# A Scaling Model of Storm Hyetograph

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## ABSTRACT

Empirical evidence suggests that statistical properties of storm rainfall at a location and within a homogeneous season have a well structured dependence on storm duration. To explain this dependence, a simple scaling model for rainfall intensity within a storm was hypothesized. It was shown both analytically and empirically that such a model can explain reasonably well the observed statistical structure in the interior of storms providing thus an efficient parametrization of storms of varying durations and total depths. This simple scaling model is also consistent with, and provides a theoretical basis for, the concept of mass curves (normalized cumulative storm depth vs. normalized cumulative time since the beginning of a storm) which are extensively used in hydrologic design. In contrast, popular stationary models of rainfall intensity are shown unable to capture the duration dependent statistical structure of storm depths and are also inconsistent with the concept of mass curves.

# 1 Introduction

This paper deals with the analysis and modeling of the stochastic structure of rainfall intensities within storms of varying duration. Storms are defined here as rainfall events which are independent of each other as based, for example, on Poisson storm arrivals. The need to parametrize the time distribution of storms which are "similar" apart from total storm depth and storm duration arose very early and the concept of mass curves, i.e., non-dimensional cumulative storm depth versus non-dimensional cumulative time since the beginning of a storm, has been extensively used for hydrologic design (e.g., Grace and Eagleson, 1966, p. 90; Huff, 1967; Eagleson, 1970, p. 194; Pilgrim and Cordery, 1975; among others). The idea behind those efforts was the recognition that for a particular location or within a meteorologically homogeneous region and for a homogeneous season, storms are expected to exhibit similarities in their internal structure despite their different durations and total storm depths. In addition, the concept of normalized mass curves was adopted in some advanced rainfall models, such as the ones of Bras and Rodriguez-Iturbe (1976), Hjelmfelt (1981), Woolhiser and Osborn (1985).

Empirical evidence from this and other studies (see sections 5 and 6) regarding the dependence of the statistical properties of incremental and total storm depths on storm duration, led us to the hypothesis of a simple scaling model for the instantaneous rainfall intensity within a storm with storm duration as the scaling parameter. This model has been thoroughly examined in this paper and the properties of the total storm depth and incremental rainfall depths have been analytically derived and have been used for model fitting and model evaluation. Another motivation for examining the simple scaling model is that it is consistent with, and provides a good theoretical basis for, the concept of mass curves which are very often used in hydrologic applications and rainfall modeling.

Most of the available continuous time rainfall models, e.g., the Neyman-Scott model (Kavvas and Delleur, 1981; Rodriguez-Iturbe et al., 1984; among others) used to describe rainfall intensities are stationary. In this paper we show that any stationary model is unable to capture the duration dependent statistical structure of rainfall intensities and is also inconsistent with the concept of mass curves.

This paper is structured as follows. Section 2 introduces notation. In section 3 the simple scaling model for instantaneous rainfall intensities within a storm is presented. The statistical properties of the total storm depth and incremental storm depths, e.g., hourly depths, are derived in section 4. In section 5 some important properties implied by the model structure are compared with features of rainfall documented in the literature. In section 6 the simple scaling model is fitted to hourly data from 89 storms in Chalara, Greece and the performance of the model is evaluated in terms of its ability to capture statistical properties not explicitly used for model fitting. In section 7 two stationary models of instantaneous rainfall intensity are examined and it is shown both analytically and empirically that these models are not able to reproduce some of the observed characteristics of storm rainfall that the simple scaling model is able to describe. In section 8 the connection of simple scaling models to mass curves is examined and it is shown that mass curves are consistent with the hypothesis of simple scaling but are inconsistent with the assumption of any stationary model for instantaneous rainfall intensities. Finally, in section 9 the scaling model is applied to generating synthetic storm hyetographs and mass curves which are shown the compare well with the corresponding empirical ones. Some concluding remarks are given in section 10.

# 2 Terminology and preliminaries

Let $D$ denote the duration of a storm and $\xi(t, D)$, $0 \leq t \leq D$ the rainfall intensity process within the storm duration. $h(t, D)$ denotes the cumulative rainfall depth process defined as

$$h(t, D) = \int_0^t \xi(s, D) ds, \quad 0 \leq t \leq D \tag{1}$$

and $X_\Delta(i, D)$ denotes the incremental rainfall depth in the interval $((i-1)\Delta, i\Delta)$ i.e.,

$$X_\Delta(i, D) = \int_{(i-1)\Delta}^{i\Delta} \xi(t, D) dt, \quad i = 1, 2, ...k \tag{2}$$

where k is the integer part of $D/\Delta$ (see Fig. 1). It is assumed that within a meteorologically homogeneous region and season every storm of duration $D$ can be considered as a realization of an ensemble characterized by that duration. Note that the process $\xi(t, D)$ is a process of finite duration $(0 \leq t \leq D)$ and thus its ensemble average is, in fact, a function of the duration $D$.

Let $\eta_\xi(t, D)$ denote the ensemble average of $\xi(t, D)$, i.e.,

$$\eta_\xi(t, D) = E[\xi(t, D)] \tag{3}$$

and $R_\xi(t_1, t_2; D)$ the second order product moment of $\xi(t, D)$ in the interval of a storm event, i.e.,

$$R_\xi(t_1, t_2; D) = E[\xi(t_1, D)\xi(t_2, D)], \quad 0 \leq t_1, t_2 \leq D \tag{4}$$

where again expectation refers to ensemble average. The covariance function of $\xi(t, D)$ is then given as

$$C_\xi(t_1, t_2; D) = Cov[\xi(t_1, D)\xi(t_2, D)] = R_\xi(t_1, t_2; D) - \eta_\xi(t_1, D)\eta_\xi(t_2, D) \tag{5}$$

In a similar manner we define the statistical properties of the cumulative depth process $h(t, D)$, i.e., $\eta_h(t, D)$, $R_h(t_1, t_2; D)$, and $C_h(t_1, t_2; D)$, and those of the incremental depth process $X_\Delta(t, D)$, i.e., $\eta_{X_\Delta}(i, D)$, $R_{X_\Delta}(i, j; D)$, and $C_{X_\Delta}(i, j; D)$.

## 3 Scaling model of storm intensities

The hypothesis is set forward that the process of instantaneous rainfall intensities within a storm, i.e., $\xi(t,D)$, $0 \le t \le D$ is a self-similar (simple scaling) process with scaling exponent $H$, i.e..

$$\{\xi(t,D)\} \stackrel{d}{=} \{\lambda^{-H}\xi(\lambda t, \lambda D)\} \tag{6}$$

where the above equality is in terms of the finite dimensional probability distribution, i.e.,

$$Pr[\xi(t_1,D) \le x_1, \ldots, \xi(t_n,D) \le x_n] = Pr[\lambda^{-H}\xi(\lambda t_1, \lambda D) \le x_1, \ldots, \lambda^{-H}\xi(\lambda t_n, \lambda D) \le x_n], \quad 0 \le t_1, \ldots, t_n \le D \tag{7}$$

(see, for example, Lamperti 1962, where, however, infinite duration stochastic processes are considered). Consequently the k-th moment of $\xi(t,D)$ is given as

$$E[\xi(t,D)^k] = \lambda^{-Hk}E[\xi(\lambda t, \lambda D)^k] \tag{8}$$

and the $(k,l)$ second product moment as

$$E[\xi(t_1,D)^k\xi(t_2,D)^l] = \lambda^{-H(k+l)}E[\xi(\lambda t_1, \lambda D)^k\xi(\lambda t_2, \lambda D)^l] \tag{9}$$

An intuitive feeling of the notion of scaling in (6) can be obtained from Figure 2 where, for example, if $D_2 = \lambda D_1$ then under appropriate scaling of time, i.e., $t_2 = \lambda t_1$, the statistical (ensemble) properties of the rainfall intensity in storms of duration $D_2$ are related to the corresponding statistical properties of the rainfall intensity in storms of duration $D_1$ according to (7).

It is noted that by setting $\lambda = 1/D$ in (6) we obtain

$$\{\xi(t,D)\} \stackrel{d}{=} \{D^H\xi(t/D, 1)\} \tag{10}$$

where $\xi(t/D,1)$ represents the intensity process of a storm event normalized to unit duration. It is then realized from (10) that the hypothesis of scaling implies that the statistical properties of the rainfall intensity in storms of any duration can be obtained by appropriate scaling of the statistical properties of the rainfall intensity in a storm normalized to unit duration.

For reasons of simplicity we will assume that the process $\xi(t,D)$ is stationary within a storm event, i.e., the finite dimensional distribution function is invariant to time translation within a storm,

$$\{\xi(t,D)\} \stackrel{d}{=} \{\xi(t+\tau, D)\}, \quad 0 \le t, t+\tau \le D \tag{11}$$

Note that this is a weak stationarity condition in that it represents stationarity of $\xi(t,D)$ only within storm events of a fixed duration and not over any storm independently of duration (or over the whole time axis), which would imply

$$\{\xi(t)\} \stackrel{d}{=} \{\xi(t+\tau)\} \tag{12}$$

as most available rainfall intensity models, e.g., the Neyman-Scott model, assume.

The weak stationarity assumption (11) should not be considered as a structural constraint of the simple scaling model but rather it is a convenient simplification. The data examined, as well as other data (e.g. Grace and Eagleson, 1966, p. 90) are not far from this assumption. Note that this assumption results to a "mean" mass curve which is a straight line. Apparently however, any mass curve derived by the use of the model as a realization of a stochastic function characterizing the mass curves (see development in section 8 and application in section 9) will not be a straight line but it will have a nonlinear shape in agreement with empirical evidence.

Under our assumption the ensemble statistical properties of the process $\xi(t, D)$ do not depend on $t$ for a given duration $D$ and the ensemble statistical properties of $\xi(t/D, 1)$ are independent of $t$ and $D$. Let us define as $c_1$ the ensemble mean of the process $\xi(t/D, 1)$, i.e.,

$$c_1 \equiv E[\xi(t/D, 1)] \tag{13}$$

Since $\xi(t/D, 1)$ is stationary we also define

$$\phi(\tau/D) \equiv E[\xi(t/D, 1)\xi((t+\tau)/D, 1)] \tag{14}$$

Based on the above relations and (6) the ensemble statistical properties of $\xi(t, D)$ can be written as

$$E[\xi(t, D)] = c_1 D^H \tag{15}$$

$$C_\xi(\tau; D) = Cov[\xi(t, D), \xi(t+\tau, D)] = (\phi(\tau/D) - c_1^2)D^{2H} \tag{16}$$

These equations imply that under the hypothesis of simple scaling (equation 6) and the assumption of stationarity within an event (equation 11) the statistical properties of $\xi(t, D)$ can be obtained from the statistical properties of the normalized to unit duration process $\xi(t/D, 1)$ and a scale changing transformation which is a power law of the storm duration. Note that the mean of the rainfall intensity process depends on the duration according to a power law with exponent $H$, while the covariance of the rainfall intensity process is also a power law of duration with exponent $2H$. Higher product moments follow similar relationships as implied by (10).

