European Geosciences Union
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Vienna | Austria | 12 – 17 April 2015

Session HS7.4/AS4.23/CL2.8
Change in climate, hydrology and society

Climate is changing, everything is flowing, stationarity is immortal

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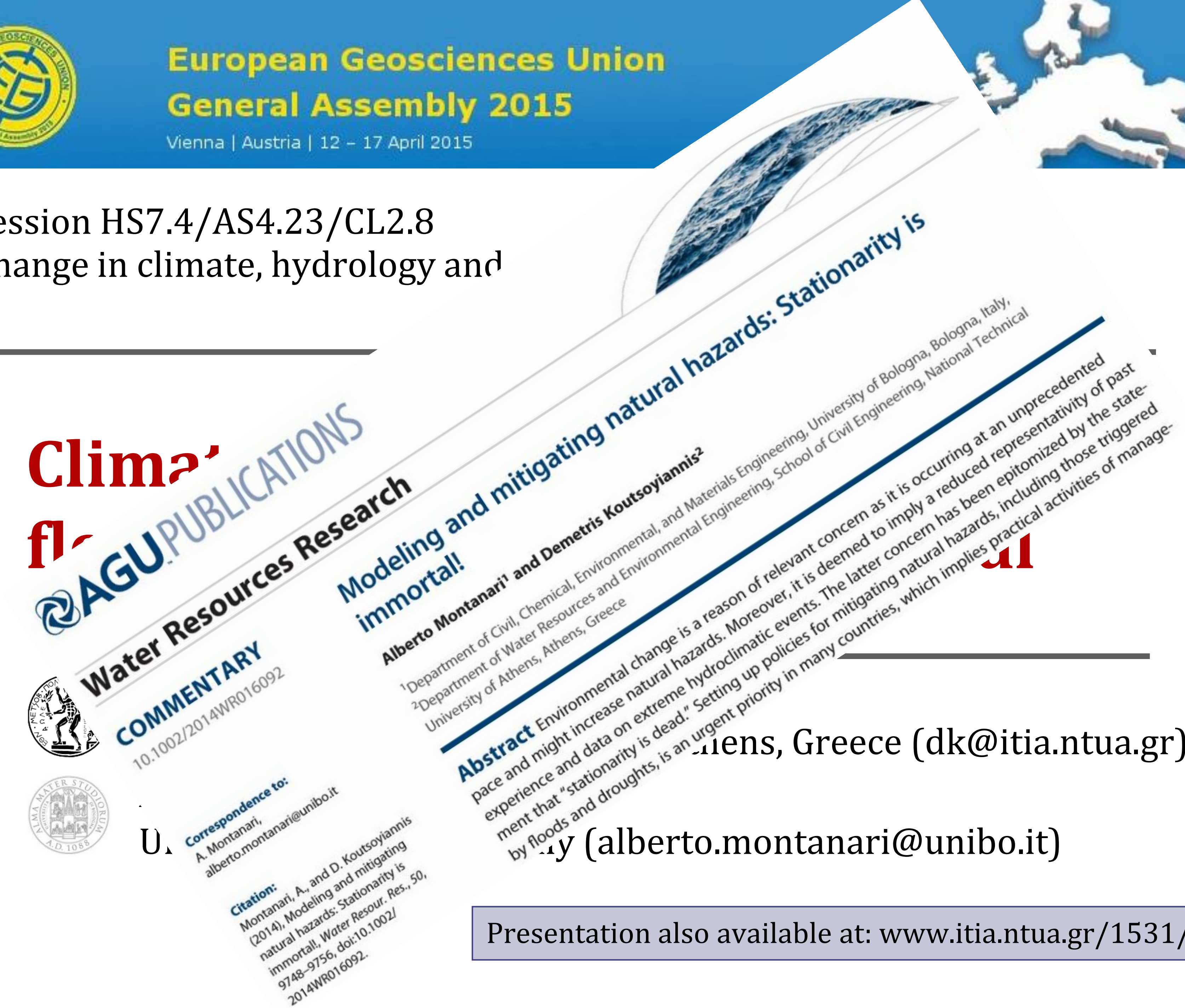
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Modeling and mitigating natural hazards: Stationarity is immortal!

