A probabilistic approach to the concept of Probable Maximum Precipitation
7th Plinius Conference on Mediterranean Storms
European Geosciences Union (EGU), Rethymnon, Greece, 5-7 October 2005
Topic 3: Meteorological, hydrological and geological risks, disaster management and mitigation strategies
S.M. Papalexiou and D. Koutsoyiannis
Department of Water Resources, School of Civil Engineering, National Technical University of Athens

Abstract
The concept of Probable Maximum Precipitation (PMP) is based on the assumption that (a) there exists an upper physical limit of the precipitation depth over a given area at a particular geographical location at a certain time of year, and (b) that this limit can be estimated based on deterministic considerations. The most representative and widespread estimation method of PMP is the so-called moisture maximization method. This method maximizes observed storms assuming that the atmospheric moisture would hypothetically rise up to a high value that is regarded as an upper limit and is estimated from historical records of dew points. In this study, it is argued that probabilistic approaches are more consistent with natural behavior and provide better grounds for estimating extreme precipitation values for design purposes.

Method overview
In this study, the most representative and widespread estimation method of PMP, the so-called moisture maximization method is applied and examined. The method is based on the following formula

\[ h_{\text{max}} = h_{\text{a}} + W \]

where \( h_{\text{a}} \) is the maximized rainfall depth and \( W \) is the precipitable water in the atmosphere during the day of precipitation, estimated by the maximum daily dew point \( T_{\text{D}} \) of the corresponding month.

The term \( T_{\text{D}} \) is estimated either as the maximum historical value from a sample of at least 50 years length, or as the value corresponding to a 100-years return period, for samples smaller than 50 years.

From the statistical theory of extremes to practice
The Gamma distribution, which is the most common probabilistic model for hydrological extremes, proved inadequate for describing the empirical distribution of the monthly maximum daily rain depths. Therefore, we attempted to apply the fundamental of the theory of extremes in a direct manner. According to (5), given a number of independent identically distributed random variables, the largest of them (more precisely, the largest order statistic), i.e. \( X_{(n:n)} = \max \{X_1, \ldots, X_n\} \), has probability distribution function

\[ F_{\text{X}(n:n)}(x) = \left( \frac{x}{b} \right)^m \prod_{i=1}^n \left( 1 - \frac{x_i}{b} \right) \]

where \( x \) is a common domain of the parent distribution, as well as the simultaneous fitting the theoretical models to the samples of daily dew points, using four parameters of the parent distribution, as well as the uncertainty related to the estimation of parameters of the Weibull distribution.

The condition of independence of random variables is not valid, as proved through the high values of autocorrelation coefficients (Figure 8). The L-moment ratio diagram of the NOA sample is illustrated in Figure 17, is an expression of the theoretical maximum distribution derived from the parent three-parameter Weibull distribution is more appropriate than the classic ones.

Model fitting to the NOA sample: A parallel optimisation approach
Given the uncertainties related to the estimation of the parameters of the parent distribution, as well as to the deviation of parameter estimates to theoretical values, we implemented a ‘parallel’ optimisation approach, by simultaneously fitting the theoretical models \( F(x) \) and \( h_{\text{max}}(x) \) to the empirical distributions of daily (Figure 9) and monthly maximum daily (Figure 10) rain depths. The objective function is written as

\[ \text{LSE} = \sum_{i=1}^{n} \left[ \frac{\text{dn}(x)}{\text{dn}(x)} - \text{dn}(x) \right] \]

where \( \text{dn}(x) \) is the empirical and the empirical data.

The strategy helped to better fit the theoretical maximum distribution derived from the parent distribution.

Application of the PMP estimation method
The estimation method of the PMP was applied to the five stations in Netherlands and Greece. The maximized precipitation time series were analyzed in comparison with the observed ones. It was concluded that the maximization process causes, sometimes, an disproportionate increase in the values of observed rainfall and that the maximized samples present a higher skewness than the observed ones, especially when the sample L-skewness values are low (Figures 11-13).

The condition of independence of random variables is not valid, as proved through the high values of autocorrelation coefficients (Figure 8). The L-moment ratio diagram of the NOA sample is illustrated in Figure 17, is an expression of the theoretical maximum distribution derived from the parent three-parameter Weibull distribution is more appropriate than the classic ones.

A probabilistic approach for the maximum daily rainfall
As concluded by the L-moment ratio diagram (Figure 15), the General Extreme Value distribution (GEV) describes appropriately the empirical distribution of the annual maximum daily values of rainfall. The GEV model was also conducted on the sample with three different methods.

Conclusions
1. There is no evidence for a physical upper bound regarding the dew points.
2. The estimation of the PMP, based on the moisture maximization concept, is somewhat uncertain and was prone to be sensitive against the available data.
3. The study proved that the existence of an upper limit on precipitation, as implied by the PMP concept, is statistically inconsistent. Moreover, such a limit cannot be specified in a deterministic way, as the method implies; instead, from a statistical point-of-view, this "limit" tends to increase.
4. According to the probabilistic analysis on the annual maximum daily rainfall depths, the hypothetical upper limit of the PMP method corresponds to a small, although realistic, exceedance probability. For example, this probability for the NOA sample is 0.27%, a value that would not be acceptable for the design of a major hydraulic structure, such as a dam.
5. A probabilistic approach, based on the GEV model, seems to be the most consistent tool for studying hydrological extremes.