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Lognormal Upper Tail of Rainfall Intensity and POT Values: Implications on the IDF Curves

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Abstract

Empirical and theoretical evidence supports the hypothesis that the upper tails of the marginal and peak over threshold (PoT) rainfall intensity distributions are lognormal, except for a multiplicative scaling factor on the probability density. We call this a scaled lognormal (SLN) tail model. The parameters of the SLN distribution are obtained by fitting the rainfall data above a given intensity threshold i^* using a least-squares method. Marginal/SLN and PoT/SLN estimates of the IDF curves are similar. They are also less data intensive and more robust against outliers than the conventional annual-maximum method with GEV distribution and PoT method with Generalized Pareto distribution.

IDF Estimation Methods

Annual maximum (AM) method:

For each averaging duration d, a GEV distribution with shape parameter k is fitted to the yearly maximum rainfall intensities $I_{max}(d)$. The T-yr return period value $I_{max}(d,T)$ is obtained as:

$$I_{max}(d,T) = F^{-1}_{GEV(d)}(1-1/T) \quad (1)$$

Peak over threshold (PoT) method :

Let $I(d)$ be the mean rainfall intensity over duration d. For each d, a Generalized Pareto (GP) distribution is fitted to the peak rainfall intensity values $I_{POT}(d;i^*)=\max\{I(d)-i^*\}$ during excursions of $I(d)$ above a given threshold i^*. If λ is the mean annual rate at which $I(d)$ upcrosses i^*, then $I_{max}(d,T)$ is estimated as

$$I_{max}(d,T) = F^{-1}_{GP(d)}(1-1/\lambda T)+i^* \quad (2)$$

Marginal Distribution (MD) method :

As shown below, above some suitable threshold i^* the distribution of $I(d)$ and $I^+(d)=[I(d)|I(d)>0]$ may be assumed to have a lognormal shape of the type

$$F_{I_d}(i) = P_1 \cdot N\left(\frac{\ln i-\mu}{\sigma}\right),\ i \ge i^*$$
$$F_{I^+_d}(i) = P^+_1 \cdot N\left(\frac{\ln i-\mu}{\sigma}\right),\quad i \ge i^* \quad (3)$$

Where P_1, $P_1^+>0$ are scaling parameters and N is the standard normal CDF. We call this a scaled lognormal (SLN) tail model. The IDF values are estimated as

$$I_{max}(d,T) = F^{-1}_{I_{max}(d)}\left(1-\frac{1}{T}\right), \text{ where} \quad (4)$$

$$F_{I_{max}(d)}(i) = F_{I_d}(i)^{1/d} \quad \text{and } d \text{ is in years.}$$

Evidence of Tail Lognormality of I(d) and $I_{PoT}(d;i^*)$

Empirical evidence:

Figures 1a and **1b** show on lognormal paper the empirical distribution of $I^+(d)$ for four rainfall records and d = 1hr and 24hr. The records are from Walnut Gulch (49 years), Porto (33 years), Heathrow Airport (51 years) and Florence (24 years). The dashed horizontal lines mark the 75% quantile for d = 1hr and the 90% quantile for d = 24hr. For the Porto and Heathrow records **Figs 2a** and **2b** show SLN distributions fitted by least-squares to these tail regions (solid lines).

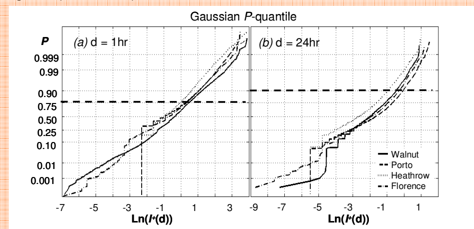

Fig. 1: Marginal distributions of positive values of $I(d)$ for averaging durations d=1hr and d=24hr.

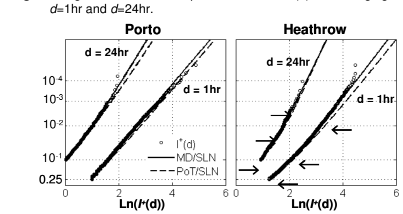

Fig. 2: SLN distributions fitted to the tail regions of Porto and Heathrow records (solid lines) and to the PoT values $I_{PoT}(d;i^*)$ above the same threshold intensity i^* (dashed lines). Arrows correspond to the thresholds used in Fig. 4.

Also shown in **Fig. 2** are SLN distributions fitted to the PoT values $I_{PoT}(d;i^*)$ above the same threshold i^* (dashed lines). It is interesting that the PoT and marginal distributions are close (this is true over a wide range of thresholds i^* and also for the other records) and that both are fitted well by SLN models.

