Entropy, recycling and macroeconomics of water resources

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Abstract

We develop a *macroeconomic model* of water quantity and quality *supply multipliers* derived from *water recycling* (Karakatsanis et al. 2013) and examine its statistical properties. Macroeconomic models that incorporate natural resource conservation have become increasingly important (European Commission et al. 2012) for national accounting. In addition, as an estimated 80% of globally used freshwater is not reused (United Nations 2012), with increasing population trends, water resource recycling becomes a solution of high priority. Recycling of water resources generates two major *conservation* effects: **(1)** conservation of water in reservoirs and aquifers and **(2)** conservation of ecosystem carrying capacity due to wastewater flux reduction. It is the properties of the distribution of *recycling efficiencies* –on quantity and quality- per sector that determine macroeconomic decoupling from geophysical *uncertainty*. Generally, uncertainty may statistically be quantified by *entropy*. Higher entropy signifies a greater dispersion of recycling efficiencies and potentially greater exposure to geophysical uncertainty; probably indicating the need for additional infrastructure for the statistical distribution's both shifting and concentration towards higher efficiencies, supply multipliers and geophysical uncertainty decoupling.

Keywords: Entropy, geophysical uncertainty, water recycling, water supply multipliers, conservation, distribution of recycling efficiencies, decoupling, macroeconomics

1. Entropy and water resource economics: Adaptability

Shannon (1948) postulated a *statistical mechanical definition of entropy*, concerning the propagation complexity of a communication signal as a random variable *(X)* within a specific time-frame. Generalizing Shannon's formula *H(X)* –considering it comprises a function of the used language's complexity as well (logarithm base)- we may write:

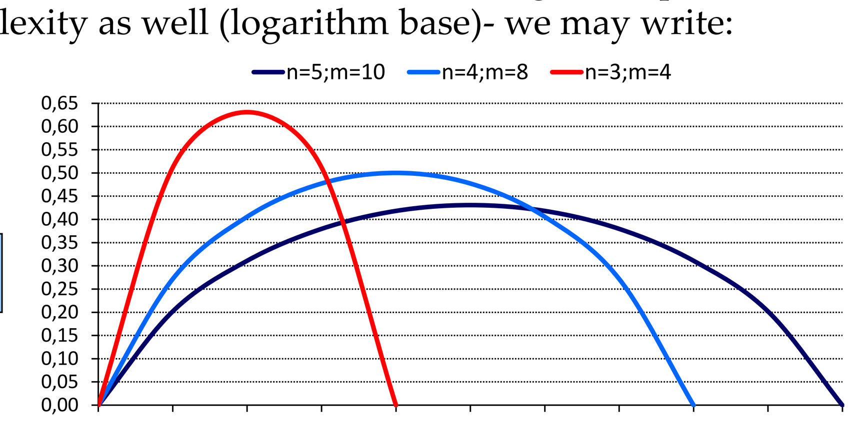
$$H(m;n) = -\sum_{i=1}^{m} P_i \cdot \log_n P_i$$

