

# Spatial interpolation of potential evapotranspiration for precision irrigation purposes

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**Abstract:** Precision irrigation constitutes a breakthrough for agricultural water management since it provides means to optimal water use. In recent years several applications of precision irrigation are implemented based on spatial data from different origins, i.e. meteorological stations networks, remote sensing data and in situ measurements. One of the factors affecting optimal irrigation system design and management is the daily potential evapotranspiration (PET). A commonly used approach is to estimate the daily PET for the representative day of each month during the irrigation period. In the present study, the implementation of the recently introduced non-parametric bilinear surface smoothing (BSS) methodology for spatial interpolation of daily PET is presented. The study area was the plain of Arta which is located at the Region of Epirus at the North West Greece. Daily PET was estimated according to the FAO Penman-Monteith methodology with data collected from a network of six agrometeorological stations, installed in early 2015 in selected locations throughout the study area. For exploration purposes, we produced PET maps for the Julian dates: 105, 135, 162, 199, 229 and 259, thus covering the entire irrigation period of 2015. Also, comparison and cross validation against the calculated FAO Penman-Monteith PET for each station, were performed between BSS and a commonly used interpolation method, i.e. inverse distance weighted (IDW). During the leave-one-out cross validation procedure, BSS yielded very good results, outperforming IDW. Given the simplicity of the BSS, its overall performance is satisfactory, providing maps that represent the spatial and temporal variation of daily PET.

**Key words:** irrigation; potential evapotranspiration; agrometeorological stations; bilinear surface smoothing; spatial modelling

## 1. INTRODUCTION

Potential evapotranspiration (PET) is a crucial parameter of several applications in hydrological modelling, irrigation and environmental studies. Especially, precision irrigation requires daily or even hourly PET estimates in order to increase the efficiency of water use through optimization procedures that include the use of sensors along with extensive soil, water and crop data. A significant concept for precision irrigation design is the spatial variability of the PET since the well-established FAO Penman-Monteith model (Allen *et al.* 1998), but also alternative frameworks (Tegos *et al.* 2015, 2017) provide point estimates.

In the present study, spatial interpolation of PET estimates based on measurements from agrometeorological stations placed in an irrigated plain of Greece is investigated in daily scale. In this context, daily PET maps for the characteristic day of each month for the 2015 irrigation period were produced, using the well-known Inverse Distance Weighting (IDW) and the recently introduced non-parametric bilinear surface smoothing (BSS) methodology (Malamos and Koutsoyiannis 2016a). Furthermore, comparison and cross validation against the calculated FAO Penman-Monteith PET for each station, were performed between the two methods, in order to evaluate the performance of BSS.

## 2. MATERIALS AND METHODS

### 2.1 Study area and meteorological stations network

The study area was located at the plain of Arta (453.29 km<sup>2</sup>, the biggest of the region), at the Region of Epirus at the north west coast of Greece. It is part of the Arachthos and Louros hydrological basins and borders with Amvrakikos Wetlands National Park. The local climate is of Mediterranean type, characterized by dry and hot summers and rainy and moderately cool winters.