# 4 Properties of total and incremental storm depths

To be able to test the hypothesis of scaling for $\xi(t, D)$ using available rainfall data, the statistical properties of incremental and total storm depths need to be derived. In this section we show that both total storm depths $h(D, D)$ and incremental storm depths $X_\Delta(i, D)$ follow simple scaling laws and expressions for their mean and covariance are derived.

## 4.1 Cumulative and total storm depths

It can be shown (see Appendix 1) that under some rather mild restrictions on the covariance of $\xi(t,D)$ the cumulative rainfall depth process $h(t,D)$ is also a simple scaling process with exponent $H+1$, i.e.,

$$\{h(t,D)\} \stackrel{d}{=} \{\lambda^{-(H+1)} h(\lambda t, \lambda D)\} \tag{17}$$

Setting $t = D$ and $\lambda = 1/D$ in the above equation we obtain

$$\{h(D,D)\} \stackrel{d}{=} \{D^{H+1} h(1,1)\} \tag{18}$$

Noting that $E[h(1,1)] = c_1$ and defining

$$c_2 \equiv var[h(1,1)] \tag{19}$$

we can write the ensemble mean and variance of the total storm depth as

$$E[h(D,D)] = c_1 D^{H+1} \tag{20}$$

$$var[h(D,D)] = c_2 D^{2(H+1)} \tag{21}$$

Note that as a result of the simple scaling model for rainfall intensities, the coefficient of variation of the total storm depth is constant and equal to $\sqrt{c_2}/c_1$.

## 4.2 Incremental storm depths

The incremental storm depth at discrete time $t = i$, i.e., $X_\Delta(i,D)$ defined in (2), can be written as the difference of cumulative storm depths as

$$X_\Delta(i,D) = h(i\Delta, D) - h((i-1)\Delta, D) \tag{22}$$

In view of the scaling of $h(t,D)$ (equation 17) the discrete-time incremental depth process $X_\Delta(i,D)$ is also scaling, i.e.,

$$\{X_\Delta(i,D)\} \stackrel{d}{=} \{\lambda^{-(H+1)} X_{\lambda\Delta}(\lambda i, \lambda D)\} \tag{23}$$

It is easy to show that the ensemble mean of $X_\Delta(i,D)$ is

$$E[X_\Delta(i,D)] = c_1 \Delta D^H = c_1 \delta D^{H+1} \tag{24}$$

where $\delta = \Delta/D$. After some algebraic manipulations (see Appendix 2) one can derive the variance of $X_\Delta(i,D)$ as

$$var[X_\Delta(i,D)] = [\psi(0,\delta) - c_1^2\delta^2]D^{2(H+1)} \tag{25}$$

where

$$\psi(0;\delta) = 2\int_0^\delta \phi(y)(\delta - y)\, dy \tag{26}$$

Similarly (see Appendix 2), the covariance can be derived as

$$C_{X_\Delta}(m;D) = Cov[X_\Delta(i,D), X_\Delta(i+m,D)] = [\psi(m;\delta) - c_1^2\delta^2]D^{2(H+1)} \tag{27}$$

where

$$\psi(m;\delta) = \int_{(m-1)\delta}^{m\delta} (y - (m-1)\delta)\phi(y)\, dy + \int_{m\delta}^{(m+1)\delta} ((m+1)\delta - y)\phi(y)\, dy \quad , m > 0 \tag{28}$$

The autocorrelation function can then be written as

$$\rho_{X_\Delta}(m;D) = \frac{\psi(m;\delta) - c_1^2\delta^2}{\psi(0;\delta) - c_1^2\delta^2} \tag{29}$$

It is interesting to note that as a manifestation of the scaling hypothesis for $\xi(t,D)$ the autocorrelation function of the incremental depth process depends on $\delta = \Delta/D$, that is, on the integration interval, normalized by storm duration, and it does not depend directly on the storm duration or the integration interval, nor on the scaling exponent $H$.

# 5 Discussion of model properties and rainfall features reported in literature

Before we embark into the details of fitting the proposed model to a specific data set and evaluating its performance (section 6) as well as theoretically and empirically comparing it to stationary models (section 7) we prefer to provide a little more insight into some important properties implied by the model structure and compare these properties with features of rainfall documented in the literature. Particularly, we will focus on the average intensity of a storm, the coefficient of variation of the total storm depth or the average intensity, and the correlation structure of incremental depths. Later in section 8 we will examine the model consequences regarding the normalized mass curves. In both sections we will illustrate that the proposed model, in spite of its novel mathematical formulation, describes adequately well known features of rainfall and is in agreement with some models while in disagreement with others.

### 5.1 Average intensity of storm

As it results from (20), the time average intensity of a storm $\bar{i}(D)$ is a function of the duration with expected value given by

$$E[\bar{i}(D)] = c_1 D^H \tag{30}$$

The model allows $H$ to take either positive, zero, or negative (but greater than -1) values. In the first case we have a mean intensity which is an increasing function of duration, while in the second the mean intensity is constant and independent of duration. The third case seems to be the most frequent, since a negative correlation of duration and mean intensity is quite common as will be discussed below. Note in that case that when $D \to 0$ it is easily shown that all the statistical moments of both the instantaneous and time average intensity tend to $\infty$. However, this is not a problem since the total depth $h(D, D) \to 0$, as it follows from (20) and (21). Thus, with $H < 0$ when $D = 0$ we have a rainfall impulse with an infinite intensity but zero total depth, which seems to be reasonable. Recall that other models (e.g., Poisson White Noise Model, Neyman-Scott White Noise Model; see Rodriguez- Iturbe et al., 1984) use the concept of rainfall impulses with zero duration.

The dependence of total storm depth or mean intensity on the duration of a storm has been investigated in several earlier studies. For example, Grace and Eagleson (1966) have studied summer storm data of Truro, Nova Scotia and St. Johnsbury, Vermont. After classifying the storms in three types (trace, moderate, and peaked storms) they established linear regression relationships between storm depth and duration of the form (keeping the notation of the present study)

$$E[h(D, D)] = aD + b \tag{31}$$

where $a$ and $b$ are parameters estimated by linear regression using all the data of each type. From this equation it follows that

$$E[\bar{i}(D)] = a + b/D \tag{32}$$

which is a hyperbolic form not practically different from (30) (as shown in their figures the power relationship might be used as well). Depending on the sign of $b$, the mean intensity can be a decreasing ($b > 0$) or increasing ($b < 0$) function of $D$. In five of the six cases studied by the authors (2 stations $\times$ 3 types) the $b$ was positive, which corresponds to a negative scaling exponent $H$, and in one case $b$ was negative, which corresponds to a positive $H$. Quite similar is the analysis of Woolhiser and Osborn (1985). Closer to the present study is the approach and the findings of Hershenhorn and Woolhiser (1977), who studied a 23-year data set of summer (July and August) storms from a raingage at Walnut Gulch Experimental Watershed, Arizona, USA. In order to determine the conditional distribution of duration given the storm depth, they adopted a linear regression relationship between logarithms of depths (minus a lower threshold) and durations. This relationship is equivalent to a power relationship of the untransformed quantities similar to (20). A conclusion on the correlation between mean intensity and duration does not result directly from their study (the regression made concerns duration versus depth; the converse regression is not seen in their paper). However, it seems that there is a positive correlation between duration and intensity (intensity increasing with duration), which corresponds to a positive scaling exponent.

The above literature findings as well as the proposed scaling model are in disagreement with any stationary model, i.e. a model which does not assume any dependence of instantaneous or incremental rainfall intensity on the duration (see also section 7). In the case of a stationary model the mean intensity is obviously a constant, independent of duration. This may seem at first view as a special case of the scaling model with zero scaling exponent. However, as it will be shown later, the scaling model is structurally different from any stationary model.

### 5.2 Coefficient of variation of storm depth or average intensity

As pointed out in section 4.1 a consequence of the scaling assumption is that the standard deviation of the total storm depth (or, equivalently, of the average intensity) is expressed as a power law of duration. This power law is exactly the same with the power law of the expected value of the depth (or average intensity) versus duration. Thus the coefficient of variation is constant and equal to $\sqrt{c_2}/c_1$. As will be shown later this property is strongly supported by the data used in this study (see Fig. 4). In addition, this property is consistent with other data sets and models of the literature.

Grace and Eagleson (1966) in order to describe the residuals from the mean storm depth given the storm duration adopted a relationship of the form

$$\frac{h(D,D) - E[h(D,D)]}{E[h(D,D)]} = cW - 1 \tag{33}$$

where $c$ is a constant and $W$ is a beta distributed random variable, independent of $D$. Obviously this form leads to a constant coefficient of variation of $h(D,D)$, independent of $D$.

Eagleson (1978) using a data set from Boston and assuming that the average intensity and duration are independent random variables with exponential distributions determined the marginal distribution of the storm depth in terms of a modified Bessel function of the first order. A similar assumption was made by Bras and Rodriguez-Iturbe (1976) in order to construct a rainfall generation model. They assumed that the distribution of the total depth (averaged over an area) conditional on duration is given by an exponential function of the average intensity. This implies that the average intensity is independent of the duration and exponentially distributed. The assumption of an average intensity independent of the duration apparently results in a constant coefficient of variation of the total storm depth as it easily obtained from $h(D, D) = \bar{i}D$ . In fact, this assumption can be considered as a special case of the scaling model with zero scaling exponent.

On the contrary, any stationary model cannot yield a constant coefficient of variation for total storm depth. Indeed, any model of this category would imply

$$E[h(D, D)] = \eta_1 D \tag{34}$$

where $\eta_1$ is the mean instantaneous intensity, and if a constant coefficient of variation is hypothesized then it is required that

$$E[h(D, D)^2] = \eta_2 D^2 \tag{35}$$

where $\eta_2$ is a constant. However, as it is proved in Appendix 3, the last equation is impossible for a stationary model, except for the case where the instantaneous intensity is constant with zero variance, which has no interest or physical meaning.

### 5.3 Autocorrelation structure of incremental depths

Another important consequence of the scaling model is that the autocorrelation coefficient for a certain lag is an increasing function of storm duration. Indeed, from (29) we obtain, for example, that $\rho_{X_\Delta}(1; D) = \rho_{X_{2\Delta}}(1; 2D)$ which means that the lag-one autocorrelation coefficient of hourly data in a storm of duration $D$ is equal to the lag-one autocorrelation coefficient of two-hour data in a storm of duration $2D$. Since, normally, the autocorrelation increases with decreasing lag it follows that the lag-one autocorrelation coefficient of the hourly data in a storm of duration $2D$ is greater than the lag-one autocorrelation coefficient of the hourly data in a storm of duration $D$. Thus, the lag-one autocorrelation coefficient is an increasing function of storm duration and this is also true for coefficients of higher lags.

