term is used to describe the following three-parameter function

$$s(t) = \left[1 - (1-t/a)^b\right]^c , \qquad t \geq 0 \tag{17}$$

The simplicity of the SSC analytical expression, as well as the one of SIUH (see Table 2) is remarkable. This form has been extracted from a similar CDF suggested by *Kumaraswamy* [1980].

7. Shifted Log-Pearson III (SLP) (left-bounded / three-parameter)

The usual logarithmic transformation, ($x = \ln t$) applied to the Pearson type III distribution,

$$f(x) = \frac{c^b (x-A)^{b-1} e^{-c(x-A)}}{\Gamma(b)}, \qquad x \geq A \tag{18}$$

is not proper for the SIUH expression, since $t = e^x$ ranges in $[e^A, \infty)$. In order to decrease the lower bound to zero, we apply the following shifted logarithmic transformation

$$x = \ln(t + e^A)$$

and we get

$$u(t) = \frac{c^b}{a\Gamma(b)} \frac{[\ln(t/a+1)]^{b-1}}{(t/a+1)^{c+1}}, \qquad t \geq 0 \tag{19}$$

where $a = e^A$ is a scale parameter.

8. Minus Log-Pearson III (MLP) (double-bounded / three-parameter)

This form is originated, also, from the Pearson type III distribution, by applying the minus logarithmic transformation, i.e. $x = -\ln t$, which gives

$$u(t) = \frac{c^b}{a\Gamma(b)} (t/a)^{c-1} [-\ln(t/a)]^{b-1}, \qquad 0 \leq t \leq a \tag{20}$$

where $a = e^A$ is a scale parameter.

TABLE 1 : TWO-PARAMETER IUH ANALYTICAL FORMS

FORM →	DOUBLE-TRIANGULAR (DT)	GAMMA (Γ)	LOG-NORMAL (LN)	WEIBULL (W)
Standardized IUH and S-curve	$u(t) = \begin{cases} \frac{2t/a}{ab}, & 0 \le t \le ba \\ \frac{2(1-t/a)}{a(1-b)}, & ba \le t \le a \end{cases}$ $s(t) = \begin{cases} \frac{(t/a)^2}{b}, & 0 \le t \le ba \\ 1-\frac{(1-t/a)^2}{1-b}, & ba \le t \le a \end{cases}$	$u(t) = \frac{(t/a)^{b-1} e^{-t/a}}{a\Gamma(b)}$ $s(t) = \frac{\gamma(t/a,b)}{\Gamma(b)}$	$u(t) = \frac{1}{t\sqrt{\pi b}} e^{-\frac{\ln^2(t/a)}{b}}$ $s(t) = G\left[\frac{\ln(t/a)}{\sqrt{b/2}}\right]$	$u(t) = (b/a)(t/a)^{b-1} e^{-(t/a)^b}$ $s(t) = 1 - e^{-(t/a)^b}$
t-range /parameters	$0 \le t \le a$ $a>0$: *scale parameter* $b<1$: *shape parameter*	$t \ge 0$ $a>0$: *scale parameter* $b>1$: *shape parameter*	$t \ge 0$ $a>0$: *scale parameter* $b>0$: *shape parameter*	$t \ge 0$ $a>0$: *scale parameter* $b>1$: *shape parameter*
Lag time t_U *Moments of area* U_n *and/or* U'_n *and/or* U''_n	$t_U = \frac{a}{3}(1+b)$ $U_2 = \frac{a^2}{18}(1-b+b^2)$ $U_3 = \frac{a^3}{135}(1+b)(1-b/2)(1-2b)$ *In general* $U'_n = \frac{2a^n}{(n+1)(n+2)} \frac{1-b^{n+1}}{1-b}$	$t_U = ab$ $U_2 = a^2 b$ $U_3 = 2a^3 b$ *In general* $U'_n = a^n b(b+1)\cdots(b+n-1)$	$t_U = ae^{b/4}$ $U_2 = a^2 e^{b/2}(e^{b/2}-1)$ $U_3 = a^3 e^{3b/4}(e^{b/2}-1)^2(e^{b/2}+2)$ *In general* $U'_n = a^n e^{n^2 b/4}$	$t_U = U_1 = a\Gamma(1+1/b)$ $U_2 = U'_2 - {}^2U'_1$ $U_3 = U'_3 - 3U'_2 U'_1 + 2U'_1$ *where* $U'_2 = a^2\Gamma(1+2/b)$ $U'_3 = a^3\Gamma(1+3/b)$ *In general* $U'_n = a^n\Gamma(1+n/b)$
Variation Cv *Skewness* Cs	$Cv = \sqrt{\frac{1-b+b^2}{2}} \frac{1}{1+b}$ $Cs = \frac{2\sqrt{2}}{5} \frac{(1+b)(1-b/2)(1-2b)}{(1-b+b^2)^{3/2}}$	$Cv = 1 / \sqrt{b}$ $Cs = 2 / \sqrt{b} = 2Cv$	$Cv = \sqrt{e^{b/2}-1}$ $Cs = \sqrt{e^{b/2}-1}\,\left(e^{b/2}+2\right)$ $= Cv^3 + 3Cv$	$Cv = \sqrt{\frac{\Gamma(1+2/b)}{\Gamma^2(1+1/b)} - 1}$ $Cs = U_3 / U_2^{3/2}$
Parameter estimation by the method of moments	$b = \frac{3 - \sqrt{24Cv^2 - 3}}{3 + \sqrt{24Cv^2 - 3}}$ $a = \frac{3t_U}{1+b}$ *restriction:* $1/\sqrt{8} \le Cv \le 1/\sqrt{2}$	$a = U_2 / t_U$ $b = t_U^2 / U_2$	$b = 2\ln(1+Cv^2)$ $a = t_U e^{-b/4} = \frac{t_U}{\sqrt{(1+Cv^2)}}$	b: *by numerical solution of the equation:* $\frac{\Gamma(1+2/b)}{\Gamma^2(1+1/b)} = Cv^2 + 1$ $a = \frac{t_U}{\Gamma(1+1/b)}$
Time to peak (mode)	$t_p = ab \quad u_p = 2/a$	$t_p = a(b-1)$	$t_p = a\, e^{-b/2}$	$t_p = a(1 - 1/b)^{1/b}$
Remarks		*Definition of Γ-function* $\Gamma(b) = \int_0^\infty x^{b-1} e^{-x}\, dx$ *Definition of the incomplete γ-function* $\gamma(y,b) = \int_0^y x^{b-1} e^{-x}\, dx$	*Definition of the normal probability distribution function G* $G(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-x^2/2} dx$	