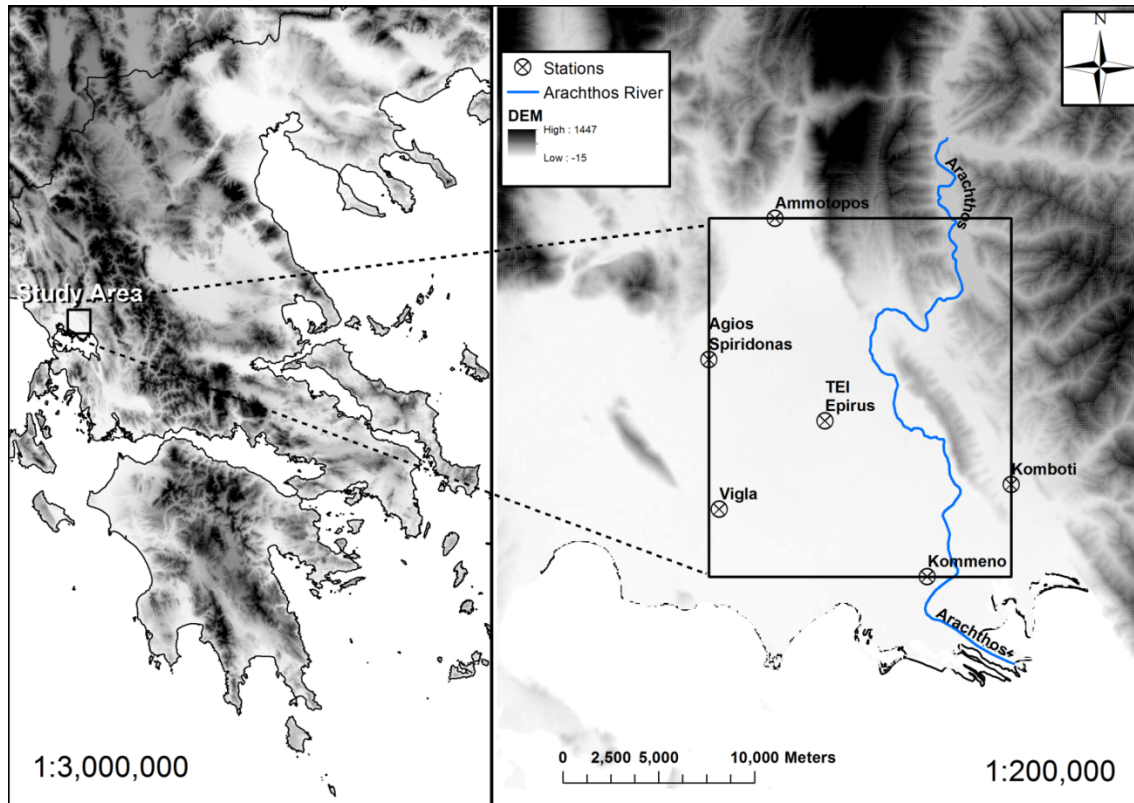


Figure 1. The Arta plain along with the study area and the agrometeorological stations network

Irrigation in the area is performed by means of surface irrigation, sprinkler irrigation and drip systems in proportions of about 40%, 40% and 20% respectively (Tsirogiannis and Triantos 2009), with a continuous diminution of surface irrigation. The vast majority of farmers irrigate based on experience and inherited practical advices. As water is plentiful and cheap, most farmers over irrigate using water by the old open canal scheme that covers part of the plain and from numerous boreholes.

An agrometeorological stations network of six fully equipped weather stations was installed in early 2015 for the implementation of IRMA\_SYS project (Malamos *et al.* 2016; <http://arta.irrigation-management.eu/>) in order to monitor evapotranspiration related parameters. The data from the six meteorological stations are available at <http://openmeteo.org/> under Database Contents License v.1.0 for individual measurements and Open Database License v.1.0 for the data series as they are published at Open Data Commons (<http://opendatacommons.org/>).

The analysis extend (mask) boundaries were defined by the coordinates of the outermost stations according to each one of the four cardinal directions. This was mandatory in order to ensure that the PET estimates adjacent to the boundaries of the study area are obtained from interpolation rather than extrapolation. The maps were produced using a 500 × 500 m grid for practical and computational reasons, covering an area of approximately 294.8 km<sup>2</sup> inside the plain of Arta.

## 2.2 The Penman-Monteith equation for potential evapotranspiration

The Penman-Monteith (PM) equation for estimating potential evapotranspiration from a vegetated surface, as formalized by Allen *et al.* (1998), is:

$$\text{PET} = \frac{1}{\lambda} \frac{\Delta(R_n - G) + \rho_a c_a (v_a^* - v_a) / r_a}{\Delta + \gamma(1 + r_s / r_a)} \quad (1)$$

where PET is the daily potential evapotranspiration;  $R_n$  is the net incoming daily radiation at the vegetated surface;  $G$  is the soil heat flux;  $\rho_a$  is the mean air density at constant pressure;  $c_a$  is the specific heat of the air;  $r_a$  is an aerodynamic or atmospheric resistance to water vapour transport for neutral conditions of stability;  $r_s$  is a surface resistance term;  $v_a^* - v_a$  is the vapour pressure deficit of the air, defined as the difference between the saturation vapour pressure  $v_a^*$  and the actual vapour pressure  $v_a$ ;  $\lambda$  is the latent heat of vaporization;  $\Delta$  is the slope of the saturation vapour pressure curve at specific air temperature; and  $\gamma$  is the psychrometric constant. Given that the typical time scale of the PM equation is daily, all associated fluxes are expressed in daily or mean daily units.

