# Evaluation of the parameterization-simulation-optimization approach for the control of reservoir systems

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[1] Most common methods used in optimal control of reservoir systems require a large number of control variables, which are typically the sequences of releases from all reservoirs and for all time steps of the control period. In contrast, the less widespread parameterization-simulation-optimization (PSO) method is a low-dimensional method. It uses a handful of control variables, which are parameters of a simple rule that is valid through the entire control period and determines the releases from different reservoirs at each time step. The parameterization of the rule is linked to simulation of the reservoir system, which enables the calculation of a performance measure of the system for given parameter values, and nonlinear optimization, which enables determination of the optimal parameter values. To evaluate the PSO method and, particularly, to investigate whether the radical reduction of the number of control variables might lead to inferior solutions or not, we compare it to two alternative methods. These methods, namely, the high-dimensional perfect foresight method and the simplified "equivalent reservoir" method that merges the reservoir system into a single hypothetical reservoir, determine "benchmark" performance measures for the comparison. The comparison is done both theoretically and by investigation of the results of the PSO against the benchmark methods in a large variety of test problems. Forty-one test problems for a hypothetical system of two reservoirs are constructed and solved for comparison. These refer to different objectives (maximization of reliable yield, minimization of cost, maximization of energy production), water uses (irrigation, water supply, energy production), characteristics of the reservoir system and hydrological scenarios. The investigation shows that PSO yields solutions that are not inferior to those of the benchmark methods and, simultaneously, it has several theoretical, computational, and practical advantages. *INDEX TERMS:* 1857 Hydrology: Reservoirs (surface); 6309 Policy Sciences: Decision making under uncertainty; 6344 Policy Sciences: System operation and management; *KEYWORDS:* parameterization, simulation, optimization, reservoir systems, water resource systems, optimal control, hydropower

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## 1. Introduction

### 1.1. Combined Simulation-Optimization: A Simple Example

[2] In contrast to other methodologies used for the control of reservoir systems, the parameterization-simulation-optimization (PSO) approach, which we will study in this paper, is not widespread. Therefore we will present it here systematically by means of examples. Initially, we will consider a very simple problem, the calculation of the firm release from a single reservoir. The problem is stated as follows: Given a reservoir with storage capacity $k$ and a sequence of inflows $i_t$ for time $t = 1, 2, \ldots, n$, where $n$ is a control horizon, find the maximum possible release $d$ that can be achieved on a year-to-year steady state basis.

[3] Let us start with an alternative, better-known formulation of the problem, which is based on linear programming and given by *ReVelle* [1999, p. 12], whose recent book deals with design and operation of single and multiple reservoirs (including problems like maximizing yield from reservoir systems, maximizing hydropower production, allocating reservoir services among different water uses, etc.) using linear programming. The linear programming problem formulation is

$$\text{maximize } d \tag{1}$$

subject to

$$s_t = s_{t-1} + i_t - d - w_t, \quad s_t \leq k, \quad s_t,\ w_t, d \geq 0,\ s_n \geq s_0 \tag{2}$$

where $s_t$ and $w_t$ are the reservoir storage and spill, respectively, at time $t$. The equality constraint in (2) represents the water balance in the reservoir whereas nonequality constraints represent physical or methodological restrictions. While the actual control (or decision) variable is only one, the steady state release (or demand) $d$, this formulation uses 2 $n$ additional control variables, $s_t$ and $w_t$, as well as a total 2 $n$ + 1 constraints, not including the


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nonnegativity constraints in (2). (The number of control variables increases by one if $s_0$ is also regarded a control variable, a case sometimes referred to as "the steady state version" because in the problem solution, the last constraint in (2) will be a binding one, i.e., $s_n = s_0$.) If a large control horizon is considered, e.g., $n = 1000$, this will mean that the control variables become 2001 instead of 1, and in addition, the number of constraints becomes 2001. Even though linear programming algorithms can solve problems with a large number of control variables, the high dimensionality is not fortunate.

[4] An alternative, low-dimensional solution is based on simulation. We observe that if, in addition to the reservoir size $k$ and the inflow series, the target release (or demand) $d$ and the initial storage $s_0$ are known, then all series related to reservoir operation can be obtained using a simple simulation model. Thus the series of release $r_t$, spill $w_t$, and reservoir storage $s_t$ are obtained by [*Pegram*, 1980]

$$r_t = \min(d, s_{t-1} + i_t - l_t), w_t = \max(0, s_{t-1} + i_t - l_t - d - k),$$
$$s_t = s_{t-1} + i_t - l_t - r_t - w_t \tag{3}$$

where for generality of equations we have included a term $l_t$ describing losses from leakage, which for this example is assumed to be zero but in subsequent problems it will be non ignorable.

[5] Now we define a performance measure as

$$L(d) := \min\{r_t; t = 1, \ldots, n\} \tag{4}$$

which we wish to maximize subject to the constraint $s_n \geq s_0$. In this way the problem became a one-dimensional maximization problem. The objective function $L(d)$ is single variate but highly nonlinear with discontinuous derivatives (because of the presence of min() and max() functions in (3)–(4)). Furthermore, it can be evaluated only by simulation (based on (3)–(4)) rather than by analytical or other numerical means. The nonlinearity and discontinuity are not serious problems though, if we have to solve a one-dimensional problem.

[6] There are several arguments favoring the second, one-dimensional formulation, over the first high-dimensional one. First, a one-dimensional problem is simpler and simplicity is always desirable. Second, the one-dimensional formulation may be computationally faster, as it contains only one control variable and one constraint, whereas, as we saw, the linear formulation contains $2n + 1$ variables and $2n + 1$ constraints. In the former the solution time increases linearly with $n$ (since it only requires a number of simulation steps equal to $n$ to evaluate the performance measure and the constraint) whereas the latter may require exponential or at least polynomial solution time (depending on the algorithm [e.g., *Chvátal*, 1983, pp. 47, 52; *Mays and Tung*, 1992, p. 90]). This is important if $n$ becomes large (and indeed does to get accurate estimates; see discussion in section 5.4). Third, the one-dimensional formulation allows a more faithful representation avoiding simplifications of the system. For example, in the above formulation we considered the inflow $i_t$ to be known and independent of the reservoir storage; this can be true if inflow is the river flow only. However, if evaporation and rainfall over the reservoir surface are not ignorable, then $i_t$ should be defined to be the sum of river flow plus rainfall minus evaporation over the reservoir area and thus it will depend nonlinearly on the reservoir storage $s_t$. Also, we assumed that the leakage $l_t$ from reservoir could be ignored. In many cases leakage may be significant and depend nonlinearly on the reservoir storage. Thus, if we include evaporation, rainfall and leakage, then the reservoir balance equation will include nonlinear expressions of storage and thus the linear formulation in (2) can no longer work, unless artificial linearization is applied, which is a simplifying and simultaneously complicating and inaccurate procedure. In contrast, the nonlinear formulation (3)–(4) implies no difficulty at all to incorporate nonlinear terms of storage.

[7] A fourth argument is related to the stochastic nature of inflow. Both the above formulations considered the problem as a deterministic one. Obviously this is not consistent with reality. In fact, the inflow series $i_t$, which was assumed known, is one likely realization of a stochastic process. Therefore it is not consistent to require that the release $r_t$ equals the demand $d$ in all time steps $t$. In fact, such an assumption will result in zero release as $n$ tends to infinity (within an infinite series there would be sequences of consecutive zero inflows that will lead to zero release); obviously this zero outcome is not what we wish to estimate. In other words, constraints (2) are not consistent with the stochastic nature of inflows but at the same time (3) are (in the latter the release $r_t$ does not necessarily equal the demand $d$). A very slight adaptation is needed in the nonlinear formulation to completely comply with the stochastic nature of the problem; specifically, it suffices to modify the definition of performance measure (4) to read

$$L(d) := r_{(\beta n)} \tag{5}$$

where $r_{(m)}$ denotes the $m$th smallest value of the series of releases $r_t$ and $\beta$ is an acceptable probability of failure (e.g., for $n = 1000$ and $\beta = 1\%$, $r_{(\beta n)} = r_{(10)}$ is the tenth smallest value of the series of releases). We will call the quantity $L(d)$ in (5) the reliable release (with reliability $1 - \beta$) rather than the firm release. This adaptation is simple and general (can perform in any situation) and does not increase computational effort (except for determining the $m$th smallest value of the series, rather than the minimum, in a series). On the contrary, it is very difficult to adapt the linear high-dimensional formulation. For the specific problem examined, such an adaptation has been described by *ReVelle* [1999, pp. 116–123]; this has been based on the idea of chance constraints [*Askew*, 1974], which is comprehensively discussed by *Loaiciga* [1988]. This adaptation is case specific as it depends on the stochastic model of inflows and cannot perform in more complicated situations (e.g., in combination with seasonal models); at the same time, it adds computational effort (requires a greater number of constraints).

[8] Related to the stochastic nature of inflows, there is another, perhaps the most important, difference of the two approaches. In the low-dimensional formulation, if a sufficiently large $n$ is chosen, the control variable $d$ is independent of the series (the sample path) that was used to derive it and depends only on the structure and parameters of the

stochastic process of inflows. Indeed, the performance measure $L(d)$ in the probabilistic formulation (5) is in fact a sample estimator of a population parameter and, as $n$ tends to infinity, $L(d)$ will tend to this population parameter. (The deterministic formulation (4) is inappropriate because, as already mentioned, $L(d)$ will tend to zero). Given that $L(d)$ is independent of the series of inflows, $d$ will be also independent of the inflows. In the high-dimensional approach, the control variables $s_t$ and $w_t$ depend on the specific series and obviously their optimal values for each $t$ will change if the inflow series changes. Thus, in the low-dimensional formulation, once the control variable is determined, it can be used to run the system for any inflow series without further optimization. On the contrary, in the high-dimensional formulation the control variables need to be determined again, by running again the optimization model, each time the inflow series is updated.

[9] The high-dimensional approach, i.e., the use of series of variables related to reservoir operation (such as storages and releases at all examined time steps) as control variables is not confined to the linear formulation described above. On the contrary, it is common in most reservoir optimization methodologies, reviews of which are given by *Yeh* [1985], *Wurbs* [1993, 1996], and *Mays and Tung* [1992, 1996], among others. These methodologies include, apart from linear programming, dynamic programming (DP) [*Buras*, 1966] and modifications of it like incremental DP [*Hall et al.*, 1969], discrete differential DP [*Heidari et al.*, 1971], and gradient DP [*Foufoula-Georgiou and Kitanidis*, 1988]. They also include stochastic dynamic programming (SDP) [*Su and Deininger*, 1974; *Askew*, 1974; *Bras et al.*, 1983; *Stedinger et al.*, 1984; *Terry et al.*, 1986; *Tejada-Guibert et al.*, 1993] and several improvements of it such as Bayesian SDP [*Karamouz and Vasiliadis*, 1992; *Kim and Palmer*, 1997], demand driven SDP [*Vasiliadis and Karzmouz*, 1994]; sampling SDP [*Kelman et al.*, 1990]. It is well known that these methods suffer from the "curse of dimensionality" [e.g., *Pereira and Pinto*, 1985]. In addition, when such high-dimensional methods, whose decision variables depend directly on inflows, are used for operating reservoirs, it is important to base the decisions on forecasts of future inflows. The value of forecasts has been demonstrated by *Stedinger et al.* [1984], *Karamouz and Vasiliadis* [1992], *Tejada-Guibert et al.* [1995], *Vasiliadis and Karzmouz* [1994], *Kim and Palmer* [1997], *Faber and Stedinger* [2001], and *Yao and Georgakakos* [2001], who showed that in general, better forecasts can improve reservoir operation.