Theoretical and model-based evidence:

To support these empirical findings, we consider a class of rainfall pulse models with storms separated by dry periods. The storms have random duration D and random average intensity. Inside the storms, rainfall intensity fluctuations exhibit multifractal scale invariance of the beta-lognormal type with parameters (C_β, C_{LN}) [1].

Two cases are considered, one with (C_β = 0, C_{LN} = 0.1) and the other with (C_β = 0.07, C_{LN} = 0.1). The lower region of the distribution of $I^+(d)$ is sensitive to C_β, but the upper tail always has a lognormal shape. This can be argued theoretically. The plots in **Fig. 3**, obtained from 460-year model simulations, support this theoretical conclusion and show similarities with the empirical distributions in **Fig. 1**.

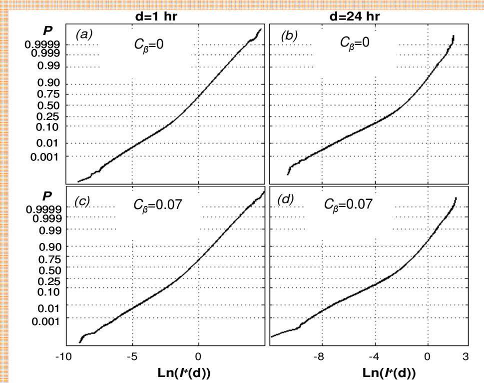

Fig. 3 : Marginal distributions of $I^+(d)$ for averaging durations d=1hr and d=24hr for the 2 simulated records with different C_β.

PoT/SLN vs. PoT/GP:

When the marginal/SLN and PoT/SLN methods are used with the same threshold i^*, the resulting IDF estimates are close. This is shown in **Fig. 4** where the estimated distributions of $I_{max}(d)$ for Porto and Heathrow are shown in stacked form for different i^*. The symbols in the figure are the observed annual maxima. This good correspondence is significant because the authors previously found that the PoT method with GP distribution performs poorly relative to the marginal/SLN method [2,3]. <u>One may conclude that the poor performance of the PoT/GP method is due to the choice of a GP distribution and can be significantly improved by using an SLN tail model.</u>

AM/GEV:

The annual-maximum method with GEV distribution performs the worst. This is due to the small sample size and the high sensitivity of the shape parameter k to the largest observed values. For example, **Fig. 5** compares marginal/SLN, PoT/GP and AM/GEV estimates of the distribution of $I_{max}(d)$ for (Heathrow, d = 1hr) and (Florence, d = 24 hr). The distributions from the AM/GEV method are strongly influenced by what one may consider outlier years.

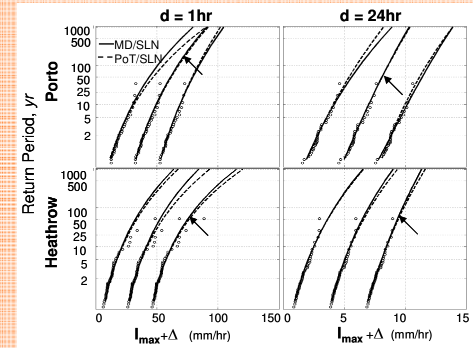

Fig. 4 : Stacked comparison of annual maximum distributions produced by the marginal/SLN (solid lines) and PoT/SLN (dashed lines) methods for different threshold values. The cores correspond to the i^* thresholds in Fig. 2 (arrows).

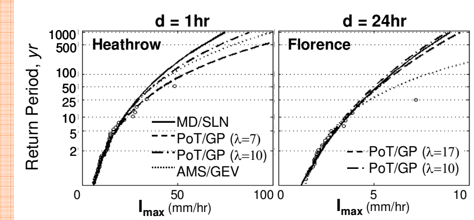

Fig. 5 : Empirical annual maxima (circles) and their GEV fittings (dotted lines), MD/SLN results from the selected models in Fig. 4 (solid lines) and PoT/GP results associated with different mean annual upcrossing rates λ.

Conclusions

- There is significant empirical evidence that the marginal and PoT distributions of rainfall intensity have a scaled lognormal (SLN) upper tail. A tail of this type is also predicted by a broad class of pulse models with multifractal scale invariance inside the storms;
- When an SLN model is fitted to the tail of the marginal distribution above a high level i^* or to the PoT values above the same threshold, the predicted IDF values are similar;
- Marginal/SLN and PoT/SLN methods perform better that the traditional PoT/GP and AM/GEV methods. It is recommended that asymptotically-justified GEV and GP distributions not be used for IDF estimation.

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