As will be seen in the next section the hourly data we analysed support this property. To the authors knowledge, this property has not been discussed elsewhere in the literature, though it is not associated with the scaling model only. This property can be considered simply as a consequence of the constant coefficient of variation of the total storm depth, which was discussed earlier. As a simplified example consider the disaggregation of the total depth into incremental depths $X_\Delta$ for a time increment $\Delta$ and assume a Markovian dependence between $X_\Delta$ with lag one correlation coefficient equal to $\rho$. Also consider that the average intensity is independent of $D$. In this case we have

$$\sum_{1\leq i,j\leq D/\Delta} Cov[X_\Delta(i)X_\Delta(j)] = (\eta_2 - \eta_1^2)D^2 \tag{36}$$

or

$$\{D/\Delta + 2\sum_{i=1}^{D/\Delta-1}\sum_{j=i+1}^{D/\Delta}\rho^{j-i}\}Var[X_\Delta] = (\eta_2 - \eta_1^2)D^2 \tag{37}$$

and after algebraic manupilations

$$\{D/\Delta + \frac{2\rho[(D/\Delta)(1-\rho) - 1 + \rho^{D/\Delta}}{(1-\rho)^2}\}Var[X_\Delta] = (\eta_2 - \eta_1^2)D^2 \tag{38}$$

In equation (38) we observe that the left hand side depends linearly on $D$ while the right hand side depends on $D^2$. Thus we conclude that either $\rho$ or $Var[X_\Delta]$ should be an increasing function of $D$.

Another interesting point to note is that the theoretical autocorrelation coefficient of the incremental process is allowed to take on negative values (see eq. (29)), a property exhibited by rainfall data of this study and others (e.g. Grace and Eagleson, 1966, pp. 91-92) but not allowed by many stationary models as will be discussed in section 7.

# 6 Model fitting and performance evaluation

## 6.1 Model fitting procedure

In section 4 the covariance function of $X_\Delta(i,D)$ was derived in terms of the covariance function of $\xi(t,D)$. In order to be able to fit the model to incremental rainfall depths a parametric form for the covariance function of $\xi(t,D)$ must be specified and the covariance of $X_\Delta(i,D)$ must be consequently derived. As it is recalled from (16) the covariance function of $\xi(t,D)$ involves a power function of duration $D$ and a function $\phi(\tau/D)$ of the normalized lag. Here we assume the following power law form for $\phi(y)$

$$\phi(y) = ky^{-\beta} \tag{39}$$

which implies the following power law second product moment for $\xi(t,D)$

$$R_\xi(\tau;D) = kD^{\beta+2H}\tau^{-\beta} \tag{40}$$

Note that this is in contrast to stationary rainfall intensity models for which the above product moment would be a function of lag $\tau$ only and not duration.

Based on this and after the computation of the integral in (26) it is shown that

$$C_{X_\Delta}(0;D) = var[X_\Delta(i,D)] = D^{2(H+1)}\delta^2\{2k/[(1-\beta)(2-\beta)]\ \ \delta^{-\beta} - c_1^2\} \tag{41}$$

By considering $C_{X_D}(0;D)$ from the above equation (by setting $\delta = 1$) and equating it to (21) one can see that the parameters $k, \beta$ of the covariance function of $\xi(t, D)$ are related to $c_1$ and $c_2$ by

$$c_2 + c_1^2 = 2k/[(1-\beta)(2-\beta)] \tag{42}$$

By computing the integral in (28) the covariance function of the incremental storm depths is

$$C_{X_\Delta}(m;D) = D^{2(H+1)}\delta^2[(c_2 + c_1^2)\delta^{-\beta} f(m,\beta) - c_1^2], \quad m \geq 0 \tag{43}$$

where

$$f(m,\beta) = [(m-1)^{2-\beta} + (m+1)^{2-\beta}]/2 - m^{2-\beta}, \quad m > 0 \tag{44}$$

and

$$f(0,\beta) = 1 \tag{45}$$

Consequently,

$$\rho_{X_\Delta}(m;D) = \frac{(c_2 + c_1^2)\delta^{-\beta} f(m,\beta) - c_1^2}{(c_2 + c_1^2)\delta^{-\beta} - c_1^2} \tag{46}$$

The model thus has four independent parameters $H, c_1, c_2$, and $\beta$ (note that $k$ is not an independent parameter, since it is related with the others by (42)) which in the empirical analysis that follows were estimated from the following relationships:

$$E[h(D,D)] = c_1 D^{H+1} \tag{47}$$

$$Var[h(D,D)] = c_2 D^{2(H+1)} \tag{48}$$

$$\rho_{X_\Delta}(1;D) = \frac{(1 + c_2/c_1^2)\delta^{-\beta}(2^{1-\beta} - 1) - 1}{(1 + c_2/c_1^2)\delta^{-\beta} - 1} \tag{49}$$

From the first relationship $c_1$, and $H$ can be estimated by least squares and $c_2$ and $\beta$ can be estimated from the second and third relationship, respectively (see also next subsection). Then using (42) the parameter $k$ can be obtained. To further evaluate the model performance based on properties not explicitly used for model fitting, the mean, variance, and autocorrelation function of the hourly rainfall depths for storms of different durations were estimated and compared to the theoretical values for the fitted model (equations 24, 41, and 46, respectively).

## 6.2 Performance evaluation

The data used to implement the scaling model for $\xi(t, D)$ consists of hourly rainfall depths for a total of 89 storm events of duration greater than or equal to two hours. All events occurred during the month of April and during 13 years of record (1971 - 1983) at the Chalara station (latitude 40° 39' N, longitude 21° 14' E, elevation 880 meters a.s.l.) in the Aliakmon river basin, province of Macedonia, Greece. The rain recorder of this station is a weekly drum chart type with a rain depth resolution of tenths of millimeters. Due to absence of tabulated data, the charts were manually digitized under the authors' supervision. The set of one month (and not the complete annual sample) was used in order to avoid possible non homogeneity of the rainfall properties due to seasonal variability. The reason for the selection of April is that this month is characterized by a sufficiently high frequency of rainfall events leading to an adequate sample size, and, at the same time, the temperatures are greater than 0°C, thus preventing the rain recorder from freezing and leading to inaccurate data, a case not valid for previous (winter) months.

Events were identified based on the assumption of independence between events. This amounts to testing for a Poisson process of storm arrivals or exponential distribution for interarrival times. A Kolmogorov-Smirnov test was used for this purpose. Thus events were allowed to include periods of zero rainfall. Starting with a trial value of the maximum zero rainfall period allowed in an event (or, equivalently, the minimum period for separating an event from the preceeding and succeeding ones), a record of interarrival times was constructed and tested for fitting an exponential distribution at a 50% significance level. With an iterative application of this method, the minimum zero rainfall period separating two events was found equal to 7 hours. This is very close to the arbitrary value adopted by Huff (1967), i.e., 6 hours. The 89 storm events had durations varying from 2 hrs to 45 hrs with a mean duration of 11.8 hrs. General characteristics of the set of storms used are given in Table 1.

The meteorological conditions responsible for the generation of the 89 storms of April belong to several types. According to a classification of the weather types in Greece by Maheras (1982, 1992), 37% of the 89 events belong to SW1 type, i.e., passage of a depression possibly accompanied by a cold front (and rarely a warm front) having SW orbit. A 24% of the events is produced by SW2 weather type, i.e., passage of a depression originated from the Sahara desert. A 13% is produced by a special weather type (DOR) characterized by a cold upper air mass (determined at the 500 mb level) producing dynamic instability. Also 11% and 6% of the total events are produced by NW1 and NW2 weather types, respectively, characterized by depressions and/or fronts with NW orbits. The remaining 9% of events is produced by the other four of the total 16 weather types of this specific classification. The orography of the region (North Pindos mountains) plays an important role in all regional rainfall phenomena. It was found that storm durations and depths of the examined data set are uniformly distributed in each of the above five most frequent weather types (SW1, SW2, DOR, NW1, and NW2), with a likely exception of the DOR type which is characterized by slightly higher durations and depths. Thus no special treatment of the events classified by weather type was done, though one could consider application of the model to different types of storms with different parameter values (obviously, this would need a large set of data).

To be able to estimate ensemble statistics, the 89 storms were grouped in five classes (1 to 5) according to their duration as shown in Table 2. For example, class 1 includes all 14 storms with duration 2 and 3 hours and class 5 all 17 events with duration between 19 and 45 hours. The basis for selecting this grouping was to have approximately the same number of events in each class. To each class a duration was assigned equal to the mean duration of all events in that class. The events were further grouped into two larger classes (A and B) were class A includes all 39 events of classes 2 and 3 and class B all 36 events of classes 4 and 5. Again the mean duration of each class was used as a representative duration of that class and events in classes A and B were used to estimate the ensemble autocorrelation functions for two different storm durations. The enlarged size of classes A and B was necessary in order to achieve reliable estimates of the autocorrelation coefficients for large lags.

Because there is variability in the durations of the events of each class around the mean duration $\overline{D}$ assigned to that class a correction procedure was applied (when necessary) in estimating the variance of the total depth in each class. This correction consisted of subtracting from the calculated variance the quantity $\sigma_D^2(k_1^2+k_2^2)$ where $\sigma_D^2$ is the variance of the durations in that class and $k_1, k_2$ are constants obtained from the linearization of the mean and standard deviation of total depths, respectively, in the neighbourhood of $\overline{D}$, i.e, $E[h(D,D)] \approx k_1\overline{D}$ and $\{Var[h(D,D)]\}^{1/2} \approx k_2\overline{D}$ (the proof for the appropriateness of the above correction is ommited). For the scaling process we have $c_1\overline{D}^{H+1} \approx k_1\overline{D}$ and $c_2\overline{D}^{2(H+1)} \approx k_2^2\overline{D}^2$ and thus the correction applied was

$$\sigma_D^2(c_1^2+c_2)\overline{D}^{2H} \tag{50}$$

It was found that this correction was negligible for all classes except the class with the larger durations (class 5). The necessity of such a correction implies an iterative process for the estimation of $c_2$ (one iteration is usually sufficient).

Based on the parameter estimation procedure discussed in the previous section the following parameter estimates were obtained for this data set:

$$\hat{H} = -0.20, \quad \hat{c}_1 = 1.05, \quad \hat{c}_2 = 0.44, \quad \hat{\beta} = 0.32 \tag{51}$$

For these parameters the value of $k$ is $\hat{k} = 0.88$. The parameters $H$ and $c_1$ were estimated by least squares on the power relationship of the mean total depth of each of the five classes versus the mean duration of the class (eq. 47). $c_2$ was then estimated as the average over all classes of $Var[h(D,D)]/D^{2(H+1)}$ (eq. 48). Finally $\beta$ was estimated with an iterative procedure for best fit of the theoretical curve of $\rho_{X_\Delta}(1;D)$ (eq. 49) to the empirical lag one correlation coefficients of all classes (see Figure 5).

The empirical mean and standard deviation of total storm depth as a function of duration as well as the theoretical curves from the fitted model are shown in Figure 3. Fig. 4 shows the empirical coefficients of variation of the total storm depth which is almost independent of duration and the theoretical coefficient of variation which is constant and equal to $\sqrt{c_2}/c_1 = 0.63$. The empirical and theoretical lag one autocorrelation coefficents of hourly rainfall depths are shown in Fig. 5 as a function of storm duration. Although deviations between the empirical and theoretical values are observed the model captures the general behavior of the empirical data and when 90% approximate confidence intervals (computed by using the Fisher-Z transformation for the autocorrelation coefficient) were positioned around the theoretical values only 1 of the 5 values was outside the confidence intervals as statistically expected. Note that the empirical autocorrelation coefficients were calculated independently of any other estimated or theoretically anticipated parameters, by considering all possible pairs (with a fixed lag) of hourly depths located in each of the events of a specific class.