[10] In contrast, as already mentioned, the low-dimensional (variable-parsimonious) formulation is not so common in water resources literature. This methodology could be characterized as combined simulation and optimization, where simulation is used to obtain values of the performance measure, which is optimized by a nonlinear optimization procedure. To avoid confusion, we must clarify that combinations of simulation and optimization techniques have been used in the water resources literature in different contexts. For example, *Lobbrecht* [1997, p. 62] describes a simultaneous simulation and optimization methodology whose modules run in parallel. The simulation module incorporates a description of the nonlinear relationships of processes in the system whereas the optimization module contains a simplified and linearized description of these processes. The optimization model runs first and its outputs are then refined (become more accurate) by the simulation model. Similar is the control-simulation framework by *Georgakakos et al.* [1999]. In this framework, a control model based on the extended linear quadratic Gaussian method [*Georgakakos and Marks*, 1987, 1989] is run and its outputs, which for example are optimal reservoir releases, are then fed to a simulation model. The latter tests the feasibility of results and makes the necessary corrections if needed. In a different context, *Johnson et al.* [1991] use heuristic operating rules to drive a simulation of a reservoir system in companion with an optimization procedure that tries to minimize the departures of the real reservoir storages from the target storages, which are set by the heuristic operation rules. Clearly, in all these cases the kind of combination of simulation and optimization is different from the one discussed in our context for the firm release calculation problem.

### 1.2. Role of Parameterization

[11] Obviously, the abovedescribed problem is too simple to be considered as a representative problem of reservoir management. Let us consider a more complex problem, i.e., the maximization of reliable release from a system of two reservoirs. In a high-dimensional system representation (based on series of variables), similar to the above linear programming model, the number of variables is at least double that of the single reservoir problem, as now in each time step we have to deal with two storages, two spills, etc. If we follow a low-dimensional approach, we may think that the addition of another reservoir introduces one degree of freedom for the simulation. This is related to the allocation of the total release to each of the two reservoirs. Normally, one can expect that one or two additional control variables suffice to deal with one degree of freedom. Indeed, *Nalbantis and Koutsoyiannis* [1997] introduced a parametric rule that can do this allocation of releases using one or two parameters. In certain simple cases, the parameters can be determined a priori by theoretical reasoning. For example, the parametric rule takes the form of the well known "space rule" [*Clark*, 1950, 1956; *Bower et al.*, 1962; *ReVelle*, 1999, p. 27] if the only concern is to minimize the spills from the system. Other such special cases have been studied in detail by *Nalbantis and Koutsoyiannis* [1997] and *Lund and Guzman* [1999]. In the general case, however, the rule must be considered as unknown and its parameters entered into the optimization model as additional control variables. This is the case in the studies by *Lund and Ferreira* [1996], *Oliveira and Loucks* [1997] and *Nalbantis and Koutsoyiannis* [1997]. In this paper we have used the mathematical formulation of rule of the latter study; a brief description of it is given in section 2.2.

[12] Thus, for a system of two or more reservoirs, the simulation itself cannot provide a unique portrait of the system evolution, unless a system parameterization comes before, which is then used within simulation to specify certain unknown quantities when the system incorporates some degrees of freedom. Adding this link to the combined simulation-optimization approach we acquire the complete methodological chain, which we have termed parameterization-simulation-optimization (for abbreviation referred to as PSO or the parametric method or the low-dimensional

method in this paper). The general idea of this methodology is not new, as it has been applied before to multireservoir systems in the references listed above. It can be also said that several organizations (e.g., the US Army Corps of Engineers) that develop operating policies, apply an empirical version of the PSO method done by hand: one specifies a rule, simulates the system based on this rule, and then adjusts it to deal with problems that emerge in simulations. In addition, the PSO method is applicable not only to reservoir systems; for example, *Schütze et al.* [1999] have applied a very similar methodology in urban wastewater systems.

[13] It is almost obvious that in the single reservoir example discussed above the two examined methodologies, the high-dimensional and the low-dimensional, will result in exactly the same value of the firm release, despite of the dramatic difference in the number of control variables. However, in a system of reservoirs it is not obvious at all that the two approaches will yield the same results. It is possible that the PSO approach will yield a suboptimal solution, as the solution depends on how good the adopted parameterization is. On the other hand, a high-dimensional nonparametric methodology is supposed to yield the optimal solution if a good optimization algorithm is used.

[14] The purpose of this paper is to evaluate the PSO method in several reservoir-related problems that deal with irrigation, water supply and power production, with several targets such as maximization of reliable release, minimization of conveyance costs, or maximization of power production. The evaluation of the method is attempted by comparing its results with those of a high-dimensional optimization approach and a simplified one-dimensional "equivalent reservoir" method that merges the reservoir system into a single hypothetical reservoir avoiding parameterization since a single reservoir involves no degrees of freedom.

[15] The PSO method is formally presented in section 2 whereas the alternative methods used for the comparisons are presented in section 3. The basic assumptions about the test system are discussed in section 4 and the test problems are described in section 5. The results of the solutions of 41 test problems are discussed in section 6 and the conclusions are given in section 7.

## 2. Parameterization-Simulation-Optimization Method

### 2.1. General Formalization

[16] According to the PSO approach, a reservoir system problem becomes a typical problem of stochastic optimization, which by definition [*Fu and Hu*, 1997, p. 1] aims at determining the setting of various "decision" parameters of a system with stochastic dynamics to optimize some performance measure of interest. Stochastic optimization is used when this performance measure cannot be obtained through analytical means and therefore must be estimated from sample paths, e.g., via stochastic simulation. The general solution procedure for such a problem is described by a typical sequence of mappings from the input parameter space and sample path to the output system performance. By adapting the typical sequence of mappings given by *Fu and Hu* [1997, p. 2] we take the following solution procedure for the reservoir system case:

$$(\boldsymbol{\theta}, \omega) \rightarrow \mathbf{X}(\boldsymbol{\mu}, \omega) \rightarrow \mathbf{Z}(\mathbf{X}(\boldsymbol{\mu}, \omega), \boldsymbol{\lambda}) \rightarrow L(\mathbf{Z}(\mathbf{X}(\boldsymbol{\mu}, \omega), \boldsymbol{\lambda}))$$
$$\rightarrow J(\boldsymbol{\theta}) := E[L(\mathbf{Z}(\mathbf{X}(\boldsymbol{\mu}, \omega), \boldsymbol{\lambda}))] \quad (6)$$

where $\boldsymbol{\theta} := (\boldsymbol{\mu}, \boldsymbol{\lambda})$ is a vector of all system parameters, which are divided into two separate vectors, the vector $\boldsymbol{\mu}$ that contains the parameters of hydrologic inputs to reservoirs (e.g., mean values, standard deviations, autocorrelations, cross-correlations, etc.) and the vector $\boldsymbol{\lambda}$ that contains parameters determining the reservoir system operation; $\omega$ denotes a sample path realization of the random variables (that is, $\omega$ can be thought of as representing the randomness in the system, e.g., all random numbers in a simulation run); $\mathbf{X}$ is the vector of hydrological inputs (river inflow, rainfall, evaporation) to the reservoir system; $\mathbf{Z}$ is the vector containing the outputs of the system (release, storage); $L$ is the sample performance measure of the system that corresponds to the sample realization represented by $\omega$; and $J$ is true (population) performance measure defined to be the expected value of $L$. This sequence is depicted in more detail in Figure 1.

[17] We must distinguish the theoretical difference between the true performance measure, which is a function of the system parameters only, from the sample performance measure, which depends on the specific simulation run, represented by $\omega$. So, to estimate $J(\boldsymbol{\theta})$, an ensemble of simulations is needed to take the ensemble average of $L(\mathbf{Z}(\mathbf{X}(\boldsymbol{\mu}, \omega), \boldsymbol{\lambda}))$. However, if the system is stationary and ergodic (in other words, if we have a steady state simulation [e.g., *Winston*, 1994, p. 1220]), $L$ will tend to $J$ as the simulation length tends to infinity. Therefore a single instance of the sample performance measure, estimated from a simulation with a large length, is an adequate estimate of the true performance measure under the stationarity and ergodicity condition. For instance, this is the case in the example problem considered in section 1.1, since the target release is constant through time and the inflow series can be assumed, as usually, stationary and ergodic. This will also be the case in all problems examined here. Conversely, if the target release is growing in time (a common situation in practice) the simulation is no more a steady state one, and numerous simulations must be performed to estimate the true performance measure.

### 2.2. Parameterization of a Reservoir System

[18] We consider a reservoir system with $q$ reservoirs, each having an active storage capacity $k^j$, $j = 1, \ldots, q$ (excluding dead volume), the sum of all being $k$. Let $s$ denote the total active storage in the system and $s^j$ be the respective active storage for reservoir $j$. (Reference to the time interval is omitted here for convenience.) Typically, the actual problem in a reservoir system is to determine the releases from all reservoirs so that their sum equals a given total demand. Equivalently, the problem is to distribute $s$ into the $q$ reservoirs. This can be done in numerous ways, as the problem has several degrees of freedom. A specific way to perform this distribution is termed an operating rule. *Nalbantis and Koutsoyiannis* [1997] introduced a parametric operating rule, with parameters determined by optimization, so that the optimal operating rule, valid through the

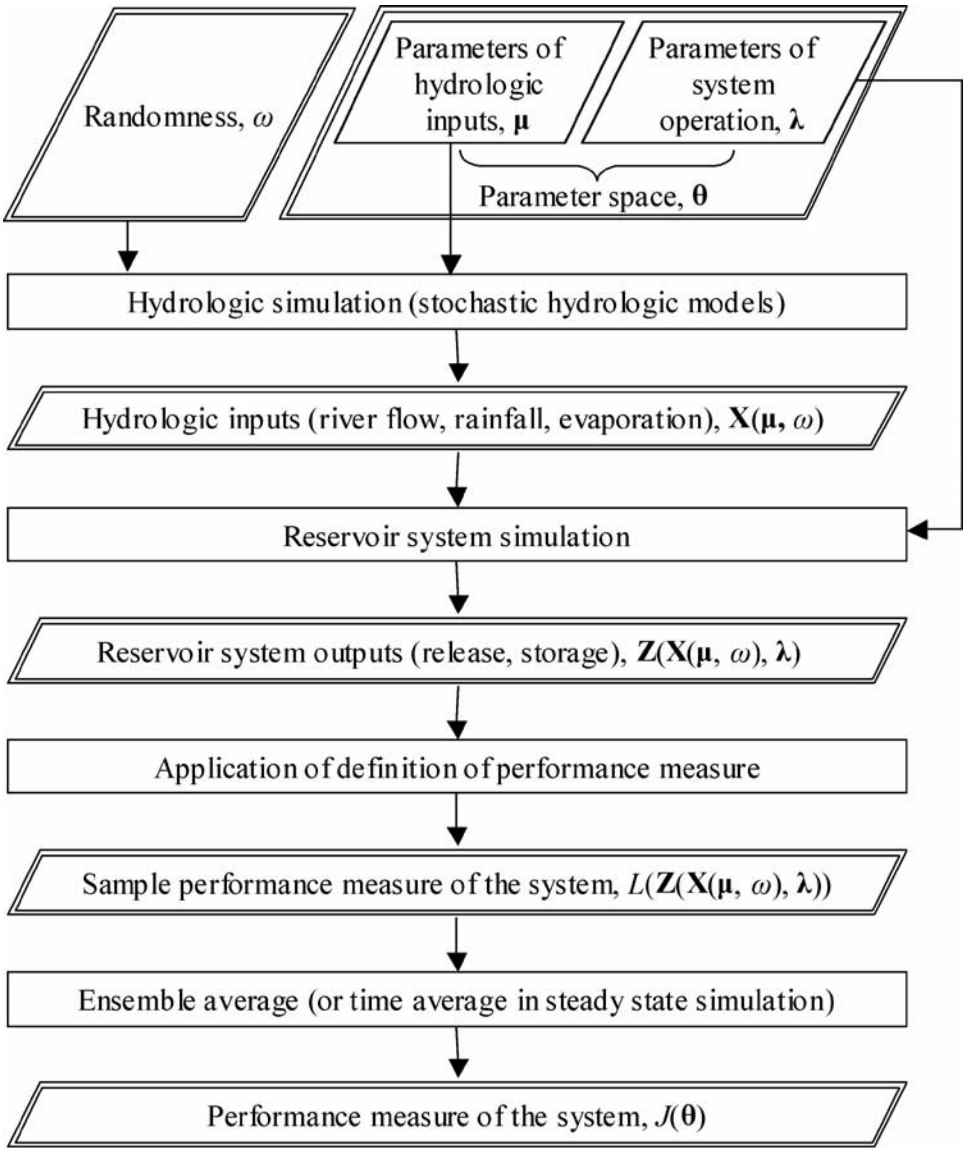


**Figure 1.** Schematic representation of the solution procedure for a reservoir system problem. Rectangles represent the steps of the solution procedure, and parallelograms represent inputs and outputs for the different steps.