For a discrete-time random variable

$$H_X(x;n) = -\int f_X(x) \cdot \log_n \left(f_X(x)\right) dx$$

For a continuous-time random variable

s.t. $\sum Pi=1$ and $\int f_X(x)dx=1$



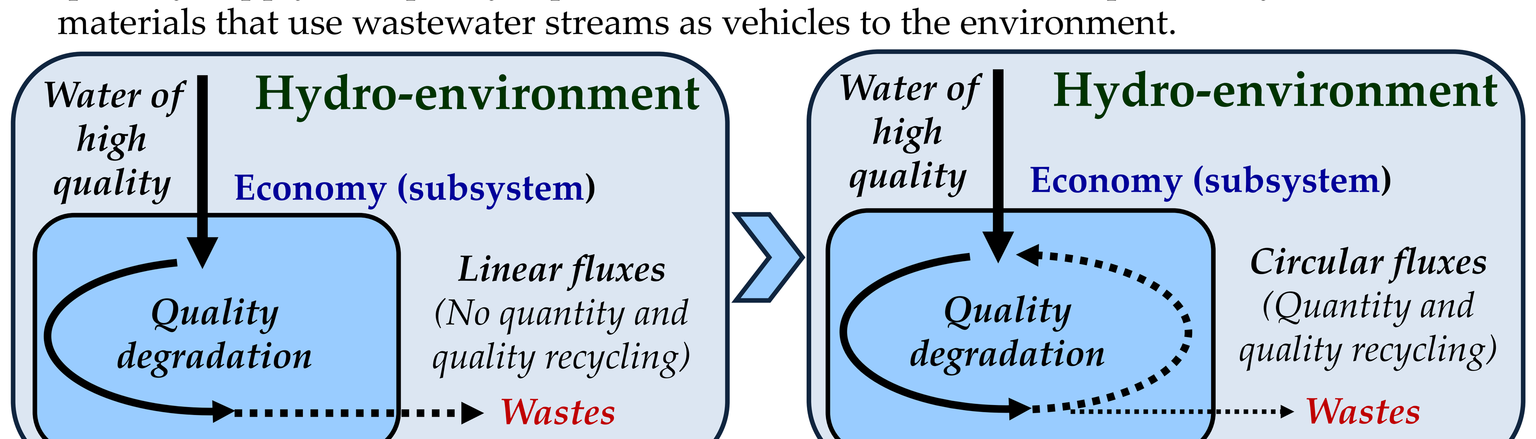
Using Shannon's paradigm, we conceptualize and simulate processes of hydrological supply as signals that incorporate *uncertainty*; statistically quantified by entropy. Uncertainty comprises either a function of pure randomness or *structured complexity*; which signifies a *wider range* of probabilities per unit time (variability). As the *economy* is considered to be a *system* of which the structural preservation relies significantly on hydrological supply, the connection of *entropy to water resource economics* lies in the effort to identify the economy's exposure to hydrological variability as well as how it can reconfigure its internal structure –via water resource recycling- in order to decouple itself from it (hydrological variability).

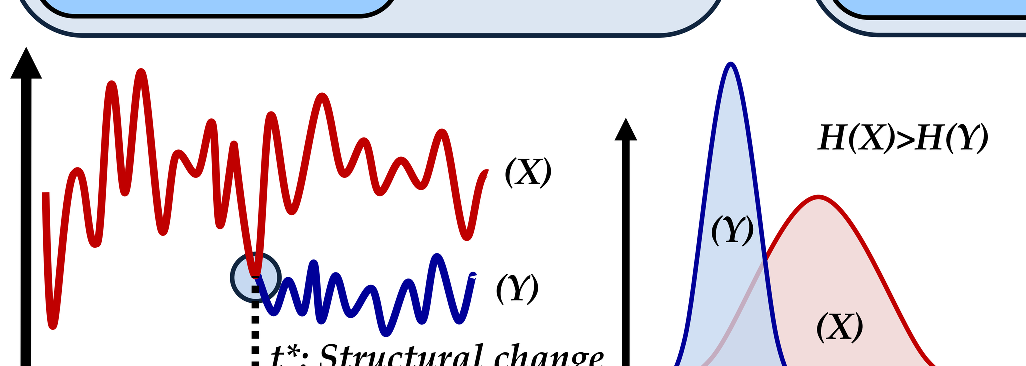
Statistical manifestations of entropy in water resource economics

- Statistically, entropy is manifested as *uncertainty* (weak form) or *structural complexity* (strong form; concerns even structured uncertainty) of water supply probability
- Statistically, *entropy buffering* is equivalent to a *structural change*; a shift of the economy's demand for natural water supply towards lower distribution parameters $\{E[X], \sigma^2, \sigma\}$
- Statistically, the utility of a water resource is *reverse proportional to its entropy*, as high entropy of natural supply signifies economic structural exposure to higher probability of water supply failure due to a wider range of hydrological events (from scarcity to floods)

2. Entropy and the challenge of a circular water economy

The challenge of the global economy consists in its *structural change* from the *linear to a circular model* of water use, where wastes are processed and reused as raw materials. For a circular water economy, entropy (uncertainty) is reduced due to: ***(a)*** *decoupling* from natural quantity supply and quality replenishment and ***(b)*** *reclamation* of potentially useful materials that use wastewater streams as vehicles to the environment.





