Stochastic rainfall forecasting by conditional simulation using a scaling model

Presentation at the XIX EGS General Assembly
Session HS2/OA13/02 "Stochastic Modelling of Rainfall in Space and Time"

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Topics of the presentation

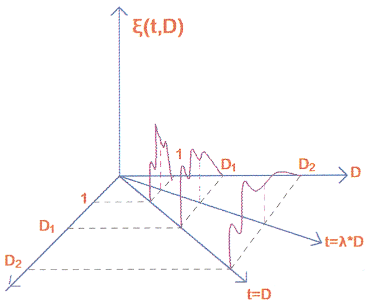
- ☆ Synopsis of the Scaling Model of Storm Hyetograph
- ☆ Data presentation and model parameters
- ☆ Performance evaluation
- ☆ General simulation scheme
- ☆ Conditional simulation scheme
- ☆ Application of the model for conditional simulation
- ☆ Conclusions

The Scaling Model of Storm Hyetograph – General Structure

Main hypothesis

$$\{\xi(t,D)\} \overset{d}{=} \{\lambda^{-H}\xi(\lambda t, \lambda D)\}$$

where $\xi()$: instantaneous rainfall intensity
D: duration of the event
t : time ($0 \le t \le D$)
H: scaling exponent



Secondary hypothesis: Weak stationarity (= stationarity within the event)

$$E[\xi(t,D)] = c_1 D^H$$

$$E[\xi(t,D)\,\xi(t+\tau,D)] = \varphi(\tau / D) D^{2H}$$

$$\varphi(\tau / D) = k(\tau / D)^{-\beta}$$

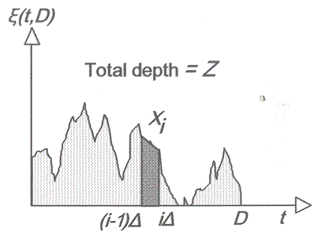
The Scaling Model of Storm Hyetograph – Main statistics

Statistics of total depth, Z

$$E[Z] = c_1 D^{H+1}$$

$$Var[Z] = c_2 D^{2(H+1)}$$

where $c_2 = c_1^2 + 2k / [(1-\beta)(2-\beta)$



Statistics of incremental depth, X

$$\mathrm{E}[X_i] = c_1 \delta D^{H+1}$$

$$\mathrm{Var}[X_i] = [(c_2 + c_1^2)\delta^{-\beta} - c_1^2]\delta^2 D^{2(H+1)}$$

$$\mathrm{Cov}[X_i, X_j] = [(c_2 + c_1^2)\delta^{-\beta} f(|j-i|, \beta) - c_1^2]\delta^2 D^{2(H+1)}$$

where $\delta = \Delta / D$, $f(m,\beta) = \frac{1}{2}[(m-1)^{2-\beta} + (m+1)^{2-\beta}] - m^{2-\beta}$ $(m > 0$

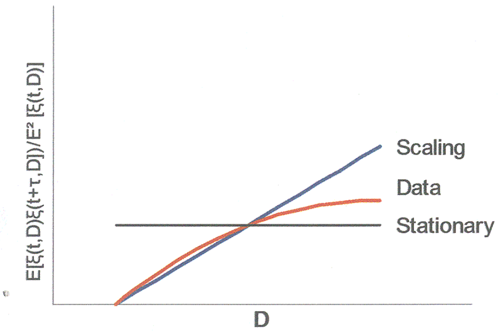
The Scaling Model of Storm Hyetograph - Estimation of parameters

Parameters

H	scaling exponent	estimated from $E[Z] = c_1 D^{H+1}$ (by least squares)
c_1	mean value parameter	
c_2	variance parameter	estimated from $c_2 = Var[Z] / D^{2(H+1)}$
β	correlation decay parameter	estimated from $\beta = 1 - \dfrac{\ln\left(E[X_i X_{i+1}] / E[X_i^2] + 1\right)}{\ln 2}$

The Scaling Model of Storm Hyetograph – Modification

Dependence of covariance structure on duration (logarithmic plot)