entire control period, defines an optimal policy in the reservoir system operation. Initially, this rule is written in the following linear form

$$s_*^j = k^j - a^j k + b^j s \tag{7}$$

where $a^j$ and $b^j$, $j = 1, \ldots, q$ are unknown parameters and $s$ stands for the target storage for the reservoir $j$, which generally may differ from the real storage $s^j$ due to the physical constraints that were not considered in this stage. Note that (7) has been written in a form slightly different from the original one by *Nalbantis and Koutsoyiannis* [1997]. In this form, parameters $a^j$ and $b^j$ are dimensionless nonnegative numbers, and since the sum of $s$ must equal $s$, each of the two sequences must add up to unity. That is, the following constraints are imposed on the parameters

$$0 \le a^j \le 1, \quad 0 \le b^j \le 1, \quad \sum_{j=1}^{q} a^j = 1, \quad \sum_{j=1}^{q} b^j = 1, \tag{8}$$

and thus the number of unknown parameters is finally $2(q - 1)$.

[19] Subsequently, because in (7) the physical constraints that the storage cannot be negative nor can it exceed the reservoir storage capacity $k^j$ are ignored, *Nalbantis and Koutsoyiannis* [1997] modified (7) to form an adjusted nonlinear parametric rule that respects these restrictions. The final nonlinear operating rule is completely determined by the initial parameters $a^j$ and $b^j$, irrespectively of adjustments. Figure 2 provides a graphical explanation of the parametric operational rules in a system of three hypothetical reservoirs both in their initial linear forms (equation (7)) and adjusted ones [*Nalbantis and Koutsoyiannis*, 1997, equation (13)].

[20] After extensive analysis, *Nalbantis and Koutsoyiannis* [1997] concluded that the operating rule (7), with parameters $a^j$ and $b^j$ obeying (8), is a convenient and efficient parameterization of the problem. Generally, the

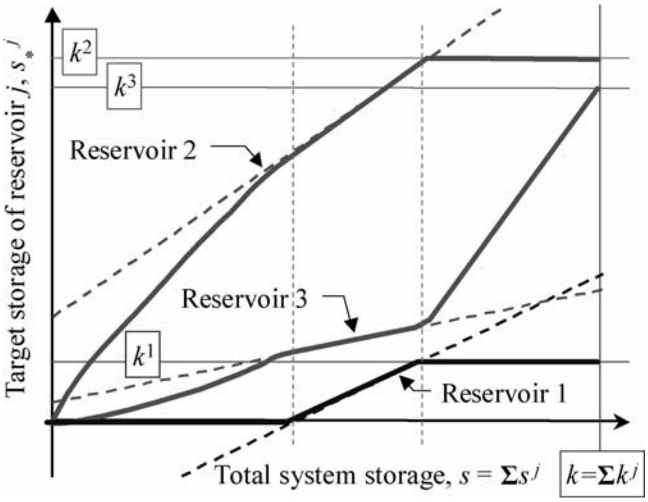


**Figure 2.** Graphical representation of operating rules for three hypothetical reservoirs; dashed lines represent the initial linear rules (equation (7)), and thick solid lines represent the adjusted nonlinear ones [*Nalbantis and Koutsoyiannis*, 1997, equation (13)].

parameters can be considered constant in time or, alternatively, as suggested by *Johnson et al.* [1991], they may be different for the refill (wet) and the drawdown (dry) season. Moreover, *Nalbantis and Koutsoyiannis* [1997] found that the parameterization is still efficient even in the simplified single-parameter form $s_*^j = b^j s$. This special form of the rule is referred to as homogenous form, in which $a^j = k^j/k$. Another interesting special case, referred to as symmetric case, is when both parameters are fixed to the same value $a^j = b^j = k^j/k$. A similar case is the well-known Clark's "space rule" [*Clark*, 1950, 1956; *Lund and Guzman*, 1999], in which $a^j = b^j = E[Q^j]/E[Q]$, where $E[Q^j]$ is the expected cumulative inflow (until the end of the refill cycle) to reservoir $j$ and $E[Q]$ is the expected cumulative inflow for all reservoirs.

[21] In addition to the parameters of the operating rule, other control variables may be introduced depending on the specific problem examined, as it will be clarified below. In any case the number of control variables in this formulation remains very limited.

### 2.3. Simulation

[22] As discussed before and also shown in Figure 1, two simulation phases are performed in a reservoir system problem. The first is the hydrologic simulation, i.e., the simulation of the hydrologic inputs such as reservoir inflows, evaporation and rainfall. This is performed using multivariate stochastic models, which in the simplest case are periodic autoregressive (PAR) models or periodic autoregressive-moving average (PARMA) models, typically operated at a monthly scale (see *Salas* [1993] for a review). In more faithful simulations, a model that reproduces long-term persistence can be used (see *Bras and Rodriguez-Iturbe* [1985] for a review and *Koutsoyiannis* [2000] for a more recent development). Such a model produces annual time series, which are subsequently disaggregated at monthly or finer scale using disaggregation models (see *Grygier and Stedinger* [1988] and *Koutsoyiannis* [1992] for an outline of such models and *Koutsoyiannis* [2001] for a more recent development).

[23] The second simulation phase is the simulation of the reservoir system. The outputs of the hydrologic stochastic model become inputs for the reservoir system simulation. In addition, the parametric reservoir rules are used in this phase. To build a simulation model for a reservoir system is a rather simple task. Such a model is based on the water balance equations of each reservoir (given in (3)) and additionally uses the physical and operational constraints of the system such as those dealing with reservoir storage capacities and aqueducts discharge capacities. The parametric rule is used in each time step to allocate the total withdrawal into the different reservoirs.

### 2.4. Optimization

[24] Clearly, in the PSO approach all problems are nonlinear and therefore linear programming algorithms must be excluded. In addition, the performance measure may incorporate multiple local optima and discontinuous derivatives. Therefore gradient-based nonlinear programming methods must be excluded, too. An appropriate choice is to use evolutionary methods, such as genetic algorithms [e.g., *Goldberg*, 1989; *Michalewicz*, 1996], which can tackle problems with discontinuities and multiple local optima, or the shuffled complex evolution method [*Duan et al.*, 1992]. Such methods have been coded in general-purpose algorithms and can be found as ready-to-use software tools (see also section 5.4).

## 3. Alternative Methods

[25] The testing of the PSO method is done by means of comparing its results with those of two alternative methods. The first, referred to as high-dimensional or perfect foresight method, is similar to the linear method discussed in the example of section 1.1, as it uses a large number of control variables. However, it is not a linear method. To make the comparison as reliable as possible, the formalization of the high-dimensional method was as close as possible to that of the parametric method adopting the same performance measure and the same global constraints. The main difference is that in the high-dimensional method we do not use parameterization. Specifically, instead of parameters, the control variables are the complete series of releases (or transformations of them) from the reservoirs (see section 5). Evolutionary optimization methods such as those discussed in section 2.4, have been used in the perfect foresight method, as well. It should be emphasized that the values of control variables in this method depend completely on the inflow series. Therefore the assumption behind this method is that the inflow series are perfectly known for the entire control horizon (hence the name "perfect foresight" method). Obviously, this is not feasible in real world and thus it cannot be regarded as a method for operating reservoirs [*Faber and Stedinger*, 2001]. However, it can be regarded as the gold standard against which any other method (in our case the PSO method) can be compared.

[26] One may argue that if the number of control variables becomes too large in the perfect foresight method, it will be too difficult to locate their optimal values even using evolutionary algorithms. Therefore we used, as an additional means of comparison, another method with one control

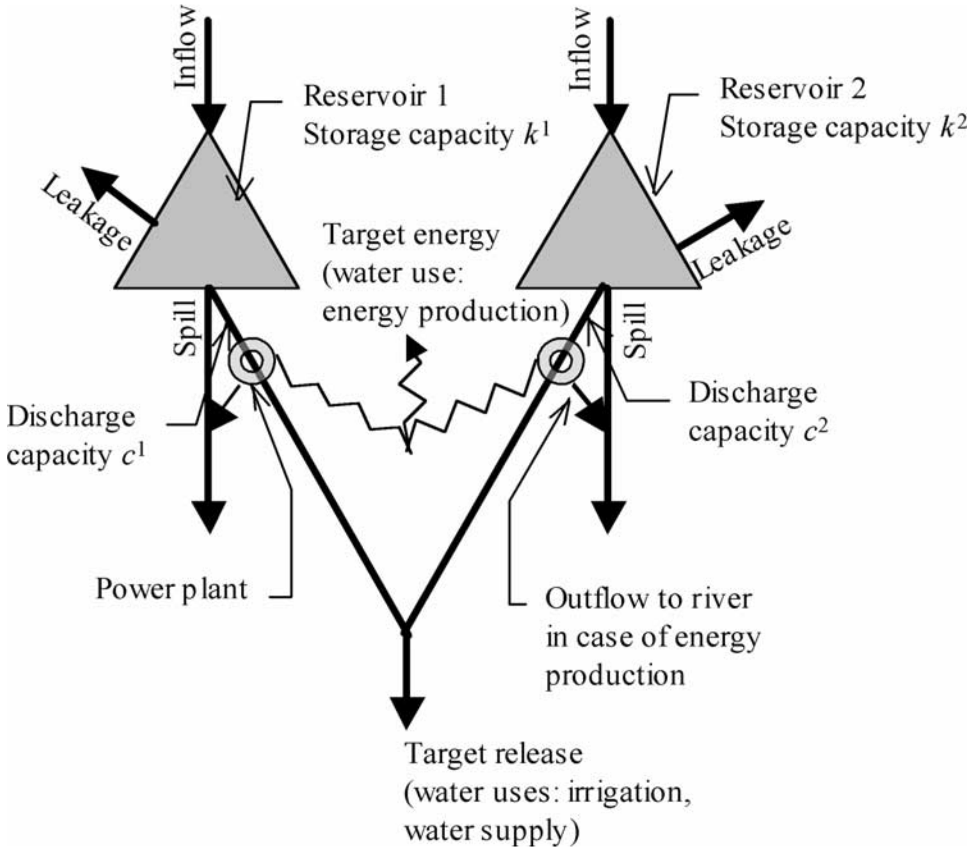


**Figure 3.** Schematic of the test reservoir system.

variable only, which is similar to the aggregation method that has been used to overcome the "curse of dimensionality" in large reservoir systems [*Pereira and Pinto*, 1985; *Terry et al.*, 1986; *Saad et al.*, 1994]. In this method, which we termed the equivalent reservoir method, the reservoir system is replaced by one hypothetical reservoir with characteristics merging those of the different reservoirs of the system. In this way, we drastically simplified the problem, making it one-dimensional and avoiding parameterization since a single reservoir involves no degrees of freedom.