Apart from the site location, expressed in terms of latitude,  $\phi$ , the PM equation requires air temperature, humidity, solar radiation and wind speed data for calculating the model variables  $R_n$ ,  $v_a^*$ ,  $v_a$ , and  $\Delta$ .

## 2.3 Spatial interpolation methods

The mathematical framework of Bilinear Surface Smoothing (Malamos and Koutsoyiannis 2016a), suggests that fit is meant in terms of minimizing the total square error among the set of original points  $z_i(x_i, y_i)$  for  $i = 1, \dots, n$  and the fitted bilinear surface, that in matrix form, can be written as:

$$p = \|\mathbf{z} - \hat{\mathbf{z}}\|^2 \quad (2)$$

where  $\mathbf{z} = [z_1, \dots, z_n]^T$  is the vector of known applicates of the given data points with size  $n$  (the superscript T denotes the transpose of a matrix or vector) and  $\hat{\mathbf{z}} = [\hat{z}_1, \dots, \hat{z}_n]^T$  is the vector of estimates with size  $n$ .

The details of the method including the algorithms and derivations of the equations are available at Malamos and Koutsoyiannis (2016a).

The parameters required to implement the methodology, are the segments of the bilinear surface, i.e.  $m_x$ ,  $m_y$  and the smoothing parameter  $\lambda$ . These parameters can be estimated by transforming the smoothing parameter  $\lambda$  in terms of tension:  $\tau_\lambda$  whose values are restricted in the interval  $[0, 1)$ , for both directions i.e.  $\tau_{\lambda_x}$  and  $\tau_{\lambda_y}$  (Malamos and Koutsoyiannis 2016a). This transformation provides a convenient search in terms of computational time and is based on the generalized cross-validation (GCV; Craven and Wahba 1978) methodology. Thus, for a given combination of segments  $m_x$ ,  $m_y$ , the minimization of GCV results in the optimal values of  $\tau_{\lambda_x}$ ,  $\tau_{\lambda_y}$ . This can be repeated for several trial combinations of  $m_x$ ,  $m_y$  values, until the global minimum of GCV is reached.

On the other hand, the Inverse Distance Weighting (IDW) method was implemented as a quick interpolator capable to address the characteristics of the study area regarding the limited number of meteorological stations. IDW is a straightforward and non-computationally intensive method (Burrough and McDonnell 1998). The IDW implementation for producing the PET maps of the study area was performed by means of ESRI's ArcGIS environment.

## 3. RESULTS AND DISCUSSION

In order to evaluate the performance of the BSS methodology for spatial interpolation of daily

PET, we interpolated the Penman-Monteith PET values acquired at the stations locations (Table 1) using both BSS and IDW for the characteristic day of each month for the 2015 irrigation period as presented in Figures 2 and 3.

Table 1. Penman-Monteith PET values at the locations of each of the six stations

Julian dates	PM PET (mm)					
	Agios Spiridonas	Vigla	Ammotopos	TEI of Epirus	Kommeno	Kompoti
105	3.4	3.2	3.4	3.2	3.2	3.1
135	4.1	4.4	4.5	4.4	4.5	4.1
162	4.8	4.4	5.2	4.9	5.5	5.4
199	6.2	5.2	6.7	6.4	6.1	6.3
229	4.6	4.7	5.0	4.7	4.9	4.8
259	3.8	3.4	3.8	3.7	3.6	3.6

