A new stochastic hydrological framework inspired by the Athens water resource system

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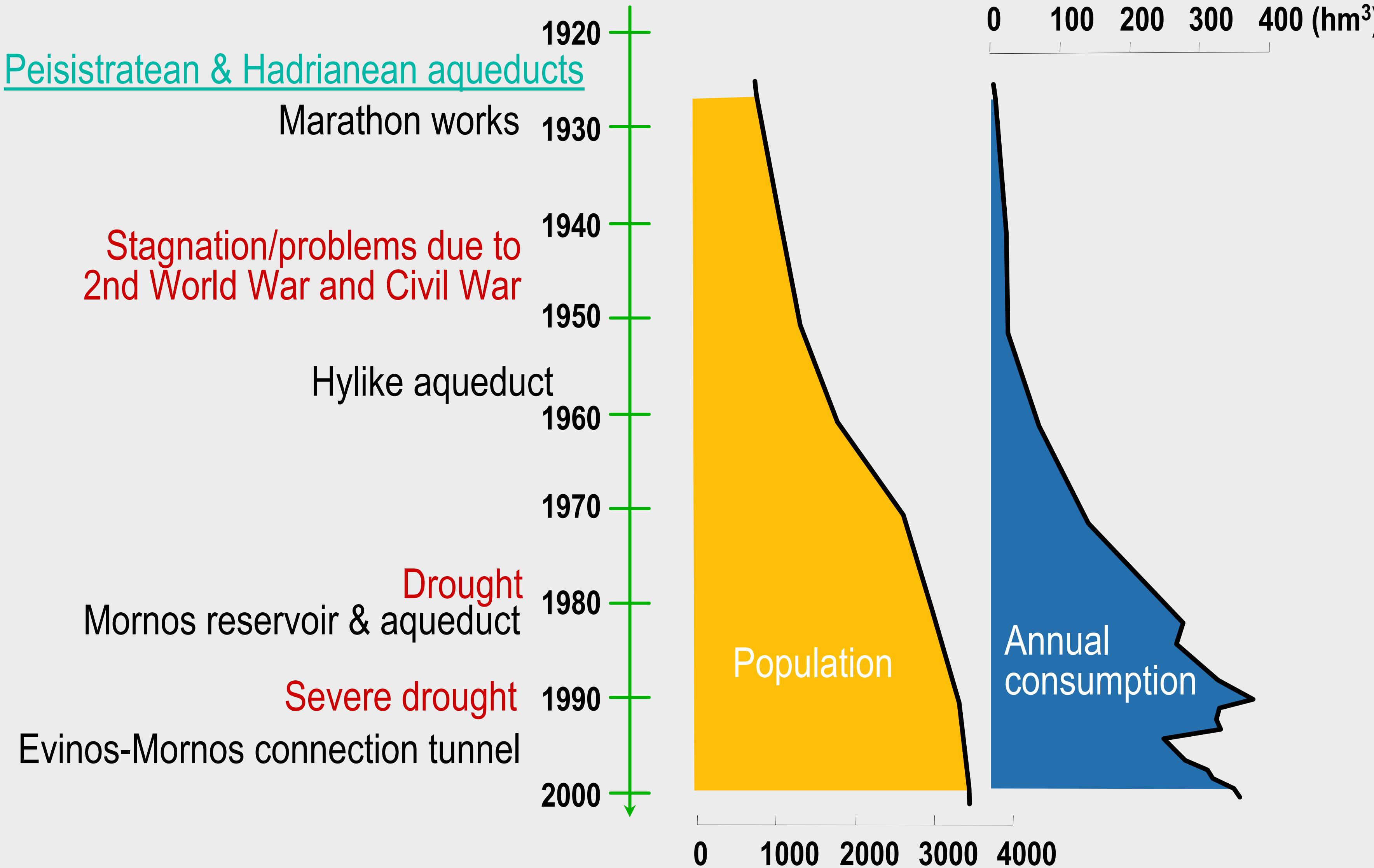
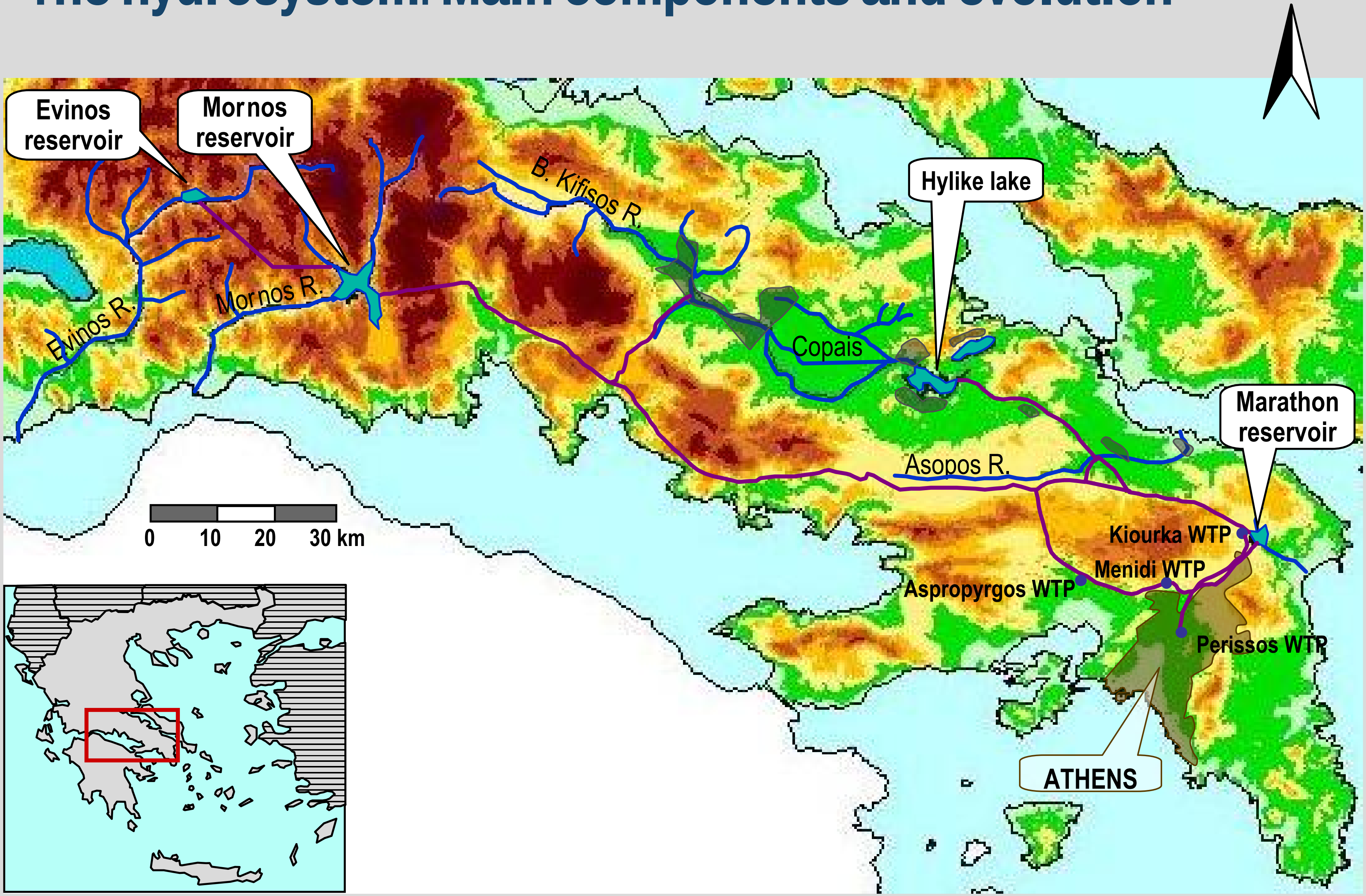
Evolution of water consumption – Milestones