Note : The above given range for the parameters corresponds to the case that u(t) is bell-shaped.

TABLE 2 : THREE-PARAMETER IUH ANALYTICAL FORMS

FORM →	BETA (B)	DOUBLE-POWER (DP)	SHIFTED LOG-PEARSON III (SLP)	MINUS LOG-PEARSON III (MLP)
Standardized IUH and S-curve	$u(t) = \dfrac{(t/a)^{b-1}(1-t/a)^{c-1}}{a\,B(b,c)}$ $s(t) = \dfrac{\beta(t/a,b,c)}{B(b,c)}$	$u(t) = (bc/a)\,(1-t/a)^{b-1} \cdot \left[1-(1-t/a)^b\right]^{c-1}$ $s(t) = \left[1-(1-t/a)^b\right]^c$	$u(t) = \dfrac{c^b}{a\Gamma(b)}\,\dfrac{\left(\ln(t/a+1)\right)^{b-1}}{(t/a+1)^{c+1}}$ $s(t) = \gamma\left(c\cdot\ln(t/a+1),b\right)/\Gamma(b)$	$u(t) = \dfrac{c^b(t/a)^{c-1}\left(-\ln(t/a)\right)^{b-1}}{a\Gamma(b)}$ $s(t) = \gamma\left(-c\cdot\ln(t/a),b\right)\,/\,\Gamma(b)$
t-range /parameters	$0\le t\le a$ $a>0$: *scale parameter* $b,c>1$: *shape parameters*	$0\le t\le a$ $a>0$: *scale parameter* $b,c>1$: *shape parameters*	$t\ge 0$ $a>0$: *scale parameter* $b>1, c>0$: *shape parameters*	$0\le t\le a$ $a>0$: *scale parameter* $b,c>1$: *shape parameters*
Lag time t_U *Moments of area* U_n *and/or* U'_n *and/or* U''_n	$t_U = \dfrac{ab}{b+c}$ $U_2 = \dfrac{a^2bc}{(b+c)^2(b+c+1)}$ $U_3 = \dfrac{2a^3bc(c-b)}{(b+c)^3(b+c+1)(b+c+2)}$ *In general* $U'_n = \dfrac{a^n b(b+1)..(b+n-1)}{(b+c)(b+c+1)..(b+c+n-1)}$	$t_U = a - U''_1$ $U_2 = U''_2 - U''^2_1$ $U_3 = 3U''_2\,U''_1 - 2U''^3_1 - U''_3$ *where* $U''_1 = a^2c\;B(1+3/b,\;c)$ $U''_2 = a^2c\;B(1+3/b,\;c)$ $U''_3 = a^3c\;B(1+3/b,\;c)$ *In general* $U''_n = a^nc\;B\left(1+\dfrac{n}{b},\;c\right)$	$t_U = U^*_1 - a$ $U_2 = U^*_2 - U^{*2}_1$ $U_3 = U^*_3 - 3U^*_2\,U^{*2}_1 + 2U^{*3}_1$ *where* $U^*_1 = a\left(\dfrac{c}{c-1}\right)^b$ $U^*_2 = a^2\left(\dfrac{c}{c-2}\right)^b$ $U^*_3 = a^3\left(\dfrac{c}{c-3}\right)^b$ *In general* $U^*_n = a^n\left(\dfrac{c}{c-n}\right)^b$	$t_U = U'_1 = a\left(\dfrac{c}{c+1}\right)^b$ $U_2 = U'_2 - U'^2_1$ $U_3 = U'_3 - 3U'_2\,U'^2_1 + 2U'^3_1$ *where* $U'_2 = a^2\left(\dfrac{c}{c+2}\right)^b$ $U'_3 = a^3\left(\dfrac{c}{c+3}\right)^b$ *In general* $U'_n = a^n\left(\dfrac{c}{c+n}\right)^b$
Variation Cv *Skewness* Cs	$Cv = \sqrt{\dfrac{c}{b(b+c+1)}}$ $Cs = \dfrac{2(c-b)}{b+c+2}\sqrt{\dfrac{b+c+1}{bc}}$	$Cv = \sqrt{U_2}\,/\,U_1$ $Cs = U_3\,/\,U_2^{3/2}$	$Cv = \sqrt{U_2}\,/\,U_1$ $Cs = U_3\,/\,U_2^{3/2}$	$Cv = \sqrt{U_2}\,/\,U_1$ $Cs = U_3\,/\,U_2^{3/2}$
Parameter estimation by the method of moments	$b = \dfrac{1-\lambda(1+Cv^2)}{Cv^2}$ $c = \dfrac{1-\lambda}{\lambda}\,b,\;\; a = t_U\,/\,\lambda$ *where* $\lambda = \dfrac{2Cv - Cs}{4Cv - Cs(1-Cv^2)}$	*By mumerical solution (e.g. Newton-Raphson method) of the above equations of moments.*	*By mumerical solution (e.g. Newton-Raphson method) of the above equations of moments.*	*By mumerical solution (e.g. Newton-Raphson method) of the above equations of moments.*
Time to peak (mode)	$t_p = \dfrac{a(b-1)}{b+c-2}$	$t_p = a - a\left(\dfrac{b-1}{bc-1}\right)^{1/b}$	$t_p = a\,e^{\frac{b-1}{c+1}} - a$	$t_p = a\,e^{-\frac{b-1}{c-1}}$
Remarks	*Definition of B function* $B(b,c) = \int_0^1 x^{b-1}(1-x)^{c-1}dx$ $= \Gamma(b)\Gamma(c)\,/\,\Gamma(b+c)$ *Definition of the incomplete* β *function* $\beta(y,b,c) = \int_0^y x^{b-1}(1-x)^{c-1}dx$		*Definition of parameter* U^*_n *(= n-th moment about the point t = -a)* $U^*_n = \int_0^\infty (t+a)^n\,u(t)\,dt$	