To check the performance of the model we computed the empirical and theoretical mean and standard deviation of the hourly rainfall depths for different durations (shown in Fig. 6) and the autocorrelation functions for classes A and B (shown in Fig. 7). It is seen that the scaling model performs reasonably well in terms of capturing statistical properties of total and incremental storm depths in storms of different durations. The largest departure of the empirical statistics from the theoretical ones are found for the standard deviation of storms of duration 2-3 hours (see, Fig. 6).

Apparently, other interpretations of the examined data set are possible and other models can be used to capture the statistical structure of the data. For example, motivating by Figure 6, one can consider that the data point from the smallest duration is anomalous and, for medium and long durations, rainfall intensity is independent of duration and rainfall depth does not scale with duration. However, the selection and fitting of the scaling model must be considered as a whole, i.e., with simultaneous regard to all properties of the total and incremental storm depths. In that respect, the model ability to capture the power function of the variance of the total depth or the constant coefficient of variation (Figures 3 and 4), and the increasing with duration autocorrelation coefficients (Figures 5 and 7), is worth noting. As it will be seen in the next section it is not easy to find an alternative simple model capable of capturing these second order properties, although any model can perform well with first order properties (i.e., expected values).

It should be noted that the above adopted parameter estimation procedure depends on the selection of classes, which raises a source of subjectivity and non-robustness. Another weakness of the procedure may be the estimation of the two parameters $H$ and $c_1$ from only the mean values of the total depth, while they also appear in the equations for variance of total and incremental depths, and autocorrelation coefficients of the incremental depths. A more robust parameter estimation procedure is a feasible future improvement of the model. Finally, it is worth noting that the developed model should not be considered as a very detailed and general model that can explain perfectly all properties of the examined data set as well as of any other data set. The authors are well aware of the fact that the rainfall structure exhibits a wide variety of patterns in different regions of the world or even in the same region under different weather conditions, thus making it impossible to develop a single model applying to all situations. The proposed model is better to be viewed as an improved alternative to the simple stationary models, still itself having a simple structure (in spite of the somewhat complicated mathematical derivations) and being characterized by parsimony of parameters. It is emphasized that the model has only four parameters while other detailed models can have even tens of parameters (e.g., the model of Woolhiser and Osborn (1985) which has a total of 26 parameters).

# 7 Comparison with stationary models

In this section we derive the statistical properties of total and incremental storm depths for two simple stationary models, i.e., models satisfying (12) and demonstrate both analytically and empirically that these models are not able to capture important statistical characteristics of storm rainfall that the simple scaling model is able to capture.

## 7.1 Derivation of statistical properties

It is easy to see that

$$E[h(D,D)] = E[h(D)] = \eta_1 D \tag{52}$$

$$E[X_\Delta(i,D)] = E[X_\Delta(i)] = \eta_1 \Delta \tag{53}$$

where $\eta_1 = E[\xi(t,D)] = E[\xi(t)]$. To derive the expressions for the variance and covariance of $h(D)$ and $X_\Delta(i)$ we need to specify functional forms for the autocorrelation function of $\xi(t)$. The following two common models (power law and markovian) are examined:

$$Model\ \ 1: \quad C_\xi(\tau, D) = C_\xi(\tau) = k_1 \tau^{-\beta_1} \tag{54}$$

$$Model\ \ 2: \quad C_\xi(\tau, D) = C_\xi(\tau) = k_2 e^{-\beta_2 \tau} \tag{55}$$

After algebraic manipulations it can be shown that for model 1

$$Var[h(D)] = \frac{2k_1}{(1-\beta_1)(2-\beta_1)} D^{2-\beta_1} \tag{56}$$

$$Var[X_\Delta(i)] = \frac{2k_1}{(1-\beta_1)(2-\beta_1)} \Delta^{2-\beta_1} \tag{57}$$

$$\rho_{X_\Delta}(m) = \frac{1}{2}[(m-1)^{2-\beta_1} + (m+1)^{2-\beta_1}] - m^{2-\beta_1} \tag{58}$$

where $0 < \beta_1 < 1$ if $k_1 > 0$ (or $1 < \beta 1 < 2$ if $k_1 < 0$), as it becomes apparent from (56) and (54). Similarly for model 2

$$Var[h(D)] = 2(k_2^2/\beta_2^2)(\beta_2 D - 1 + e^{-\beta_2 D}) \tag{59}$$

$$Var[X_\Delta(i)] = 2(k_2^2/\beta_2^2)(\beta_2 \Delta - 1 + e^{-\beta_2 \Delta}) \tag{60}$$

$$\rho_{X_\Delta}(m) = \frac{(1-e^{-\beta_2\Delta})^2}{2(\beta_2\Delta - 1 + e^{-\beta_2\Delta})} e^{-\beta_2(m-1)\Delta} \tag{61}$$

Note that in both of the above models the coefficient of variation of the total storm depth is not constant but is a function of the storm duration. For example, for model 1 the coefficient of variation is $(\sqrt{2k_1/[(1-\beta_1)(2-\beta_1]}/\eta_1)D^{-\beta_1/2}$. This property of the model is in disagreement with the empirical evidence (see section 5 and Fig. 9) that the coefficient of variation of total strom depths is constant and independent of storm duration.

In the next section these two models are fitted to the data from the 89 storms described earlier.

### 7.2 Model fitting and performance evaluation

Both models have three parameters. Equation (52) can be used to estimate $\eta_1$ using the sample of total depths. Equations (58) and (61), when setting $m = 1$, can be used to estimate $\beta_1$ and $\beta_2$, respectively. The empirical lag-one autocorrelation coefficient used in these equations can be calculated from the whole sample of hourly data. Finally $k_1$ and $k_2$ are estimated from equations (57) and (60), respectively, by using the sample of total depths. The following parameters were estimated for the above two models:

$$\begin{aligned} &Model\ \ 1: \hat{\eta}_1 = 0.65, \ \ \hat{k}_1 = 0.61, \ \ \hat{\beta}_1 = 0.51 \\ &Model\ \ 2: \hat{\eta}_1 = 0.65, \ \ \hat{k}_2 = 1.25, \ \ \hat{\beta}_2 = 1.58 \end{aligned} \tag{62}$$

Fig. 8 shows the empirical and theoretical mean and standard deviation of the total storm depths. It is observed that both stationary models are not able to capture the duration dependent structure of these statistics. This is further verified by Fig. 9 which shows the empirical and theoretical coefficient of variation of the total storm depths as a function of duration. The empirical and theoretical first autocorrelation coefficient of the hourly rainfall depths is shown in Fig. 10 as a function of duration. As was analytically seen from (58) and (61) the autocorrelation of hourly rainfall depths is independent of the duration and cannot obtain negative values. As the lag increases $\rho_{X_\Delta}(m; D)$ is always positive in (61) and if the ranges of $\beta_1$ and $k_1$ are as given in the previous subsection, this is also the case for $\rho_{X_\Delta}(m; D)$ in (58). This is in disagreement with the empirical observations (see, for example, Fig. 10).

To further evaluate the model performance based on properties not explicitly used in model fitting we evaluated the empirical and theoretical mean and standard deviation of the hourly rainfall depths (equations 53, 57, and 60) and autocorrelation functions (equations 58 and 61) for model 1 and model 2, respectively. These figures together with Figs. 8, 9, and 10 demonstrate the superiority of the scaling model and the inability of the stationary models to capture important statistical properties of storm rainfall.

# 8 Mass curves

In this section we examine the concept of normalized mass curves in reference to the scaling model and, also for comparison, in reference to the stationary models. We will see that the stationary models are incompatible with this concept, while a scaling model can be compatible and, thus, can provide a means for stochastically generating mass curves for storms with independently generated totals. In the next section we will see how the model can be practically applied for the stochastic generation of storm hyetographs and, as a result of this application, we will observe that the proposed model with only four parameters can be a relatively good representation of the traditional mass curves determined as a set of curves each corresponding to a specific probability level.

The use of dimensionless mass curves, i.e., normalized rainfall depth $h^*(t/D)$ versus normalized time $t/D$, implies that a stochastic function $h^*(.)$ can be found such that

$$h(t, D) = h^*(t/D)h(D, D) \tag{63}$$

where $h(D, D)$ is a stochastic variable (the total storm depth) apparently independent of $t$, whereas $h^*(t/D)$ is a stochastic function independent of both $D$ and $h(D, D)$ satisfying $h^*(0) = 0$ and $h^*(1) = 1$. A similar relationship holds for the instantaneous intensity, that is,

$$\xi(t, D) = \xi^*(t/D)\bar{i}(D) \tag{64}$$

where $\xi^*(.)$ denotes the derivative of $h^*(.)$ and $\bar{i}(D) = h(D,D)/D$. Taking $k$ moments in (63) and (64) we obtain respectively

$$E[h(t,D)^k] = E[h^*(t/D)^k]E[h(D,D)^k] \tag{65}$$

and

$$E[\xi(t,D)^k] = E[\xi^*(t/D)^k]E[\bar{i}(D)^k] \tag{66}$$

Similar relationships hold for the $(k,\ell)$ second product moments, i.e.,

$$E[h(t_1,D)^k h(t_2,D)^\ell] = E[h^*(t_1/D)^k h^*(t_2/D)^\ell]E[h(D,D)^{k+\ell}] \tag{67}$$

and

$$E[\xi(t_1,D)^k \xi(t_2,D)^\ell] = E[\xi^*(t_1/D)^k \xi^*(t_2/D)^\ell]E[\bar{i}(D)^{k+\ell}] \tag{68}$$

Under the assumption of stationarity over time none of the above relationships can hold. Consider, for example, model 1 for which

$$E[h(t,D)] = \eta_1 t = \eta_1(t/D)D \tag{69}$$

and

$$\begin{aligned} E[h(t,D)^2] &= \eta_1^2 t^2 + \{2k_1/[(1-\beta_1)(2-\beta_1)]\}t^{2-\beta_1} \\ &= \eta_1^2(t/D)^2 D^2 + \{2k_1/[(1-\beta_1)(2-\beta_1)]\}\,(t/D)^{2-\beta_1} D^{2-\beta_1} \end{aligned} \tag{70}$$

It becomes apparent that for $k$ =2 no function $h^*(t/D)$ can be found to satisfy (70). A generalized proof of the incompatibility of any stationary model with the concept of mass curves is found in Appendix 3.