The introduction of water recycling shifts the original time-series for freshwater demand (X) towards both lower values and lower variability. The new distribution (Y) as less dependent from natural supply is more reliable as subject to lower uncertainty (more resistant against a wider probability range); hence it has lower entropy

3. Modelling water quantity and quality recycling

The primary modelling elements concern the efficiency among economic sectors that exchange wastewater (Karakatsanis 2010): ***(a)*** *Efficiency Index* ***(b)*** *Efficiency Topology Matrix* and ***(c)*** *Supply Multiplier*.

$$m_{ij} = M_{ij}^{o} / M_{ij}^{I}$$

Efficiency Index: M^0 = wastewater outflow, M^I = wastewater inflow, where m_{ij} (with $0<m<1$) is a flux of water quantity or (and) quality from sector *i* to sector *j*, with m_{ij} = or ≠ m_{ji}

$$W_{M_{ij}} = \frac{W_{t_{ij}}}{(1-m_{ij})}$$

$$\begin{vmatrix} m_{11} & m_{12} & m_{13} & \ldots & m_{1j} \\ m_{21} & m_{22} & m_{23} & \ldots & m_{2j} \\ m_{31} & m_{32} & m_{33} & \ldots & m_{3j} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ m_{i1} & m_{i2} & m_{i3} & \ldots & m_{ij} \end{vmatrix}$$

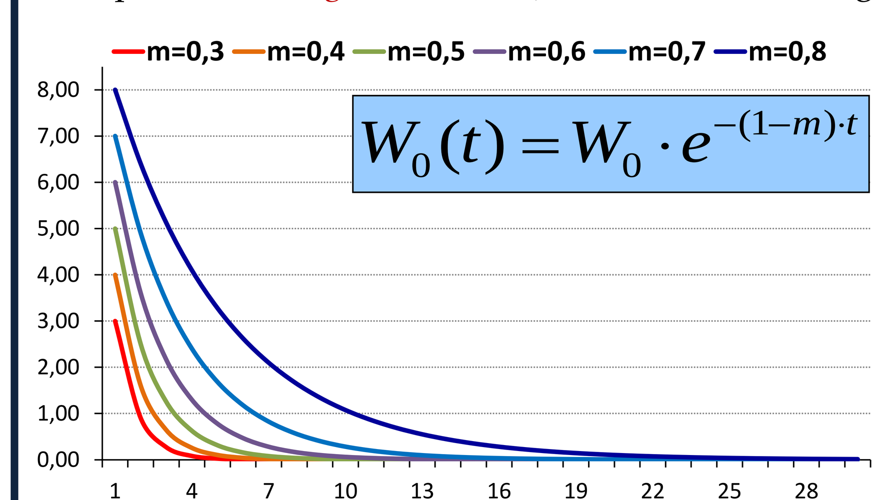
Efficiency Topology Matrix: A Leontief-type input-output $n \cdot n$ matrix that reveals the efficiency indexes of all probable wastewater combinations *w* between sectors. Non-zero values of m_{ij} with $i=j$ signify a single closed loop. For *b* the number of hubs, possible combinations vary from $1 \le w \le b^2$

Supply Multiplier: W_{tij} is an amount of water at any time *t* and $m_{ij}=m_{ij}$ is the *common* Efficiency Index between sectors *i* and *j* (for i = or ≠ j)

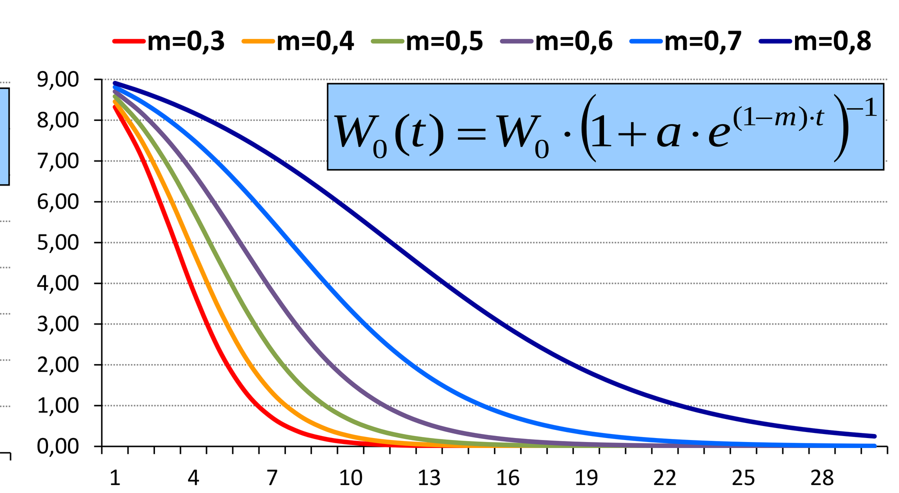
4. Modelling economic residence times of water

