A probabilistic approach to the concept of Probable Maximum Precipitation

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Abstract

The concept of Probable Maximum Precipitation (PMP) is based on the assumptions that: (a) there exists an upper physical limit of the precipitation depth over a given area at a particular geographical location at a certain time of year, and (b) that this limit can be estimated based on deterministic considerations. The most representative and widespread estimation method of PMP is the so-called moisture maximization method. This method maximizes observed storms assuming that the atmospheric moisture would hypothetically rise up to a high value that is regarded as an upper limit and is estimated from historical records of dew points. In this study, it is argued that fundamental aspects of the method may be flawed or illogical. Furthermore, historical time series of dew points and "constructed" time series of maximized precipitation depths (according to the moisture maximization method) are analyzed. The analyses do not provide any evidence of an upper bound either in atmospheric moisture or maximized precipitation depth. Therefore, it is argued that a probabilistic approach is more consistent to natural behaviour and provides better grounds for estimating extreme precipitation values for design purposes.

Method overview

In this study, the most representative and widespread estimation method of PMP, the so-called *moisture maximization method* is applied and examined. The method is based on the following formula:

$$h_m = \frac{W_m}{W} h$$

where $\boldsymbol{h_m}$ is the maximized rainfall depth; $\boldsymbol{h}$ is the observed precipitation; $\boldsymbol{W}$ is the precipitable water in the atmosphere during the day of rain, estimated by the corresponding daily dew point $\boldsymbol{T_d}$; and $\boldsymbol{W_m}$ is the maximized precipitable water, estimated by the maximum daily dew point $\boldsymbol{T_{d,m}}$ of the corresponding month. The term $\boldsymbol{T_{d,m}}$ is estimated either as the maximum historical value from a sample of at least 50 years length, or as the value corresponding to a 100-years return period, for samples smaller than 50 years.

Statistical analysis of daily dew points

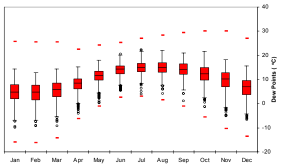

Figure 1: Box plot of daily dew points of NOA station.

Daily dew point time series, taken from four stations in Netherlands and one in Greece (the National Observatory of Athens, NOA), were analyzed. A common remark for all stations is that the range of dew points during the summer months is much shorter, compared to the winter ones (Figure 1).

The values of L-skewness of daily dew points were, in general, negative. Moreover, during the summer months they were higher than in winter months (Figure 2).

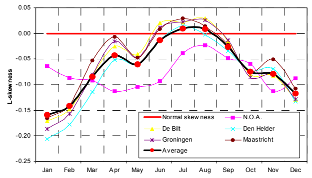

Figure 2: L-skewness of daily dew points.

Through the frequency analysis of the samples, the three-parameter Weibull distribution was considered as the most appropriate theoretical model for the empirical distribution of the daily dew points (Figure 3).

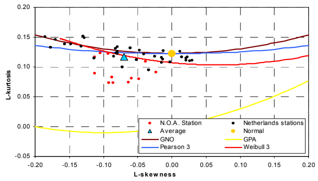

Figure 3: L-moment ratio diagram of daily dew points.

From the statistical theory of extremes to practice

The Gumbel distribution, which is the most common probabilistic model for hydrological extremes, proved inadequate for describing the empirical distribution of the monthly maximum daily dew points. Therefore, we attempted to apply the fundamentals of the theory of extremes in a direct manner. According to it, given a number of *n* independent identically distributed random variables, the largest of them (more precisely, the largest order statistic), i.e. $X = \max(Y_1, \dots, Y_n)$ has probability distribution function $H_n(x) = [F(x)]^n$, where $F(x) = P(Y_i \le y)$ is the common probability distribution function, known as the *parent distribution* of Y_i. The frequency analysis for the daily dew points indicated that the three-parameter Weibull model is a sufficient probabilistic model for describing the empirical distribution of them; hence $F(x)$ can be used as the parent distribution. Consequently, the theoretical maximum distribution of the monthly maximum daily dew point is described by $H_n(x)$, where *n* stands for the days of each month.

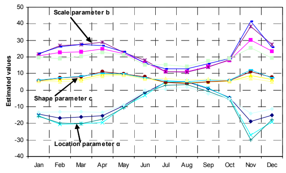

Figure 4: Estimated parameters of the three parameter Weibull for NOA station.

The thee parameter Weibull distribution was fitted to the samples of daily dew points, using four different estimation methods. The uncertainty of the estimated parameter is obvious (Figure 5).