On the contrary, the self similar models are not incompatible with normalized mass curves. It is easy to show that if the process $\xi(t,D)$ is defined by (64) (or, equivalently, if $h(t,D)$ is defined by (63)) and at the same time the dependence between total depth and duration is of a power type, i.e.,

$$h(D,D) = D^H W \tag{71}$$

where $W$ is a random variable independent of $D$ (or, equivalently, the logarithm of the total depth is linearly dependent on duration), then $\xi(t,D)$ is a self-similar (simple-scaling) process, as defined by (6). The proof is obvious and will be omitted. As we will see below, the reverse is not valid in all cases, i.e., not any scaling model can satisfy (63) or (64) in a strict and complete way. Nevertheless, equations (65) and (66) are satisfied for any simple scaling model. Indeed, for a scaling model

$$E[h(t,D)^k] = D^{k(H+1)} E[h(t/D,1)^k] \tag{72}$$

while

$$E[h(D,D)^k] = D^{k(H+1)} E[h(1,1)^k] \tag{73}$$

Hence

$$E[h(t,D)^k] = \frac{E[h(t/D,1)^k]}{E[h(1,1)^k]} E[h(D,D)^k] \tag{74}$$

which is consistent with (65) since it results to

$$E[h^*(t/D)^k] = \frac{E[h(t/D,1)^k]}{E[h(1,1)^k]} \tag{75}$$

which is a function of only $t/D$. The above equation defines completely the marginal distribution of $h^*(.)$ at every dimensionless time position. Concerning the multivariate distribution and joint product moments the situation is more complicated. It can be shown that there exist simple scaling models that satisfy (67) and (68) but this is not true for any model. The problem originates from the constraint $h^*(1) = 1$ along with the requirement for full independence of $h^*(t/D)$ and $h(D,D)$. In Appendix 4 it is proved that the assumption of weak stationarity which was made for reasons of simplicity (eq. (11)) is inconsistent with (68). The task to build a model fully consistent with the requirement of complete statistical independence of $h^*(t/D)$ and $h(D,D)$ is possible but implies mathematical complexity and inflexibility. So we preferred in this study to build a simple and easily applicable model by reducing the requirement of complete independence to the that of orthogonality of $h^*(t/D)^k$ and $h(D,D)^k$ (for $k = 1,2,...$). Apparently, the condition of orthogonality is assured by (65) which is valid for any scaling model. As will be shown later (Section 9 and Figures 13-14) this compromise is practically negligible.

# 9 Generating storm hyetographs

The scaling model can be applied for generating storm hyetographs at an incremental basis for any time step $\Delta$. One can recognize that the correlation structure implied by the scaling model, even in the case of the weak stationarity, is somewhat complicated and differs from the structure of a typical linear model, i.e., an ARMA(p, q) model. However, the introduction of a nonlinear model for the generation is not necessary. Since the consecutive events are isolated and the number of generation steps in each event is limited, a proper linear model can be established to carry out the generation. Two possible procedures are discussed below both presuming a given storm duration $D$. The first is a typical sequential procedure where the incremental depths $X_\Delta(i,D)$

are generated one at a time and the total storm depth $h(D,D)$ is then obtained by summation. The second is a disaggregation procedure where a given total storm depth is disaggregated into incremental depths. In both cases the scaling model is utilized to determine the parameters of the generation model. Denoting $\mathbf{X} = [X_\Delta(1,D), X_\Delta(2,D), ..., X_\Delta(k,D)]^T$, where $k = D/\Delta$ (assumed to be an integer) the parameters required are the first moments $E[\mathbf{X}]$ given by (24) and the second moments $Cov[\mathbf{X},\mathbf{X}]$ given by (27) or more specifically by (43). Also required is an assumption about the marginal distribution. Here after examination of the data set of this study and in light of other studies the two-parameter gamma distribution was adopted. The generation scheme for the sequential procedure can be based on

$$\mathbf{X} = \mathbf{\Omega V} \tag{76}$$

where $\mathbf{\Omega} = [\omega_{ij}]$ is a $k \times k$ matrix of coefficients and $\mathbf{V} = [\mathbf{V}_1, ..., \mathbf{V}_k]^T$ is a vector of mutually independent random variables with unit variance and a three- parameter gamma distribution function. The parameters of this scheme are determined by the following equations which are easily obtained

$$\mathbf{\Omega\Omega}^T = Cov[\mathbf{X},\mathbf{X}] \tag{77}$$

$$\omega_{ij}E[V_i] = E[X_i] - \sum_{\ell=1}^{i-1}\omega_{i\ell}E[V_\ell] \tag{78}$$

$$\omega_{ij}^3\mu_3[V_i] = \mu_3[X_\Delta(i,D)] - \sum_{\ell=1}^{i-1}\omega_{i\ell}^3\mu_3[V_\ell] \tag{79}$$

where $\mu_3[V_i]$ is the third moment of $V_i$ and $\mu_3[X_\Delta(i,D)]$ is the third moment of $X_\Delta(i,D)$ determined analytically from the assumed marginal distribution. The $\mathbf{\Omega}$ matrix is considered as lower triangular and is computed by deconvolution of $\mathbf{\Omega\Omega}^T$. In the case of the disaggregation procedure, first one might have to generate $h(D,D)$ (if it is not already known). This can be done by using (20) and (21) after assuming a distribution function (a two-parameter gamma distribution was adopted here).

Motivated by the concept of normalized mass curves, the following procedure was adopted for the disaggregation:

1. Apply the sequential procedure as described above to obtain an initial sequence $X'_\Delta(i,D), i = 1, ..., k$;

2. Determine a normalized sequence $X^*_\Delta(i,D) = X'_\Delta(i,D)/S'$, where $S' = \sum_{i=1}^{k} X'_\Delta(i,D)$ . This sequence determines a realization of a dimensionless mass curve;

3. Calculate the final sequence $X_\Delta(i,D) = X^*_\Delta(i,D)h(D,D)$.

Both the above procedures have some sources of inaccuracy. The generated by the sequential procedure values of $X_\Delta(i,D)$ can be negative, a possibility arising either from the three-parameter gamma distribution of $V_i$ or from possibly negative values $\omega_{ij}$. To avoid this when negative values $X_\Delta(i,D)$ are generated they can be set zero, a correction consistent with the definition of a storm which allows for zero incremental depths. Furthermore, the sum of three-parameter gamma variables implied by (76) theoretically is not gamma distributed, though a good approximation can be obtained by the introduction of third moments. Finally, a third source of inaccuracy is expected in the case of the disaggregation procedure due to the non-complete independence of the total depth and normalized mass curve discussed in Section 8. To delimit such an effect during the execution of the generation we can reject sequences $X'_\Delta(i,D)$ leading to a ratio $h(D,D)/S'$ quite far from unity.

By using the parameter set of Section 6 we applied both the above procedures for generating 10,000 synthetic hyetographs in a hourly basis for a storm of duration of 20 hours. A series of comparisons between theoretical values of several statistics with the corresponding values obtained by simulation were made. The examined statistics are first, second and third order marginal moments, marginal distributions, and autocorrelation coefficients of hourly depths. All the comparisons (which are not presented here, expect for the following three examples) had satisfactory results. Originating from this exercise, Figure 13 indicates the degree of inaccuracy due to the first two of the above discussed sources of inaccuracy in reproducing the distribution of the hourly depths. It is shown that the deviation of the simulated frequency curves from the theoretical ones are confined to values of $X_1(10,20) \leq 0.5$ mm. Remarkable are the smaller departures of the disaggregation model simulated curves as compared with the ones of the sequential model. Figure 14 shows that both (sequential and disaggregation) procedures perform well in reproducing the covariance structure of hourly depths as theoretically determined by the scaling model. Note that Figure 14(b) corresponding to the disaggregation procedure does not differ in performance from 14(a) corresponding to the sequential procedure. This means that the potentially expected inaccuracy due to the previously discussed violation of the complete independence of $h^*(t/D)$ and $h(D,D)$ (we only satisfied orthogonality) is not important and, consequently, this weakness of the model in being fully compatible with mass curves is not substantial.

Finally, Figure 15 referring to the normalized mass curves was constructed from hyetographs of the so called (after Huff, 1967) second quartile group (i.e. hyetographs having their the heaviest part in the second quarter of their duration). The curves presented are similar and were drawn with the same method proposed by Huff (1967) and correspond to the 50% (median) as well as 10% and

90% probability levels. Three groups of curves appear in Figure 15. First are the synthetic curves computed at the step 2 of the disaggregation procedure from that portion of the hyetographs that belong to the second quartile group. Second are the curves computed from the historical data of this study. Specifically, from the total historical sample, 19 storms of a total of 75 (about 1/4) were found to belong to the second quartile type (note that the storms of class 1, i.e., those of duration less than 4 h, were discarded since it was not possible to identify the quartile they belong to). Due to the lack of a sufficient sample size of historical data in the month of April, we plotted also a third group of curves from historical data of 140 second quartile storms recorded at the same station Chalara but for all months of the year. The third group of curves originates from another study (Stylianidou, 1985). The comparison plot shows that all three synthetic and historical groups of curves are very close to each other without any remarkable deviation (perhaps, except for the lower part of the 90% synthetic curve). Thus Figure 15 gives a good indication that the scaling model with as few as four parameters can represent or summarize effectively the statistical characteristics of a storm population otherwise given by a family of curves. Additionally, note that the curves of Figure 15 are based on the assumption of the weak stationarity, i.e., a "mean" mass curve which is a straight line of uniform mean intensity. However, as observed from Figure 15, the synthetic curves (even the median curve) have nonlinear shape in accordance with the historical curves. To understand this one must consider that the curves correspond to a portion of the totally generated hyetographs conditionally selected so as to have the main slope located at the second quarter of their duration.

It must be emphasized that the above model is not a complete rainfall generator but rather is a generator of hyetographs of individual storms. However, it can be easily extended to a complete generator by appending a component for the storm and interarrival time durations.

# 10 Concluding remarks

The developed simple scaling model for the temporal structure of storm rainfall has a simple mathematical structure with only four parameters but it explains reasonably well the statistical properties of the examined historical data providing thus an efficient parametrization of storms of varying durations and total depths. In addition, it is consistent with, and provides a theoretical basis for, the concept of normalized mass curves.

It was found that the scaling model is superior to the examined stationary models, which were unable to capture important statistical properties of storm rainfall and were inconsistent with the concept of normalized mass curves. Furthermore, the scaling model provides a stochastic nondimensionalization approach which is apparently superior to the popular use of mass curves, because of the contraction in a few parameters of all the information otherwise given by a family of curves and the implication of a stochastic approach to storm hyetograph generation, which is not possible by the traditional method of mass curves.

The proposed model, when combined with a stochastic process of the storm arrivals (e.g. a Poisson process) and a set of distribution functions for the rainfall duration and intensity can give a complete rainfall generator at a point or on an areal basis. Moreover, merely the scaling model can be useful in hydrologic applications, such as in evaluation of design storms, as an evolution of the concept of mass curves.

Different configurations of the model can be obtained by using e.g. different forms of the covariance function of the rainfall intensity. In addition, the weak stationarity condition, used here as a convenient assumption, is not a necessary structural constraint and it can be removed or substituted in cases where the historical data exhibit nonstationarities within each event. A more robust parameter estimation technique and model evaluation at time scales different than the hourly is a feasible future improvement of the model.

