



XIth Scientific Assembly of the International Association of Hydrological Sciences (IAHS 2022)  
***Extremes in hydroclimatic systems***

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# **Investigating the clustering mechanisms of hydroclimatic extremes: from identification to modelling strategies**

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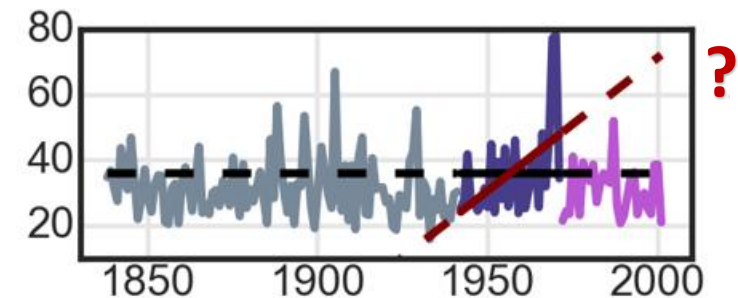
# Importance of temporal clustering of hydrological extremes

Temporal properties of extreme events are less understood compared to marginal distribution properties.

□ **Temporal dependence properties of hydrological extremes affect:**

- Duration of flood and extreme rainfall events
- Failures due to 'medium-magnitude' yet prolonged/consecutive events, e.g. bridge scour, dam failures, landslides
- Droughts
- Insurance and re-insurance strategies
- Climatic simulations & projections
- Risk perception & communication

Small to large scales



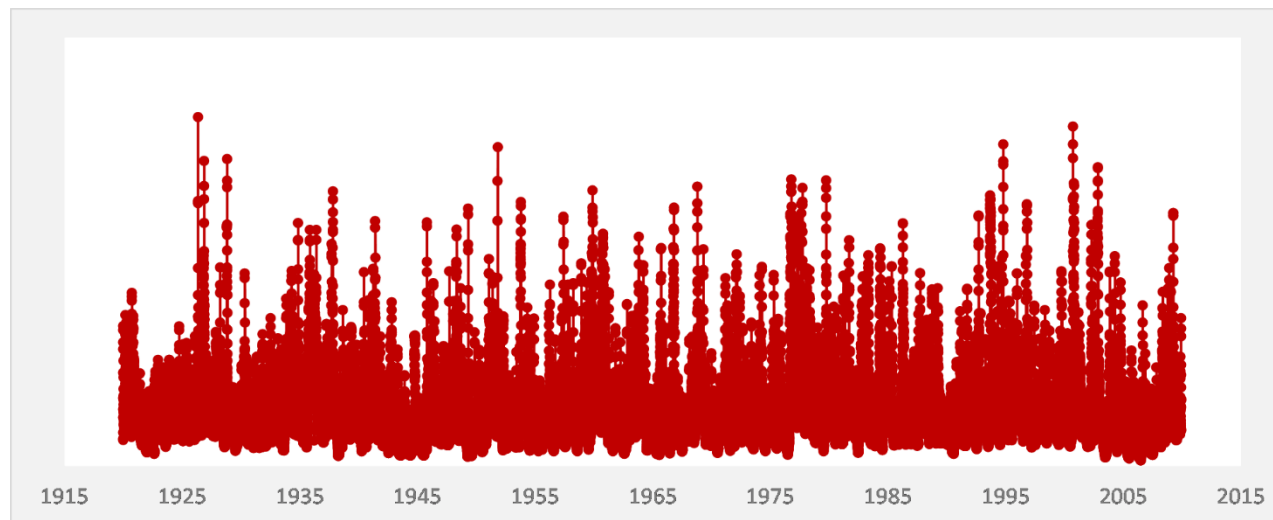
□ As continue-time simulation is progressively adopted in hazard models, the need to identify and reproduce dependence of hydroclimatic extremes becomes more relevant.

# Temporal clustering of extremes: a 'reverse' stochastic approach

- How can we **identify** and characterize **multi-scale clustering** of extremes?
  - **Seasonal**, i.e. multitude of days to multitude of months
  - **Annual and multi-annual**, i.e.  $\sim 365$  days to multitude of years
  - **Climatic**, i.e. 30 years
- What is a parsimonious approach for modelling multi-scale clustering?

**We study clustering of extremes through the study of the parent process' properties.**

- The two are typically decoupled in the literature under asymptotic assumptions.
- Extremes are treated as independent and clustering is studied either under a multi-variate perspective or via non-stationary reasoning.



# Stochastic properties of the parent process

- Scaling of the second-order properties in long-time horizons
  - was first observed in the water level data of the Nile River by **Hurst** (1951)
  - was first mathematically described by **Kolmogorov** (1940)
  - is a ubiquitous behaviour known as long-range dependence/persistence/Hurst phenomenon/Hurst-Kolmogorov (HK) dynamics associated with increased long-term variability (Koutsoyiannis, 2010).

Climacogram (Koutsoyiannis, 2010) variance of the averaged process as a function of the timescale of averaging:

$$\gamma(k) := \text{var} \left[ \frac{\underline{X}(k)}{k} \right] \quad \text{where} \quad \underline{X}(k) := \int_0^k \underline{x}(t) dt \quad \text{aggregated process at timescale } k$$

**Climacogram with asymptotic scaling at  $t \rightarrow \infty$  defines an HK process in continuous-time** (Koutsoyiannis, 2021):

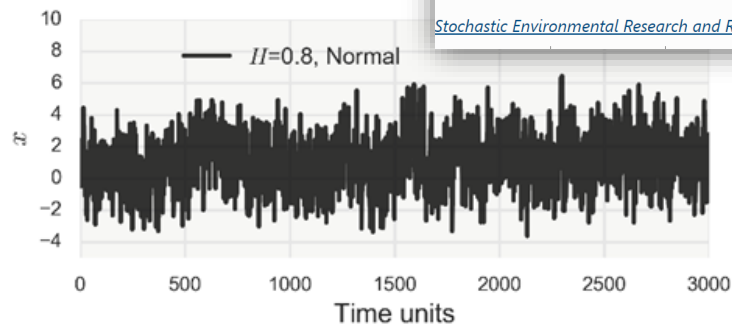
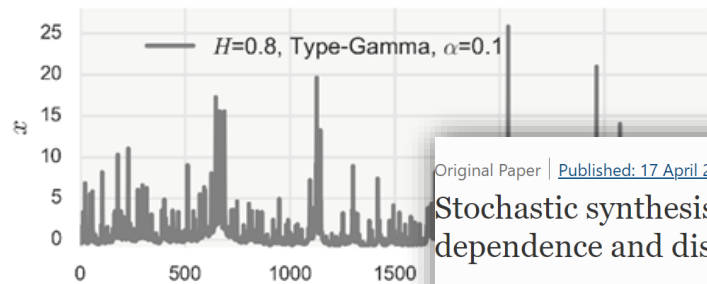
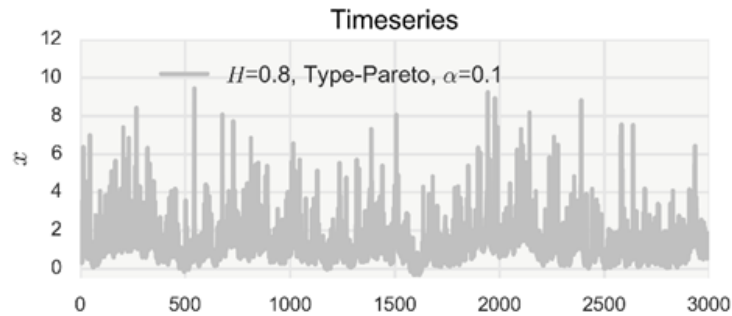
$$\gamma(k) := \lambda \left( \frac{\alpha}{k} \right)^{2-2H}$$

where  $\alpha$  and  $\lambda$  are scale parameters, with dimensions  $[t]$  and  $[x^2]$  while  $H$  is the so-called Hurst parameter ranging in the interval  $(0,1)$ .