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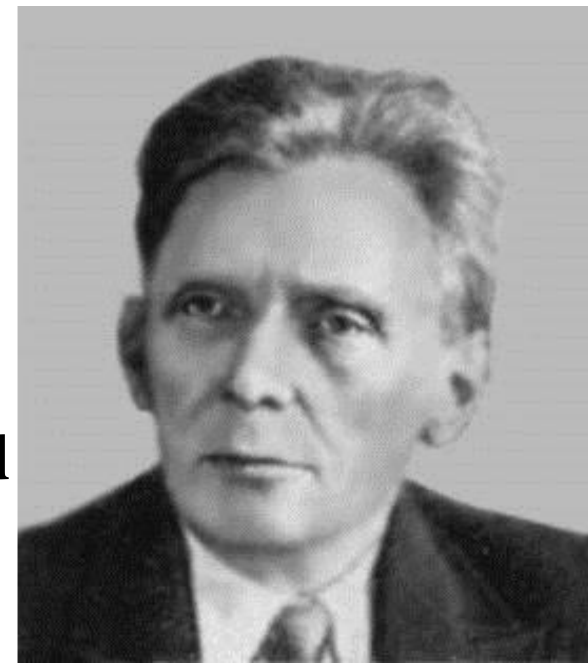
Abstract Environmental change is a reason of relevant concern as it is occurring at an unprecedented pace and might increase natural hazards. Moreover, it is deemed to imply a reduced representativity of past experience and data on extreme hydroclimatic events. The latter concern has been epitomized by the statement that "stationarity is dead." Setting up policies for mitigating natural hazards, including those triggered by floods and droughts, is an urgent priority in many countries, which implies practical activities of manage-

Presentation also available at: www.itia.ntua.gr/1531/

The meaning of stationarity —A historical note

Andrey Kolmogorov
(1903 – 1987)

- Kolmogorov (1931)
  - clarified that the term *process* means *change of a certain system*;
  - introduced the term *stochastic process*;
  - used the term *stationary* to describe a probability density function that is unchanged in time.
- Khinchin (1934) gave more formal definitions of a *stochastic process* and of *stationarity*.



Aleksandr Khinchin
(1894 – 1959)

Definition of stationarity

- Kolmogorov (1938) gave a concise presentation of the definition as follows:

  a stationary stochastic process [...] is a set of random variables x_t depending on the parameter t, $-\infty < t < +\infty$, such that the distributions of the systems

  $$(x_{t_1}, x_{t_2}, \ldots, x_{t_n}) \text{ and } (x_{t_1+\tau}, x_{t_2+\tau}, \ldots, x_{t_n+\tau})$$

  coincide for any n, t_1, t_2, … , t_n, and τ.

- Processes that are not stationary are called **nonstationary**; their statistical properties (at least some of them) change in time being **deterministic** functions of time.
- As far as we know:
  - This definition of stationarity has never been disputed.
  - There has never been an alternative formal definition of stationarity.
  - The terms stationary and stationarity are often misused.

Implications

- Stationarity refers to stochastic processes.
- This is not pure speculation: it implies relevant practical implications.
- If a process is deterministic, it is meaningless to refer to stationarity. The process would be perfectly predictable, its changing properties could be figured out with precision without the need to call stationarity into play.
- Conversely, a process which is not fully predictable implies the presence of a stochastic component. Such component must be predicted in statistical terms.
- It becomes necessary to call stationarity into play, to assume that the statistical behaviors do not change in time.
- If we admit that statistics change in time we imply that the process is non-stationary, but we need to define the change in statistics in deterministic terms.

In which world do stationarity and nonstationarity belong?

Real world

Physical system

Unique evolution

Time series (observed)

Perpetual change

Many different models can be constructed

Mental copies depend on model constructed

The observed time series is unique; the simulated can be arbitrarily many

An important consequence:
Stationarity is immortal

Abstract representation

Model (Stochastic process)