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# 11 Appendix 1: Self-similarity of $h(t, D)$

Let us consider the $(k, \ell)$ second product moment of $h(t, D)$

$$\begin{aligned}
& E\{h(t_1, D)^k h(t_2, D)^\ell\} \\
&= E\{[\int_0^{t_1} \xi(s, D)\, ds]^k [\int_0^{t_2} \xi(q, D)\, dq]^\ell\} \\
&= \int_0^{t_1} \cdots \int_0^{t_1} \int_0^{t_2} \cdots \int_0^{t_2} E\{\xi(s_1, D) \cdots \xi(s_k, D) \xi(q_1, D) \cdots \xi(q_\ell, D)\} \\
&\qquad ds_1 \cdots ds_k\, dq_1 \cdots dq_\ell
\end{aligned} \tag{80}$$

Similarly,

$$\begin{aligned}
& E\{h(\lambda t_1, \lambda D)^k h(\lambda t_2, \lambda D)^\ell\} \\
&= \int_0^{\lambda t_1} \cdots \int_0^{\lambda t_1} \int_0^{\lambda t_2} \cdots \int_0^{\lambda t_2} E\{\xi(s_1, \lambda D) \cdots \xi(s_k, \lambda D) \xi(q_1, \lambda D) \cdots \xi(q_\ell, \lambda D)\} \\
&\qquad ds_1 \cdots ds_k\, dq_1 \cdots dq_\ell \\
&= \int_0^{t_1} \cdots \int_0^{t_1} \int_0^{t_2} \cdots \int_0^{t_2} E\{\xi(\lambda\sigma_1, \lambda D) \cdots \xi(\lambda\sigma_k, \lambda D) \xi(\lambda\psi_1, \lambda D) \cdots \xi(\lambda\psi_\ell, \lambda D)\} \lambda^{k+\ell} \\
&\qquad d\sigma_1 \cdots d\sigma_k\, d\psi_1 \cdots d\psi_\ell
\end{aligned} \tag{81}$$

where the last equality has been obtained by setting $s_i = \lambda\sigma_i$ and $q_i = \lambda\psi_i$. Note that this last equality would not hold if any product moment contained dirac delta terms. This can be seen by considering for simplicity one term only, say $E[\xi(s, \lambda D)]$, and observing that if that term had the form $f(s)\delta(s - s_0)$ then $\int_0^{\lambda t} E[\xi(s, \lambda D)]\, ds = \int_0^{\lambda t} f(s)\delta(s - s_0)\, ds = f(s_0)$ while the term obtained by substituting $s = \lambda\sigma$ would give $\int_0^t E[\xi(\lambda\sigma, \lambda D)]\lambda\, d\sigma = \int_0^t f(\lambda\sigma)\delta(\lambda\sigma - s_0)\lambda\, d\sigma = \lambda f(s_0) \neq f(s_0)$.

In view of (9) the above equality can be further written as

$$\begin{aligned}
& E\{h(\lambda t_1, \lambda D)^k h(\lambda t_2, \lambda D)^\ell\} \\
&= \lambda^{k+\ell} \lambda^{H(k+\ell)} \int_0^{t_1} \cdots \int_0^{t_1} \int_0^{t_2} \cdots \int_0^{t_2} E\{\xi(\lambda\sigma_1, \lambda D) \cdots \xi(\lambda\sigma_k, \lambda D) \xi(\lambda\psi_1, \lambda D) \cdots \xi(\lambda\psi_\ell, \lambda D)\} \\
&\qquad d\sigma_1 \cdots d\sigma_k\, d\psi_1 \cdots d\psi_\ell
\end{aligned} \tag{82}$$

By comparing (80) and (82) we obtain

$$\lambda^{-(H+1)(k+\ell)} E\{h(\lambda t_1, \lambda D)^k h(\lambda t_2, \lambda D)^\ell\} = E\{h(t_1, D)^k h(t_2, D)^\ell\} \tag{83}$$

This result can be similarly extended to the product moments of any order and thus we conclude that

$$\{h(t, D)\} \stackrel{d}{=} \{\lambda^{-(H+1)} h(\lambda t, \lambda D)\} \tag{84}$$

# 12 Appendix 2: Covariance function of incremental depths

From the definition of $X_\Delta(i,D)$ in (2) we obtain

$$\begin{aligned}
E\{[X_\Delta(i,D)]^2]\} &= \int_{(i-1)\Delta}^{i\Delta}\int_{(i-1)\Delta}^{i\Delta} E[\xi(t_1,D)\xi(t_2,D)]dt_1\,dt_2 \\
&= \int_{(i-1)\Delta}^{i\Delta}\int_{(i-1)\Delta}^{i\Delta} R_\xi(t_1-t_2;D)\,dt_1\,dt_2 \\
&= \int_{-\Delta}^{\Delta} R_\xi(\tau;D)|\Delta-\tau|\,d\tau \\
&= 2\int_0^{\Delta} R_\xi(\tau;D)(\Delta-\tau)\,d\tau \qquad (85)
\end{aligned}$$

where the last but one equality results from simplification of the double integral by setting $\tau = t_1 - t_2$ and observing that the integration area is equivalent to $|\Delta - \tau|\,d\tau$ with $\tau$ varying from $-\Delta$ to $\Delta$, whereas the last equality comes from recognizing that $R_\xi(\tau, D)$ is an even function of $\tau$ contaning no concentrated masses. Substituting $R_\xi(\tau; D)$ from (16) in the above expression we obtain

$$E\{[X_\Delta(i,D)]^2\} = D^{2(H+1)}\psi(0;\delta) \qquad (86)$$

where $\delta = \Delta/D$ and $\psi(0;\delta)$ is as defined in (26). From the above the expression (25) can be easily obtained.

Similarly, the second product moment of $X_\Delta(i,D)$ is given as

$$\begin{aligned}
R_{X_\Delta}(m;D) &= E[X_\Delta(m+1;D)X_\Delta(1;D)] \\
&= \int_0^{\Delta}\int_{m\Delta}^{(m+1)\Delta} E[\xi(t_1,D)\xi(t_2,D)\,dt_1\,dt_2 \\
&= \int_0^{\Delta}\int_{m\Delta}^{(m+1)\Delta} R_\xi(t_1-t_2;D)\,dt_1\,dt_2 \\
&= \int_{(m-1)\Delta}^{m\Delta} R_\xi(\tau;D)(\tau-(m-1)\Delta)\,d\tau \\
&\quad + \int_{m\Delta}^{(m+1)\Delta} R_\xi(\tau;D)((m+1)\Delta-\tau)\,d\tau \qquad (87)
\end{aligned}$$

Substituting $R_\xi(\tau, D)$ from (16) and setting $\delta = \Delta/D$ we obtain

$$R_X(m;D) = D^{2(H+1)}\psi(m;\delta) \qquad (88)$$

where $\psi(m;\delta)$ is defined in (28). Equation (27) for the covariance function of $X(i,\Delta)$ is then easily obtained from the above.

## 13 Appendix 3: Incompatibility of stationary models with scaling properties

The expected value of the storm depth in any stationary model is given by

$$E[h(D, D)] = \eta_1 D \tag{89}$$

where $\eta_1$ is the mean instantaneous intensity. Let us examine the possibility that the second marginal moment is given by a power function of $D$, i.e.,

$$E[h(D, D)^2] = \eta_2 D^\theta \tag{90}$$

where $\eta_2$ and $\theta$ are constants. In the case of a stationary model we have

$$E[h(D, D)^2] = \int_0^D \int_0^D E[\xi(t_1)\xi(t_2)]dt_1 dt_2 = \int_0^D \int_0^D R_\xi(t_1 - t_2)dt_1 dt_2 \tag{91}$$

where $R_\xi(.)$ is the second product moment for the instantaneous intensity $\xi(t)$ (which is not a function of $D$). The last double integral can be simplified (e.g., as in Papoulis, 1965, p. 325), and then equated to (90) to give

$$2\int_0^D R_\xi(\tau)(D - \tau)d\tau = \eta_2 D^\theta \tag{92}$$

Taking derivative of the above equation with respect to $D$ we get

$$2\int_0^D R_\xi(\tau)d\tau = \eta_2 \theta D^{\theta-1} \tag{93}$$

Taking once more derivatives and substituting $D$ with $\tau$ we obtain the form of $R_\xi(\tau)$, that is,

$$R_\xi(\tau) = \frac{\eta_2\theta(\theta - 1)}{2}\tau^{\theta-2}, \quad \tau > 0 \tag{94}$$

Besides, the variance of the storm depth is

$$Var[h(D, D)] = \eta_2 D^\theta - \eta_1^2 D^2 = \eta_1^2 D^2[(\eta_2/\eta_1^2)D^{\theta-2} - 1] \tag{95}$$

Now we can observe that the case where $\theta > 2$ is impossible since it implies that $Var[h(D, D)]$ would be negative for some large $D$ and, also would yield a correlation function of the instantaneous intensity increasing with lag $\tau$, which is unreasonable. Likewise, the case where $\theta < 2$ is also impossible since it implies a negative $Var[h(D, D)]$ for some small $D$ (though in this case we don't have any problem with the autocorrelation function). Finally, the only possibility with mathematical meaning is the case where $\theta = 2$. But, as results from (94) in that case $R_\xi(.)$ is constant and, consequently, the instantaneous intensity would be constant with zero variance, a case with no interest or physical meaning.

Now let us examine the compatibility of the stationary model with mass curves in the general case. From (65) for $k = 2$ we get

$$E[h(t,D)^2] = E[h^*(t/D)^2]E[h(D,D)^2] \tag{96}$$

Note that the left-hand side of the above equation, in the case of a stationary model, is in fact a function of only $t$. Thus denoting $\phi(t) = E[h(t,D)^2]$ and $\psi(\lambda) = E[h^*(\lambda)^2]$ we can rewrite (96) as

$$\phi(\lambda D) = \psi(\lambda)\phi(D) \tag{97}$$

and since

$$\phi(\lambda\mu D) = \psi(\lambda)\phi(\mu D) = \psi(\lambda)\psi(\mu)\phi(D) \tag{98}$$

while at the same time

$$\phi(\lambda\mu D) = \psi(\lambda\mu)\phi(D) \tag{99}$$

we conclude that

$$\psi(\lambda\mu) = \psi(\lambda)\psi(\mu) \tag{100}$$

Thus

$$\psi(\lambda) = \lambda^\theta \tag{101}$$

for some constant $\theta$. Furthermore, with the substitution of the above into (97) and after setting $\lambda = 1/D$ we get

$$\phi(D) = \eta_2 D^\theta \tag{102}$$

where the constant $\eta_2 = \phi(1)$. The above equation is equivalent to (90) and thus it cannot be valid with the exception of the case that $\theta = 2$, which was described above. We conclude that any stationary model is incompatible with the concept of normalized mass curves.