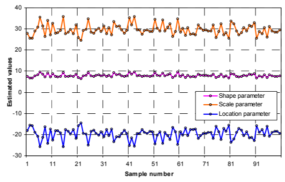

Figure 6: Monthly average autocorrelation coefficient of daily dew points.

The condition of independence of random variables is not valid, as proved by the high values of autocorrelation coefficients (Figure 6).

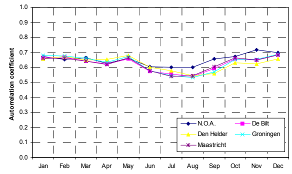

Figure 7: Estimated values for parameter *n*.

As the condition of independence of random variables is not valid, the parameter *n* was inspected to be and proved, indeed, lower than the theoretically expected value (Figure 7).

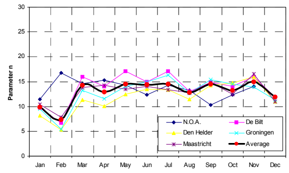

Figure 5: Parameters of the Weibull distribution, estimated by the maximum likelihood method.

100 synthetic samples of 3000 values, with a priori statistical structure, were generated (Figure 7); next, their parameters were re-evaluated using various methods. The Monte Carlo simulation approach proved the uncertainty related to the estimation of parameters of the Weibull distribution.

The L-moment ratio diagram (Figure 8) illustrates that the theoretical maximum distribution derived from the parent three-parameter Weibull distribution is more appropriate than the classic ones.

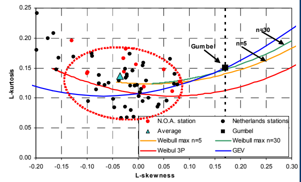

Figure 8: L-moment ratio diagram of maximum daily dew points for each month.

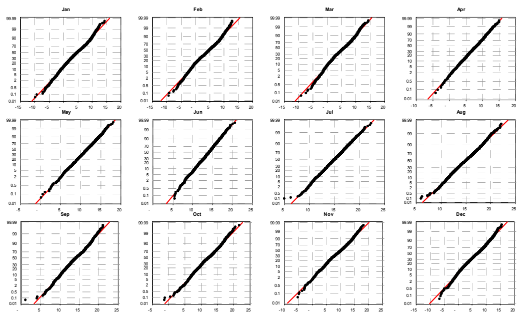

Figure 9: Probability plots of daily dew points of NOA station on three parameter Weibull paper.

Model fitting to the NOA sample: A parallel optimisation approach

Given the uncertainties related to the estimation of the parameters of the parent distribution, as well as the deviation of parameter *n* from its theoretical value, we implemented a "parallel" optimization approach, by simultaneously fitting the theoretical models $F(x)$ and $H_n(x)$ to the empirical distributions of daily (Figure 9) and monthly maximum daily (Figure 10) dew points. The objective function is written as:

$$\mathrm{LSE}_{tot} = \mathrm{LSE}[F(x)] + [\mathrm{LSE}[H_n(x)]]^2$$

where LSE is the least square error between the theoretical and the empirical data.

This strategy helped to better fit the theoretical maximum distribution derived by the parent distribution.

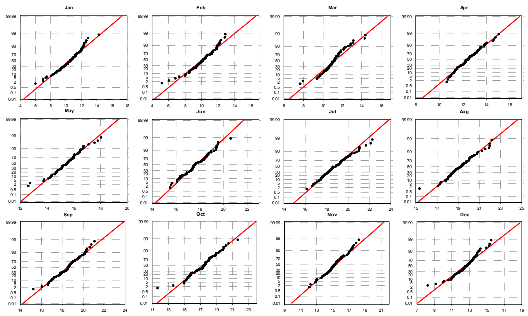

Figure 10: Probability plots of monthly maximum daily dew points of NOA station on theoretical maximum distribution paper.

Application of the PMP estimation method

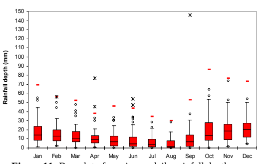

Figure 11: Box plot of maximum daily rainfall depths per month.

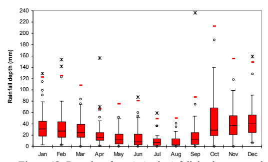

Figure 12: Box plot of maximized rainfall depths per month.

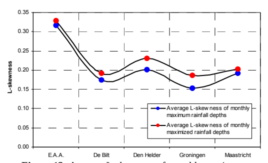

Figure 13: Average L-skewness of monthly maximum and maximized daily rainfall depths.