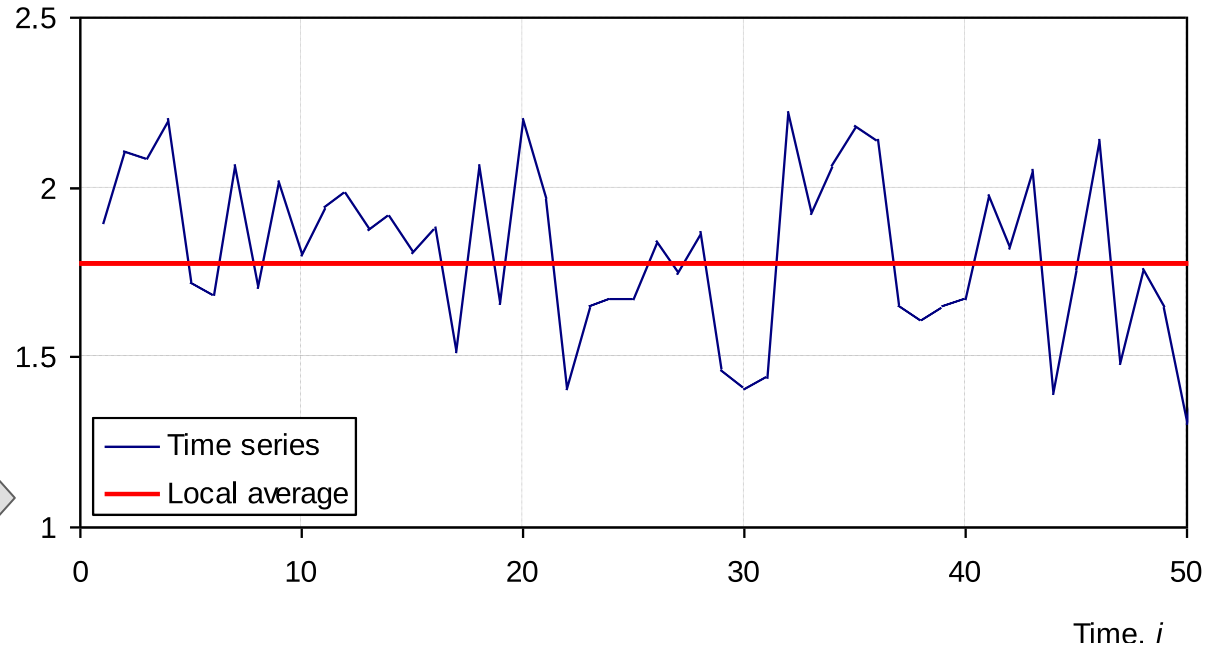
Ensemble (Gibbs's idea): **mental copies** of natural system

Time series (simulated)

Both stationarity and nonstationarity apply here (not in the real world)

Does a time series tell us if it is stationary or nonstationary?

- Not actually.
- Actually, a time series is neither stationary nor nonstationary.
- These are properties of the stochastic process that generated the time series.