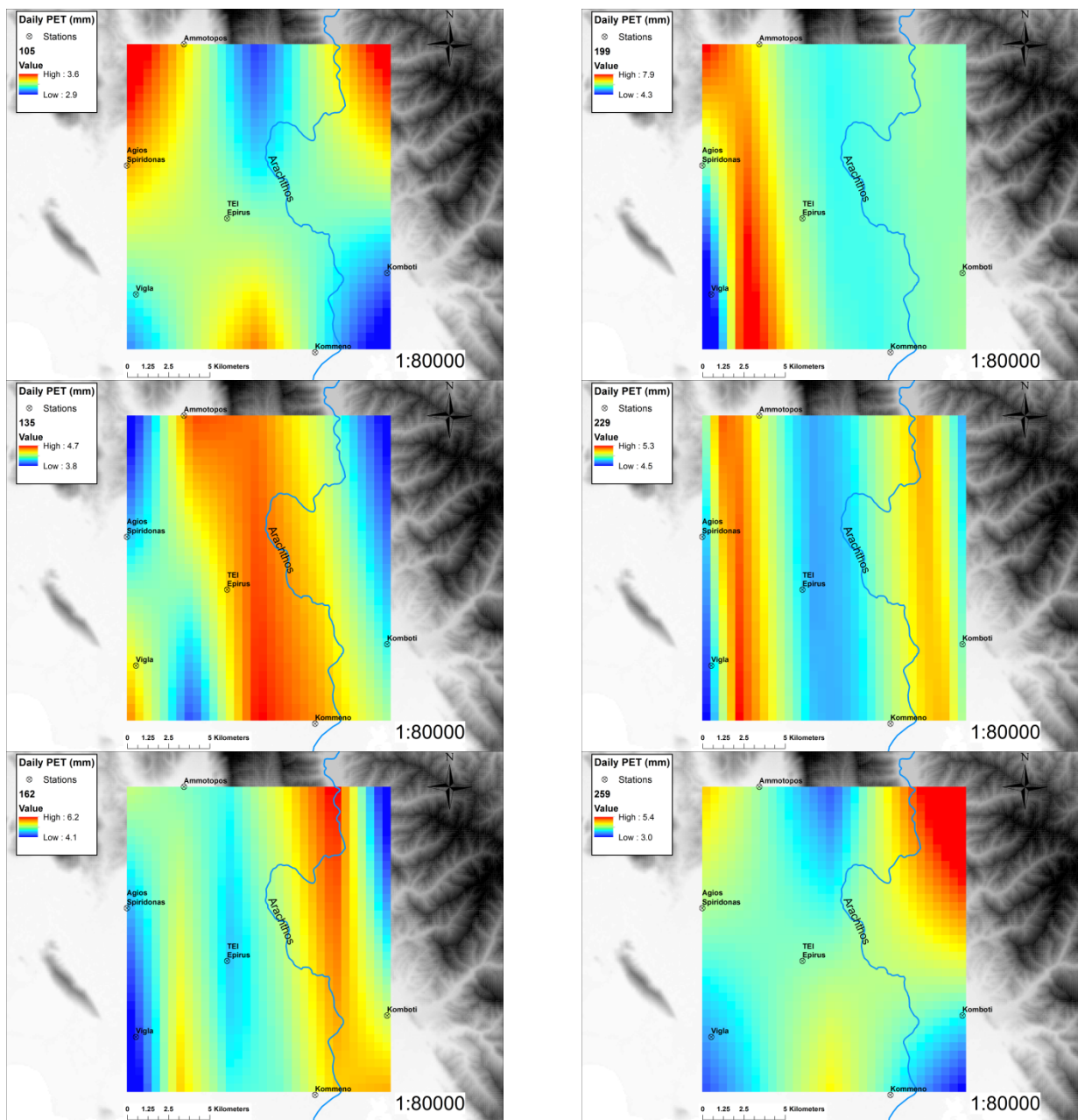


Figure 2. BSS PET maps for Julian dates 105, 135, 162, 199, 229, 259 of year 2015