The estimation method of the PMP was applied to the five stations in Netherlands and Greece. The maximized precipitation time series were analysed in comparison with the observed ones. It was concluded that the maximization process causes, sometimes, a disproportional increase to the range of the values of observed rainfall and that the maximized samples present a higher skewness than the observed ones, especially when the sample L-skewness values are low (Figures 11-13).

Figure 14 illustrates the values of maximized rainfall depths of the station of National Observatory of Athens in descending order, the respective daily recorded rainfall depths, the respective daily precipitable water and the maximum monthly precipitable water. The 120 maximized rainfall depths that are illustrated in Figure 14 are the overall of the 10 maximum rainfalls of each month. It is evident that the estimated PMP point is located in a very uncertain area of the curve, where the slope is very high.

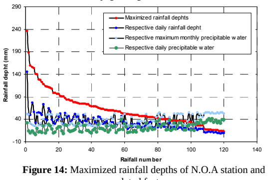

Figure 14: Maximized rainfall depths of N.O.A station and related factors.

The maximized sample of rainfall depths was examined in respect with the factors that may affect them. The maximized rainfalls seem to be directly related to the observed rainfalls, while there is some doubt about their correlation with the maximum precipitable water and no evidence of any correlation with the daily precipitable water (Figure 15).

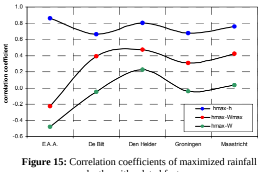

Figure 15: Correlation coefficients of maximized rainfall depths with related factors.

The sample of the 120 maximized rainfall depths was probabilistically approached, and in comparison with the respective maximum monthly and annual rainfall samples. The above data regarding the National Observatory of Athens are illustrated in Figure 16. Probably, if a longer rainfall record was available, the estimation of the PMP would be higher. Furthermore, if the distribution of maximized rainfalls is the only component taken under consideration, then the estimation of PMP comes out as an uncertain estimation of the first order statistic.

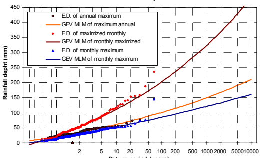

Figure 16: Probability diagram of annual maximum, monthly maximum and monthly maximized daily rainfall depths.

The PMP estimation method was applied in regards to the monthly maximum daily dew point, for a wide range of return periods. The assumed model of the monthly maximum daily dew point was the one derived by the three-parameter Weibull distribution. The estimation of the PMP, as illustrated in Figure 17, is an ascent function of the monthly maximum daily dew point.

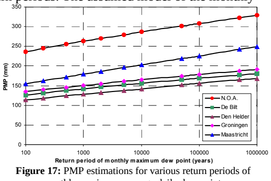

Figure 17: PMP estimations for various return periods of monthly maximum average daily dew point.

A probabilistic approach for the annual maximum daily rainfall

As concluded by the L-moment ratio diagram (Figure 18), the General Extreme Value distribution (GEV) describes appropriately the empirical distribution of the annual maximum daily values of rainfall. The GEV model was fitted on the sample with three different methods.

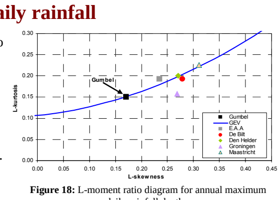

Figure 18: L-moment ratio diagram for annual maximum daily rainfall depths.

The estimation of the PMP, according to the fitted model, was associated to a return period or, equivalently, a probability of exceedance. It was concluded that this probability is not negligible. The analysis was also conducted using the Gumbel model, which obviously underestimates the exceedance probability of the PMP (Figure 19).

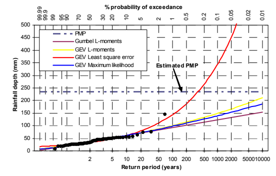

Figure 19: Probability plot of annual maximum daily rainfall values of NOA station on Gumbel paper.

Conclusions

1. There is no evidence for a physical upper bound regarding the dew points.
2. The estimation of the PMP, based on the moisture maximization concept, is considerably uncertain and was proved too sensitive against the available data.
3. The study proved that the existence of an upper limit on precipitation, as implied by the PMP concept, is statistically inconsistent. Moreover, such a limit cannot be specified in a deterministic way, as the method implies; in reality, from a statistical point-of-view, this "limit" tends to infinite.
4. According to the probabilistic analysis on the annual daily maximum rainfall depths, the hypothetical upper limit of the PMP method corresponds to a small, although realistic, exceedance probability. For example, this probability for the NOA sample is 0.27%, a value that would not be acceptable for the design of a major hydraulic structure, such as a dam.
5. A probabilistic approach, based on the GEV model, seems to be the most consistent tool for studying hydrological extremes.