Estimation of hydraulic parameters of a confined aquifer from slug test in fully penetrating well using a complete Quasi-Steady flow model in an inverse optimal estimation procedure

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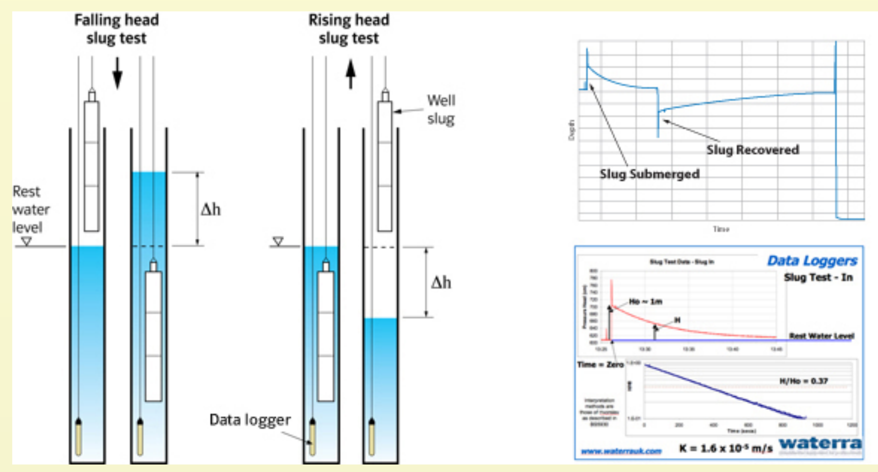
MOTIVATION

Slug tests allow fast and economical estimation of an aquifer's hydraulic parameters. When the radial hydraulic conductivity K_r and the specific storage coefficient S_s are estimated by slug tests performed in wells fully penetrating confined aquifers, the data are evaluated with the transient-flow model of Cooper et al. (1967). That analytical solution, however, is computationally involved and awkward, so groundwater professionals fit the data by rough visual matching procedure using few type-curves.



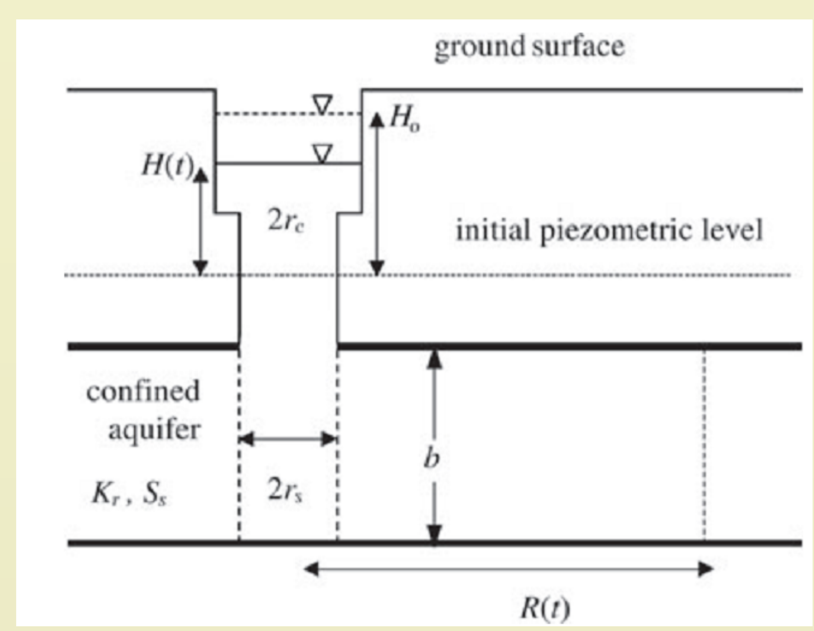
Slug test (figure adapted from www.waterra-in-situ.com)

SCOPE: Presentation of test results of **automated inverse procedure for estimating optimally the hydraulic formation parameters K_r and S_s from slug test observations** for the over-damped case.

This procedure is embedded in a **stand-alone application that is based on the Octave 3.6.2 engine** [it does not require installing Octave or Matlab]. The software couples the Shuffled Complex Evolution (SCE) optimization method of Duan et al. (1992) with the mathematically simple quasi-steady flow solution of Koussis & Akylas (2012) that is computationally efficient, yet approximates the solution of Cooper et al. (1967) closely.