- $H = 0.5$  yields a White-Noise process,
- $H > 0.5$  indicates enhanced change/clustering/variability at large-scales.

# Simulation of synthetic timeseries reproducing HK dynamics

- ✓ Stochastic synthesis of timeseries preserving the **second-order scaling** and the first **4 moments** of the process.



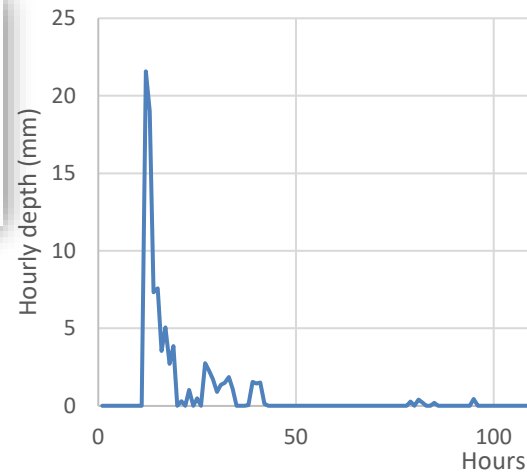
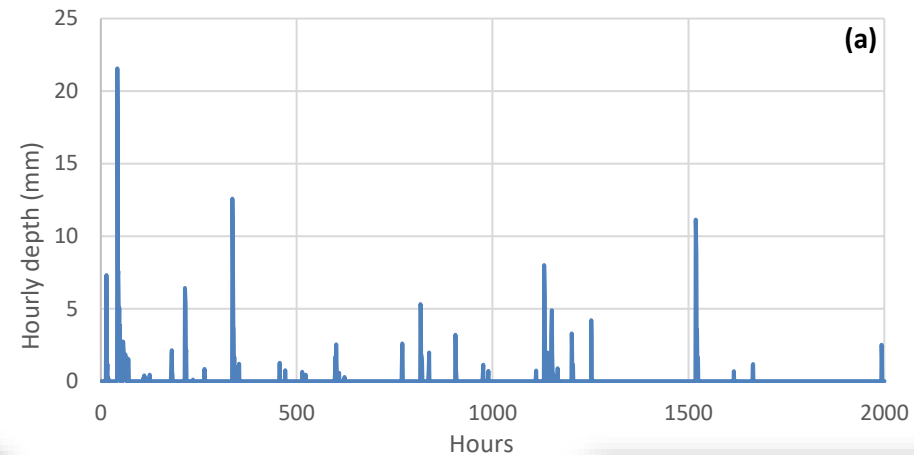
Original Paper | [Published: 17 April 2018](#)

Stochastic synthesis approximating any process dependence and distribution

[Panayiotis Dimitriadis](#) & [Demetris Koutsoyiannis](#)

[Stochastic Environmental Research and Risk Assessment](#) **32**, 1493–1515 (2018) | [Cite this article](#)

- ✓ Extended frameworks also **preserving time irreversibility** and **higher-order moments**.



Article

**Towards Generic Simulation for Demanding Stochastic Processes**

Demetris Koutsoyiannis\* and Panayiotis Dimitriadis



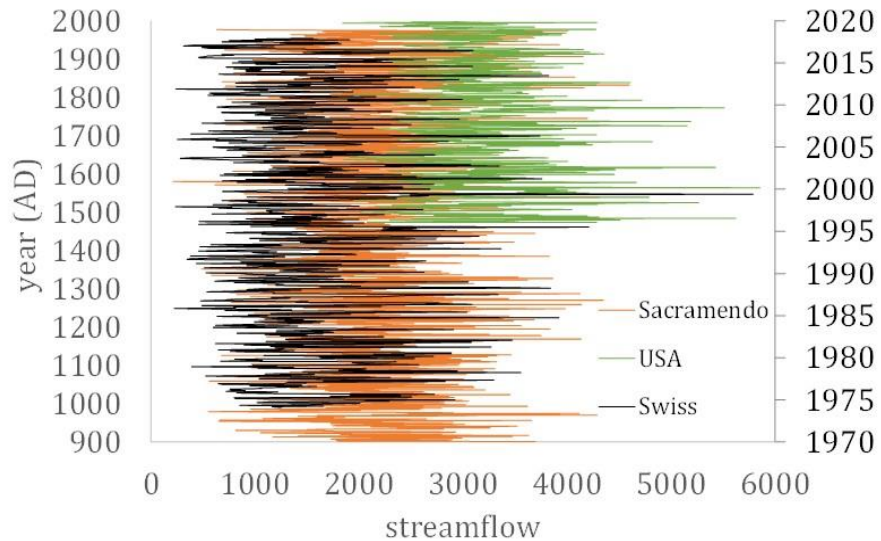
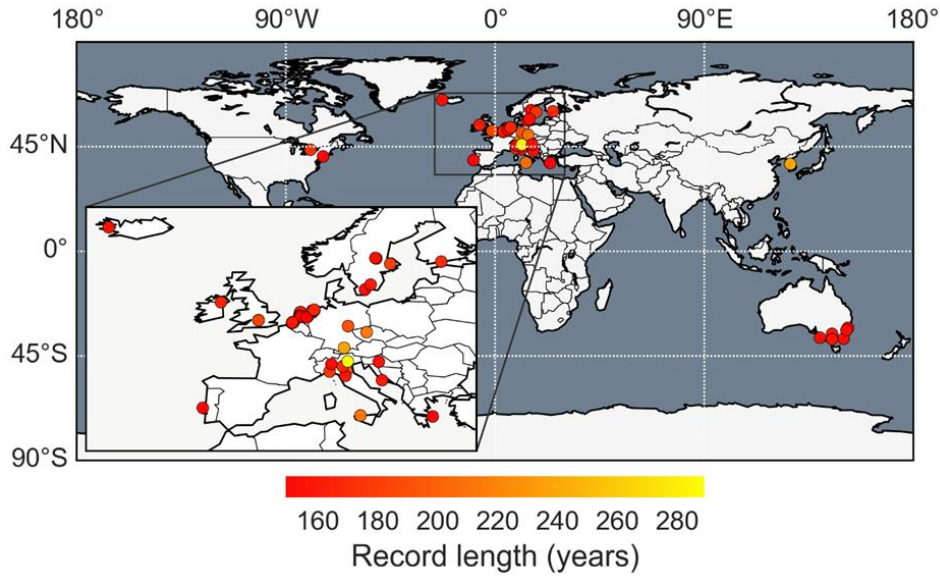
Article