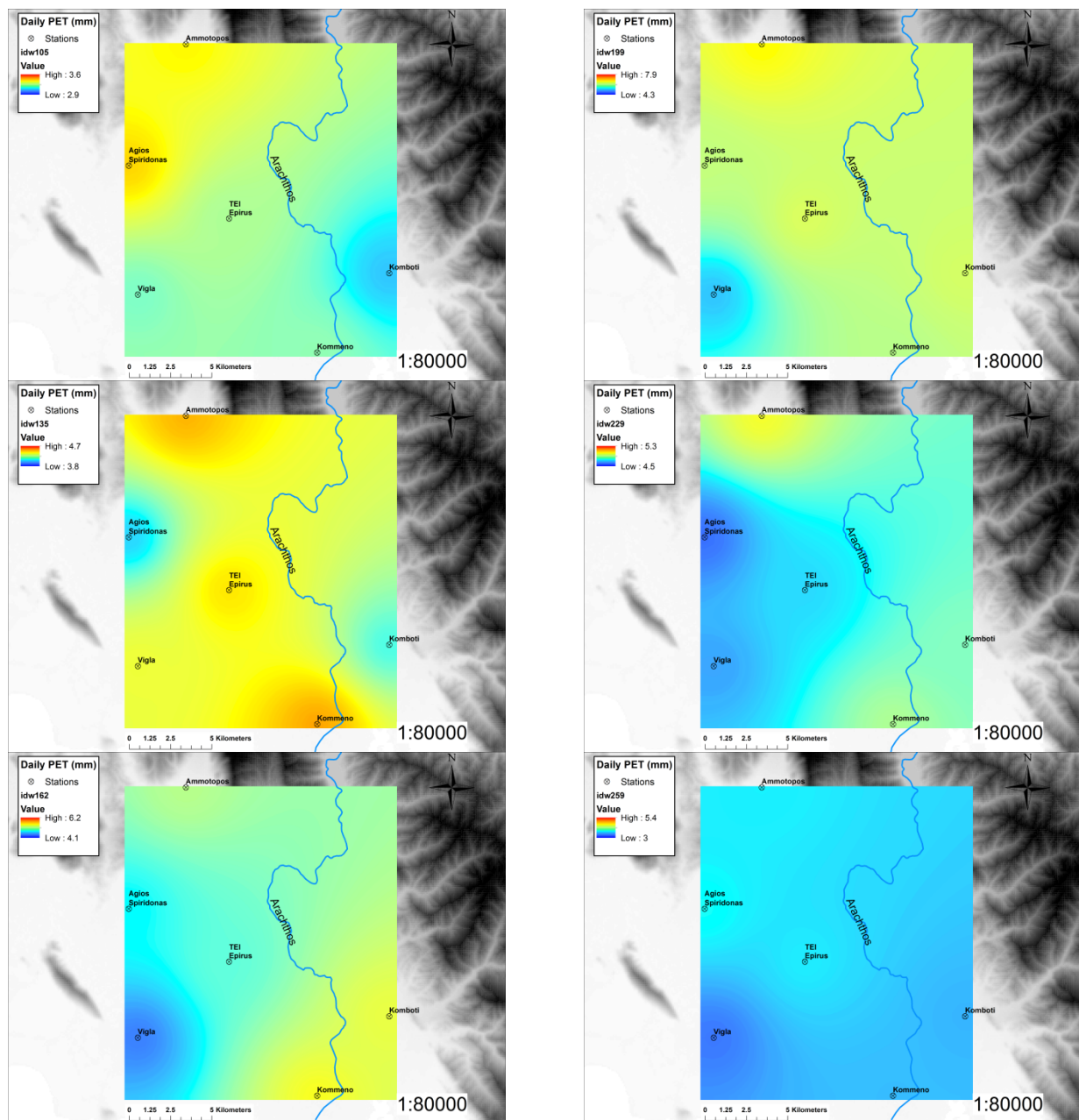


Figure 3. IDW PET maps for Julian dates 105, 135, 162, 199, 229, 259 of year 2015

From the comparison of the produced maps it is obvious that the BSS modelled PET values, have wider range than those of IDW, thus producing very plausible interpolation surfaces that respect the variation due to terrain, avoiding the characteristic IDW's bull's eye shaped artefacts.

Considering BSS's implementation, the global minimum of GCV for each day was reached by implementing it for different numbers of segments  $m_x$  and  $m_y$  ( $1 \leq m_x \leq 15$  and  $1 \leq m_y \leq 15$ , while  $m_x + m_y + 1 \geq 6$ ) and minimizing GCV for each one, by altering the adjustable parameters.

The results of the above procedure are presented in Table 2, along with the corresponding mean square error values. It is obvious that the BSS implementation resulted in very small mean square error values, respecting the estimated PM PET values at the stations locations. We note that there is no need to present the corresponding mean square error values for the IDW at the given data points i.e. the stations locations, since it is an exact method of interpolation so its results respect the data points exactly.