[27] The initial thinking behind the equivalent reservoir approach is that a single reservoir merging the characteristics of the complete system would possibly exhibit noninferior performance in comparison with the system of separate reservoirs. Indeed, it can be proved [*Koutsoyiannis and Economou*, 2003] that the single equivalent reservoir whose storage capacity, inflows and losses equal the respective sums of those of the separate reservoirs, has always noninferior performance in comparison with the system of separate reservoirs, if the objective is maximization of release or reliability. Therefore the performance of the single equivalent reservoir is an upper bound for that of the system, and thus it is very interesting to know this limit; furthermore, it is easy to calculate it accurately because of the simplicity of the calculations (one-dimensional problem). If the objective is maximization of power production, it turns out that the equivalent reservoir is not necessarily superior in performance (see in section 6). Even in this case, where the performance of the single equivalent reservoir is not an upper bound for that of the system, it provides a good and easy-to-find means of comparison.

[28] It should be emphasized that the equivalent reservoir method is not a method to operate a reservoir system. It only provides a means for comparison of the performance of the real reservoir system as it determines in a simple manner a "benchmark" performance measure that in certain cases is an upper bound for the reservoir system.

## 4. Basic Assumptions

[29] For generality and flexibility regarding the system characteristics and services, we assembled a hypothetical reservoir system for this study, based on experience from several existing reservoir systems and avoiding using a specific real-world system, which would confine the system characteristics and water uses. For simplicity we assumed that our hypothetical system comprises two reservoirs, as shown in Figure 3. To keep our calculations as simple as possible, we used a sequence of simple simulation models, describing the essentials of the system. It is reminded that our purpose in this exercise is a comparison of different methodologies, and not the most detailed representation of a reservoir system. For all simulations we adopted a monthly time step, which is a good compromise between simplicity and accuracy, also assuming that all months are equal in duration.

[30] For hydrological simulation we neglected rainfall and evaporation at the reservoir area and modeled only the monthly river flow. For the latter we adopted a periodic

**Table 1.** Monthly Distribution of Inflows and Demands

| Month | Month Number | Mean Inflow,[a] | | Demand, and % | |
|---|---|---|---|---|---|
| | | Reservoir 1 | Reservoir 2 | Irrigation | Water Supply |
| November | 1 | 2.2 | 2.7 | 0.0 | 7.7 |
| December | 2 | 8.2 | 8.2 | 0.0 | 7.7 |
| January | 3 | 22.0 | 21.0 | 0.0 | 7.7 |
| February | 4 | 25.3 | 24.0 | 0.0 | 7.1 |
| March | 5 | 20.2 | 19.3 | 0.0 | 7.8 |
| April | 6 | 10.3 | 10.2 | 5.0 | 7.7 |
| May | 7 | 5.2 | 5.4 | 10.0 | 8.6 |
| June | 8 | 2.6 | 3.0 | 20.0 | 9.2 |
| July | 9 | 1.3 | 1.8 | 23.0 | 9.6 |
| August | 10 | 0.6 | 1.2 | 22.0 | 9.0 |
| September | 11 | 0.3 | 0.9 | 15.0 | 9.3 |
| October | 12 | 1.8 | 2.3 | 5.0 | 8.6 |
| Year | | 100.0 | 100.0 | 100.0 | 100.0 |

[a]The mean annual inflow expressed in equivalent water depth is assumed 225.0 mm and 316.5 mm for the catchment of reservoirs 1 and 2, respectively.

(or seasonal) autoregressive process of order 1 (PAR(1)). The assumed monthly distribution of the mean inflow is shown in Table 1. The coefficients of variation, skewness, lag-one autocorrelations and lag-zero cross-correlations of inflows for simplicity were assumed constant for all months with values shown in Table 2. Especially for the monthly coefficients of variation, two alternative values were adopted for both reservoirs, thus forming two hydrological scenarios. These are referred to as low variation (LV) and high variation (HV) scenarios, with monthly coefficient of variation 0.50 and 0.70, respectively. These values result in annual coefficients of variation of about 0.35 and 0.50 for the LV and HV scenarios, respectively. The other annual statistics are almost equal for both scenarios (skewness coefficients 0.56–0.59 and 0.68–0.69 for locations 1 and 2 respectively; lag one autocorrelation coefficients 0.07 and 0.09 for locations 1 and 2 respectively; lag zero cross-correlation coefficient 0.62).

[31] For the simulation of the reservoir system operation, it is assumed that the relationship between water elevation, $z$, and storage, $s$, is

$$s = k(z/z_{\max})^{\zeta} \tag{9}$$

where $z_{\max}$ is the maximum water elevation (measured from the elevation where $s = 0$), which corresponds to the storage capacity $k$, and $\zeta$ is a constant of the reservoir.

[32] The water balance is given by (3) applied either to each reservoir or to the entire system. In addition to spill, leakage losses are also assumed, given as a function of storage as

$$l = \xi_0 + \xi_1 s \tag{10}$$

where $\xi_0$ and $\xi_1$ are characteristic constants of each reservoir. As explained before, the linear form of (10) is not plausible. Apparently, in the PSO approach there is no restriction about the form of loss equation; however the linear form was necessary to apply the concept of equivalent reservoir (otherwise the equivalent reservoir losses cannot be identical to the sum of the losses of the two reservoirs for any values of storage in them; see *Koutsoyiannis and Economou* [2003]).

[33] Three alternative water uses are assumed: irrigation (IR), urban water supply (WS) and hydropower energy production (HP). Water for irrigation is supplied seven months per year, with the monthly percentages given in Table 1. Urban water supply is assumed to vary from month to month (for all months of the year) with the monthly percentages given again in Table 1. Hydroelectric energy is attempted to be steady in each month and year (see section 5.3) but to avoid spill when a reservoir is full, energy is produced in excess of the steady value by increasing the release from the reservoir. The reservoir spills only when release reaches an upper limit, or discharge capacity $c$ ($hm^3$/month).

[34] Hydroelectric energy is proportional to the release $r$ and the net hydraulic head $h$, i.e., $e = \eta \rho g r h$ where $\eta$ is the efficiency (<1), $\rho$ the density of water and $g$ the gravity acceleration. We define the specific energy

$$\psi := \eta \rho g h/(z_0 + z) \tag{11}$$

where $z_0$ is the elevation difference from the elevation where $s = 0$ to the penstock outlet. This quantity takes a maximum value $\psi = 0.2725$ GWh/hm$^4$ (= 9810 kg m$^{-2}$ s$^{-2}$) when the energy conversion losses and the hydraulic losses are insignificant ($\eta = 1$ and $h = z_0 + z$, respectively). Here we assume that $\psi$ has a constant value smaller than $\psi = 0.2725$ GWh/hm$^4$. Combining (11) and (9) we get

$$e = \psi r \left[z_0 + z_{\max}(s/k)^{1/\zeta}\right] \tag{12}$$

which is the final equation used in simulations.

[35] Two versions or types of the reservoir system are studied, which are named symmetric (S) and nonsymmetric (NS). All characteristics of the system including constants for the power production equation are given in Table 3. The basic difference between them is that in type S the extensive characteristics, such as storage capacity and discharge capacity, are proportional to the inflows and the intensive characteristics, such as constants of several relationships, are equal for both reservoirs. The equivalent reservoir approach was used with the type S system only and the characteristics of the equivalent reservoir are shown in Table 3.

## 5. Test Problems

[36] The problems studied fall into three categories, whose objectives are, respectively, the maximization of

**Table 2.** Basic Characteristics of the PAR(1) Model for Reservoir Inflows (Constant For All Months)

| | Scenario LV | | Scenario HV | |
|---|---|---|---|---|
| | Reservoir 1 | Reservoir 2 | Reservoir 1 | Reservoir 2 |
| Coefficient of variation | 0.5 | 0.5 | 0.7 | 0.7 |
| Coefficient of skewness | 1.0 | 1.5 | 1.0 | 1.5 |
| Lag one autocorrelation | 0.7 | 0.8 | 0.7 | 0.8 |
| Lag zero cross-correlation | 0.6 | 0.6 | 0.6 | 0.6 |

**Table 3.** Basic Characteristics of the Reservoir Systems

| | System NS | | System S | | |
|---|---|---|---|---|---|
| | Reservoir 1 | Reservoir 2 | Reservoir 1 | Reservoir 2 | Equivalent Reservoir |
| Catchment area (km$^2$) | 500 | 600 | 500 | 600 | 1100 |
| Storage capacity $k$ (hm$^3$) | 150 | 300 | 150 | 253.2 | 403.2 |
| Max water level $z_{\max}$ (m) | 60 | 70 | 60 | 60 | 60 |
| Coefficient of stage-storage relationship (equation (9)) | | | | | |
| Exponent $\zeta$ | 3 | 3 | 3 | 3 | 3 |
| Coefficients of leakage relationships (equation (10)) | | | | | |
| $\xi_0$ (hm$^3$/month) | 1 | 0 | 0 | 0 | 0 |
| $\xi_1$ (hm$^3$/hm$^3$/month) | 0.01 | 0 | 0.01 | 0.01 | 0.01 |
| Characteristics of power plants (equation (12)) | | | | | |
| $\psi$ (GWh/hm$^4$) | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| $z_0$ (m) | 30 | 20 | 30 | 30 | 30 |
| $c$ (hm$^3$/month) | 50 | 100 | 28 | 47.3 | 75.3 |

the reliable release for a certain reliability level (for consumptive uses, i.e. irrigation and water supply), the minimization of the cost to convey water into consumption (again for consumptive uses), and the maximization of the benefit from energy production. A description of the problems of each category follows.

## 5.1. Maximization of Reliable Release

[37] The problem of maximization of the reliable release has been already posed in the introduction on an annual basis. Here we describe it on a monthly basis for a reservoir system with two reservoirs.

### 5.1.1. Parametric Approach

[38] Considering two instances of the parametric reservoir rule, one for the refill period and one for the drawdown period, the unknown parameters will be four, namely the parameters $a^1$ and $b^1$ of the first reservoir for both periods (those of the second reservoir are determined from (8)). The annual demand or target release, $d_{\text{ann}}$, from the system is an additional unknown; the monthly demands $d_t$ are determined in terms of $d_{\text{ann}}$ and the monthly percentages given in Table 1. Thus the problem involves five control variables to be determined by optimization, which compose the vector

$$\boldsymbol{\lambda} = \left(d_{\text{ann}}, a_{\text{r}}^1, b_{\text{r}}^1, a_{\text{d}}^1, b_{\text{d}}^1\right) \qquad (13)$$

where the subscripts "r" and "d" refer to the refill and drawdown period, respectively. If we assume the homogeneous form of the reservoir rule ($a_{\text{r}}^1 = a_{\text{d}}^1 = k^1/k$), or if we do not distinguish between refill and drawdown period, then $\boldsymbol{\lambda}$ will have three items. Furthermore, if we assume one period and simultaneously the homogeneous form of the reservoir rule, $\boldsymbol{\lambda}$ will have two items.