THE QUASI-STEADY (QS) METHOD (Koussis & Akylas, 2012)

By including a storage balance inside the confined aquifer, Koussis and Akylas derived a complete **Quasi-Steady*** flow solution for the response of a slug-test well fully penetrating the aquifer, $H(t)/H_0 = f(\alpha, \beta)$, where $\beta = K_r bt/r_c^2$ is dimensionless time and $\alpha = r_s^2 S_s b/r_c^2$ a dimensionless storage parameter; b is the formation thickness, r_s the effective radius of well screen and r_c the effective radius of well casing.



The operational equations of the QS flow model are given below, in which $R(t)$ is the unknown time-varying radial distance to the edge of the QS flow domain, where the head in the aquifer vanishes, $h(R) = 0$.

$$\frac{H_o - H(t)}{H(t)} = \frac{bS_s r_s^2}{r_c^2}\left(\frac{\left(R(t)/r_s\right)^2 - 2\ln\left(R(t)/r_s\right) - 1}{2\ln\left(R(t)/r_s\right)}\right) \qquad \frac{d\ln\left[H(t)/H_o\right]}{dt} = -\frac{2K_r b}{r_c^2 \ln\left[R(t)/r_s\right]}$$

*** In a quasi-steady solution, a series of steady states substitutes the transient process, considering the physical system to evolve in abrupt steps from one steady state to the next one.**

Solution Algorithm

The above system of two non-linear equations is solved numerically with the following algorithm:

- Set the variable $y := R/r_s$, which is computed in n non-constant increments, i.e., $y_j = y_{j-1} + \Delta y_j$ with $y_0 = 1$, $\Delta y_j = 0.015 \exp(0.015\, j)$ and $j = 0, 1, ...n$ (n should be large enough to ensure that β_n is greater than the maximum dimensionless time of measurements).
- For each value y_j, calculate an $F(y_j)$ value by the formula $F(y_j) = 0.5\,(y_j^2 - 2 \ln y_j - 1)/\ln y_j$ (this is the term in parentheses on the right-hand side of the first operational equation above).
- Set $HH0_j := H(\beta_j)/H_o$, i.e., the value of H/H_o at the dimensionless time β_j. Then, estimate the pairs ($HH0_j$, β_j), which give the curves of the Quasi-Steady (QS) solution, with the following formulae:

$HH0_j = [\alpha F(y_j) + 1]^{-1}$; $\quad \beta_j = \beta_{j-1} + \Delta\beta_j$; $\Delta\beta_j = 0.5 \ln[(y_{j-1} + y_j)/2]\,(\ln[\alpha F(y_{j-1}) + 1] - \ln[\alpha F(y_j) + 1])$ and $\beta_0 = 0$.

PROCEDURE

Real measurements: The set of real measurements were taken from Butler (1998), who studied the results of a slug test at the Lincoln county monitoring site.

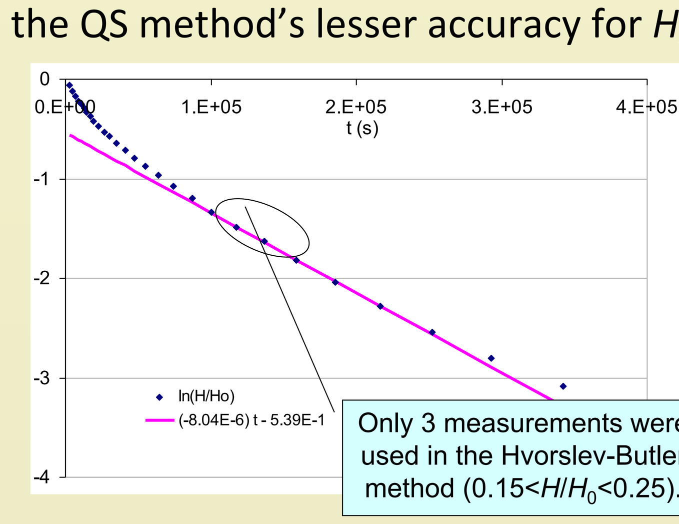
Synthetic measurements: Synthetic data were produced using Cooper's type curves for a combination of seven α values and five a priori assumed K_r. The result was the 35 pairs of references values that are shown in the table bellow.