**Multiscale Temporal Irreversibility of Streamflow and Its Stochastic Modelling**

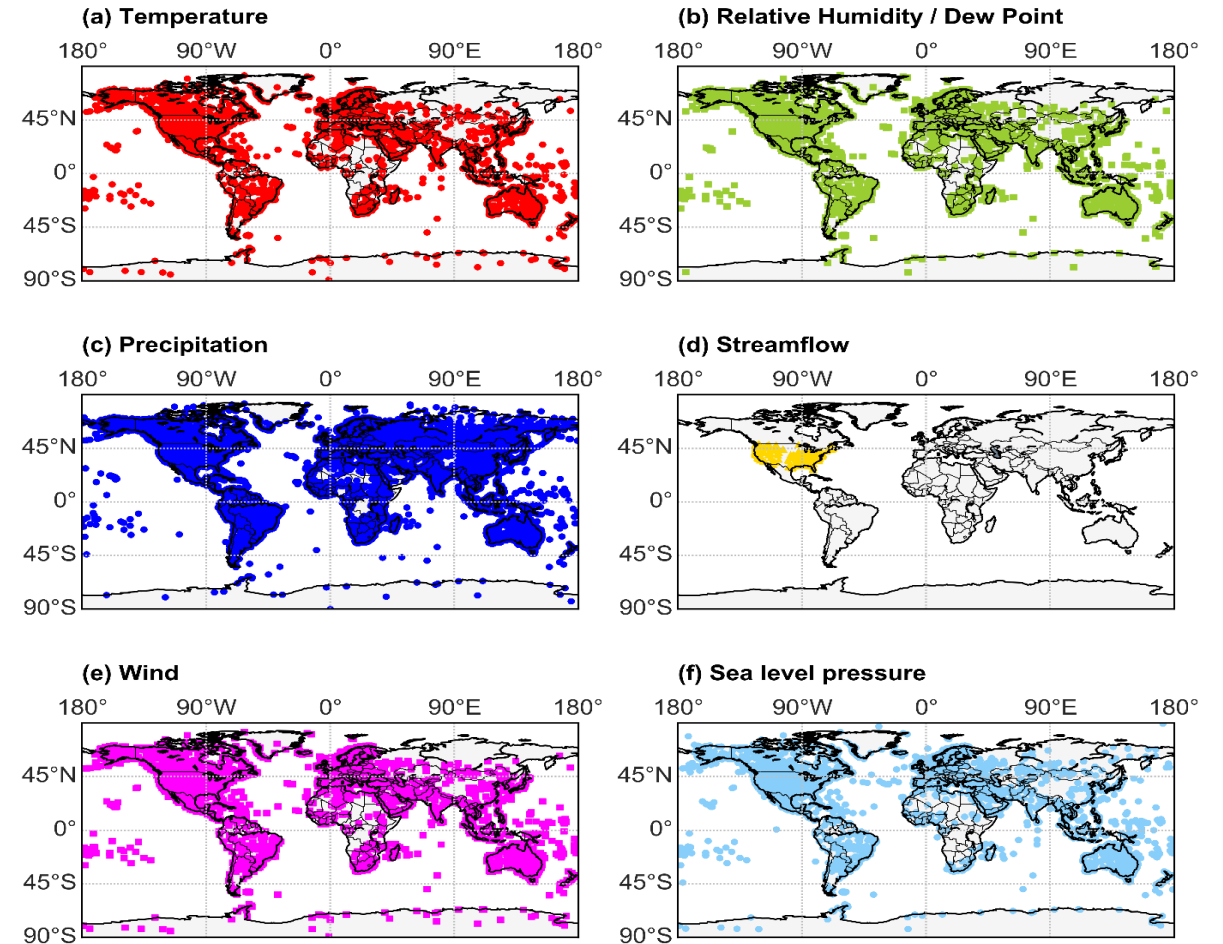
Stelios Vavoulogiannis\*, Theano Iliopoulou, Panayiotis Dimitriadis and Demetris Koutsoyiannis

# Long-term records and global-scale analysis

## Investigation of long-term rainfall and streamflow records



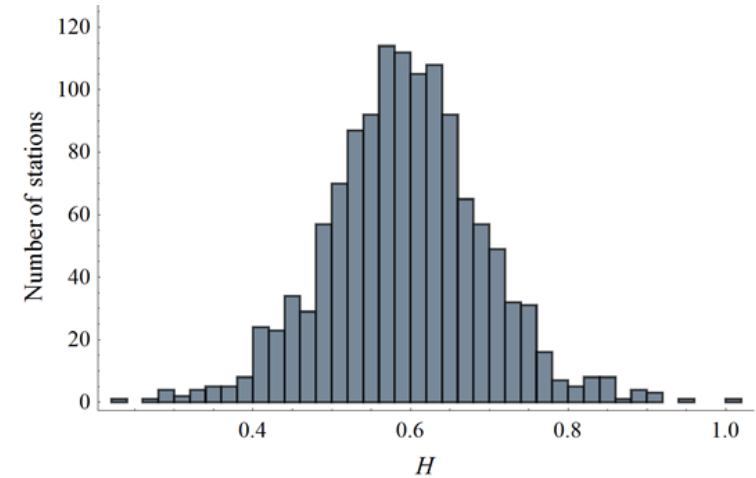
## Investigation of hydroclimatic records at a global-scale



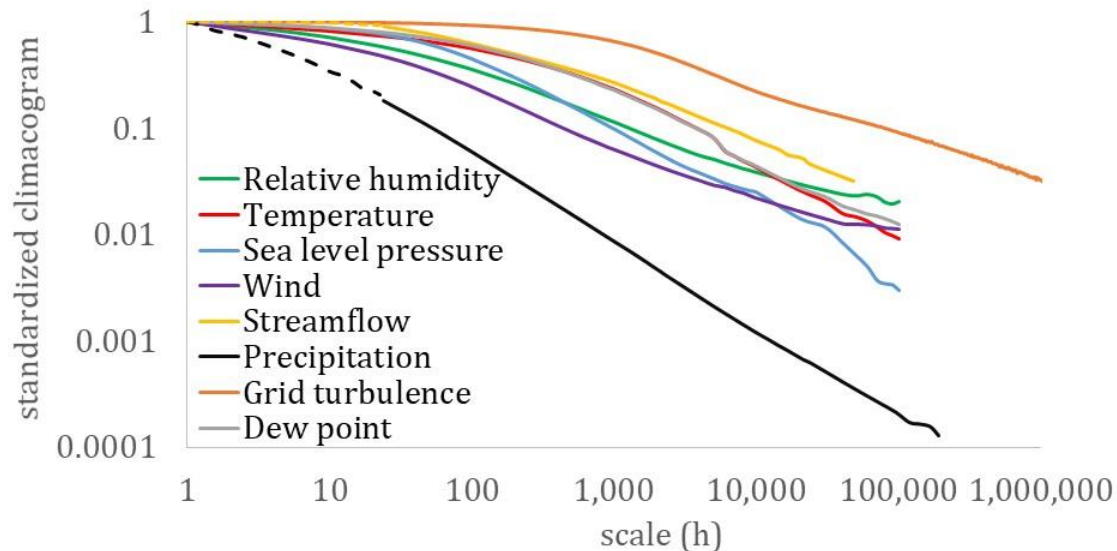
# Persistence in the parent hydroclimatic process

## Precipitation:

- A common  $H \approx 0.6$  is representative for the majority of the records above 100 years;
- 50% of the records  $H \geq 0.59$
- 2.5% of the records  $H \geq 0.8$
- 15% no evidence of persistence.