[39] An appropriate performance measure can be based on the adjusted average release from the system, i.e.,

$$L = \frac{1}{n}\sum_{t=1}^{n} r_t + \frac{1}{n}(s_n - s_0) \qquad (14)$$

where the release $r_t$ and storage $s_t$ are the sums of the respective quantities of both reservoirs. The adjustment $(1/n)$ $(s_n - s_0)$ expresses the mean annual increase (or decrease) of the storage of the system throughout the simulation period, and its incorporation is required for a fair comparison of different solutions. $L$ is evaluated by means of simulation.

[40] The parameters obey the inequality constraints in (8), which in fact facilitate calculations as they narrow the domain of the control variables. There is one additional constraint, the restriction of the reliability (in satisfying demand) above an accepted limit $\alpha$, which in yearly basis is expressed by

$$\frac{12}{n}\sum_{p=1}^{n/12} \min\{U(r_t - d_t); t = 12(p-1)+1, \ldots, 12p\} \geq \alpha \qquad (15)$$

where $U(x)$ is the Heaviside's unit step function, with $U(x) = 1$ for $x \geq 0$ and $U(x) = 0$ for $x < 0$. The sum in the left-hand side of (15) counts the years where the monthly release $r_t$ (the sum of both reservoirs) meets the demand $d_t$ in all months of the year. Note that there is a fundamental difference between constraints (8) and (15). The former is a parameterization constraint and is evaluated directly at the parameterization phase, while the latter is a global constraint that can be evaluated only after the simulation is complete. A third category of constraints, referred to as simulation constraints, includes upper bounds for releases, storages etc. (as described in section 4), which are handled directly by the simulation model and are not entered into the optimization problem formulation at all.

[41] In brief, the statement of this problem is to maximize $L(\boldsymbol{\lambda})$, as defined in (14), subject to (8) and (15). It is a low-dimensional nonlinear optimization problem. The problem statement and algorithm is exactly the same for both consumptive water uses, irrigation and water supply.

### 5.1.2. Perfect Foresight Approach

[42] The perfect foresight approach followed here is to split the demand $d_t$ of each time step (month) $t$ into two partial demands $d_t^1$ and $d_t^2$, one for each reservoir. Utilizing the fact that $d_t^1 + d_t^2 = d_t$, we introduce the sequence of numbers $x_t$ with

$$0 \leq x_t \leq 1, \qquad t = 1, 2, \ldots, n \qquad (16)$$

such that $d_t^1 = x_t d_t$ and $d_t^2 = (1 - x_t)\, d_t$. Thus the control variables to be optimized are the $n$ numbers $x_t$ (rather than the 2 $n$ variables $d_t^1$ and $d_t^2$) plus the annual release $d_{\text{ann}}$. Given the values thereof and consequently those of $d_t^1$ and

$d_t^2$, we can estimate the sequences of release, storage and spill from (3) applied separately for each reservoir.

[43] The performance measure is the same as in the parametric approach, i.e., as in (14), and the constraint (15) is valid here as well. The essential difference is that in the high dimensional approach the vector of control variables contains the $n$ variables $x_t$ plus the annual demand $d_{\text{ann}}$. The performance measure $L$ and the left-hand side of the constraint (15) are functions of this vector. In brief, in its high dimensional form the problem is to maximize $L(d_{\text{ann}}, x_1, \ldots, x_n)$ as defined in (14), subject to (16) and (15).

#### 5.1.3. Equivalent Reservoir Approach

[44] The equivalent reservoir approach is a one-dimensional approach, as the only unknown to be optimized is the annual demand $d_{\text{ann}}$. The formulation is very similar to that already described in the introduction, but with a monthly rather than annual time step. In brief, in its one-dimensional form the problem is to maximize $L(d_{\text{ann}})$ as defined in (14), subject to (15).

### 5.2. Minimization of Cost

[45] If the maximum reliable release from the system exceeds the demand, then we can apply a different operating policy, which will result in a smaller release from the system. This problem becomes interesting if there is some conveyance cost $\kappa^j$ per unit water, different for each reservoir $j$. Here we have assumed that the demand is about 90% of the maximum reliable release and the conveyance cost for reservoir 1 is 30% higher than that of reservoir 2 (i.e., $\kappa^1 = 1.3$, $\kappa^2 = 1$, in arbitrary units). As this problem relies on the different characteristics of the two reservoirs, the equivalent reservoir approach is not applicable here.

#### 5.2.1. Parametric Approach

[46] The vector of parameters $\boldsymbol{\lambda}$ contains one parameter less than in equation (13) as now the annual demand $d_{\text{ann}}$ is a known constant. The performance measure is the mean annual cost, i.e.,

$$L = \kappa^1 \frac{12}{n} \sum_{t=1}^{n} r_t^1 + \kappa^2 \frac{12}{n} \sum_{t=1}^{n} r_t^2 \tag{17}$$

whereas constraint (15) remains valid. Thus our problem is to minimize $L(\boldsymbol{\lambda})$ as defined in (17), subject to (8) and (15).

#### 5.2.2. Perfect Foresight Approach

[47] The control variables are the $n$ numbers $x_t$ as defined in section 5.1.2. The problem is to maximize $L(x_1, \ldots, x_n)$ as defined in (17), subject to (16) and (15).

### 5.3. Maximization of Benefit From Energy Production

[48] In general, the value of produced energy may vary through the months of the year, as demonstrated by *Kelman et al.* [1990], who introduced the concept of monthly "subjective coefficients" that adjust the monthly generated energies assuming low values for months with plentiful of hydroelectric energy and high values otherwise. This concept is directly applicable to the PSO approach by setting different energy targets and values in different months; for simplicity of the demonstration, however, we assumed constant targets and values for all months. It is also known [e.g., *Georgakakos et al.*, 1997] that the value of hydroelectric energy vary in real time subject to various water- and power-demand states and constraints, so that real time operation of hydropower plants requires optimal allocation of turbine load to maximize the energy value; this, however, is beyond the scope of this paper.

[49] In the simplified approach followed here, we distinguish between two kinds of energy with different prices [e.g., *Stevens and Davis*, 1969, p. 24.8; *Mays and Tung*, 1992, p. 283; *ReVelle*, 1999, p. 59]. The firm or primary energy is that produced at a constant rate (target energy) throughout the entire simulation period and has a higher price $\pi_f$. The energy produced in excess of primary energy is the secondary or excess energy and has a lower price $\pi_s$; here we assume that $\pi_f = 1$ and $\pi_s = 0.5$ in arbitrary units. Energy production in deficit of the set monthly energy target is penalized by applying the price $\pi_s$, rather than $\pi_f$, for all produced energy of that month. The problem is to maximize the total benefit from both primary and secondary energy minus the penalties for deficit.

#### 5.3.1. Parametric Approach

[50] With reference to the problem of maximization of reliable release, as formulated in section 5.1.1, the essential difference here is that we do not have a constant annual water demand $d_{\text{ann}}$ but rather an annual energy target $\delta_{\text{ann}}$ which is equally distributed over all months. Therefore, if we assume different parameter sets for the refill and the drawdown period, the vector of unknown parameters is similar to that in equation (13) but with $\delta_{\text{ann}}$ replacing $d_{\text{ann}}$.

[51] The simulation procedure in this case is somewhat more complicated than in the case of maximization of the reliable release. In each simulation step, given the target energy $\delta_{\text{mon}} = \delta_{\text{ann}}/12$, the required releases $r_t^1$ and $r_t^2$ from each reservoir are estimated by an iterative procedure which terminates when the target energy is met (first condition) and simultaneously the reservoir storages fulfill the parametric rule (second condition). These two conditions, in conjunction with (12), which transforms release to energy, are sufficient to determine a unique set of releases $r_t^1$ and $r_t^2$ at each time step $t$. The procedure does some special processing when the reservoirs become empty (the produced energy is less than the target energy) or are about to spill (the produced energy is greater than the target energy).

[52] The performance measure will be the mean annual benefit minus penalty, i.e.,

$$L = \pi_f \delta_{\text{mon}} \frac{12}{n} \sum_{t=1}^{n} U(e_t - \delta_{\text{mon}}) + \pi_s \frac{12}{n} \sum_{t=1}^{n} (e_t - \delta_{\text{mon}}) \cdot U(e_t - \delta_{\text{mon}}) + \pi_s \frac{12}{n} \sum_{t=1}^{n} e_t [1 - U(e_t - \delta_{\text{mon}})] \tag{18}$$

where the energy $e_t$ is the sum of the energies produced from each reservoir, as estimated from (12). The first sum of Heaviside's functions $U()$ in the right-hand side of (18) counts the months with produced energy $e_t$ greater than or equal to the target $\delta_{\text{mon}}$; thus the first term represents the value of primary energy. Likewise, the second term represents the value of secondary energy whereas the third term represents the penalized value of energy during months with deficit of energy production.

[53] To avoid intentional emptying of the reservoirs (by the optimization algorithm) at the end of the simulation period, a constraint is required, i.e.,

$$s_n \geq s_0 \tag{19}$$

Thus the problem formulation is to maximize $L(\boldsymbol{\lambda})$ as defined in (18), subject to (8) and (19).

#### 5.3.2. Perfect Foresight Approach

[54] We follow an approach similar to that in section 5.1.2 based on the partial demands $d_t^1$ and $d_t^2$, one for each reservoir. However, in this case the sum of demands is not known and therefore we cannot use the fractions $x_t$ as control variables. Rather, we use both series $d_t^1$ and $d_t^2$ as control variables, a total of 2 $n$ variables, satisfying

$$0 \leq d_t^1 \leq c^1, \quad 0 \leq d_t^2 \leq c^2, \quad t = 1, \ldots, n \tag{20}$$

Given the values of $d_t^1$ and $d_t^2$, we can estimate the sequences of release $r_t^j$ using (3), then the energies $e_t^j$ from (12) and their sum $e_t$.

[55] The performance measure is the same as in the parametric approach, i.e., as in (18), and the constraint (19) is valid here as well. The essential difference is that in the perfect foresight approach the vector of control variables contains the 2 $n$ variables $d_t^j$ plus the target energy $\delta_{\text{ann}}$. Thus the problem statement is to maximize $L(\delta_{\text{ann}}, d_t^1, d_t^2, \ldots, d_n^1, d_n^2)$ as defined in (18), subject to (20) and (19).

#### 5.3.3. Equivalent Reservoir Approach

[56] The equivalent reservoir approach is again a one-dimensional approach, as the only unknown to be optimized is the annual target energy $\delta_{\text{ann}}$. Given the monthly target energy $\delta_{\text{mon}} = \delta_{\text{ann}}/12$, we can determine the corresponding water demand from (12). Then the problem becomes very similar to that already described in the introduction, but with a varying rather than constant demand at each time step. In brief, in its one-dimensional form the problem is to maximize $L(\delta_{\text{ann}})$ as defined in (18), subject to (19).

### 5.4. Other Considerations

[57] It can be shown with simple statistical calculations that the required years of simulation $m$ to obtain an accurate estimate of the failure probability $\beta$ on an annual basis (in satisfying a certain demand) with an acceptable error $\pm\varepsilon\beta$ and confidence $\gamma$, is $m = (z_{(1 + \gamma)/2}/\varepsilon)^2 \, (1/\beta - 1)$, where $z_p$ is the $p$-quantile of the standard normal distribution. Assuming $\beta = 6\%$ (or reliability $\alpha = 1 - \beta = 94\%$), $\varepsilon = 10\%$ and $\gamma = 95\%$ (so that $z_{(1 + \gamma)/2} = 1.96$) we get $m \approx 6000$ years or $n \approx 72\,000$ months. This indicates that the required simulation lengths in reservoir problems must be of the orders of thousands to tens of thousand years.