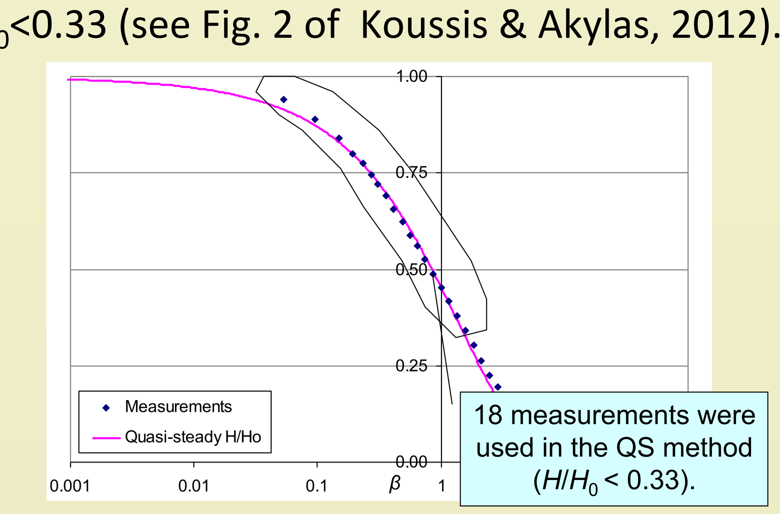
α \ K (m/d)	2.02×10^{-4}	2.02×10^{-2}	1	10	100
10^{-1}					
10^{-2}					
10^{-3}					
10^{-4}		(2.02×10^{-2} m/d, 4.07×10^{-6} m^{-1})			
10^{-5}					
10^{-6}					
10^{-7}					

Reference S_s calculated from α values i.e. $S_s = \alpha\, r_c^2 /(r_s^2\, b)$.
The values of r_s and r_c were 0.071 and 0.025 m respectively.

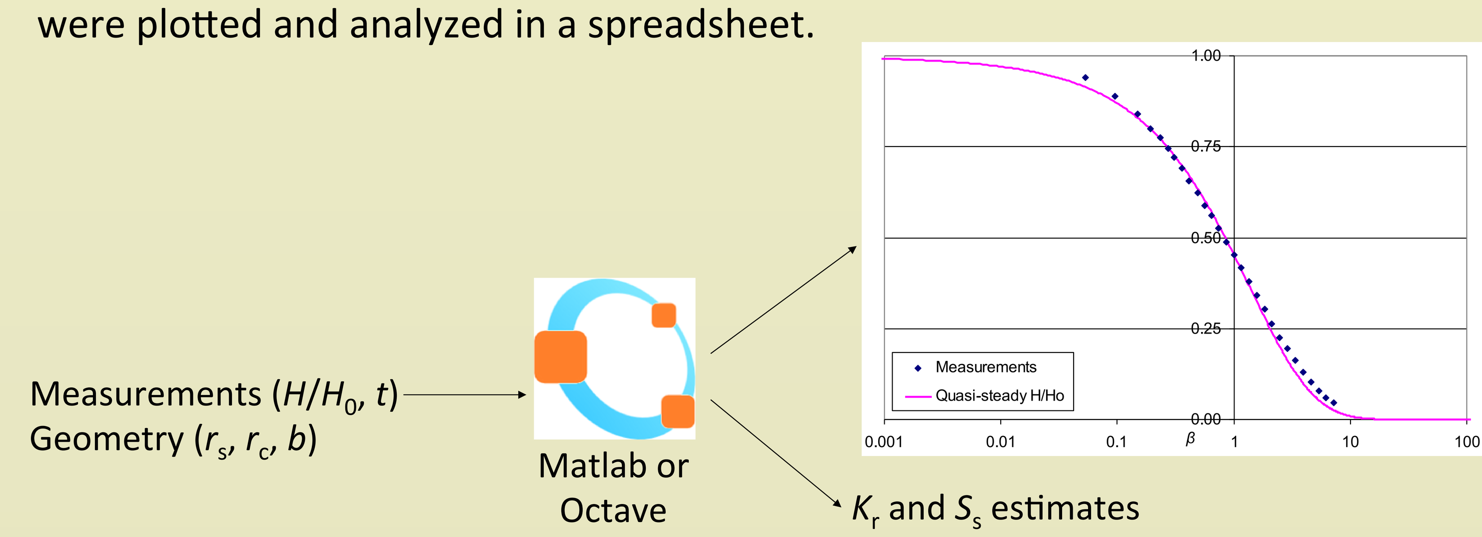
H/H_0 values were obtained from Table 5.14.1a of Todd and Mays (2005). A priori assumed K_r values were used to calculate the time t of H/H_0 values i.e. $t = \beta\, r_c^2/(Kr\ b)$. The value of b was 3.05 m.