Note : The above given range for the parameters corresponds to the case that u(t) is bell-shaped.

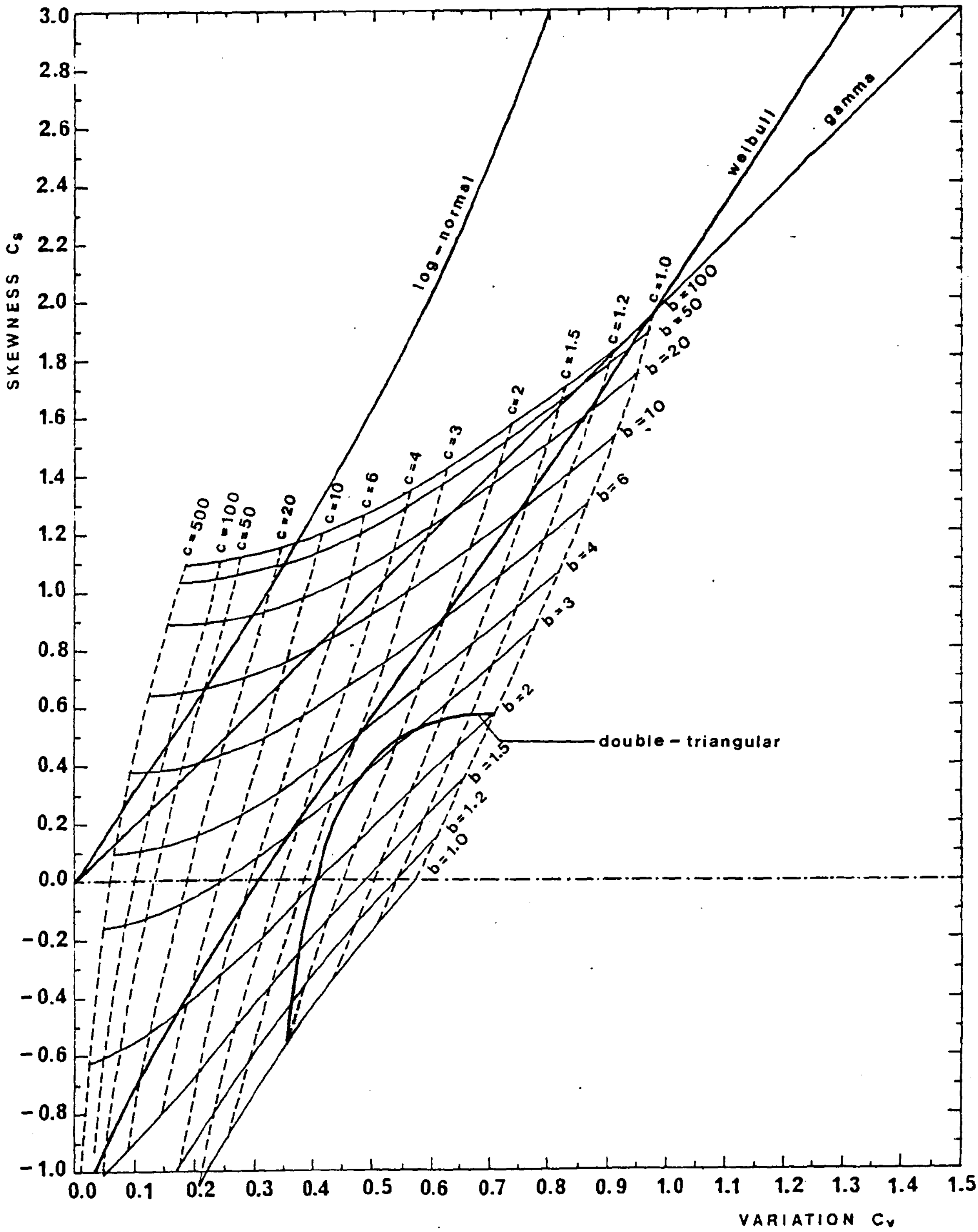

Figure 1: Skewness coefficient versus coefficient of variation, for the two parameter forms.