Correction to the correlation decay parameter

$$\beta = \beta_0 + \beta_1 \ln(D) \quad (\beta_1 < 0)$$

The Scaling Model of Storm Hyetograph – General properties

- Not description of the structure of a specific storm
- Statistical description and efficient parametrisation of a population of storms
- This population can include:
  - All storms,
  - Storms of a specific season,
  - Storms with intensity and/or depth greater than a given threshold, etc.
  - Point rainfall or areal (average) rainfall
- Simple construction of generation schemes for simulation (sequential, disaggregation, conditional)
- Consistency with, and parametrisation of, normalised mass curves

Data presentation and model parameters

Data sets

River Basin	Aliakmon (Greece)	Reno (Italy)	Evinos (Greece)	Evinos (Greece)
Point or areal rainfall	Point	Areal	Point	Point
Event type	All	hourly depth >1 mm	hourly depth > 7 mm or daily depth > 25 mm	hourly depth > 7 mm or daily depth > 25 mm
Season	April	All year	Oct. - Apr.	May - Sep.
Record period	13 years (1971-1983)	2 years (1990-1991)	20 years (1971-1990)	20 years (1971-1990)
Number of events	89	149	200	93

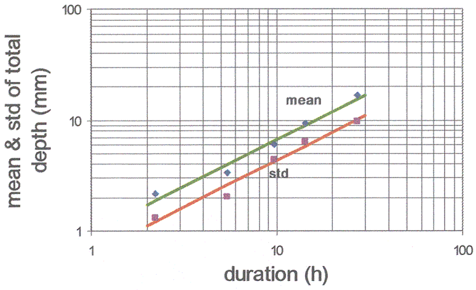
Model parameters

H	-0.163	-0.051	-0.332	-0.604
c1	0.964	0.518	5.475	10.042
c2	0.392	0.190	8.373	19.232
β_0	0.635	0.434	0.620	0.608
β_1	-0.1	-0.065	-0.109	-0.020

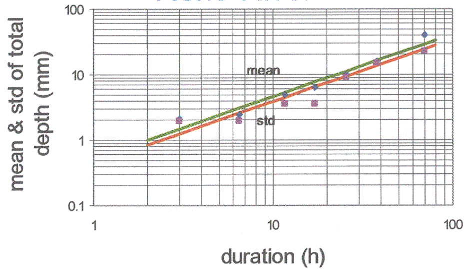
Performance evaluation

Mean and standard deviation of total depth

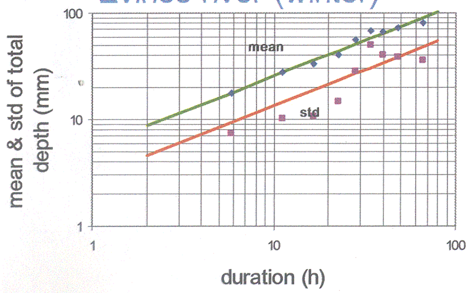
Aliakmon River



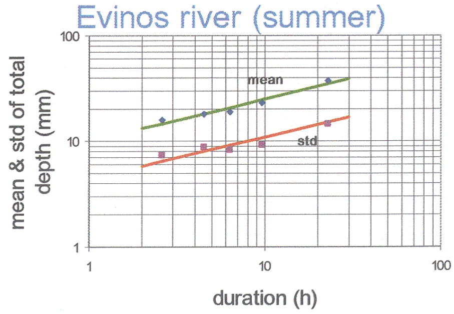
Reno River



Evinos river (winter)



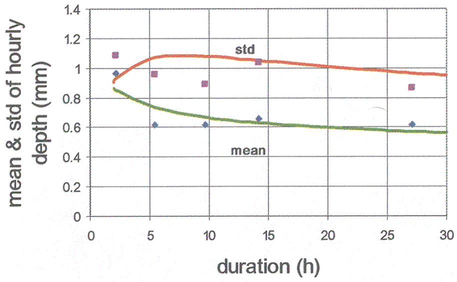
Evinos river (summer)