[58] It is a matter of a second to perform a simulation of that length using the PSO method or the method of equivalent reservoir when applicable. However it is intractable to use a high dimensional methodology because in this case the number of control variables would be 72 000 or 144 000 depending on the type of the problem. Apparently, this is a serious advantage of the parametric method over the high dimensional one. However, our purpose here is not to obtain as accurate estimates of some quantities as possible, but rather to compare the results of different methods. Naturally, this can be done using much smaller simulation lengths, in order for the high dimensional method to be applicable.

[59] As an optimization tool, a commercial solver developed by Frontline Systems (http://frontsys.com), which uses a number of methods from the literature on genetic and evolutionary algorithms, was adopted for all three methods used in this paper. It uses a real-number representation rather than a bit-string or encoded representation of the problem, and it handles both integer and general continuous variables. The solver uses both mutation and crossover or recombination to generate new points. Some of the specific methods are proprietary, but uniform mutation, bounds mutation, and convex combination in crossover are all used. Most-fit members are selected through tournament selection and least-fit members are selected for elimination via a proprietary method. The population is updated incrementally rather than entirely replaced at each generation. Constraints are handled by a combination of penalty functions and "constraint repair" methods. Some parameters of the algorithm are adapted during the solution process (D. H. Fylstra, personal communication). This solver can handle problems with up to 400 variables. This means that a simulation period of 50 years is tractable for the irrigation problems (50 years $\times$ 7 variables per year = 350 variables; note that irrigation lasts 7 months per year) and 16 years for the energy production problems (16 years $\times$ (2 $\times$ 12) variables per year = 384 variables); for the water supply problems a period of 16 years was chosen, too (16 years $\times$ 12 variables per year = 192 variables).

## 6. Application and Results

[60] Combining the three categories of problems described above, the three solution methods, the three water uses, the two versions of the reservoir system and the two hydrological scenarios, we constructed and resolved 41 different problems marked as 1 to 41 (not exhausting all possible combinations), whose characteristics are listed in the first two columns of Tables 4 (first category, maximization of release), 5 (second category, minimization of cost) and 6 (third category, maximization of benefit from energy production). The problems are forming 13 groups marked as I to XIII, so that the problems of each group have exactly the same characteristics apart from the method followed to solve them and the number of control variables. For example, as shown in Table 4, problems 1–5 belong to group I; the objective in all of them is to maximize the reliable release (MR); the water use is irrigation (IR); the reservoir system is type NS; and the hydrologic scenario is LV. For problem 5 the perfect foresight method is followed whereas problems 1–4 are remedied using the PSO methodology, with varying number of seasons (one or two) and parameters per season (one or two) in each case, as shown in column 2 of Table 4.

[61] The results of calculations are shown in Tables 4–6 as well. The results for the problem group I in Table 4 show that the PSO methodology with 5 control variables (problem 1; 2 parameters per season $\times$ 2 seasons + annual demand) resulted in practically the same performance as in the perfect foresight method with 351 control variables (problem 5). When the number of parameters of the PSO methodology becomes smaller than 5 (problems 2–4) there is a slight reduction in performance, but even with one parameter (problem 4; two control variables) the parameterization is very effective as the reduction in performance is only 1.68%. Similar are the results for the other problem groups of the same category regardless of the water use (irrigation or water supply). The reduction in performance is 0–0.2% for the parameterized schemes with 4 parameters and up to 2.9% for the parameterized scheme with one

**Table 4.** Definition and Results of Problems of the First Category (Maximization of Release)

| Problem Group[a] | Problem[b] | Mean Annual Volumes, $hm^3$ Inflow | Demand | Spill | Leakage | Storage Difference[c] | Release | Deficit[d] | Adjusted Release[e] | Attained Reliability, % | Performance Reduction,[f] % |
|---|---|---|---|---|---|---|---|---|---|---|---|
| I (IR/NS/LV) | 1 (PSO/2/5) | 311.9 | 244.4 | 48.7 | 20.3 | −0.7 | 243.7 | 0.7 | 243.0 | 94.00 | 0.00 |
| I (IR/NS/LV) | 2 (PSO/1/3) | 311.9 | 242.7 | 48.5 | 22.2 | −0.8 | 242.0 | 0.7 | 241.2 | 94.00 | −0.72 |
| I (IR/NS/LV) | 3 (PSO/2/3) | 311.9 | 241.3 | 55.7 | 16.0 | −0.4 | 240.7 | 0.7 | 240.3 | 94.00 | −1.11 |
| I (IR/NS/LV) | 4 (PSO/1/2) | 311.9 | 240.1 | 52.2 | 20.8 | −0.5 | 239.3 | 0.7 | 238.9 | 94.00 | −1.68 |
| I (IR/NS/LV) | 5 (PF/-/351) | 311.9 | 244.4 | 48.7 | 20.3 | −0.7 | 243.7 | 0.7 | 243.0 | 94.00 | 0.00 |
| II (IR/NS/HV) | 6 (PSO/2/5) | 317.1 | 217.5 | 85.3 | 16.0 | −0.2 | 216.0 | 1.5 | 215.8 | 94.00 | −0.20 |
| II (IR/NS/HV) | 7 (PF/-/351) | 317.1 | 217.5 | 84.3 | 16.5 | −0.2 | 216.5 | 1.1 | 216.2 | 94.00 | 0.00 |
| III (IR/S/LV) | 8 (PSO/2/5) | 311.9 | 235.7 | 49.7 | 29.3 | −1.1 | 234.0 | 1.7 | 232.9 | 94.00 | −0.12 |
| III (IR/S/LV) | 9 (ER/-/1) | 311.9 | 235.7 | 48.5 | 30.3 | −1.1 | 234.2 | 1.5 | 233.1 | 94.00 | −0.05 |
| III (IR/S/LV) | 10(PF/-/351) | 311.9 | 236.8 | 49.3 | 29.4 | −1.1 | 234.3 | 2.4 | 233.2 | 94.00 | 0.00 |
| IV (WS/NS/LV) | 11 (PSO/2/5) | 298.2 | 258.3 | 23.8 | 11.0 | 5.7 | 257.6 | 0.6 | 263.3 | 93.75 | 0.00 |
| IV (WS/NS/LV) | 12 (PSO/1/3) | 298.2 | 256.8 | 29.6 | 8.4 | 4.0 | 256.2 | 0.6 | 260.2 | 93.75 | −1.18 |
| IV (WS/NS/LV) | 13 (PSO/2/3) | 298.2 | 254.6 | 18.8 | 19.3 | 6.1 | 253.9 | 0.6 | 260.1 | 93.75 | −1.25 |
| IV (WS/NS/LV) | 14 (PSO/1/2) | 298.2 | 250.0 | 20.9 | 21.7 | 6.3 | 249.4 | 0.6 | 255.6 | 93.75 | −2.93 |
| IV (WS/NS/LV) | 15 (PF/-/193) | 298.2 | 258.3 | 23.8 | 11.0 | 5.7 | 257.6 | 0.6 | 263.3 | 93.75 | 0.00 |
| V (WS/NS/HV) | 16 (PSO/2/5) | 297.6 | 228.2 | 48.7 | 14.4 | 6.7 | 227.8 | 0.4 | 234.5 | 93.75 | 0.00 |
| V (WS/NS/HV) | 17 (PF/-/193) | 297.6 | 228.2 | 48.7 | 14.4 | 6.7 | 227.8 | 0.4 | 234.5 | 93.75 | 0.00 |
| VI (WS/S/LV) | 18 (PSO/2/5) | 298.2 | 240.6 | 23.5 | 30.0 | 4.7 | 240.1 | 0.6 | 244.7 | 93.75 | 0.00 |
| VI (WS/S/LV) | 19 (ER/-/1) | 298.2 | 240.6 | 23.5 | 30.0 | 4.7 | 240.1 | 0.6 | 244.7 | 93.75 | 0.00 |
| VI (WS/S/LV) | 20 (PF/-/193) | 298.2 | 240.6 | 23.5 | 29.9 | 4.7 | 240.1 | 0.6 | 244.7 | 93.75 | 0.00 |

[a]Explanation of symbols in parentheses: (1) water use (IR, irrigation; WS, water supply; HP, hydropower production), (2) type of reservoir system (S, symmetric; NS, nonsymmetric; see Table 3), and (3) hydrologic scenario (LV, lower variation; HV, higher variation; see Table 2).
[b]Explanation of symbols in parentheses: (1) method (PSO, parameterization-simulation-optimization; ER, equivalent reservoir; PF, perfect foresight), (2) number of seasons, and (3) total number of control variables.
[c]$(s_n - s_0)/n$.
[d]Demand − release.
[e]Performance measure = release − storage difference.
[f]With regard to performance measure of the PF problem.

parameter only. Notably, the equivalent reservoir methodology, which was applied to problem groups III and VI (problems 9 and 19, respectively) resulted in performance practically as good as the perfect foresight method and the PSO method with 4 parameters. It is noted that in group III the slight (by 0.05%), superiority of the perfect foresight method (problem 10) in comparison to the equivalent reservoir method (problem 9) is artificial: during a year with failure, the perfect foresight method increased artificially the deficit of that year by saving some water in the reservoir, thus avoiding the failure in the next year and subsequently increasing slightly the target release for the entire period.

[62] Furthermore, in Table 5 we may observe that the results of the parametric method with 4 parameters are almost identical to those of the perfect foresight method with 350 variables (irrigation) or 192 variables (water supply). The biggest difference in performance (minimum cost) appears in problem group IX and is 0.24%, an insignificant value.

**Table 5.** Definition and Results of Problems of the Second Category (Minimization of Cost)

| Problem Group[a] | Problem[b] | Mean Annual Volumes, $hm^3$ Inflow | Demand | Spill | Leakage | Storage Difference[c] | Release | Deficit[d] | Adjusted Release[e] | Attained Reliability, % | Cost[f] | Performance Reduction,[g] % |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| VII (IR/NS/LV) | 21 (PSO/2/4) | 311.9 | 218.9 | 70.2 | 27.3 | −2.3 | 216.7 | 2.3 | 214.4 | 96.00 | 223.9 | 0.00 |
| VII (IR/NS/LV) | 22 (PF/-/350) | 311.9 | 218.9 | 70.2 | 27.3 | −2.3 | 216.7 | 2.3 | 214.4 | 94.00 | 223.9 | 0.00 |
| VIII (IR/NS/HV) | 23 (PSO/2/4) | 317.1 | 191.7 | 101.4 | 27.7 | −0.7 | 188.7 | 3.0 | 187.9 | 96.00 | 192.6 | 0.00 |
| VIII (IR/NS/HV) | 24 (PF/-/350) | 317.1 | 191.7 | 101.4 | 27.7 | −0.7 | 188.7 | 3.0 | 187.9 | 94.00 | 192.6 | 0.00 |
| IX (WS/NS/LV) | 25 (PSO/2/4) | 298.2 | 230.9 | 39.6 | 25.5 | 2.3 | 230.8 | 0.2 | 233.1 | 93.75 | 242.2 | 0.24 |
| IX (WS/NS/LV) | 26 (PF/-/192) | 298.2 | 230.9 | 39.0 | 26.8 | 1.8 | 230.4 | 0.5 | 232.3 | 93.75 | 241.6 | 0.00 |
| X (WS/NS/HV) | 27 (PSO/2/4) | 297.6 | 205.4 | 58.8 | 27.3 | 6.4 | 205.1 | 0.3 | 211.5 | 93.75 | 213.3 | 0.00 |
| X (WS/NS/HV) | 28 (PF/-/192) | 297.6 | 205.4 | 58.8 | 27.3 | 6.4 | 205.1 | 0.3 | 211.5 | 93.75 | 213.3 | 0.00 |

[a]Explanation of symbols in parentheses: (1) water use (IR, irrigation; WS, water supply; HP, hydropower production), (2) type of reservoir system (S, symmetric; NS, nonsymmetric; see Table 3), and (3) hydrologic scenario (LV, lower variation; HV: higher variation; see Table 2).
[b]Explanation of symbols in parentheses: (1) method (PSO, parameterization-simulation-optimization; ER, equivalent reservoir; PF, perfect foresight), (2) number of seasons, and (3) total number of control variables.
[c]$(s_n - s_0)/n$.
[d]Demand − release.
[e]Release − storage difference.
[f]Minimized performance measure. Cost is in arbitrary units.
[g]With regard to performance measure of the PF problem.