Objective function of the optimization: Objective function was the RMS error between measurements and H/H_0 values calculated by the QS method. Because the dimensionless times β_j of the H/H_0 values and the measurements are not expected to coincide, we used linear interpolation. The arguments of the objective function were the log-values of K_r and S_s, instead of plain values, to tackle the wide parameter range, extending over several orders of magnitude (Efstratiadis et al., 2008). Only measurements with $H/H_0 > 0.33$ were used in the optimization. This was dictated by the QS method's lesser accuracy for $H/H_0 < 0.33$ (see Fig. 2 of Koussis & Akylas, 2012).





Implementation: The QS method, the objective function and support scripts (~100 lines code) were implemented in Matlab m files, compatible with GNU Octave. The SCE algorithm was obtained from MATLAB CENTRAL (2013). For results visualization (fit of QS curve to measurements), comma-separated files were prepared, which were plotted and analyzed in a spreadsheet.

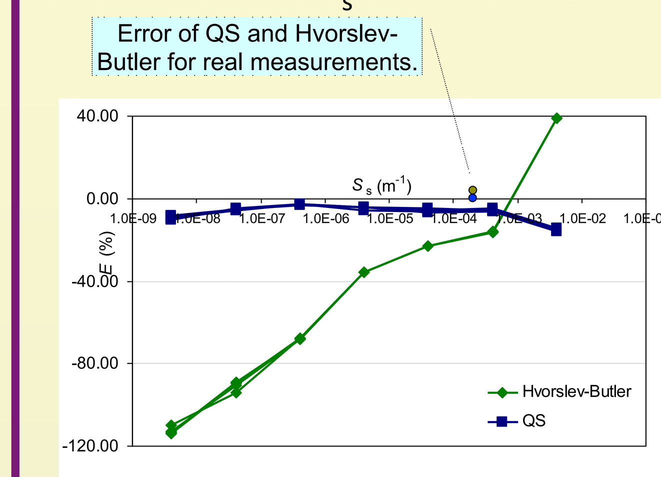


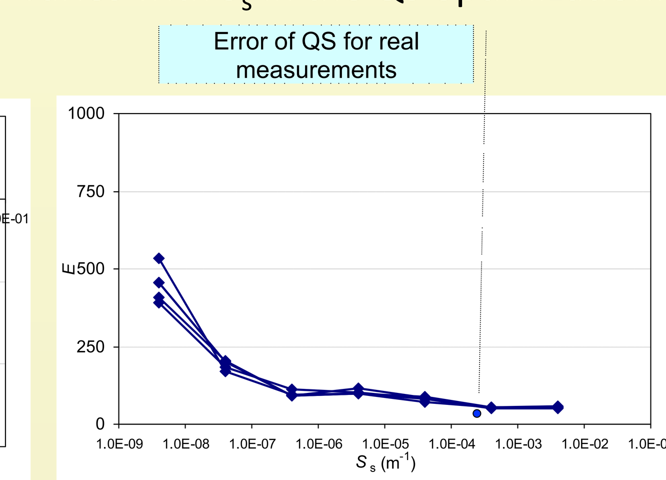
Estimation error: For each one of the 35 synthetic measurements plus the real observations, the optimization algorithm estimated the K_r, S_s pair that minimized an objective function. The estimated pair was compared against the reference (*a priori* known) values of the corresponding data. The estimation error was calculated as:

$$\text{Error} = \frac{\text{Estimation-Reference}}{\min(\text{Reference, Estimation})}$$

RESULTS

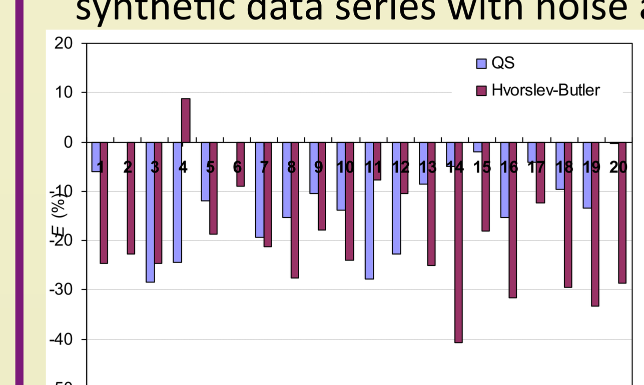
The left figure below shows how the estimation error of K_r varies with S_s for the QS-optimization & Hvorslev-Butler methods. The right figure shows how the S_s estimation error varies with S_s in the QS optimization.