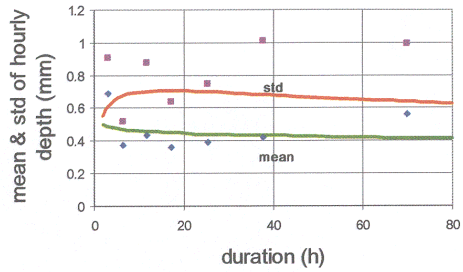
Performance evaluation

Mean and standard deviation of hourly depth

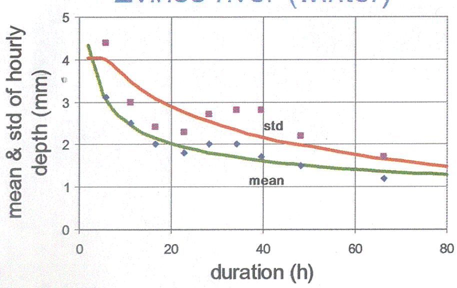
Aliakmon River



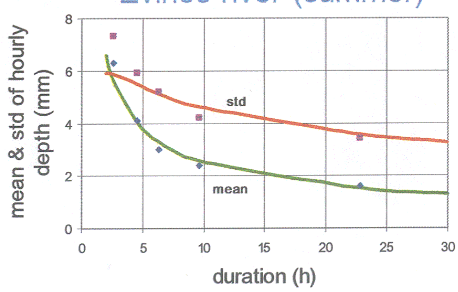
Reno River



Evinos river (winter)

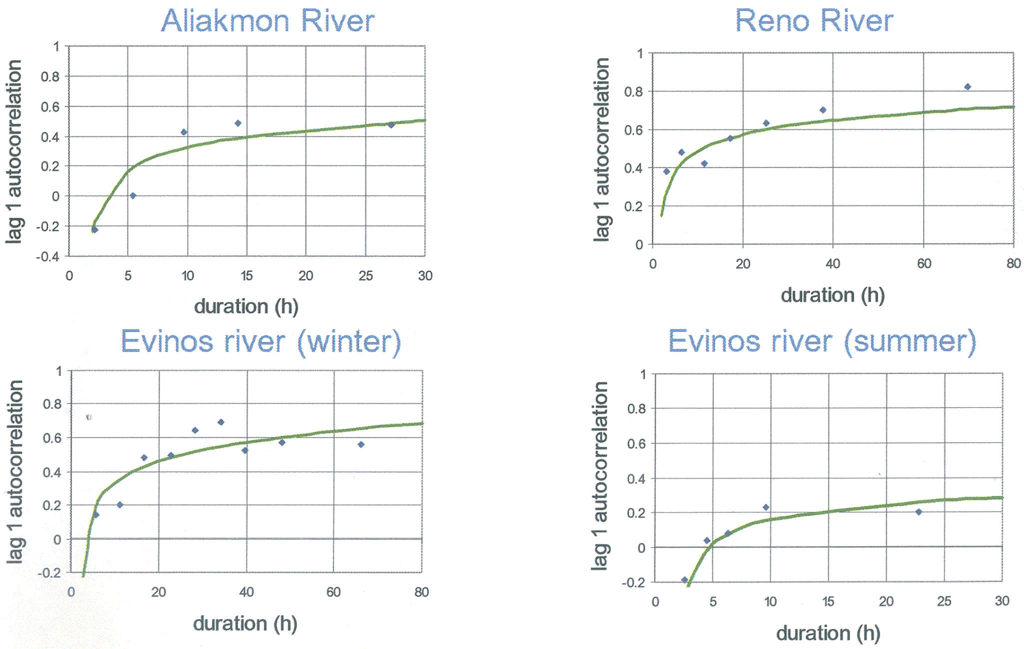


Evinos river (summer)