**Table 6.** Definition and Results of Problems of the Third Category (Maximization of Benefit From Energy Production)

| Problem Group[a] | Problem[b] | Mean Annual Volumes, hm$^3$ | | | | | | Annual Energy, GWh | | | Benefit[e] | Performance Reduction,[f] % |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Inflow | Spill | Leakage | Storage Difference[c] | Release | Adjusted Release[d] | Target | Produced Primary | Produced Secondary | | |
| XI (HP/NS/LV) | 29 (PSO/2/5) | 298.2 | 0.1 | 21.1 | 7.6 | 269.4 | 276.9 | 48.7 | 48.7 | 7.9 | 52.7 | −1.35 |
| XI (HP/NS/LV) | 30 (PSO/1/3) | 298.2 | 0.1 | 23.9 | 7.9 | 266.2 | 274.1 | 48.2 | 48.2 | 7.7 | 52.1 | −2.48 |
| XI (HP/NS/LV) | 31 (PSO/2/3) | 298.2 | 0.0 | 21.1 | 7.6 | 269.4 | 277.0 | 48.9 | 48.9 | 7.5 | 52.6 | −1.44 |
| XI (HP/NS/LV) | 32 (PSO/1/2) | 298.2 | 0.1 | 23.9 | 7.9 | 266.2 | 274.2 | 48.2 | 48.2 | 7.7 | 52.1 | −2.49 |
| XI (HP/NS/LV) | 33 (PF/-/385) | 298.2 | 0.0 | 20.9 | 0.0 | 277.3 | 277.3 | 48.7 | 48.7 | 9.4 | 53.4 | 0.00 |
| XII (HP/NS/HV) | 34 (PSO/2/5) | 297.6 | 0.9 | 20.4 | 8.0 | 268.3 | 276.3 | 43.8 | 43.8 | 12.3 | 50.0 | −1.66 |
| XII (HP/NS/HV) | 35 (PSO/1/3) | 297.6 | 0.9 | 23.2 | 8.4 | 265.1 | 273.5 | 43.0 | 43.0 | 12.5 | 49.3 | −2.99 |
| XII (HP/NS/HV) | 36 (PSO/2/3) | 297.6 | 0.9 | 20.4 | 8.0 | 268.3 | 276.3 | 43.8 | 43.8 | 12.3 | 50.0 | −1.66 |
| XII (HP/NS/HV) | 37 (PSO/1/2) | 297.6 | 0.9 | 23.2 | 8.4 | 265.1 | 273.5 | 43.0 | 43.0 | 12.5 | 49.3 | −3.00 |
| XII (HP/NS/HV) | 38 (PF/-/385) | 297.6 | 0.0 | 20.2 | 0.0 | 277.5 | 277.5 | 43.8 | 43.8 | 14.0 | 50.8 | 0.00 |
| XIII (HP/S/LV) | 39 (PSO/2/5) | 298.2 | 4.2 | 33.9 | 6.1 | 254.0 | 260.1 | 46.4 | 46.4 | 6.3 | 49.6 | −1.92 |
| XIII (HP/S/LV) | 40 (ER/-/1) | 298.2 | 4.0 | 34.1 | 6.0 | 253.9 | 260.1 | 46.2 | 46.2 | 6.4 | 49.4 | −2.33 |
| XIII (HP/S/LV) | 41 (PF/-/385) | 298.2 | 0.0 | 32.1 | 0.0 | 266.0 | 266.0 | 46.6 | 46.6 | 7.9 | 50.6 | 0.00 |

[a]Explanation of symbols in parentheses: (1) water use (IR, irrigation; WS, water supply; HP, hydropower production), (2) type of reservoir system (S, symmetric; NS, nonsymmetric; see Table 3), and (3) hydrologic scenario (LV, lower variation; HV, higher variation; see Table 2).
[b]Explanation of symbols in parentheses: (1) method (PSO, parameterization-simulation-optimization; ER, equivalent reservoir; PF, perfect foresight), (2) number of seasons, and (3) total number of control variables.
[c]$(s_n - s_0)/n$.
[d]Release − storage difference.
[e]Maximized performance measure. Benefit is in arbitrary units.
[f]With regard to performance measure of the PF problem.

[63] Some more significant differences in performance appear in the problems of the third category, those dealing with the maximization of the benefit from energy production, as shown in Table 6. It is noted that in all cases all methods managed to meet the target release, thus avoiding penalization. The biggest difference in performance of the parametric method with four parameters with regard to that of the perfect foresight method is 1.9%, and appears in problem group XIII (problems 39–41). Notably, however, the parametric method (problem 39) performs slightly better than the method of the equivalent reservoir (problem 40).

[64] To acquire a better insight of the behavior of the three methods we give more detailed results for problems 39–41 of group XIII, which seems the most interesting, as it gave the biggest differences in the performance of the three methods. Thus Figure 4 provides a graphical representation of parametric operating rules for problem 39. We may observe that the optimization procedure resulted in different rules for the refill period (November to April) and the drawdown period (May to October), which both depart from Clark's space rule. In this case, the latter rule coincides with the symmetric rule (see section 2.2). We remind that the reservoir system used in this problem is characterized by symmetry and the latter rule represents the symmetry in the reservoir operation. Had we adopted the space rule for the system operation, the resulting benefit would be 49.4 units, the same value resulted from the equivalent reservoir method (problem 40). Interestingly, by breaking the symmetry and using different parameters for the refill and drawdown periods, we were able to slightly increase benefit to 49.6 units. Specifically, the increase is caused by the significant change of the rule during the drawdown period. In this period the probability of spill is insignificant and it may be better to store as much water as possible in the smaller reservoir 1, because in this case we will have greater hydraulic head for the same amount of water. This is exactly represented by the rule of the drawdown period shown in the middle of Figure 4. Even though the increase of performance of 0.2 units is small (0.4%), it indicates that a reservoir system operated with the parametric rule can be more efficient than a single equivalent reservoir merging the characteristics of the different reservoirs of the system.

[65] Figure 5 depicts the evolution of the storage, release, leakage, spill, and produced energy from the reservoir system in problem group XIII (problems 39–41). The curves for the PSO method (problem 39) are almost indistinguishable from those of the equivalent reservoir method (problem 40), which means that the parametric rule guides the system very close to the "symmetric" evolution of the equivalent reservoir, although some slight differences exist (not distinguishable in the figure) which are responsible for the already discussed slight (0.4%) improvement of performance. Furthermore, the curves for the PSO method (problem 39) are almost indistinguishable from those of the perfect foresight method (problem 41) for most of the time apart from months 140–192. Specifically, we observe that both PSO and the equivalent reservoir methods resulted in spill from the system at months 161 and 172, whereas the perfect foresight method avoided that spill by increasing the releases of previous months, and thus it was able to produce some additional secondary energy. In addition the perfect foresight method increased the releases at the last months of the simulation period so as to yield a total storage at the end of simulation ($s_n$) equal to that of the beginning ($s_0$). We remind that in the problems of this category we have posed the constraint $s_n \geq s_0$ (relationship (19)). Thus, in the perfect foresight method this became a binding constraint whereas in the other two methods this was not a binding constraint (there is a surplus $s_n - s_0$). These two facts explain how the perfect foresight method was able to increase the system performance.

[66] Interestingly, inspecting the evolution of the separate reservoirs 1 and 2 (not shown in Figure 5, which depicts aggregate quantities of both reservoirs), it is observed that

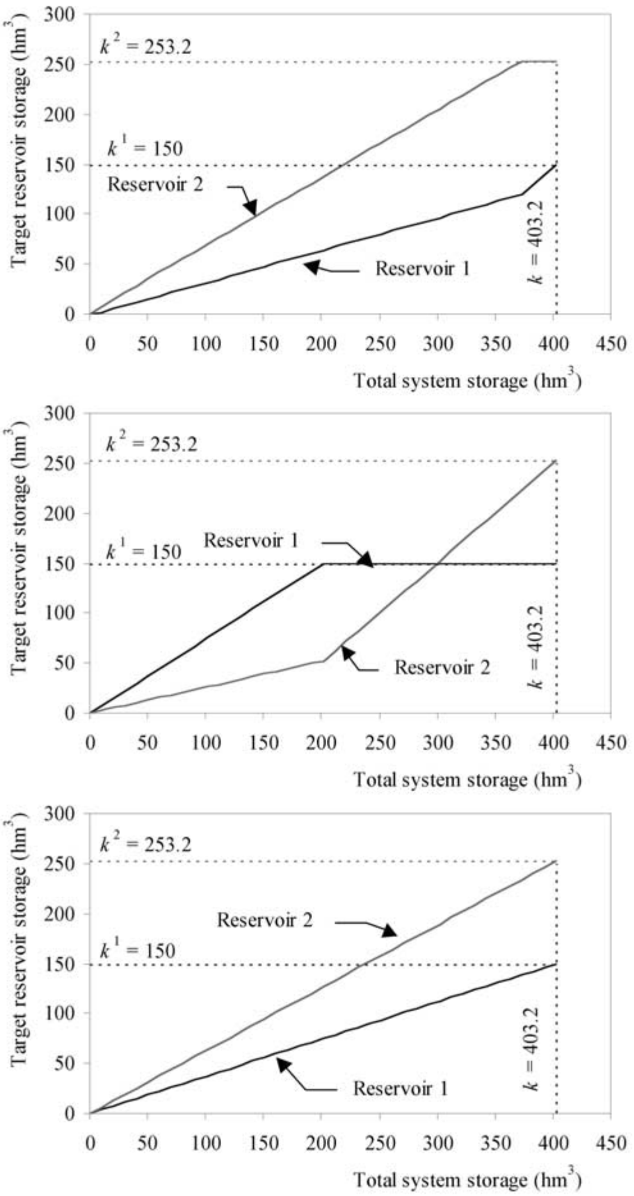


**Figure 4.** Graphical representation of parametric operating rules for problem 39; top panel: rule for the refill period (November to April); middle panel: rule for the drawdown period (May to October); bottom panel: Clark's space rule coinciding with the symmetric rule (for comparison only). The parameters of the rules are $a^1 = 1 - a^2 = 0.375$ and $b^1 = 1 - b^2 = 0.350$ for the refill period, $a^1 = 1 - a^2 = 0.372$ and $b^1 = 1 - b^2 = 1$ for the drawdown period, and $a^1 = b^1 = 1 - a^2 = 1 - b^2 = 0.372$ for the space rule.

the parametric and perfect foresight methods guided the separate reservoirs to very different evolutions, although the aggregate quantities of the reservoir system are close as discussed above. Furthermore, it is observed that in PSO both reservoirs spilled simultaneously at the two spill periods discussed above. If only one reservoir had spilled whereas the other had free space to store water, this would indicate inappropriateness or misspecification of the parametric rule in allocating the storage in each reservoir. Conversely, the simultaneous spill confirms the good performance of the parametric rule.

