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Annual Rainfall Maxima: Large-Deviation Alternative to Extreme-Value and Extreme-Excess Methods

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Contrary to common belief, Gumbel's extreme value (EV) and Pickands' extreme excess (EE) theories do not generally apply to rainfall maxima at the annual level. This is true not just for long averaging durations d, as one would expect, but also in the high-resolution limit as $d \to 0$. We reach these conclusions by studying the annual maxima of scale-invariant rainfall models with a multiplicative structure. We find that for $d \to 0$ the annual maximum rainfall intensity in d, $I_{year}(d)$, has a generalized extreme value (GEV) distribution with a shape parameter k that is significantly higher than that predicted by Gumbel's theory and is always in the EV2 range. Under the same conditions, the excess above levels close to the annual maximum has generalized Pareto (GP) distribution with a parameter k that is always higher than that predicted by Pickands' theory. The proper tool to obtain these results is large deviation (LD) theory, a branch of probability that has been largely ignored in stochastic hydrology.

In the classic EV and EE settings one considers a single random variable X and studies either the distribution of the maximum of n independent copies of X as $n \to \infty$ or the distribution of the excess $X_u = (X - u|X \geq u)$ as the threshold $u \to \infty$. A well known result is that, if under renormalization these distributions approach non-degenerate limits, then the distribution of the maximum is GEV(k), the distribution of the excess above u is GP(k), and the common shape parameter k depends on the tail behavior of X.

When applied to rainfall extremes, X is typically taken to be $I(d)$, the rainfall intensity in a generic d interval. The problem with the EV approach is that the number of d intervals in one year, $n(d) = 1yr/d$, may be too small for convergence of $I_{year}(d)$ to the asymptotic GEV distribution. Likewise, in the EE approach, thresholds u on the order of the annual maximum may be too low for convergence of the excess to the asymptotic GP distribution. This is indeed what happens in multifractal (and likely other multiplicative) models of rainfall.

In contrast to EV and EE theories, LD theory considers a sequence of random variables $\{X_n = \prod_{i=1}^{n} Y_i,\ n = 1, 2, \dots\}$ where $Y_1,\ Y_2, \dots$ are independent copies of a non-negative random variable Y and evaluates the probability $P[X_n > e^{\gamma n}]$ for given $\gamma > 0$ as $n \to \infty$. For application to scale-invariant rainfall one writes $I(d) = I(D)X_{n=\ln(D/d)}$, where D is the outer limit of the scale-invariant behavior. By using LD results, one finds that as $d \to 0$ the distribution of $I_{year}(d)$ is EV2(k) with k that depends on the body (not the tail) of the distribution of Y.

One can use LD theory also to extend Pickands' EE results. In this case one considers the excess $X_{n,e^{\gamma n}} = (X_n - e^{\gamma n}|X_n \geq e^{\gamma n})$ where the threshold u is made to vary with n as $u = e^{\gamma n}$ for some $\gamma > 0$. One finds that, when the threshold is set to values on the order of the annual maximum and $d \to 0$, the excess of $I(d) = I(D)X_{n=\ln(D/d)}$ approaches a GP(k) distribution where k is the same as in the EV2(k) distribution of $I_{year}(d)$. For d finite, $I_{year}(d)$ does not have a GEV distribution, but one can use LD theory to find the best-fitting EV2(k) distribution within a given range of quantiles.

The use of large-deviation theory rather than extreme-value or extreme-excess theories represents a significant conceptual change in the way annual rainfall maxima are viewed and evaluated. There are also practical implications. Use of LD theory to calculate the distribution of the annual maximum does not require knowledge of the upper tail behavior of $I(d)$. Rather one needs to know the distribution of $I(d)$ in a less extreme region with significant gains

in estimation accuracy and robustness. Some of the practical implications are considered in a companion study (Lepore *et al*., “Annual Rainfall Maxima: Practical Estimation Based on Large-deviation Results,” EGU 2009).

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