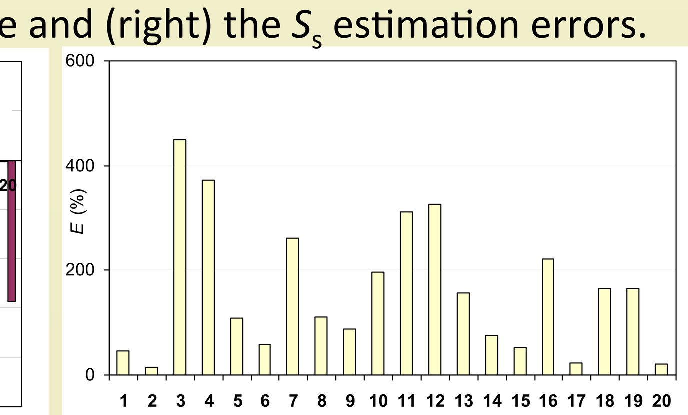




To test the influence of measurement noise on the efficiency of the automated method, we used 20 time series of white noise ξ on the synthetic data produced from the reference values $K_r = 2.02\times10^{-4}$ m/d and $S_s = 4.07\times10^{-5}$ m^{-1}. ξ follows the normal distribution N (0, 0.025) normalized by H_0, taken as 2 m; σ = 0.025 m corresponds to the worst precision mentioned by Sorensen & Butcher (2011). The noisy series were calculated as $HN_{noise} = H/H_0 + \xi_0$; H/H_0 from Cooper et al (1967).

Figure below displays (left) the K_r estimation errors when using the 20 synthetic data series with noise and (right) the S_s estimation errors.





CONCLUSIONS

- The reliability of the K_r-estimates of the Quasi-Steady method coupled with the SCE algorithm is influenced only marginally by S_s.
- The reliability of the Hvorslev-Butler K_r-estimates depends on S_s.
- The magnitude of K_r does not influence K_r-estimates of either method.
- The Quasi-Steady method exploits much more of available information.
- The error of S_s-estimates of Quasi-Steady method increases sharply (especially for low K_r) beyond a specific threshold (~5×10^{-7} m^{-1}) .

References

Butler, J.J. Jr. 1998.The Design, Performance, and Analysis of Slug Tests.Boca Raton, Florida: Lewis Publishers.

Cooper, H.H., J.D. Bredehoeft and I.S. Papadopulos (1967) Response of a finite-diameter well to an instantaneous charge of water, *Water Resour. Res.* **3**(1): 263–269.

Duan, Q., S. Sorooshian and V. Gupta (1992) Effective and efficient global optimization for conceptual rainfall-runoff models, *Water Resour. Res.*, **28**(4), pp. 1015–1031.

Efstratiadis, A., Nalbantis, I. Koukouvinos, A. Rozos, E. and Koutsoyiannis, D. 2008. HYDROGEIOS: A semi-distributed GIS-based hydrological model for modified river basins, Hydrology and Earth System Sciences, 12, 989–1006.

Koussis A.D. and E. Akylas (2012) Slug test analysis for confined aquifers in the over-damped case: Quasi-steady flow model, with estimation of the specific storage coefficient, *Ground Water*, **50**(4): 608–613.

MATLAB CENTRAL. 2013. Shuffled Complex Evolution (SCE-UA) Method by Qingyun Duan, http://www.mathworks.com/matlabcentral/fileexchange/7671, accessed January 2013.

Sorensen, J.P.R., and Butcher, A.S. (2011), Water Level Monitoring Pressure Transducers, A Need for Industry-Wide Standards, Ground Water Monitoring & Remed., 31, no. 4, pp. 56-62.

Todd, D.K. and L.W. Mays (2005) Groundwater Hydrology, J. Wiley & Sons, New York.