## Other hydroclimatic processes:

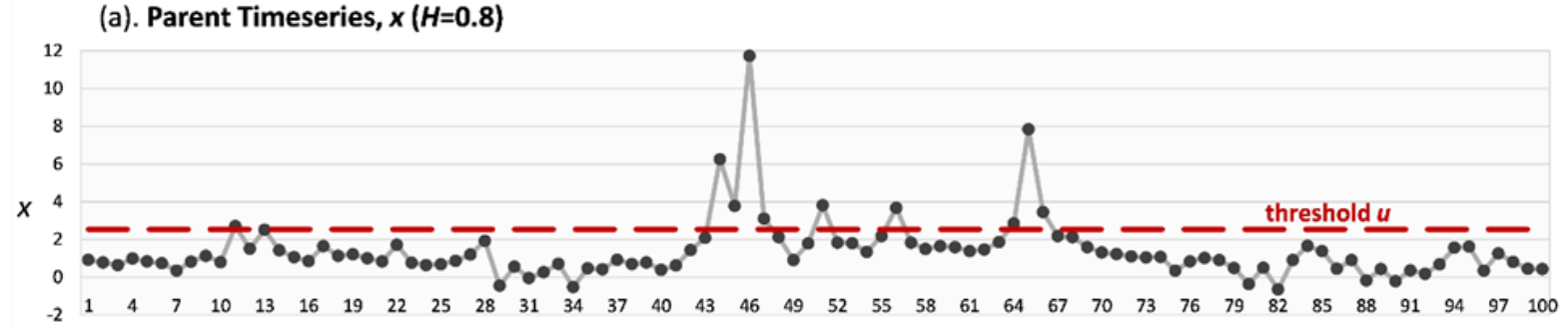


### Average $H$ parameter

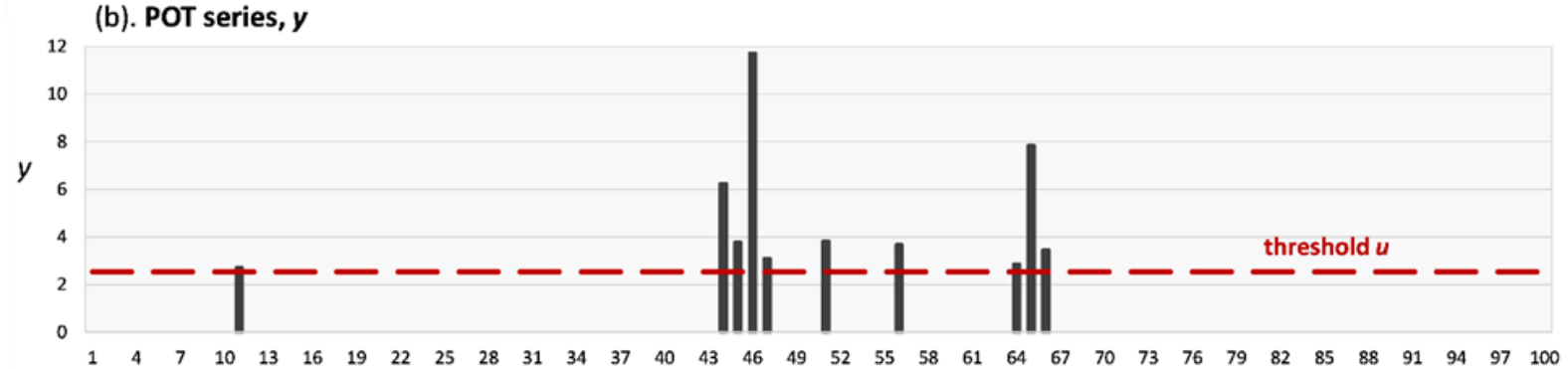
Near-surface temperature	<b>0.81</b>
Relative humidity	<b>0.83</b>
Dew point	<b>0.77</b>
Sea level pressure	<b>0.7</b>
Wind speed	<b>0.85</b>
Streamflow	<b>0.78</b>
Precipitation	<b>0.61</b>

# Does persistence **propagate** to the extremes?

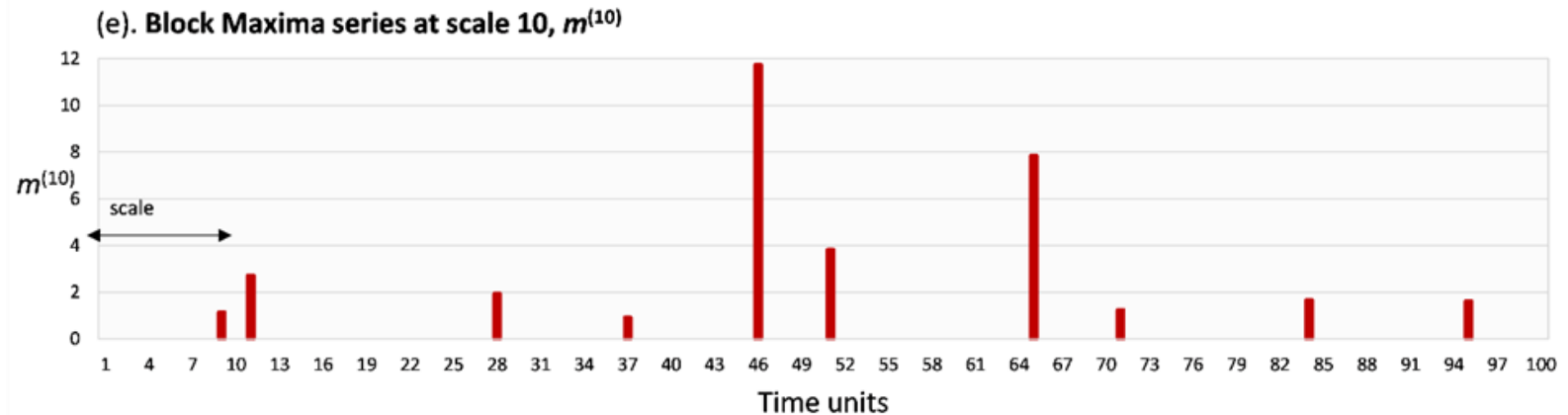
Parent timeseries



Peaks Over Threshold (POT)



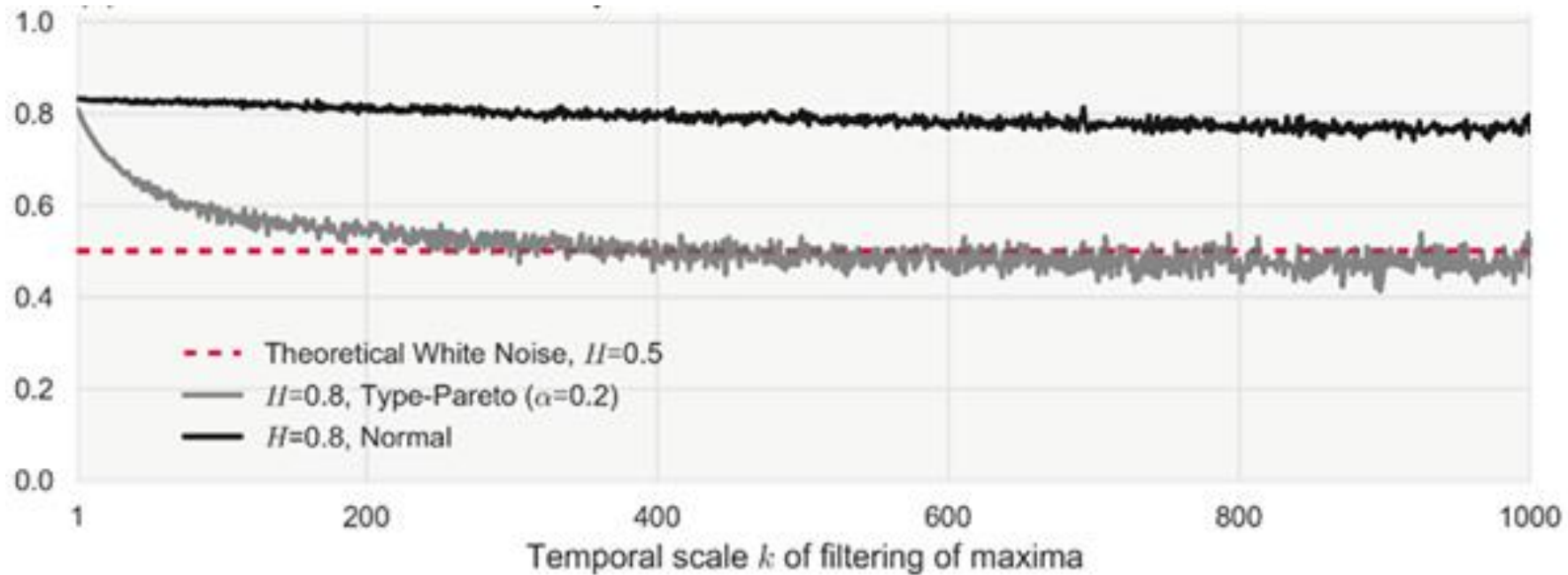
Block Maxima





# Can we use the Hurst parameter for block maxima?

Estimated Hurst parameter of maxima,  $H$



- The climacogram estimator **falsely indicates independence after a few scales of filtering** of extremes from non-Gaussian processes. Similar results are obtained from the dispersion index.