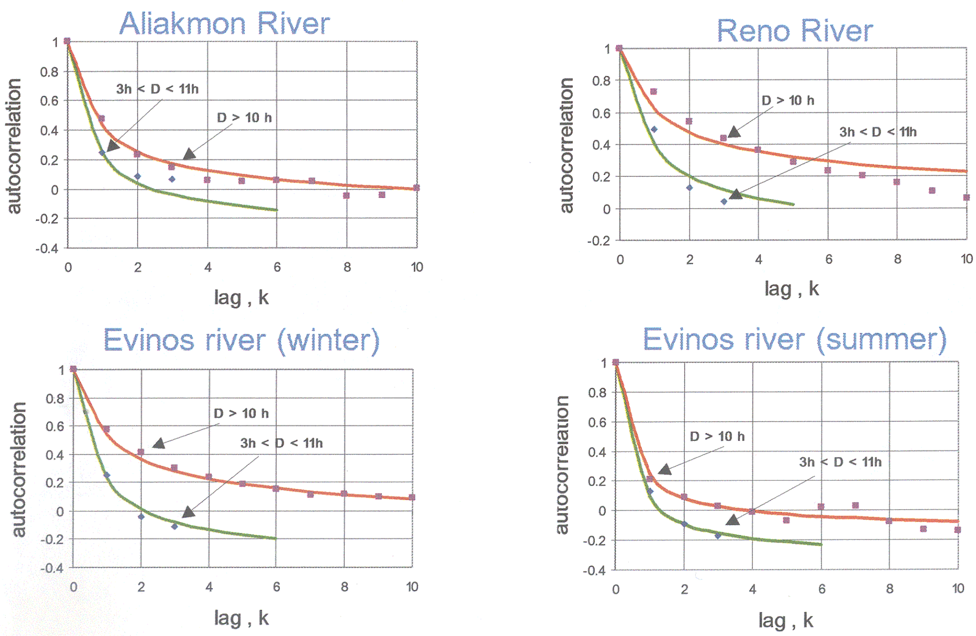
Performance evaluation

Lag 1 autocorellation coef. of hourly depth



Performance evaluation

Autocorrelation function of hourly depth



General simulation scheme

Sequential scheme

1. **Calibration of scaling model**: Estimation of parameters c_1, c_2, β(or β_0, β_1),H
2. **Calculation of** E[X], Cov[X,X], μ_3[X]
3. **Formulation of generating scheme**

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_k \end{bmatrix} = \begin{bmatrix} \omega_{11} & 0 & \cdots & 0 \\ \omega_{21} & \omega_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{k1} & \omega_{k2} & \cdots & \omega_{kk} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_k \end{bmatrix}$$

or $\mathbf{X} = \Omega \mathbf{V}$ (V_i independent, appr. 3-par. gamma)

4. **Estimation of parameters of the generating scheme**
   a. Coefficient matrix

   $\Omega\Omega^T = \mathrm{Cov}[\mathbf{X}, \mathbf{X}] \Rightarrow \Omega$ by decomposition (lower triangular)

   b. Statistics of V_i

   $\omega_{ii}\, \mathrm{E}[V_i] = \mathrm{E}[X_i] - \sum_{l=1}^{i-1} \omega_{il}\, \mathrm{E}[V_l]$

   $\mathrm{Var}[V_i] = 1$

   $\omega_{ii}^3\, \mu_3[V_i] = \mu_3[X_i] - \sum_{l=1}^{i-1} \omega_{il}^3\, \mu_3[V_l]$

5. **Generation of V_i**
6. **Calculation of Xi**

General simulation scheme

Disaggregation scheme

1. **Generation of total depth** Z
2. **Application of the sequential procedure to obtain an initial sequence** X_i'
3. **Determination of the final (adjusted) sequence** $X_i = \dfrac{X_i'}{\sum_{j=1}^{k} X_j'} Z$

Conditional simulation scheme

Step 1 Generation of duration D

Conditions

- Known past (Total duration > current duration)
- Predicted future (Total duration is given approximately from meteorological forecasts)

Step2 Generation of hourly depths Xj

Lead time

- Fixed, L (adaptation of parameters every L steps)
- Not fixed (Generation of all remaining steps)

Conditions

- Known past
- Known past + Predicted future Total depth is given approximately from meteororological forecasts
- Known past + Predicted future Total depth for a future time period (6 hours) is given approximately from meteororological forecasts