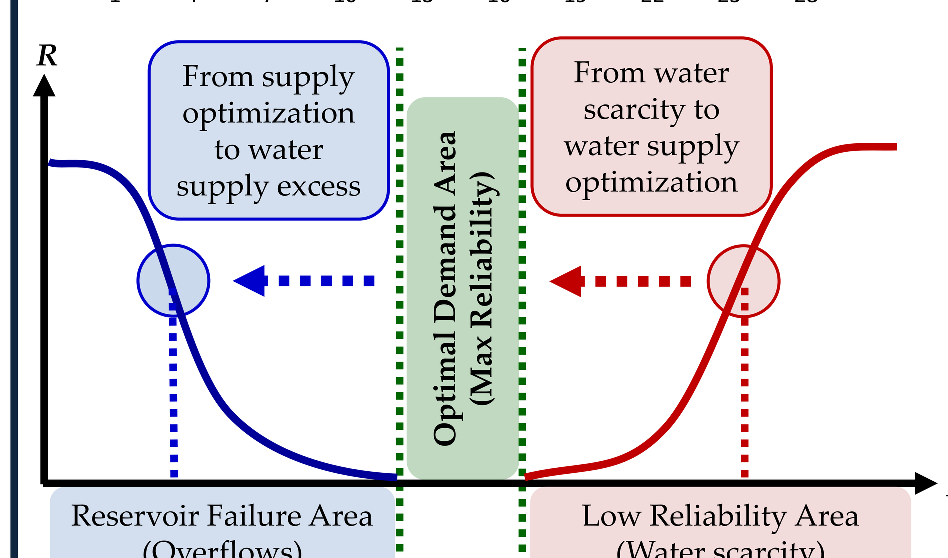
Water resource recycling, extends the time that an initial amount of extracted freshwater W_0 resides in the economic system with the ability to produce economic value- until the latter decays completely (due to the 2nd Law)– defined as *economic residence time of water*. Assuming a constant efficiency rate *m*, we model residence times with two (2) decay models: ***(a)*** the one-parameter *Exponential* and ***(b)*** the two-parameter *Logistic* model (with $a>0$, as a scaling parameter of proximity to W_0. Here, $a=0,1$).

$$W_0(t) = W_0 \cdot e^{-(1-m)\cdot t}$$



$$W_0(t) = W_0 \cdot \left(1 + a \cdot e^{(1-m)\cdot t}\right)^{-1}$$





The problem of excessive residence times

Water resource infrastructure aims at increasing reliability (*R*) against both scarcity and floods. As water demand (*D*) increases, it finds an area of joint reliability optimization. However, if the recycling of water resources results to be too efficient, it may increase excessively the residence time of an initial water amount W_0, so that (*D*) for naturally supplied freshwater will minimize enough to increase the probability of overflows and loss of naturally supplied economic value.

5. Entropy and stochastic modelling of water quality mortality

We model the evolution of water quality across water use with the two-parameter *Weibull distribution* as a *failure (to preserve water quality) concentration process*. Furthermore, as abundant quantitatively but degraded qualitatively water is of less economic value, we should examine the *Mutual Information* (*I*) of probability density of recycled quality (*X*) and quantity (*Y*) to identify deviations from the optimal path, in order to target efficiency enhancements of wastewater paths and recycling processes.

The Weibull distribution

CDF $F(X;c,k) = 1 - e^{(x/c)^k}$

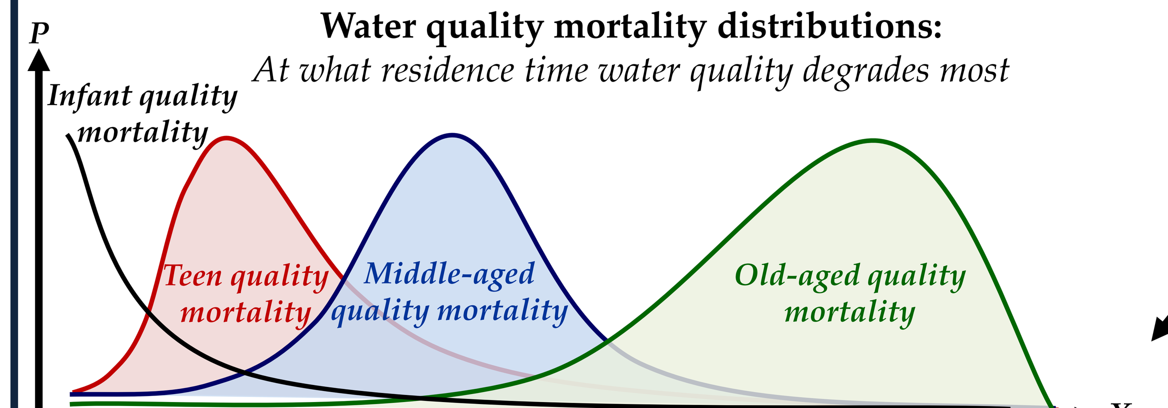
PDF $f(x;c,k) = \frac{k}{c^k} \cdot x^{k-1} \cdot e^{(x/c)^k}$