Second-order characterizations are not suited for identification of clustering of non-Gaussian extremes, but may be effective for near-Gaussian processes (e.g. air temperature, see Glynis et al. 2021).

# A new probabilistic multi-scale clustering index

1 POT series:

$$\underline{y}_{-i} := \begin{cases} x_{i'}, & x_i > u \\ 0, & x_i \leq u \end{cases}$$

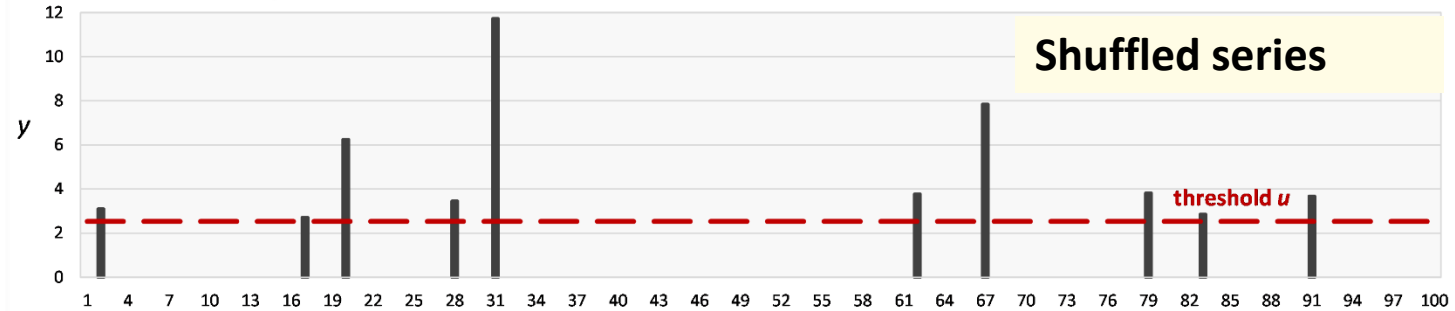
2 Counts of POT occurrences at timescale  $k$ :

$$z_q^{(k)} := \underline{N}(qk) - \underline{N}((q-1)k)$$

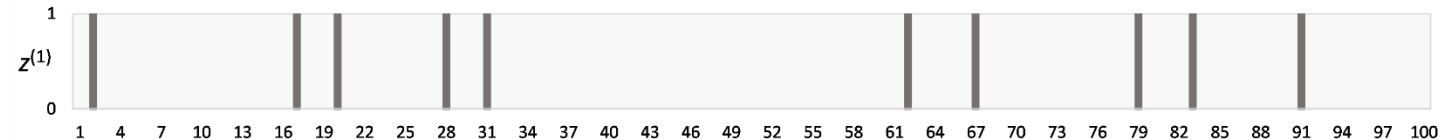
3 Indicator function of  $z$ :

$$r_{-q}^{(k)} := \begin{cases} 1, & z_{-q}^{(k)} > 0 \\ 0, & z_{-q}^{(k)} = 0 \end{cases}$$

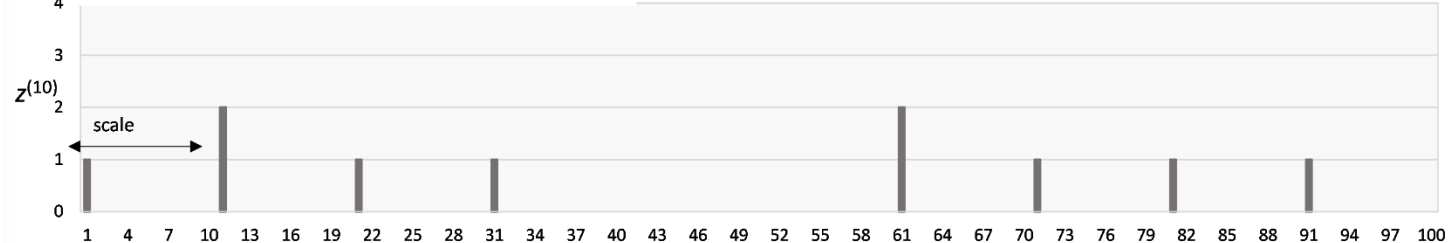
(b). POT series,  $y$



(c). Counts of POT occurrences at basic scale,  $z^{(1)}$



(d). Counts of POT occurrences at scale 10,  $z^{(10)}$



4 Probability of exceedance of the threshold for timescale  $k$ :

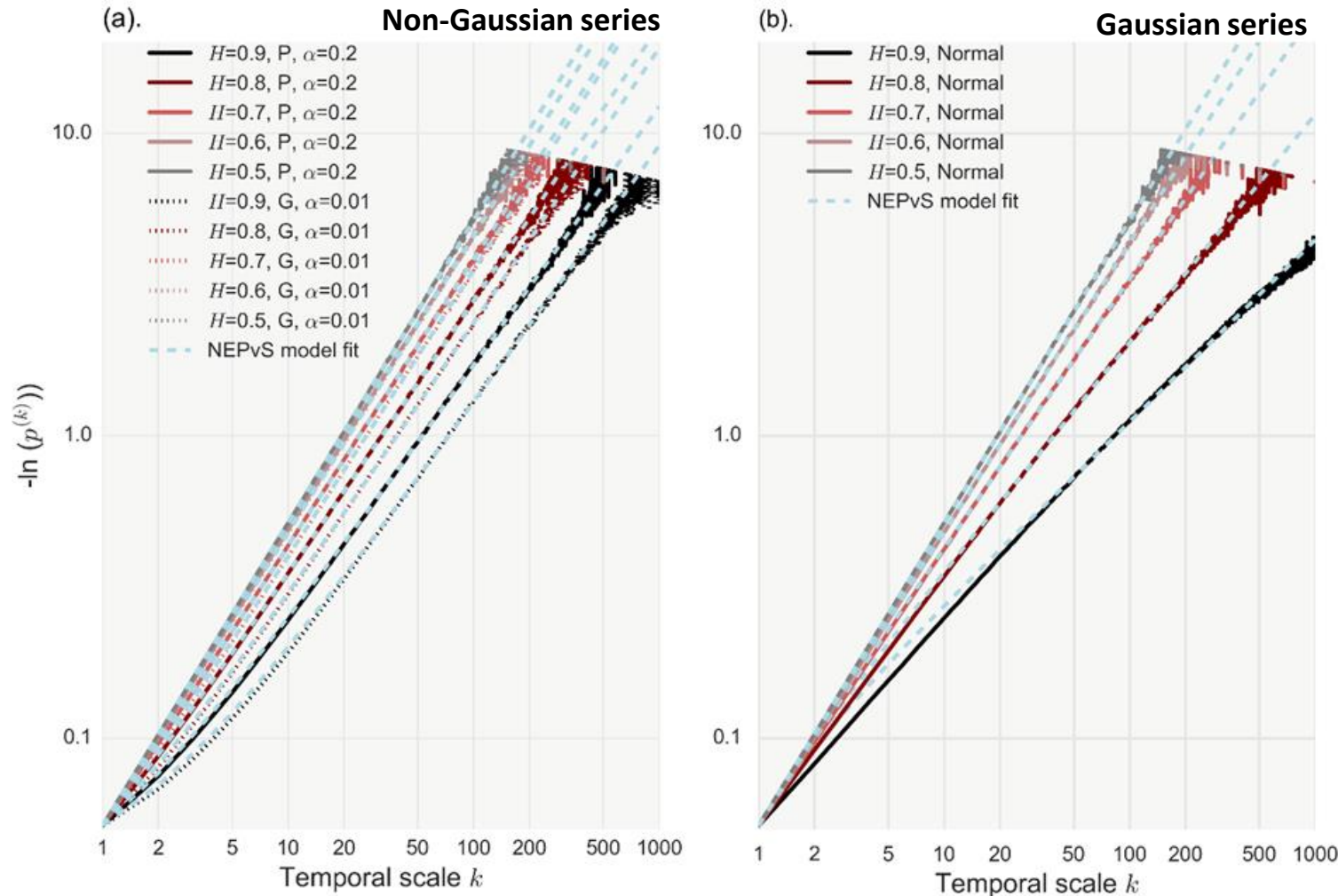
$$\bar{p}^{(k)} = \frac{\sum_1^{\lfloor n/k \rfloor} r_{-q}^{(k)}}{\lfloor n/k \rfloor}$$