Relation between definition parameters and coefficients of variation and skewness, for the double-power (DP) form.

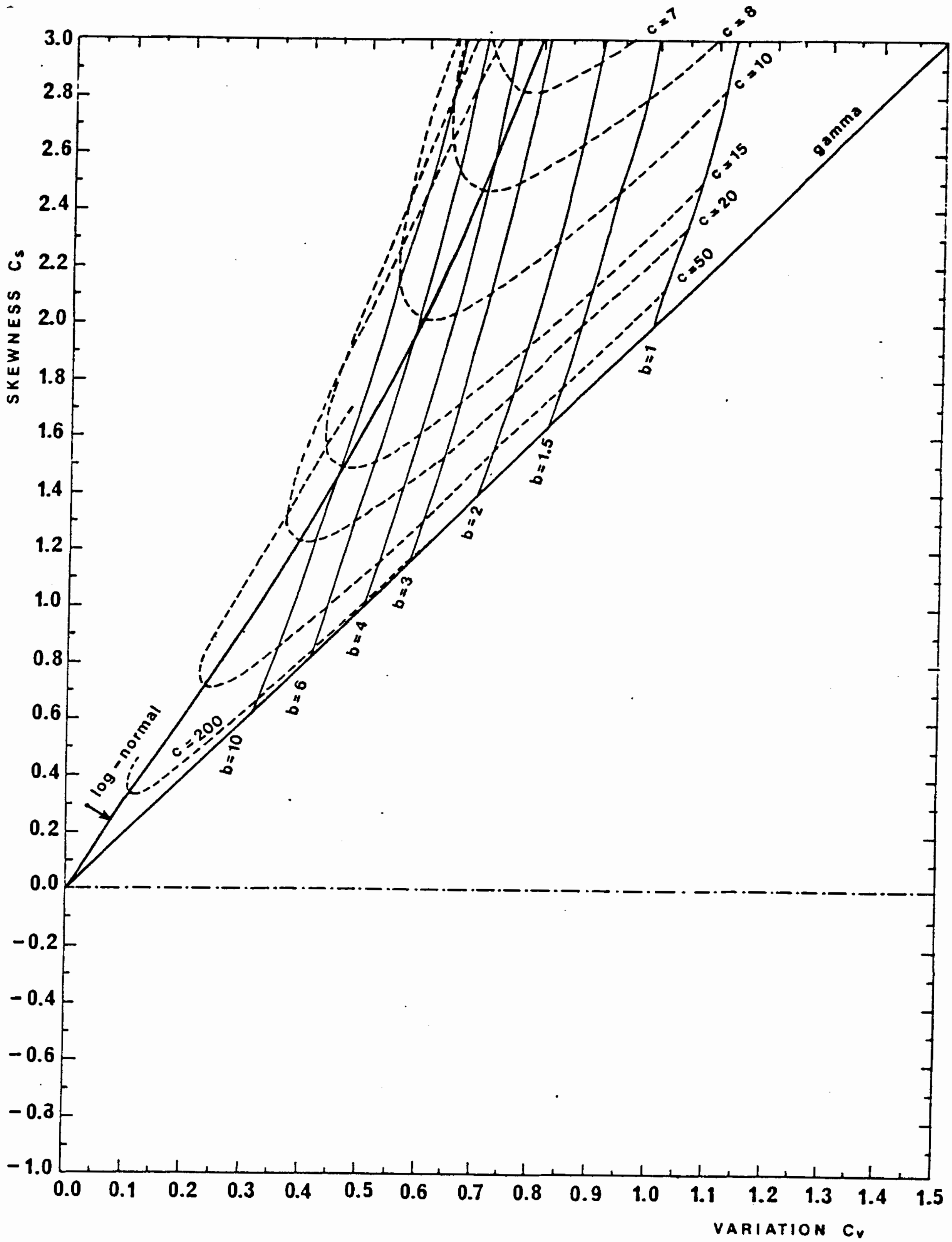

Figure 2: Relation between definition parameters and coefficients of variation and skewness, for the shifted log-Pearson (SLP) form.