H(X) $H_W = \gamma \cdot (1 - 1/k) + \ln(c/k) + 1$

For constant scale parameter *c*, the mortality type is defined by the position parameter *k*. Lower values of *k* signify greater dispersion and entropy of the water quality state per unit time

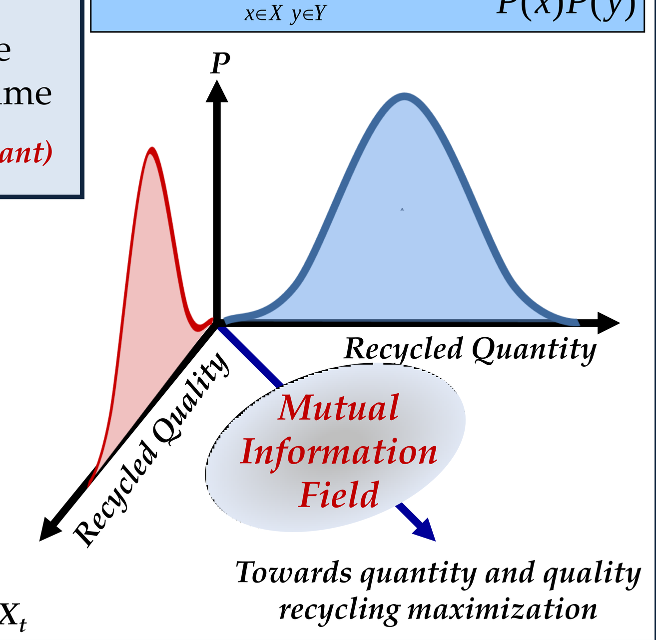
(γ~0,5722; Euler-Mascheroni constant)

Water quality mortality distributions:
At what residence time water quality degrades most



Mutual Information I(X;Y)