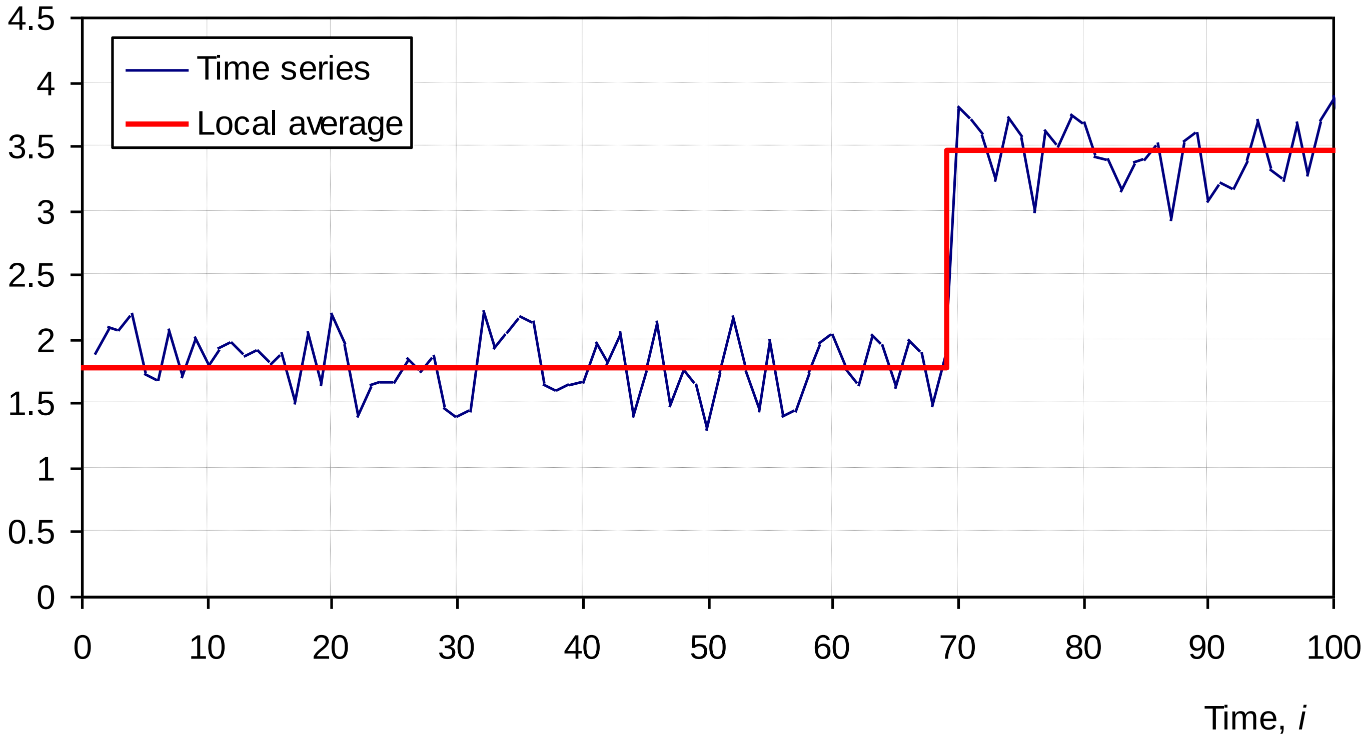


Example:
50 terms of a synthetic time series

See details of this example in Koutsoyiannis (2011)

Does this example suggest stationarity or nonstationarity?

Example time series extended up to time 100

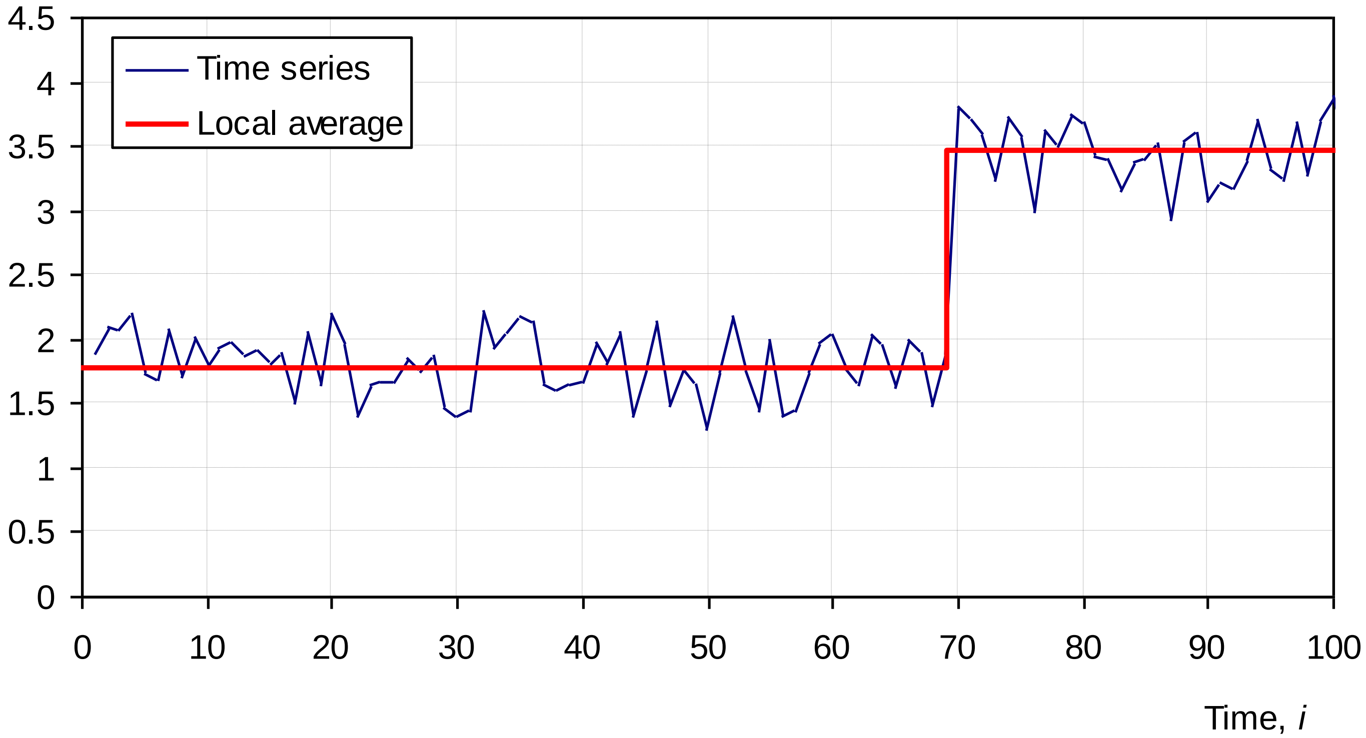


Mean m (red line) of time series (blue line) is:

$m = 1.8$ for $i < 70$

$m = 3.5$ for $i \geq 70$

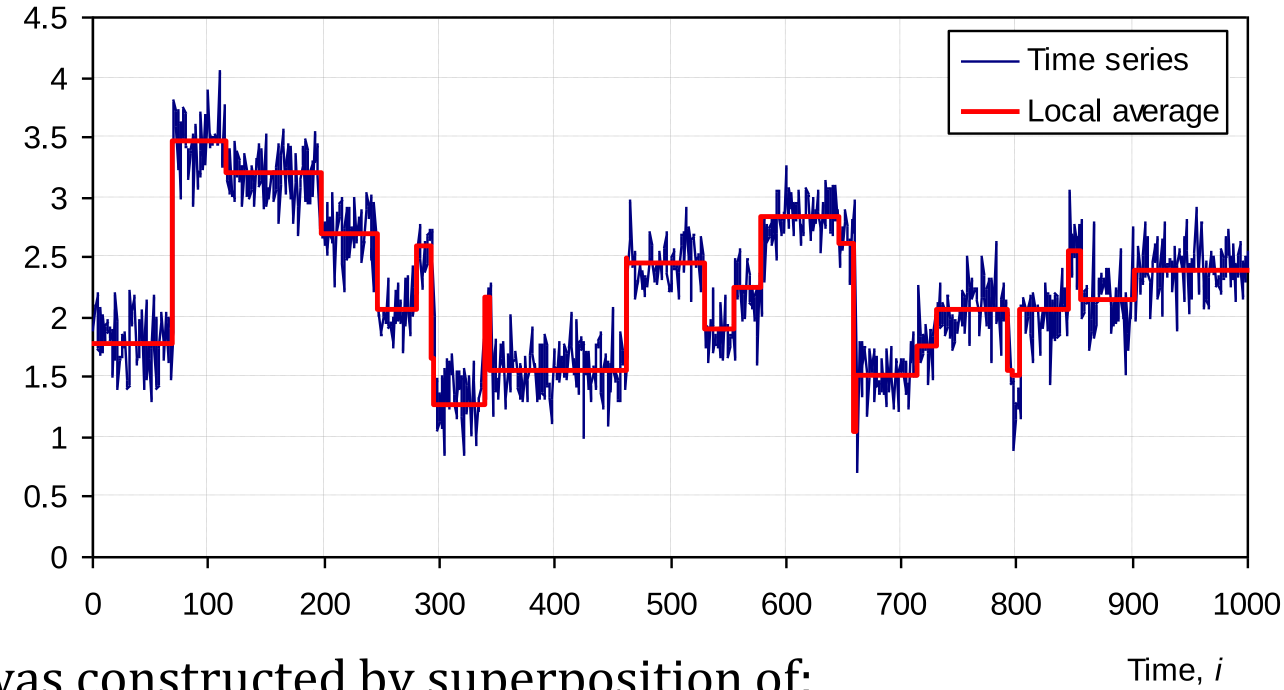
Reformulation of question: Does the red line reflect a **deterministic** function?



Example time series extended up to time 100

- If the red line is a deterministic function of time: → **nonstationarity**.
- If the red line is a random function (realization of a stationary stochastic process) → **stationarity**.