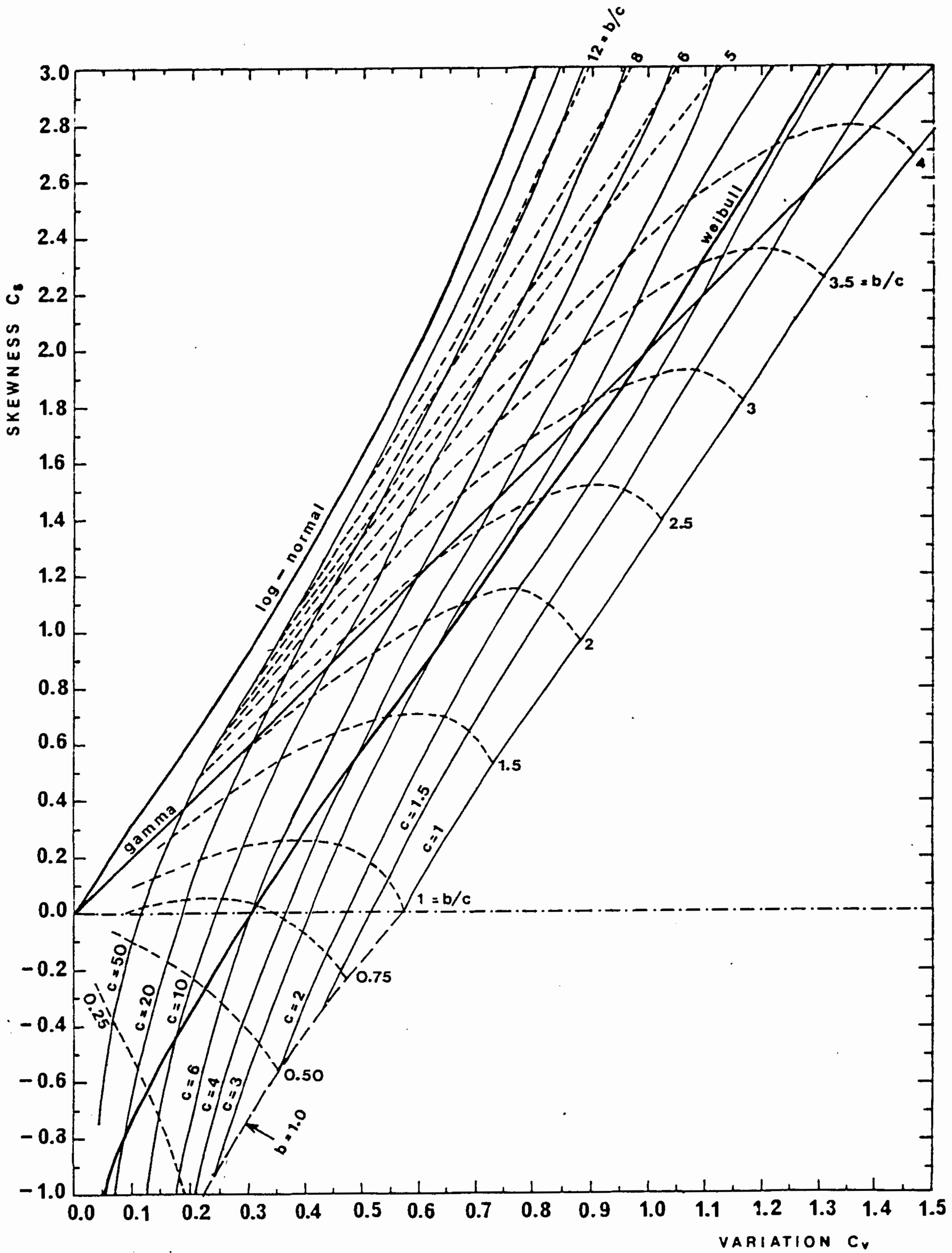

Figure 3: Relation between definition parameters and coefficients of variation and skewness, for the minus log-Pearson (MLP) form.

4. COMPARISON OF THE ANALYTICAL FORMS

Since the definition parameters of the above described forms (a, b, c) are not comparable, it is preferable to use the first three moments (t_U, U_2, U_3) instead, which can be expressed in terms of the definition parameters (Tables 1 and 2). The derivative dimensionless descriptors

coefficient of variation $C_v = (U_2)^{1/2} / t_U$, and

skewness coefficient $C_s = U_3 / (U_2)^{3/2}$

are best indicators for the comparison.

The two-parameter forms have a fixed relation between C_v and C_s, that is

$$C_s = 2C_v \tag{21}$$

for the gamma form,

$$C_s = 3C_v + C_v^3 \tag{22}$$

for the log-normal form, while the relations for the other two forms have not simple analytical expressions. All the four relations are plotted in Fig. 1, from which we conclude that the double triangular form gives the lowest value of C_s for a given value of C_v, and the log-normal the higher one.