Also, the above criteria of performance may not be representative with respect to the validity of the interpolation results in other locations, except for those incorporated in the interpolation procedure. So, an alternative technique was implemented for the evaluation of the bilinear surface

smoothing method efficiency, in terms of performing a leave-one-out cross validation procedure of the two methods. The procedure was implemented in MS-Excel<sup>®</sup> and included a loop in which the PET values at the location of each station were estimated using the remaining five as the input dataset to each one of the two interpolation methods. In this way, we acquired a total of six, one for every day, PET estimates at the stations locations which were compared against the already acquired daily PM PET values. We should note that BSS was implemented using the previously obtained parameter values as presented in Table 2.

Table 2. BSS optimal parameter values and performance indices

Julian dates	mx	my	$\tau_{\lambda x}$	$\tau_{\lambda y}$	Mean square error	Global minimum GCV
105	2	3	0.902	0.154	$1.75 \times 10^{-5}$	$2.93 \times 10^{-3}$
135	4	5	0.003	0.765	$1.42 \times 10^{-5}$	$3.76 \times 10^{-3}$
162	5	4	0.99	0.001	$1.13 \times 10^{-5}$	$4.95 \times 10^{-3}$
199	6	12	0.76	0.019	$7.42 \times 10^{-5}$	$3.15 \times 10^{-2}$
229	14	6	0.201	0.001	$3.77 \times 10^{-6}$	$6.35 \times 10^{-4}$
259	2	6	0.784	0.067	$2.47 \times 10^{-5}$	$5.11 \times 10^{-3}$

The performance of each method was evaluated by using statistical criteria such as: mean bias error (MBE), mean absolute error (MAE), root mean square error (RMSE), mean square error (MSE) and modelling efficiency (EF) which is calculated on the basis of the relationship between the observed and predicted mean deviations (Malamos and Koutsoyiannis 2016b). As Table 3 states, BSS clearly outperformed IDW in all circumstances, apart from the EF and RMSE criteria at the Kompoti station. In this case, both methods failed to provide satisfactory estimates of PET values. An explanation to this behaviour is the fact that the Kompoti station is placed on the east side of the study area close to the mountains (see Figure 1) so when it is missing, the available information is inadequate to describe the orography effects, thus resulting in insufficient estimates.

Table 3. Performance of BSS and IDW against PM PET values in the leave-one-out cross validation procedure

	Agios Spiridonas		Vigla		Ammotopos		TEI of Epirus		Kommeno		Kompoti	
	BSS	IDW	BSS	IDW	BSS	IDW	BSS	IDW	BSS	IDW	BSS	IDW
MBE (mm)	0.0	0.8	0.0	0.4	0.0	0.4	0.0	0.1	-0.5	1.6	-1.7	1.7
MAE (mm)	0.1	0.8	0.1	0.4	0.0	0.4	0.1	0.1	0.5	1.6	1.7	1.7
RMSE (mm)	0.1	0.8	0.2	0.5	0.1	0.4	0.1	0.1	0.8	1.6	1.9	1.7
EF	0.99	0.18	0.95	0.47	1.00	0.85	0.99	0.98	0.44	-1.55	-2.02	-1.44

## 4. CONCLUSIONS

Two different approaches for spatial interpolation of daily potential evapotranspiration were implemented using data from six meteorological stations located in the plain of Arta, at the Region of Epirus. Both approaches were implemented for the characteristic day of each month for the 2015 irrigation period, i.e. Julian dates: 105, 135, 162, 199, 229 and 259. The objective was to evaluate the performance of bilinear surface smoothing (BSS) method against the inverse distance weighting (IDW) method. The comparison against the estimated values of the FAO Penman-Monteith (PM) PET for each station showed that BSS yielded very good results with very small mean square error values, respecting the given PM PET values.

Also, a leave-one-out cross-validation procedure per station was used for validating the performance of both spatial interpolators. Thus we acquired a total of six, one for every day, PET estimates at the stations locations which were compared against the already acquired daily PM PET values. During this cross validation procedure BSS clearly outperformed IDW in almost every case,

respecting the variation of the terrain and also avoiding the characteristic IDW's bull's eye shaped artefacts. Given the simplicity of the BSS methodology, its overall performance is satisfactory, providing maps that represent the spatial and temporal variation of daily PET, thus granting the necessary tools for implementing precision irrigation on daily or finer time scale.

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