$$I(X;Y) = \sum_{x \in X} \sum_{y \in Y} P(x,y) \log \frac{P(x,y)}{P(x)P(y)}$$

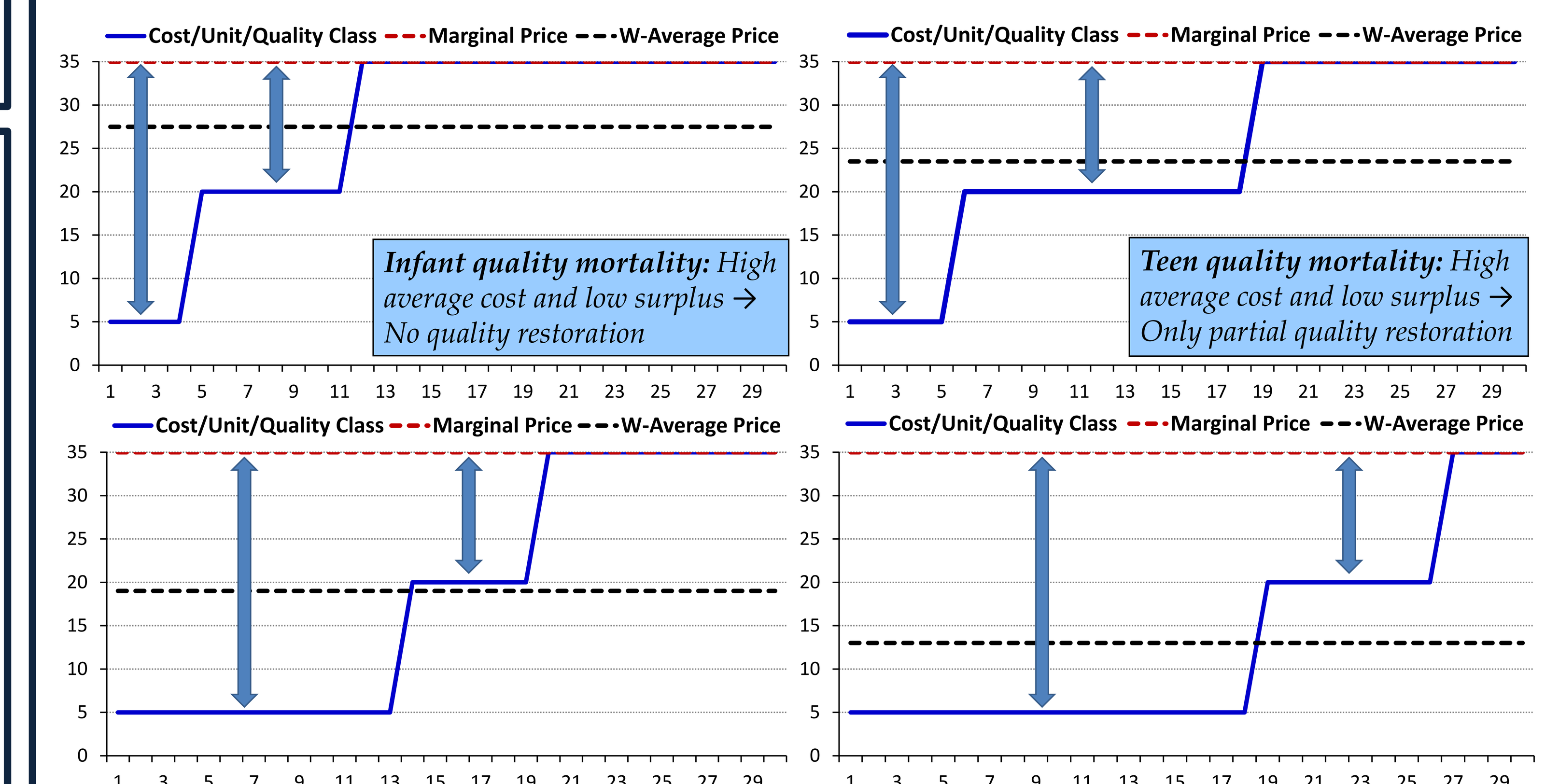


6. Water as a "ricardian" resource

Biophysical surpluses: From "land-rent" to "hydro-rent"

A natural resource of finite quantity, which incorporates finite qualitative characteristics that are distributed in more than one classes η ($\eta>1$), generates a ***biophysical surplus*** of which the economic value does not derive from any kind of economic activity but purely from its own natural attributes.

Water quality could concern a measure of chemical potential for diluting compounds (Huang et al. 2007), which comprises a common biophysical surplus of water. We model the cost and value surplus patterns of a typical ricardian water resource with four (4) cases of quality mortality structure that reflect patterns described by the respective Weibull distributions:



Infant quality mortality: *High average cost and low surplus → No quality restoration*

Teen quality mortality: *High average cost and low surplus → Only partial quality restoration*

Middle-aged quality mortality: *Average cost drops and surpluses rise → Deployment of quality restoration*

Old-aged quality mortality: *Maximization of quality restoration → Cost minimized and surpluses maximized*

Pricing methodologies of ricardian water

(a) Marginal Pricing

The final price (P_M) of water is determined by the cost to extract, process and dispose of the ***worst quality class*** (*i*) of available water. All biophysical surpluses (BS_{P_M}) derived from all better quality classes are directly distributed to the owners of water.

$$P_M = Max(C_i)$$

$$BS_{P_M} = \sum_{i=1}^{n} (C_{Max} - C_i) \cdot Q_i$$

(b) Weighted-Average (W-A) Pricing

The final price ($P_{W\text{-}A}$) of water is formed by the weighted-average (*W-A*) of the costs per quality class (*i*). The distribution of water quality mortalities is of vital importance. Biophysical surpluses ($BS_{P_{W\text{-}A}}$) are zero as they are already distributed as subsidies of cost reduction to water of lower quality classes. Suitable for social water policy.

$$P_{W-A} = \frac{\sum_{i=1}^{n} (Q_i / Q_T) \cdot C_i}{Q_T}$$

$$BS_{P_{W-A}} = 0$$

7. Conclusions

- ✓ The connection of entropy to water economics consists in the economy's decoupling from (exogenous) hydrological supply uncertainty
- ✓ Demand for freshwater after water recycling must fall within the optimal reliability area in order to avoid the cost of managing systematic supply excesses or (residual) shortages
- ✓ The parameters of the water quality mortality distribution determine fundamentally the mean (expected) cost level of available water resources in the economy

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