The domain of C_v and C_s for the three-parameter forms differs from one form to another. In particular, in the case of beta form, it is easily shown that this domain extends below the curve (21) corresponding to the gamma form; in the shifted log-Pearson form the domain extends above the curve (21) and exceeds the curve (22), corresponding to the log-normal form (see Fig. 2); in the double-power form the domain is quite similar to the one of the beta form, but with an extension above the curve (21) (see Fig. 1); finally, the minus log-Pearson form has the widest domain, extending below the curve (22) (see Fig. 3). The above observations and Figures 1 through 3 are quite helpful for the selection of the proper form.

Systematic examination of the various SIUH shapes, for specified values of the first three moments (or parameters t_U, C_v and C_s), showed that, generally, the shapes are quite similar. Fig. 4 and 5 illustrate the variation of two main characteristics, i.e. the time to peak and the peak discharge, of the SIUH and of the DUH for duration $D=t_U$, respectively, versus the variation of coefficients C_v and C_s. Due to the similarity of the various shapes, it is difficult to distinguish the curves for each separate form. Thus, in most cases, one single curve, for each value of C_v, has been drawn in Fig. 4 and 5; this curve represents all the three-parameter forms. The characteristics of the two-parameter forms are also in agreement with these curves. An exception to that is the double-triangular form, yielding to a deviating higher peak of SIUH, due to the discontinuity in its derivative; however the deviation reduces in the case of DUH, which is of more practical interest.

The above discussion shows that all the examined forms are of similar performance for the SIUH representation. It is obvious that the three-parameter forms are more adjustable, while the two-parameter are simpler. The selection of a specific form may be based on the values of the descriptors C_v and C_s (see next paragraph). The simplicity of the form may, also, be considered. We notice that the double-triangular, Weibull and double-power forms have the simpler expressions for both u(t) and s(t) functions;. calculations require no use of computers or statistical tables. From the other side the double-triangular, gamma, log-normal and beta forms have the simpler relations between their moments and their definition parameters; this is of interest when parameters are estimated from moments.

A final observation at this point, drawn from Fig. 5, is that the magnitude of the peak discharge of the IUH or DUH increases with the decrease of the coefficient of variation as well as with the increase of the skewness coefficient.

5. PARAMETER ESTIMATION BY THE METHOD OF MOMENTS

We assume that the IUH identification is based on recorded rainfall and runoff data of the catchment and not on various catchment characteristics.

Let I(t) be the net hyetograph of a recorded rainfall event and Q(t) is the corresponding surface runoff hydrograph, derived from the recorded data. Dividing the above functions with the total net rainfall depth, H, and the total surface runoff volume, V, respectively, we get the **standardized net hyetograph** i(t) and the **standardized surface runoff hydrograph** q(t). Furthermore let t_I and t_Q be the times from the origin to the centers of area of i(t) and q(t), respectively, and I_n and Q_n the n-th central moments of i(t) and q(t).