Answer of the last question: the red line is a realization of a stochastic process

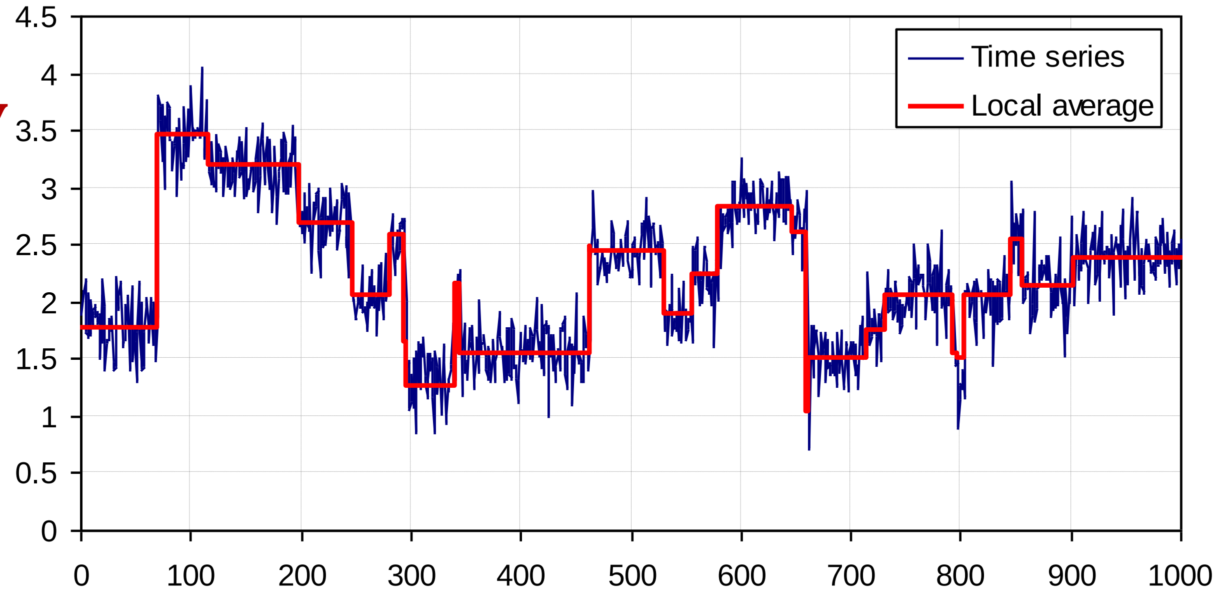


Example 1 extended up to time 1000

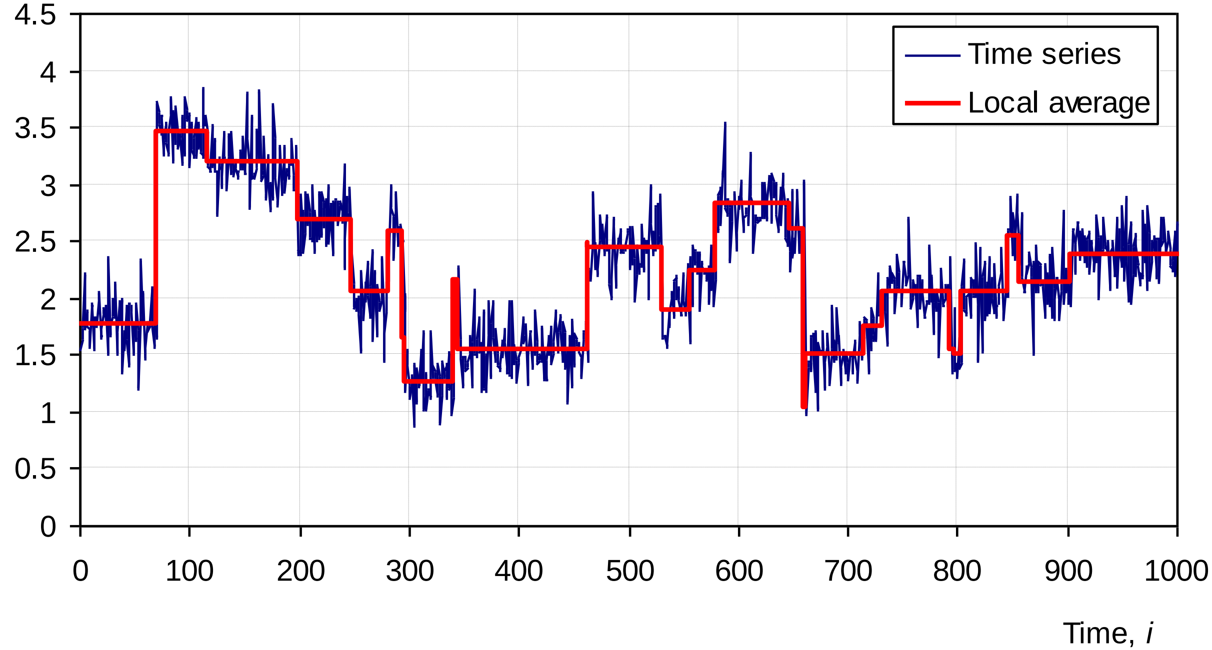
- The time series was constructed by superposition of:
  - A stochastic process with values $\underline{m}_j \sim N(2, 0.5)$ each lasting a period $\underline{\tau}_j$ exponentially distributed with $E[\underline{\tau}_j] = 50$ (red line);
  - White noise $N(0, 0.2)$.
- Nothing in the model is nonstationary.
- The process of our example is **stationary.**