[67] Following the observations regarding how the perfect foresight method improved by 1.9% the performance of the system, it is almost clear that this is due to the fact that it modified the releases at past times to better exploit the inflows at future times. This, however, is the result of the assumption that the future inflows are perfectly known, which is unrealistic. For example, it cannot be accurately predicted at month 150 that the system will spill at month 161 so as to increase the release at time 150, as actually the perfect foresight method did. Therefore the improvement is artificial and cannot be implemented in practice.

[68] On the other hand, the parametric method does not necessarily rely on forecasts of future inflows. To better investigate this, we applied the parametric and the equivalent reservoir methods 20 more times using different inflow series with the same length (16 years) generated with the same stochastic model. The series were chosen in a manner that the average annual inflow of the 16 years does not differ from that of the initial series more than 5%. We did not perform any optimization but rather we used the values of the control variables estimated in problems 39–41. The average performance measure (benefit) of the 20 simulations was 49.6 and 49.5 for the PSO method and the equivalent reservoir method, respectively, i.e., almost identical to the values 49.6 and 49.4 shown in Table 6 for problems 39 and 41, respectively. The application of this technique with the perfect foresight method would not have a sense, as the decision variables of the latter are tightly connected to inflows and the change of inflow series would cancel the validity of the results.

[69] Similar investigations, using 20 alternative inflow series, were done for other problems as well and in all cases they showed a good performance of the PSO method. Thus, in problem group V of the maximum release category, the average (over the 20 runs) attained reliability for the parametric method was 94.1% (the minimum acceptable is 93.75%) and the performance measure (adjusted release) was 229.3, 2.2% smaller than the optimized performance for the original series. Furthermore, in the problem group IX, which belongs to the cost minimization category, the performance measure of the parametric method was 242.3, almost equal to the optimized performance for the original series, and the attained reliability 97.5%, higher than the acceptable 93.75%.

[70] These results do not necessarily mean that forecasts of inflows do not have any value for the PSO method. Available forecasts (extending, e.g., a few months) can be utilized in several ways. First, the forecasts should be incorporated in the early part of the longer inflow series that are used for the reservoir system simulation. Then, during that early simulation period, a separate set of parameters of the parametric rule could be used, which are entered into the optimization procedure as additional control variables into the vector $\boldsymbol{\lambda}$. Another possibility is to use a hybrid approach in which for the first few months the releases from different reservoirs are not determined by the parametric rule but rather entered directly into the vector of control variables $\boldsymbol{\lambda}$ whereas beyond the forecast lead-time the releases obey the parametric rule. The above results show that such modifications would lead to only marginal

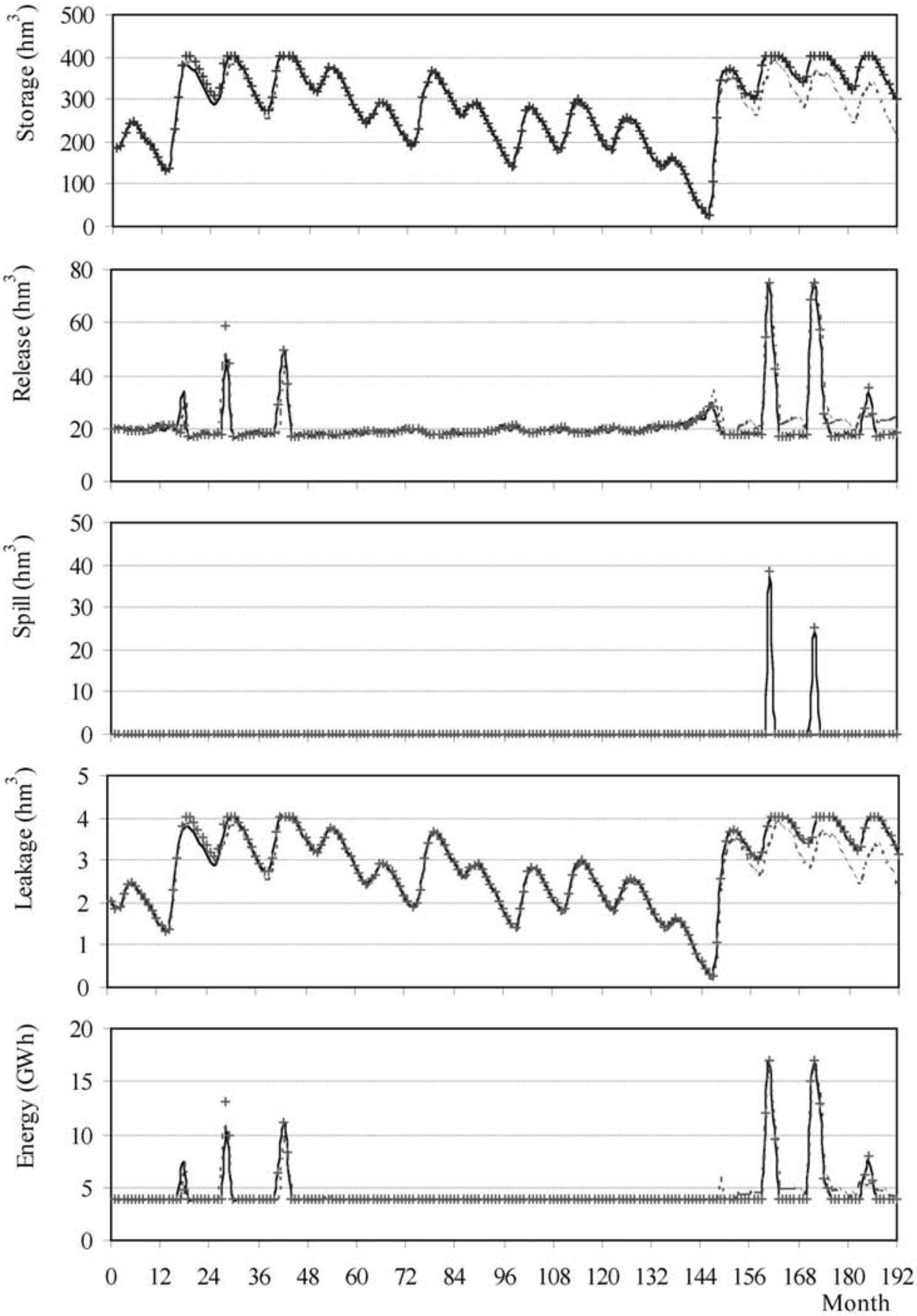


**Figure 5.** Evolution of the storage, release, spill, leakage, and produced energy in the reservoir system of problem group XIII as obtained through the PSO method (problem 39; solid lines), the equivalent reservoir method (problem 40; crosses), and the perfect foresight method (problem 41; dashed lines).

improvement of the performance a system. However, it may be interesting to study such modifications in critical situations, e.g., in cases where initial system storage is extremely low (near emptying) or extremely high (near spilling). Such a study is beyond the scope of this paper.

## 7. Summary and Conclusions

[71] In contrast to most common methods for optimal control of reservoir systems, which require a large number of control variables, PSO uses a handful of control variables. Specifically, the set of control variables consists of a "target variable" depending on the objective of the problem examined (e.g., target release or target energy) and a few parameters that determine a simple expression for allocating the degrees of freedom of the reservoir system operation, known as the parametric reservoir rule. The performance measure (or objective function) of the reservoir system operation, which is to be optimized, is a function of these control variables. Its value for a given set of values of the

control variables is obtained by simulation of the reservoir system. The control problem involves also constraints (e.g., dealing with the acceptable system reliability), which are either handled within simulation or evaluated at the end of simulation. An optimization algorithm is used to obtain the optimal values of control variables; for each trial set of values of the control variables it runs the simulation model, evaluates the performance measure and the constraints, and subsequently modifies the control variables guiding them toward their optimal values.

[72] To evaluate the parameterization-simulation optimization method we have compared it to the high-dimensional perfect foresight method, which, although perfect foresight is not feasible in real world, can be regarded as the gold standard against which any other method can be compared. As an additional means of comparison, a simplified method that merges the reservoir system into a single hypothetical "equivalent reservoir" was used, which, although does not suffice for the system control, can determine in a simple manner a "benchmark performance measure that in certain cases is an upper bound for the system. The comparison is done both by theoretical reasoning and by empirical investigation of the results of the alternative methods in a large variety of test problems.

[73] In the theoretical level, the PSO method exhibits several advantages over a high-dimensional method. First is its simplicity due to the low dimensionality, i.e., the fact that it uses a handful of control variables only, in comparison with hundreds or thousands control variables that may be required in a typical high-dimensional method for the same simulation period. Because of the low dimensionality, PSO is very effective and efficient in locating its optimal solution. Second is the fact that the required computing time in PSO increases only linearly with the number of simulation steps $n$ whereas in a high-dimensional method this time may increase even exponentially with $n$. Because of this, the performance measure in PSO can be based on a large simulation period, thus avoiding situations of defining it on a short (e.g., one- or two-year) basis in which case the system operation ignores future impacts of management of today. Third is the fact that PSO is directly (by definition) combined with a simulation model of the system, incorporating stochastic and deterministic components, and describing the system dynamics as accurately as possible, thus avoiding simplifications of the system (e.g., linearization of equations or discretization of the state-space). Fourth, the parametric method is theoretically consistent with the stochastic nature of the reservoir problems and very easily incorporates concepts like probability, reliability, expected value, etc., also assigning values to such quantities. Fifth, the optimal values of the control variables do not depend on a specific realization (sequence) of inflows (or any other quantity that has a stochastic behavior) and they do not have to be changed if this realization changes (unless the system characteristics, the inflow statistics, or the operational objectives and constraints changed). Sixth, once the system is optimized with the PSO method, it can be very easily operated applying the parametric reservoir rule (even in its graphical form) without model runs at all. Similar to this, the model parameters and, consequently, the operation policy do not depend on forecasted values of inflows, which could be highly uncertain.

[74] In the empirical level, the comparison is done in terms of the results of 41 test problems that combine the three solution methods, three categories of objectives, three water uses, two versions of the reservoir system and two hydrological scenarios. The results show that the PSO method if used with two pairs of parameters per reservoir, one for the refill period and one for the drawdown period, yields solutions that are not inferior to those of the high-dimensional perfect foresight method, despite of the huge difference of the number of control variables of the two methods. Specifically, if the objective is to maximize the reliable release or to minimize the conveyance cost, the performance measures obtained by the two methods are almost identical. When applicable, the equivalent reservoir method yielded results identical to the other methods as well. If the objective is the maximization of the benefit from energy production, the perfect foresight method was able to seemingly improve the solution of the parametric method (with two pairs of parameters per reservoir) by up to 1.9% and that of the equivalent reservoir method by 2.3%. However, a more thorough investigation of the results of all methods shows that this improvement relies on the assumption that future inflows are perfectly predicted for an arbitrary large lead-time, an assumption that is obviously unrealistic.

[75] The problems examined in this study were intentionally simple in order to serve as a convenient means for comparisons. What these simple problems did not demonstrate is the high flexibility of the PSO approach and its ability to model very complex reservoir systems with a large number of reservoirs, a complicated topology of aqueducts and a variety of simultaneous water uses and operational goals. This ability, which has been demonstrated elsewhere [*Koutsoyiannis et al.*, 2002], is owing to the small number of control variables and mainly to the incorporation of the simulation with its well-known competence in analyzing complex systems.

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