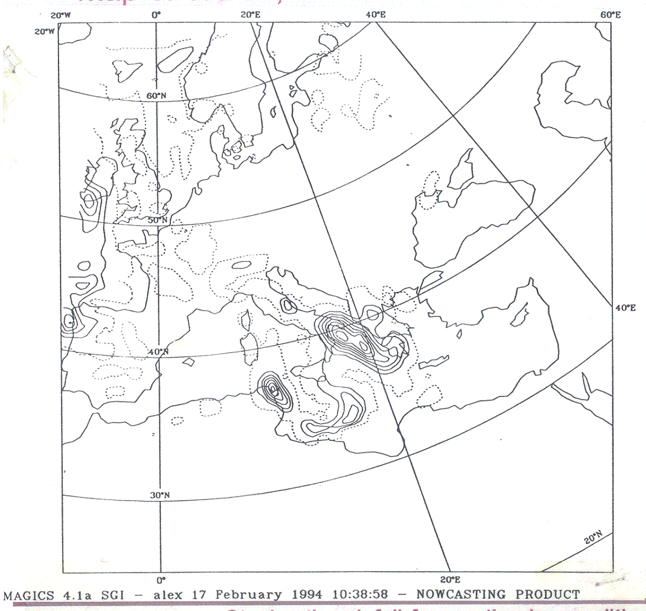
Generation scheme

- Sequential scheme
- Disaggregation scheme

Coupling of meteorological forecast

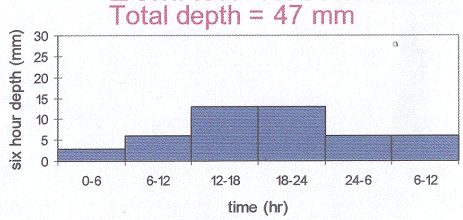
Example of ECMRWF quantitative precipitation forecast

Map of 9/1/94, 06-12 G.M.T.

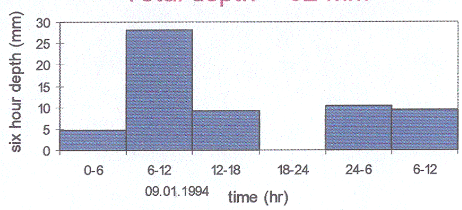


Comparison of forecast and observed precipitation at Evinos River Basin (6 hours intervals)

ECMRWF forecast
Total depth = 47 mm

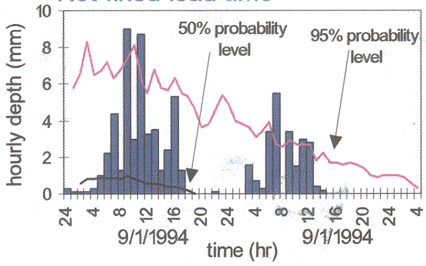


Observed areal precipitation
Total depth = 62 mm

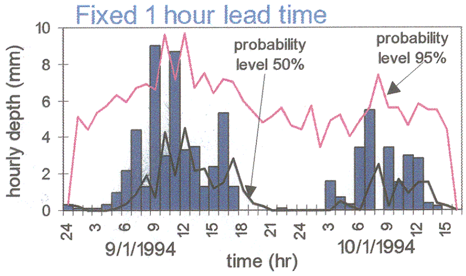


Application of the model for simulation

Known past for duration and depth
Not fixed lead time

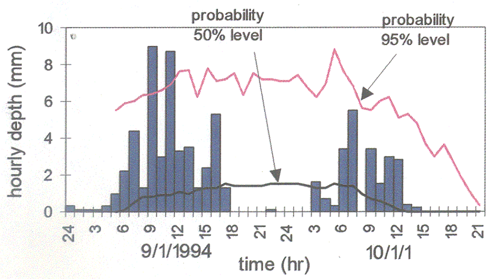


Known past for duration and depth
Fixed 1 hour lead time



Known past for duration and depth. Not fixed lead time

Estimates for future: $Z_l<Z<Z_h$ where $|Z_{l,h} - Z|=0.3*Z$
$D_l<D<D_h$ where $|D_{l,h} - D|=0.2*D$



Conclusions

1. The Scaling Model of Storm Hyetograph is suitable for a variety of data sets regardless of season and rain type.

2. It can support a variety of stochastic simulation schemes taking into account any information (condition) for the past or future of rainfall.

3. Specifically, it can be combined with a meteorological forecast to disaggregate it into smaller time steps, also adding a stochastic component to the deterministic forecast.