The big difference of **nonstationarity** and stationarity (1)

The initial time series



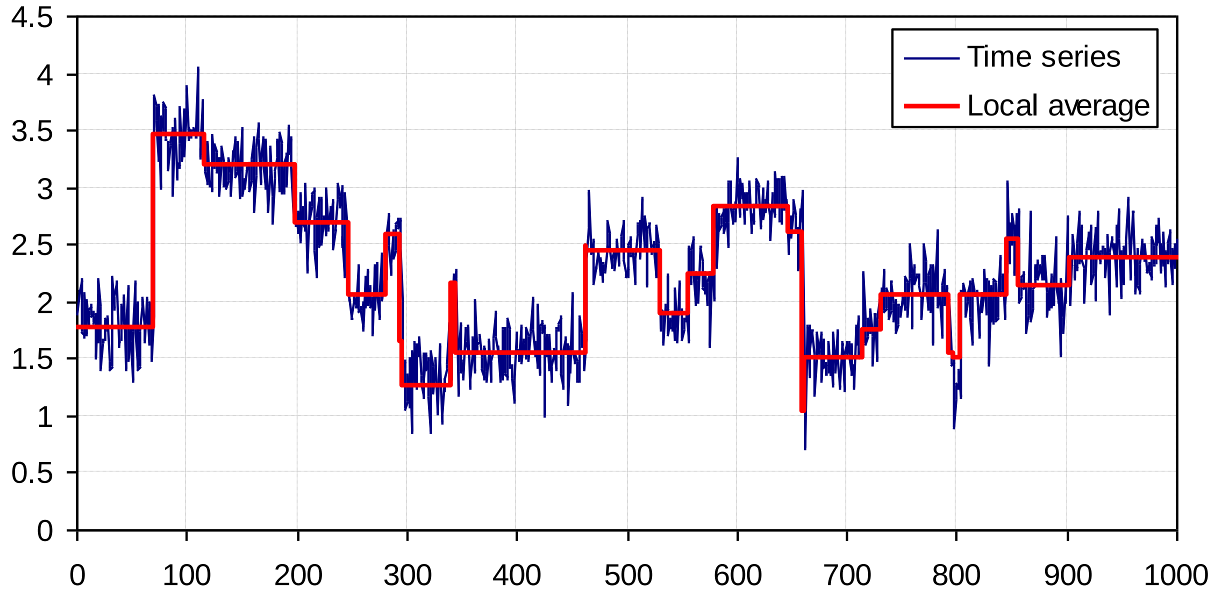
A mental copy generated by a nonstationary model (assuming the red line is a deterministic function)