# 14 Appendix 4: Incompatibility of the independence of normalized and total depth with the weak stationarity condition

Starting with the obvious relation

$$\int_0^1 \xi^*(u)du = 1 \tag{103}$$

written in the form

$$\int_0^\delta \xi^*(u)du + \int_\delta^1 \xi^*(u)du = 1 \tag{104}$$

where $\delta$ is an arbitrary number ($0 \leq \delta \leq 1$) we obtain that

$$\int_0^\delta \int_0^\delta E[\xi^*(u)\xi^*(s)]duds - \int_\delta^1 \int_\delta^1 E[\xi^*(u)\xi^*(s)]duds = \int_0^\delta E[\xi^*(u)]du - \int_\delta^1 E[\xi^*(u)]du \tag{105}$$

To prove the above equation multiply (104) successively by the first and second integral terms of its left-hand side, then subtract the two obtained equations, and take expected values. It is easy to show that equation (105) is inconsistent with the following concurrent equations

$$E[\xi^*(u)] = c_1^* \tag{106}$$

$$E[\xi^*(u)\xi^*(s)] = \phi^*(|u-s|) \tag{107}$$

where $c_1^*$ is an arbitrary constant and $\phi^*(.)$ is an arbitrary function. Indeed, (107) implies that (see analogous cases in Appendix 2 and 3)

$$\int_0^\delta \int_0^\delta E[\xi^*(u)\xi^*(s)]duds = 2\int_0^\delta \phi^*(\tau)(\delta-\tau)d\tau \tag{108}$$

$$\int_\delta^1 \int_\delta^1 E[\xi^*(u)\xi^*(s)]duds = 2\int_0^{1-\delta} \phi^*(\tau)(1-\delta-\tau)d\tau \tag{109}$$

Thus (105) becomes

$$\int_0^\delta \phi^*(\tau)(\delta-\tau)d\tau - \int_0^{1-\delta} \phi^*(\tau)(1-\delta-\tau)d\tau = c_1^*(\delta - 1/2) \tag{110}$$

and, after taking derivatives with respect to $\delta$

$$\int_0^\delta \phi^*(\tau)d\tau - \int_0^{1-\delta} \phi^*(\tau)d\tau = c_1^* \tag{111}$$

or

$$\int_{1-\delta}^\delta \phi^*(\tau)d\tau = c_1^* \tag{112}$$

Apparently there is no function $\phi^*(.)$ consistent with the above equation (except for the case $\phi^*(\tau) = 0$). Thus the function $\xi^*(\tau)$ cannot have concurrently both properties (106) and (107). At the same time the assumption of weak stationarity (eq. (13) -(14)) along with (64) implies that

$$E[\xi(t_1, D)\xi(t_2, D)] = D^{2H}\phi(|t_1 - t_2|/D) = E[\xi^*(t_1/D)\xi^*(t_2/D)\bar{i}(D)^2] \tag{113}$$

and, if $\xi^*(t/D)$ and $\bar{i}(D)$ are hypothesized independent, then

$$E[\xi^*(t_1/D)\xi^*(t_2/D)] = \phi(|t_1 - t_2|/D)/(c_2 + c_1^2) \tag{114}$$

which is equivalent to (107) with $\phi^*(u) = \phi(u)/(c_2 + c_1^2)$. We conclude that either $\xi^*(t/D)$ and $\bar{i}(D)$ should not be hypothesized independent (but only orthogonal) or the covariance function $C_\xi(t_1, t_2; D)$ should not be considered as a function of $(|t1 - t2|/D)$. If one wants to keep the complete independence assumption he/she has to adopt a complicated covariance function which adds considerable complexity to the model.

| | Min | Max | Mean | Std |
|---|---|---|---|---|
| Duration (h) | 2 | 45 | 11.8 | 8.9 |
| Interarrival time (h) | 10 | 470 | 101.3 | 106.2 |
| Total depth (mm) | 0.3 | 38.9 | 7.5 | 7.7 |
| Mean intensity (mm/h) | 0.1 | 2.55 | 0.69 | 0.48 |
| Hourly depth (mm) | 0.0 | 8.2 | 0.64 | 0.93 |

Table 1: General characteristics of the 89 storms used in the analysis.

| class | $D_{min}$ | $D_{max}$ | $\overline{D}$ | $\sigma_D$ | No. of events $N_1$ | Total no. of hourly depths $N_2$ |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 2.2 | 0.4 | 14 | 31 |
| 2 | 4 | 7 | 5.4 | 1.2 | 20 | 108 |
| 3 | 8 | 11 | 9.7 | 1.1 | 19 | 184 |
| 4 | 12 | 18 | 14.2 | 1.9 | 19 | 269 |
| 5 | 19 | 45 | 27.1 | 6.2 | 17 | 461 |
| A=(2+3) | 4 | 11 | 7.4 | 2.4 | 39 | 292 |
| B=(4+5) | 12 | 45 | 20.3 | 7.9 | 36 | 730 |
| Total | 2 | 45 | 11.8 | 8.9 | 89 | 1053 |

Table 2: Classification of storms according to their duration. The storms in each class were used to estimate the ensemble statistics of that class.

Figure 1: Definition of terms.

Figure 2: Schematic for explanation of scaling.

Figure 3: Scaling model: empirical and theoretical means (squares and solid line, respectively) and standard deviations (triangles and dotted line, respectively) of total storm depths as a function of storm duration (log-log plot).

Figure 4: Scaling model: empirical (squares) and theoretical (solid line) coefficient of variation of total storm depths as a function of duration.

Figure 5: Scaling model: empirical (squares) and theoretical (solid line) first autocorrelation coefficient of hourly rainfall depths as a function of duration. Dotted lines represent the 90% approximate confidence limits.

Figure 6: Scaling model: empirical and theoretical mean (squares and solid line, respectively) and standard deviation (triangles and dotted line, respectively) of hourly rainfall depths as a function of duration.

Figure 7: Scaling model: empirical and theoretical autocorrelation function of hourly rainfall depths as a function of duration. Squares and thin line represent small durations ($4h \leq D \leq 11h$ and $D = 7h$, respectively), while triangles and thick lines represent large durations ($12h \leq D \leq 45h$ and $D = 20h$, respectively).

Figure 8: Stationary models: empirical and theoretical means (squares and dashed line, respectively) and standard deviations (triangles and dotted line for model 1, solid line for model 2, respectively) of total storm depths as a function of storm duration (log-log plot).

Figure 9: Stationary models: empirical (squares) and theoretical (dotted line for model 1, solid line for model 2) coefficient of variation of total storm depths as a function of duration.

Figure 10: Stationary models: empirical (squares) and theoretical (solid line) first autocorrelation coefficient of hourly rainfall depths as a function of duration. Dotted lines represent the 90% approximate confidence limits.

Figure 11: Stationary models: empirical and theoretical mean (squares and dashed line) and standard deviation (triangles and dotted line for model 1, solid line for model 2, respectively) of hourly rainfall depths as a function of duration.

Figure 12: Stationary models: empirical and theoretical autocorrelation function of hourly rainfall depths as a function of duration. Squares and triangles represent empirical values for small ($4h \leq D \leq 11h$) and large durations ($12h \leq D \leq 45h$) respectively, while thick and thin lines represent models 1 and 2, respectively (same for all durations).

Figure 13: Theoretical (solid line) and simulated (triangles) distribution function of the incremental depth $X_1(10, 20)$ (the tenth hourly depth of a storm with duration 20 hrs). The simulated distribution is obtained (a) by the sequential model and (b) by the disaggregation model.

Figure 14: Theoretical (solid lines) and simulated (points) correlation structure of the incremental (hourly) depths for a storm of duration 20 hrs. The simulated structure is obtained (a) by the sequential model and (b) by the disaggregation model.

Figure 15: Comparison of historical and synthetic normalized mass curves of second quartile storms at Chalara station, Greece, for 10%, 50% (median) and 90% probability levels. Synthetic curves (thick solid lines) are obtained from a simulated sample by using the disaggregation procedure. The two groups of historical curves correspond to the records of April (circles) and all months of the year (squares) respectively.

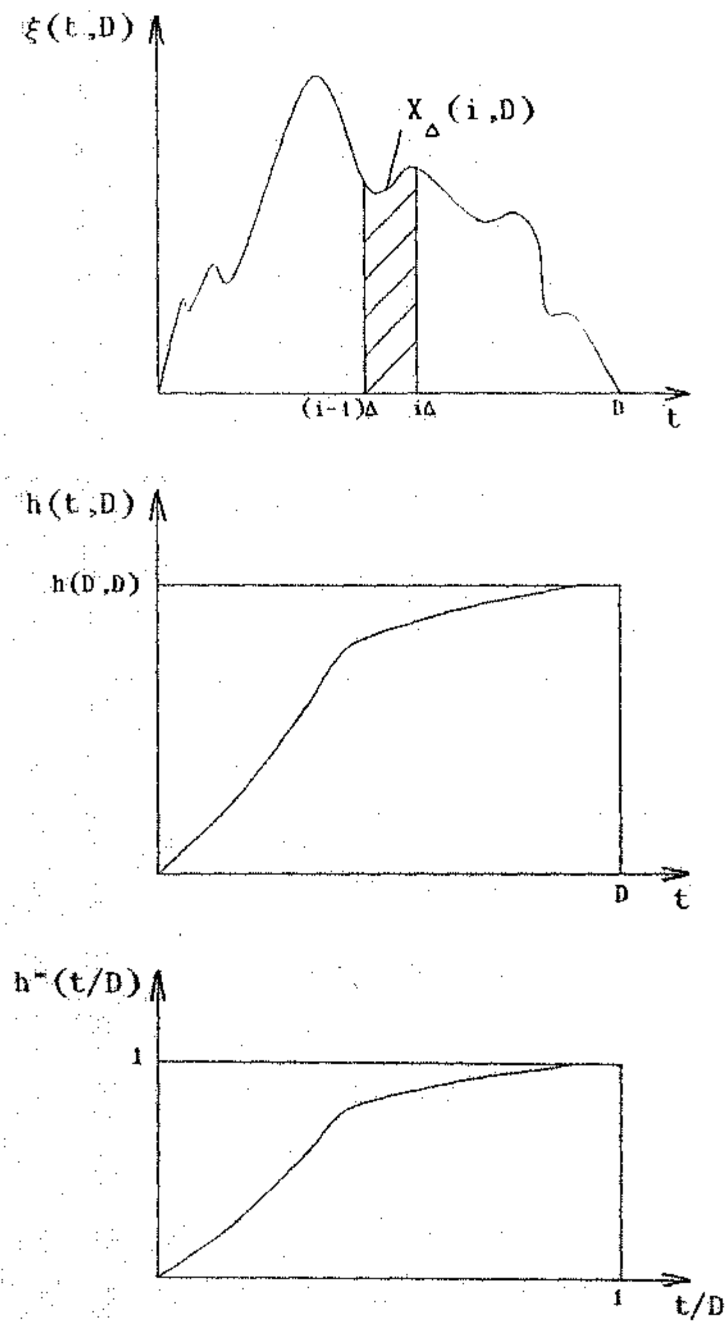


FIG. 1.