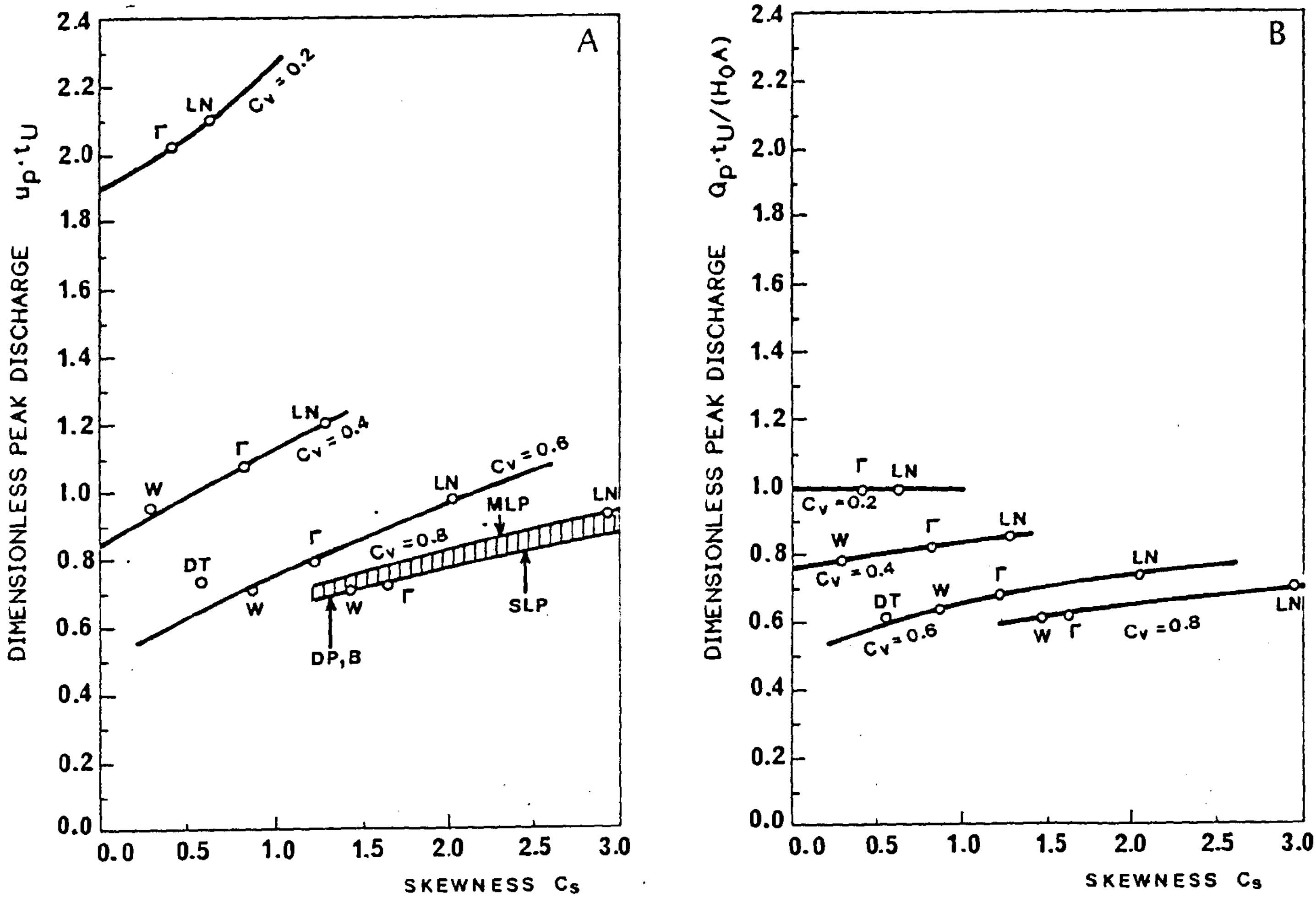

Figure 4: Dimensionless peak discharge of (A) standardized instantaneous unit hydrograph (SIUH) and (B) unit hydrograph with duration $D = t_U$, versus the coefficients of variation and skewness, for various analytical forms.

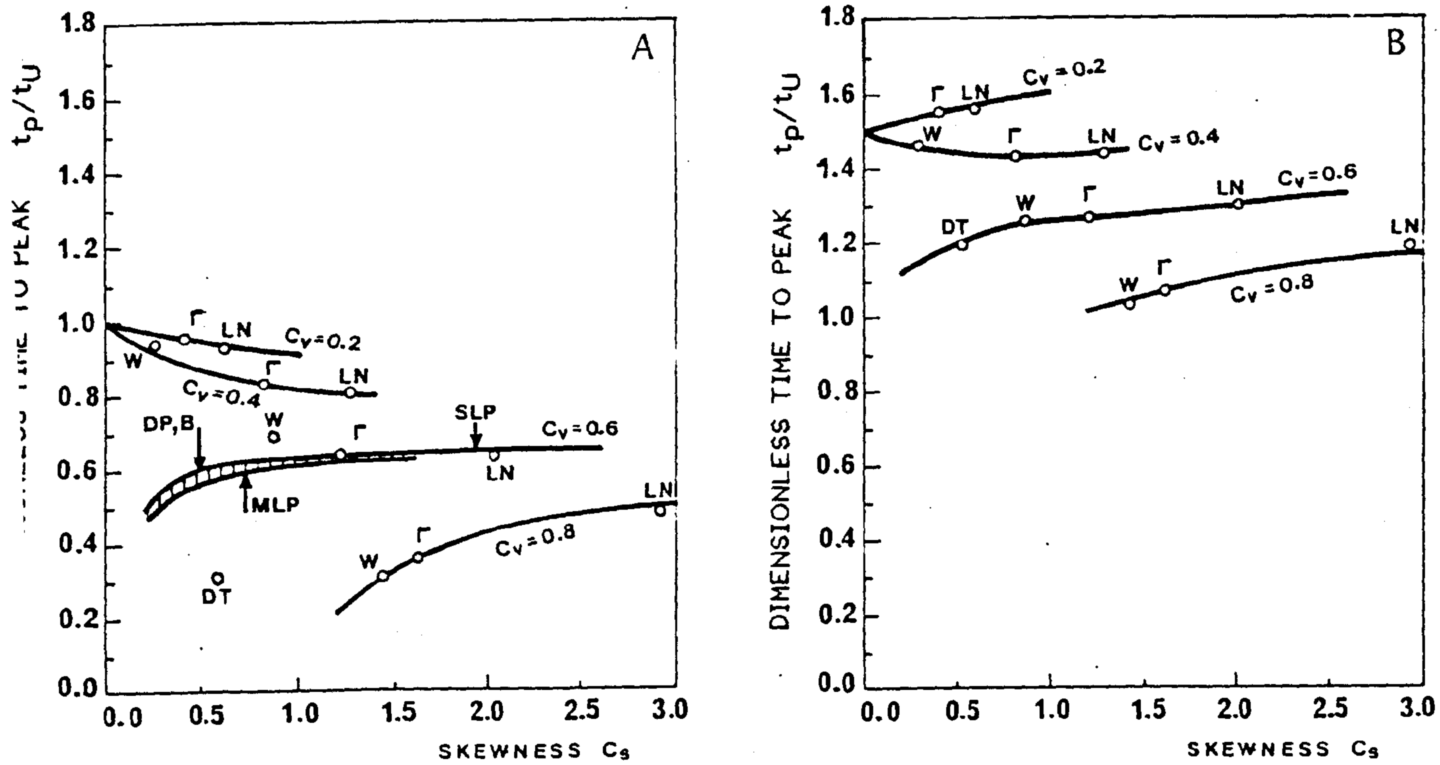

Figure 5: Dimensionless time to peak of (A) standardized instantaneous unit hydrograph (SIUH) and (B) unit hydrograph with duration $D = t_U$, versus the coefficients of variation and skewness, for various analytical forms.