5 Non-exceedance probability versus scale (NEPvS):

$$p^{(k)} = 1 - \bar{p}^{(k)}$$

# Multi-scale clustering of extremes under persistence

Persistence increases the probability of non-occurrence of extremes at multiple scales, i.e. **prolonged periods of absence of extreme events are more probable than in an IID process.**



□ For a White-Noise process

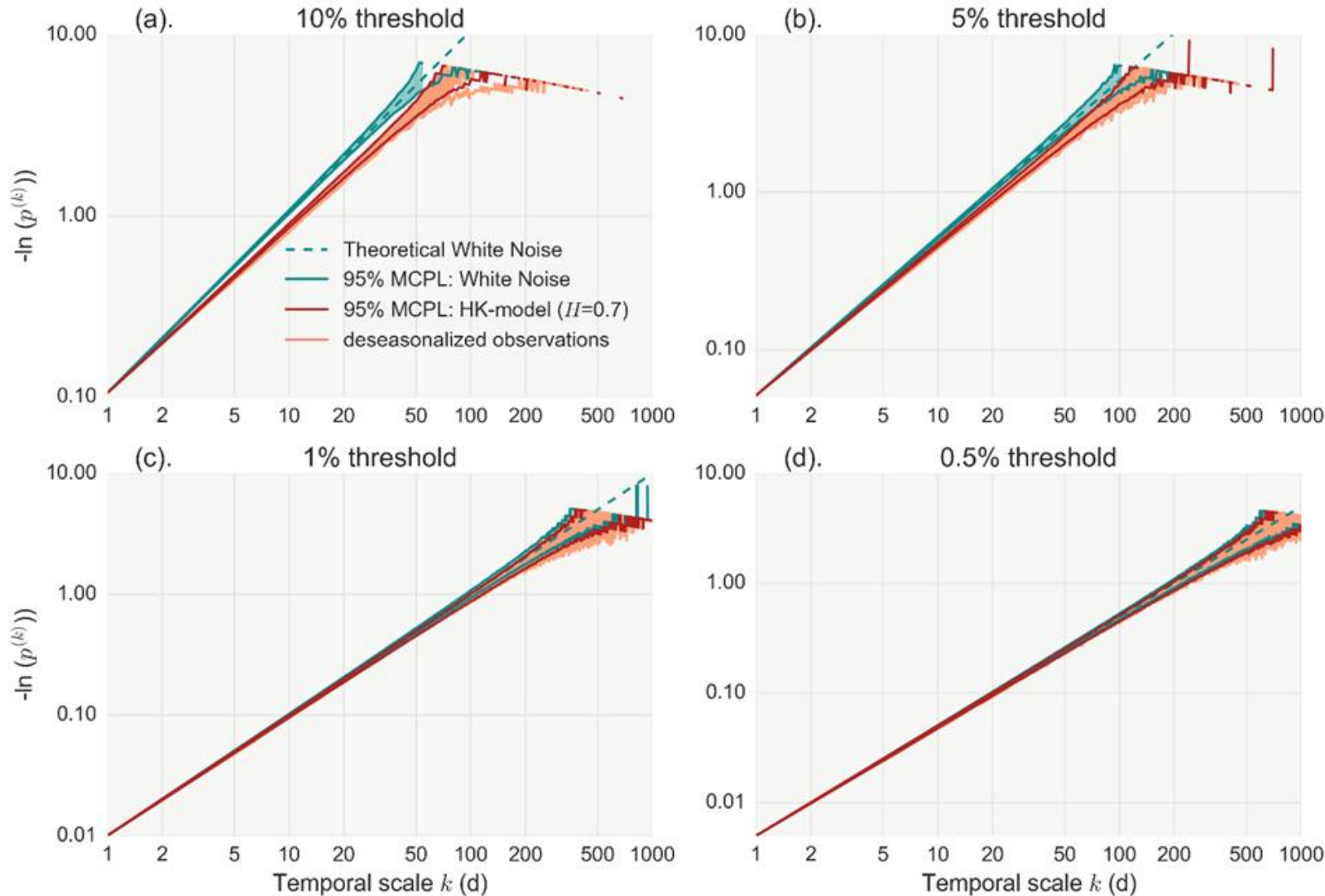
$$p^{(k)} = p^k$$

□ Two-parameter model

$$p^{(k)} = p^{(1+(\xi^{-1/\eta}-1)(k-1))^\eta} \quad (\text{adapted from Koutsoyiannis, 2006})$$

- For  $\eta = 1$  and  $\xi = 0.5$ , the model describes Poisson occurrences.
- For  $\eta < 1$  and  $\xi > 0.5$  clustering emerges.