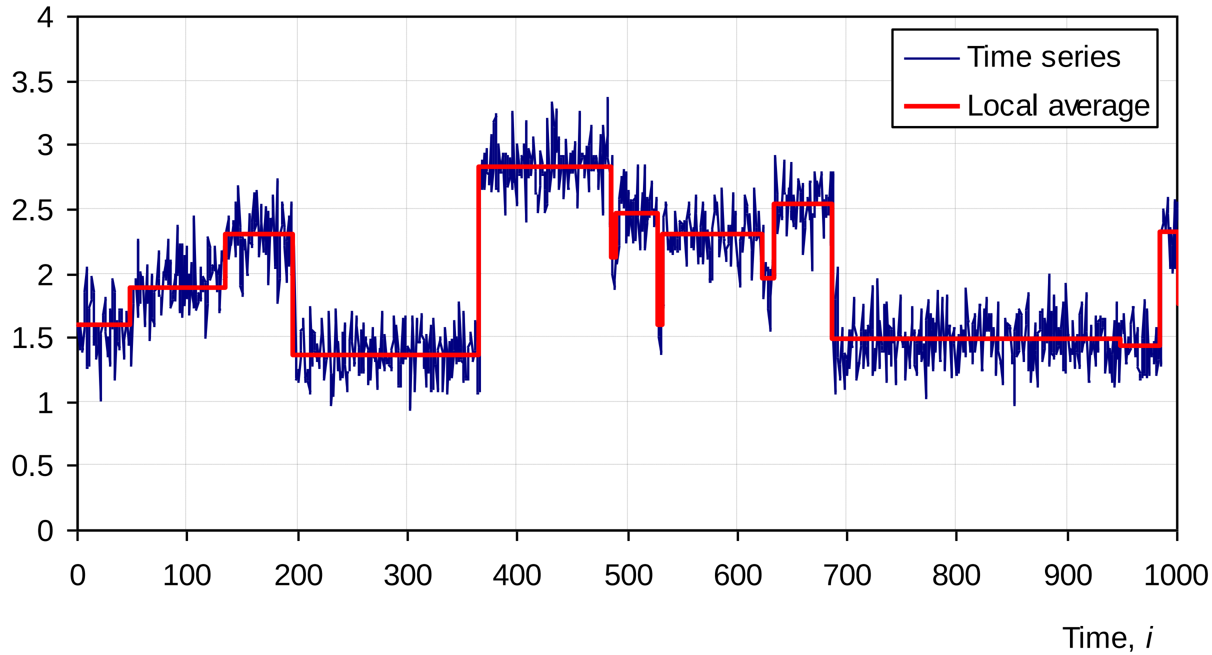
Unexplained variance (differences between blue and red line): $0.2^2 = 0.04$.

The big difference of nonstationarity and **stationarity** (2)

The initial time series



A mental copy generated by a stationary model (assuming the red line is a stationary stochastic process)



Unexplained variance (the "undecomposed" time series): 0.38 (~10 times greater).

Justified use of nonstationary descriptions: Models for the past

- Changes in catchments happen all the time, including in quantifiable characteristics of catchments and conceptual parameters of models.
- If we know the evolution of these characteristics and parameters (e.g. we have information about how the percent of urban area changed in time), then we build a nonstationary model:
  - Information → Reduced uncertainty → Nonstationarity.
- If we do not have this quantitative information, then:
  - We treat catchment characteristics and parameters as random variables.
  - We build stationary models entailing larger uncertainty.

In nonstationary models stationarity is again important

- Even if we have a good deterministic model applicable for future times, we can never hope that it will describe the future in full detail and precision.
- Uncertainty will ever be present.
- That uncertainty (unexplained variability) should be represented as a random component superimposed to the deterministic change given by the deterministic model; that random component is necessarily stationary.
- Thus, even if a process is nonstationary, it will necessarily include a stationary component, and therefore any future prediction needs to ultimately rely on the assumption of stationarity of that random part.

The discussion about stationarity is beyond semantics

- For mitigation of natural hazards, solving practical problems implies the design of management policies and engineering structures that need to be based on the estimation of design variables and their uncertainty, which is also related to economical feasibility of solutions.
- The stationarity concept is useful because it highlights the fact that, whatever deterministic controls and mechanisms are identified and whatever progress is made in deterministic modelling, there will always be unexplainable variability in any system for which a probabilistic description assuming stationarity is needed.
- Both **exact predictability** (particularly for distant times) and **inference without data** are **impossible**.
- Only (physically-based) stochastic modelling using real-world data offers a pragmatic solution.
- Thus, it is not paradoxical to conclude that **stationarity is immortal**, as immortal is the need for statistical descriptions and the need to seek robust solutions to practical problems.

Panta Rhei: Does change entail nonstationarity?

Reply: No
See justification in a series of papers

Change in Hydrology and Society
IAHS Scientific Decade 2013-2022

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Concluding remarks

- Panta Rhei (or: Change is Nature's style).
- Change occurs at all time scales.
- Stationarity is a property of a process and a **process** is synonymous to **change**.
- Nonstationarity should not be confused with change, nor with dependence of a process in time.
- Stationarity and nonstationarity apply to models, not to the real world, and are defined within stochastics.
- Nonstationary descriptions are justified only if the future can be predicted in deterministic terms.
- Unjustified/inappropriate claim of nonstationarity results in underestimation of variability, uncertainty and risk!!!

Stationarity is not dead. It is immortal!!!

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