Nash [1959] showed that the moments of SIUH are related to the above moments by

$$t_U = t_Q - t_I \tag{23}$$

$$U_2 = Q_2 - I_2 \tag{24}$$

$$U_3 = Q_3 - I_3 \tag{25}$$

Somewhat more complex relations exist for moments of higher order.

These relations permit a simple calculation of the SIUH moments, which can then determine the definition parameters of a specific SIUH form.

The whole process of the SIUH identification, with this method, consists of the following steps:

1. Calculate the moments of i(t) and q(t);
2. Calculate the moments of u(t) using (23) through (25);
3. Calculate the descriptors C_V and C_S;
4. Select an analytical form, which is proper for C_V and C_S;
5. Calculate the definition parameters of the selected form;

The equations in Tables 1 and 2, relating the SIUH's moments and descriptors, to the definition parameters should be used for the step 5 of the process. Double-triangular, gamma, log-normal and beta forms are the simplest to be used with this method. The other analytical forms require the use of a numerical procedure for the solution of equations. The nomographs of Fig. 1, 2 and 3 support the form selection in step 4 and may replace numerical methods of equation solving in step 5, when there is no need for high accuracy.

The real problem with this method is that the SIUH's moments calculated from separate recorded rainfall/runoff events usually differ at a remarkable degree. One solution to that problem, oriented towards the derivation of a mean unit hydrograph, is obtained by taking the average of each moment. The obtained parameters of the SIUH can then be modified, in order to get more severe unit hydrographs, to be used in design floods.

6. PARAMETER ESTIMATION BY THE METHOD OF LEAST INTEGRAL SQUARE ERROR

The method of moments is the simplest parameter estimation procedure, but it is not the only obtainable one, since other methods could also be set up for this aim.

The method proposed here is oriented towards the minimization of an objective function, defined as the integral square error between the recorded runoff hydrograph, and the derived, with the use of the selected analytical form, convoluted runoff hydrograph. This objective function can be formulated as

$$g(a, b, c) = \sum_{i=1}^{m} (Q_i - Q^*_i)^2 \qquad (26)$$

where **g()** is the objective function, **a**, **b**, **c** are the parameters of the SIUH, which in this point are considered as decision variables, (the method can be applied for more than 3 parameters, as well), i is a time index, Q_i is the ordinate of the recorded surface runoff hydrograph, Q^*_i is the corresponding ordinate of the convoluted surface runoff hydrograph, and **m** is a sufficiently large integer constant.

We note that **g()** is a convex function, and neither itself, neither its derivatives can have simple analytical forms; thus the analytical optimization methods can not be applied here. The minimization of **g()** is carried out through a proper iterative numerical procedure. In each iteration a set of values of the parameters is assumed and the value of the objective function is calculated, as described in the following steps:

1. Calculate SIUH and SSC for the assumed set of parameters;
2. Calculate unit hydrograph for the appropriate rain duration, using (5) and (3);
3. Calculate flood hydrograph by convolution of the unit hydrograph and the recorded net hyetograph;
4. Calculate the integral error between recorded and convoluted flood hydrographs, by (26).

A fully general algorithm, in pascal programming language, has been developed for the above minimization procedure, which executes systematically the required iterations. It looks like the **bisection algorithm**, used for the equation solving, but uses three successive points of each (decision) variable, in the way that the middle point corresponds to the lowest value of **g()**. There is not any restriction about the number of decision variables, but the addition of more variables increases exponentially the required number of computations. Thus the algorithm is time consuming, but its generality is considerable. In the examined problem, the use of double-triangular, Weibull, or double-power forms speeds up computations, because of the simplicity of functions s(t) in these forms.

This method produces better results than the method of moments, and this will be verified later on. The method can be easily extended to the case that more than one recorded flood hydrographs are available; in that case the objective function should be defined as the total error for all hydrographs.

7. ADVANTAGES OF THE PARAMETRIC APPROACH

As it was pointed out in the introduction, the parametric approach is in general less accurate than the standard linear approach, but it has certain advantages, the first being its simple and secure numerical computations (though the mathematical background may seem somewhat complicated).

Not only the limited number of parameters is a handicap, but it is also a superiority in some cases, especially when we need to establish the relationship between unit hydrograph and catchment characteristics; such a relationship assist the unit hydrograph derivation in neighbouring ungauged catchments.

The pre-selection of a smooth analytical form for the representation of IUH, guarantees smooth unit hydrograph and S-curve. Thus the parametric method can be applied also to the smoothing of a DUH, derived by the usual linear method, in order to avoid problems, that frequently appear when a DUH is converted from one