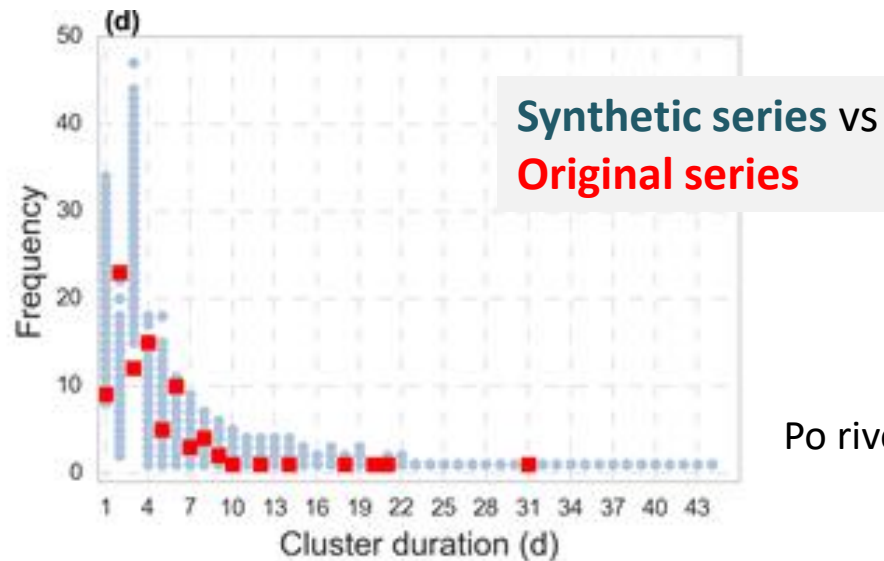
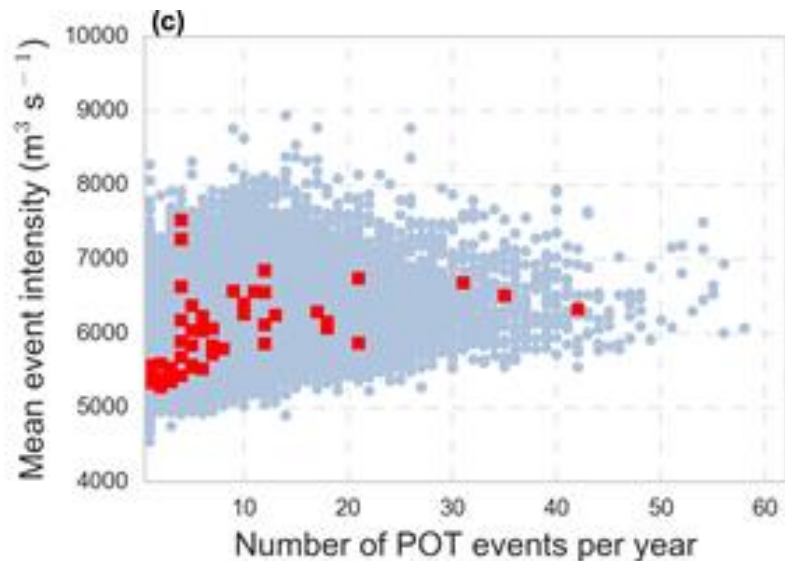
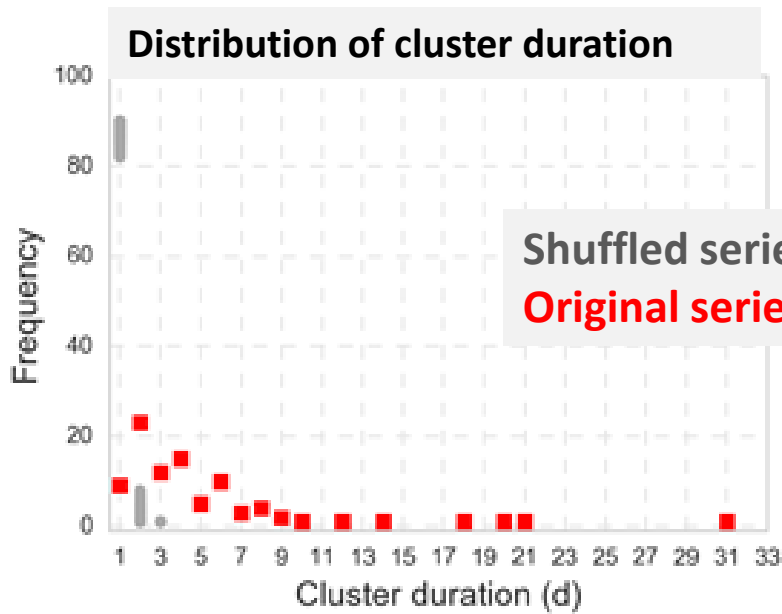
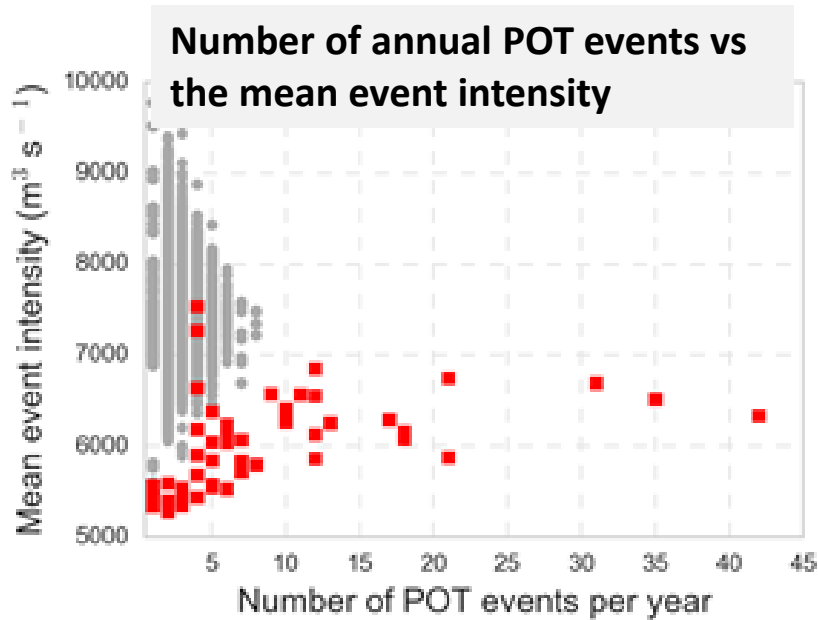
# Clustering of rainfall POT occurrences in the Netherlands



$10^3$  Monte Carlo simulations from an HK-model preserving the empirical  $H$  parameter and first four moments compared to the 28 deseasonalized records.

- HK model is consistent with the majority of the records; a few stations exhibiting even stronger clustering outside of the 95% region of the model.
- **As the threshold increases the probabilistic behaviour of POT occurrences resembles a purely random one.**

# Clustering properties of daily POT streamflow at the annual scale

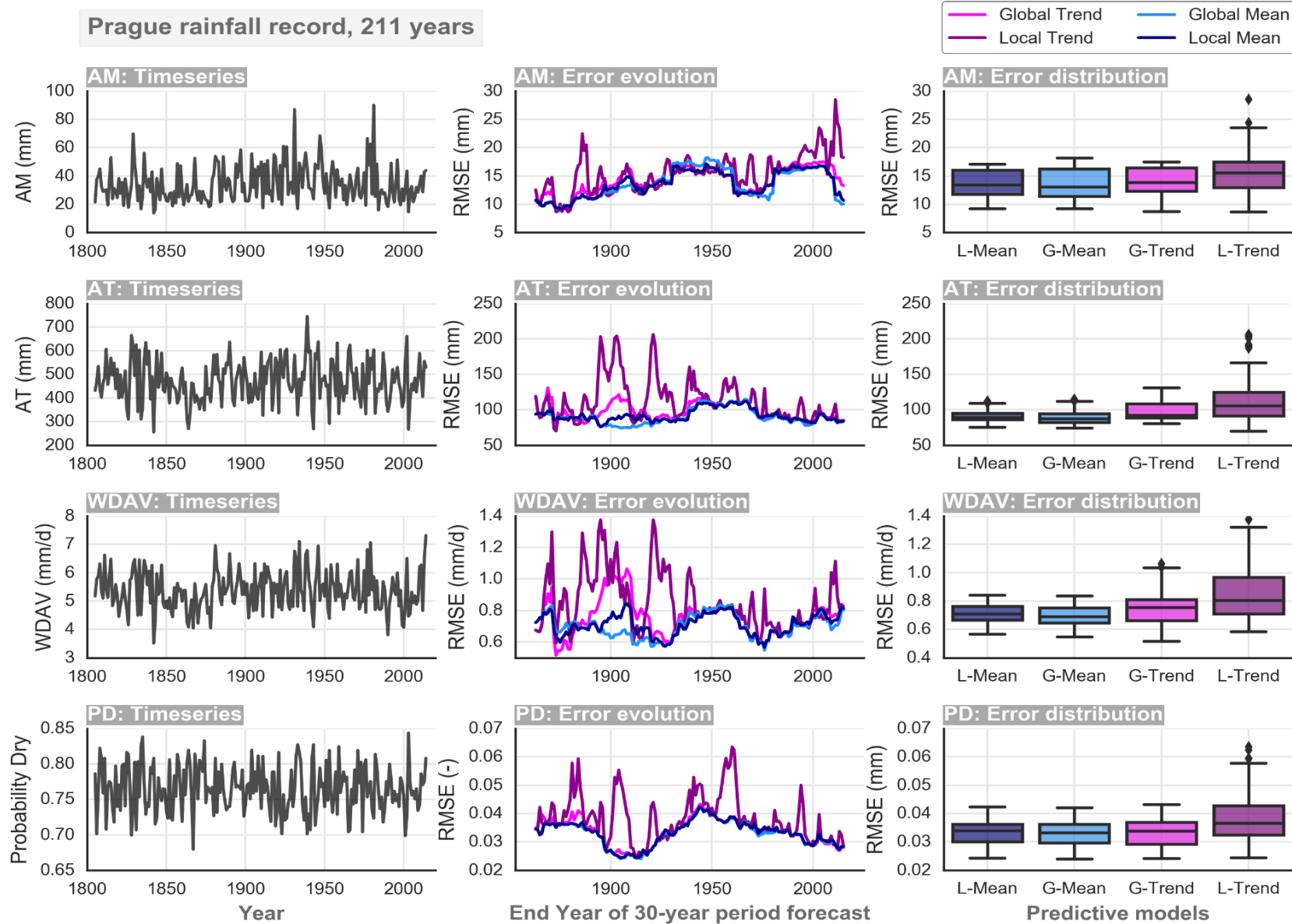


Dependence of POT events at the annual scale is manifested by:

- positive association between **the mean intensity and the number of POT events per year.**
- **longer cluster duration** compared to the IID case

Po river daily streamflow series (90 years)

# Climatic-scale rainfall clustering – Are trends an effective descriptor?

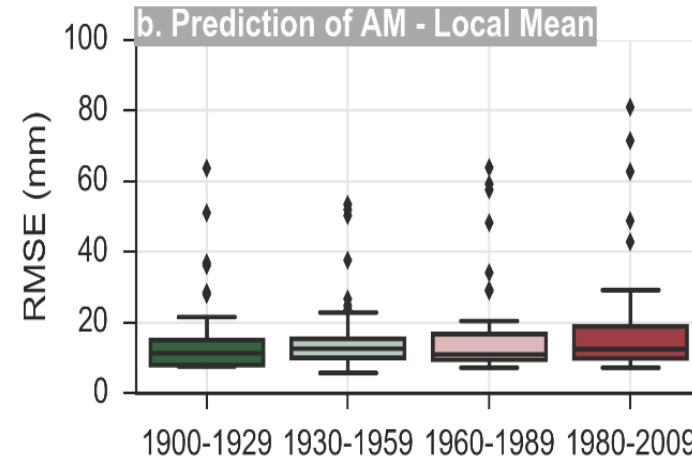
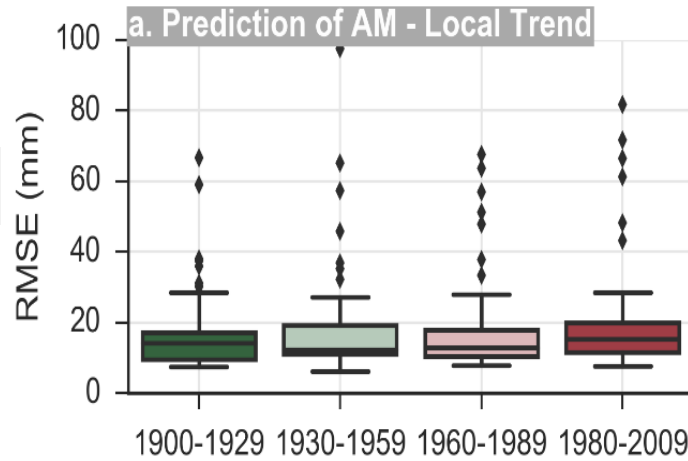


We assess the ‘trends’ effectiveness in long-term projections via a **prediction-oriented evaluation framework**.

We compare the predictive performance of **global and local trend models** over climatic periods (30 years) to the one obtained by **global and local mean models**.

# Extreme rainfall projections – Local mean models prevail

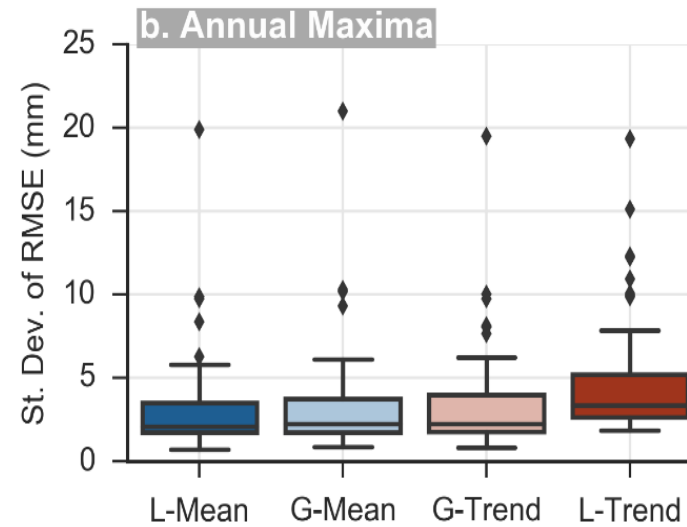
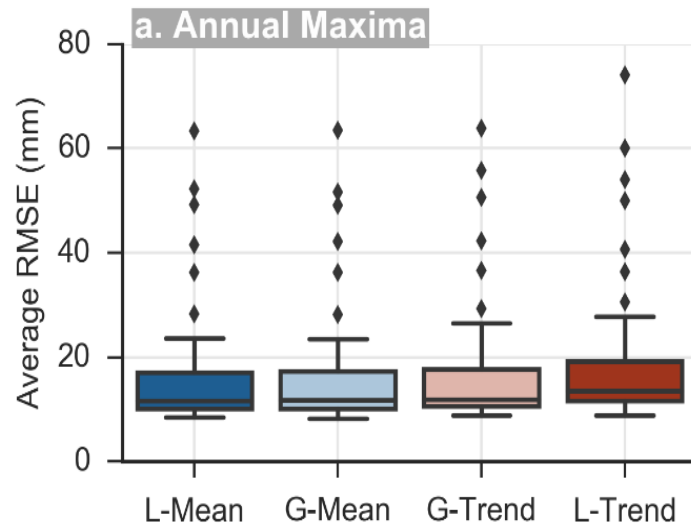
Fixed periods



The models' predictive performance **ranks from best to worse** as follows:

1. L-Mean
2. G-Mean
3. G-Trend
4. L-Trend

Moving-window periods



In persistent process, where clustering arises, local information is likely to be more relevant for prediction.

# Conclusions

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- ❑ Linking clustering of extremes to the stochastic properties of the parent process offering probabilistic insights otherwise difficult to obtain.
- ❑ Extremes from skewed processes tend to 'hide' the persistence of the parent process, often falsely signalling independence. **Identifiability of persistence weakens as the threshold increases.**
- ❑ A **new probabilistic index** (NEPvS) is proposed to represent clustering based on the multi-scale probability of non-exceedance of a given threshold.
- ❑ POT events from persistent processes have a bilateral character compared to IID processes:
  - absence of POT events in a scale is more likely; persistence increases the multi-scale probability of non-occurrence of extremes.
  - yet if occurrence of extremes is triggered, a higher cluster duration and greater mean and aggregate intensity thereof should be expected.
- ❖ Deviations from IID behaviour are prominent in observational records of extremes.
- ❖ Understanding and modelling the temporal properties of the parent process is a parsimonious way to reproduce observed temporal clustering under a univariate stochastic approach.



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# Thank you!

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Επιχειρησιακό Πρόγραμμα  
Ανάπτυξη Ανθρώπινου Δυναμικού,  
Εκπαίδευση και Διά Βίου Μάθηση

Ειδική Υπηρεσία Διαχείρισης

Με τη συγχρηματοδότηση της Ελλάδας και της Ευρωπαϊκής Ένωσης



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