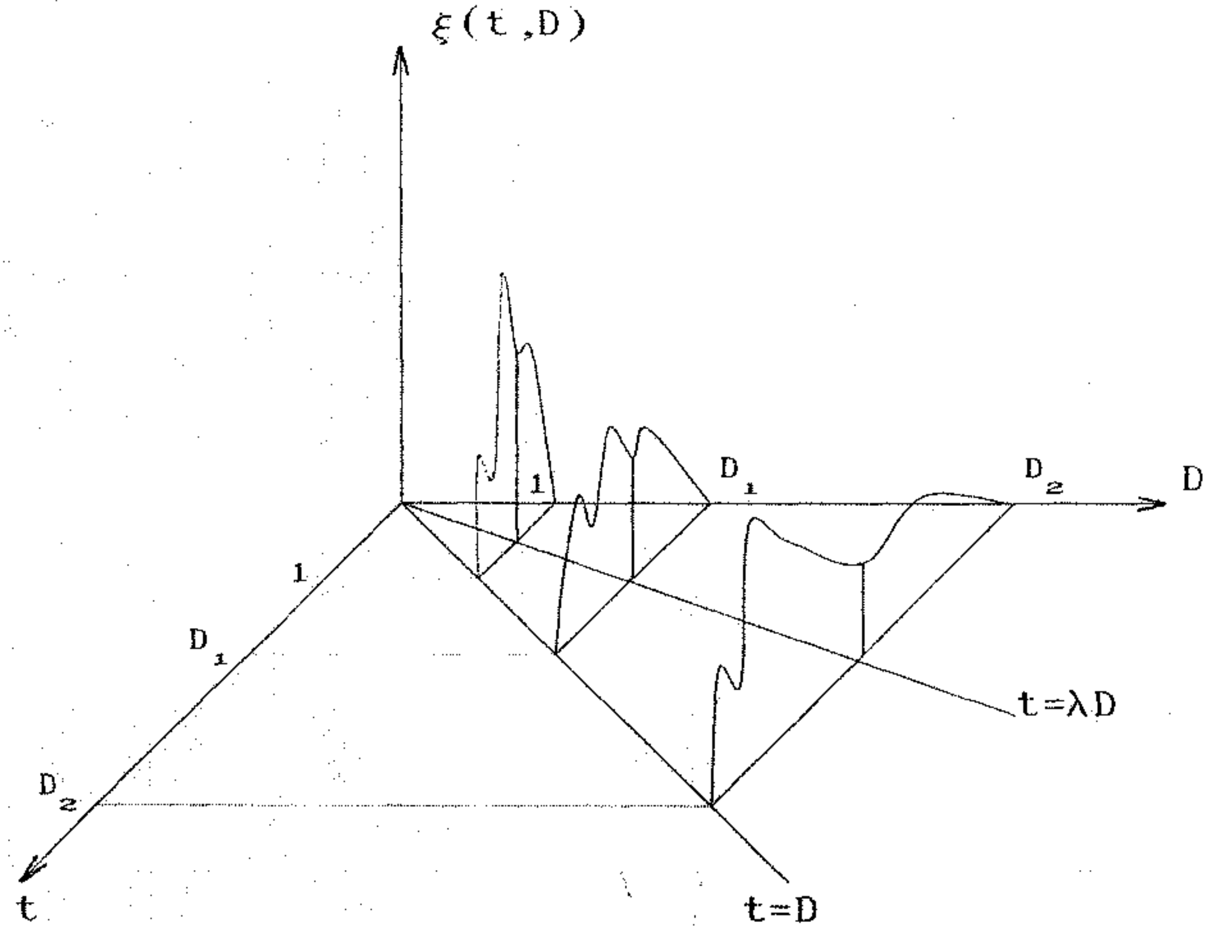


FIG. 2

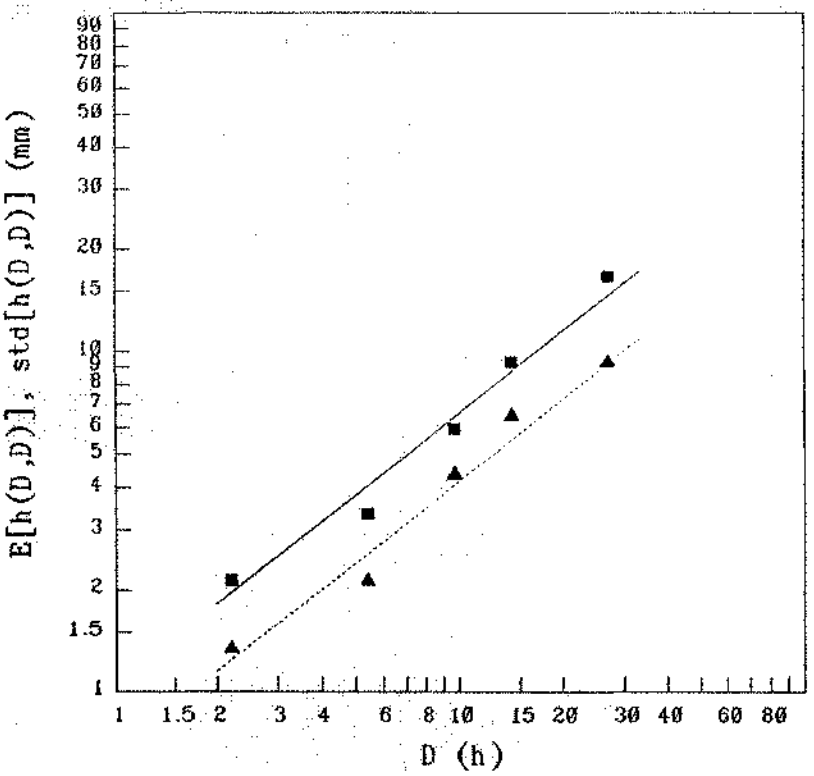


FIG. 3

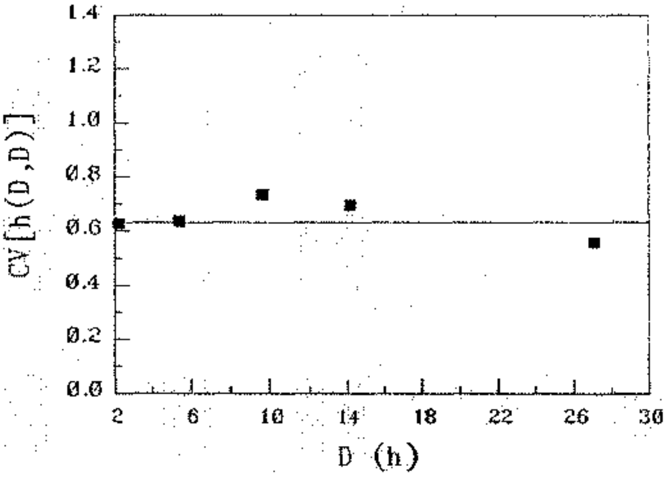


FIG. 4

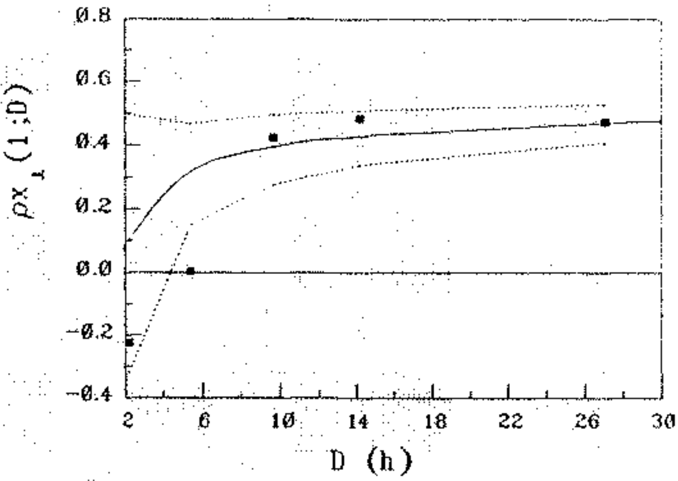


FIG. 5

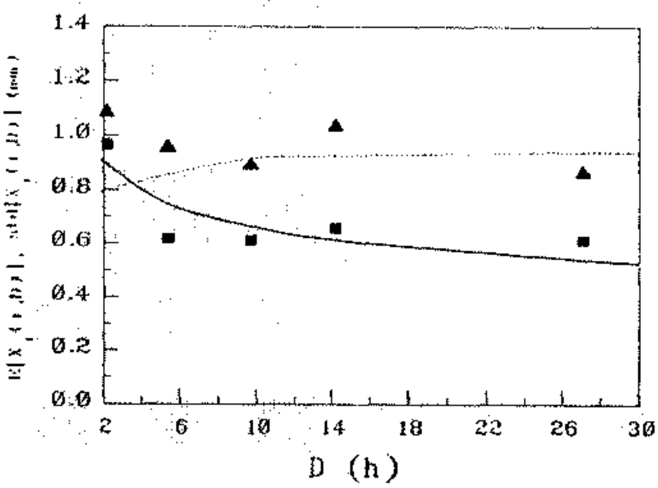


FIG. 6

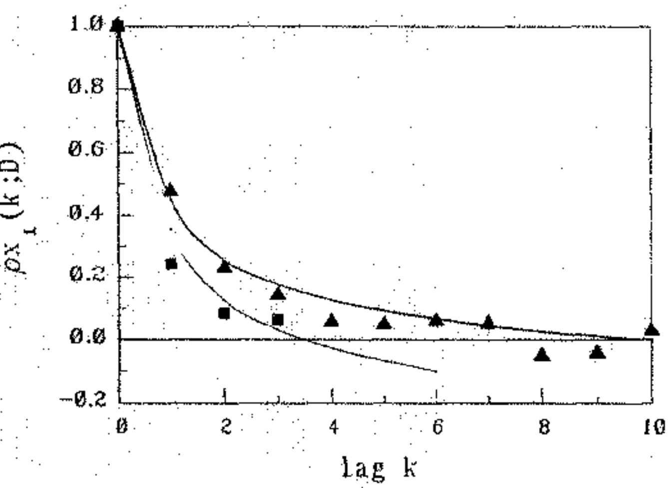


FIG. 7

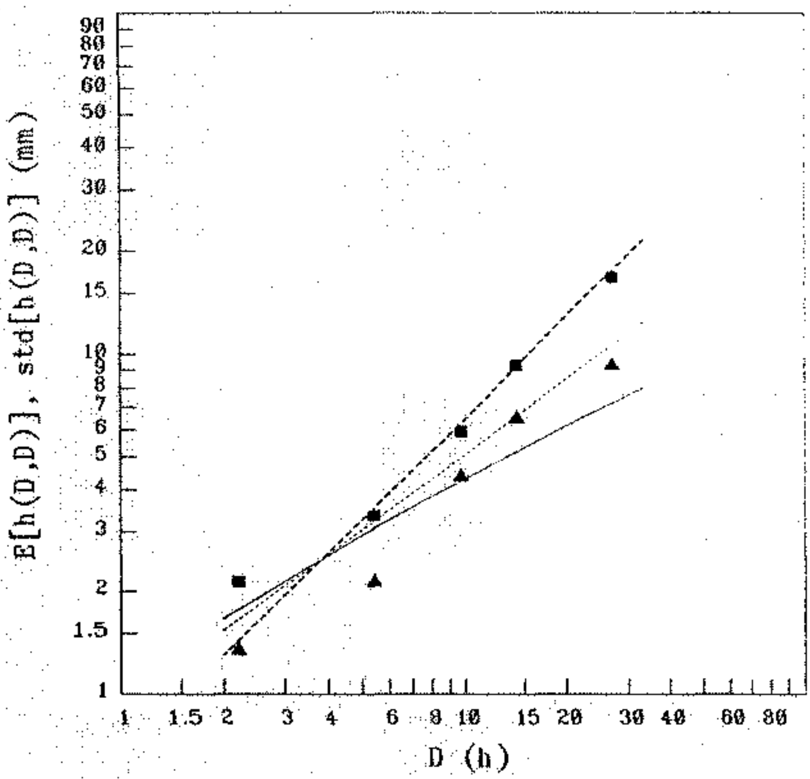
E[h(D,D)], std[h(D,D)] (mm)
D (h)
90
80
70
60
50
40
30
20
15
10
9
8
7
6
5
4
3
2
1.5
1
1
1.5
2
3
4
6
8
10
15
20
30
40
60
80

FIG. 8