The hydrosystem: Main components and evolution



Parts of the presentation

1. Diagnosis

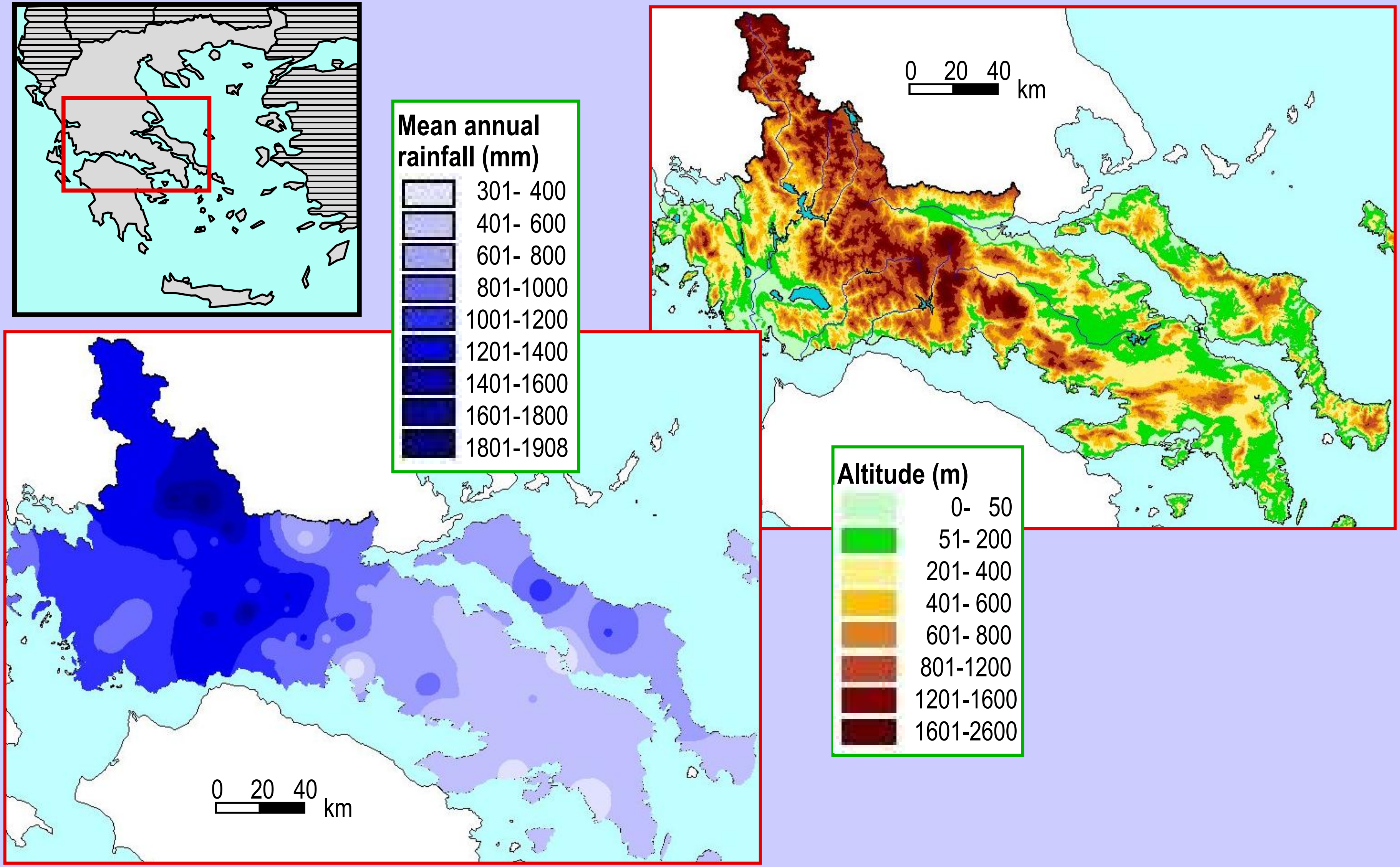
2. Explanation

3. Operational synthesis

1. Diagnosis

Ὕσον, ὕσον Ζεῦ κατὰ τῆς ἀρούρης τῶν Ἀθηναίων

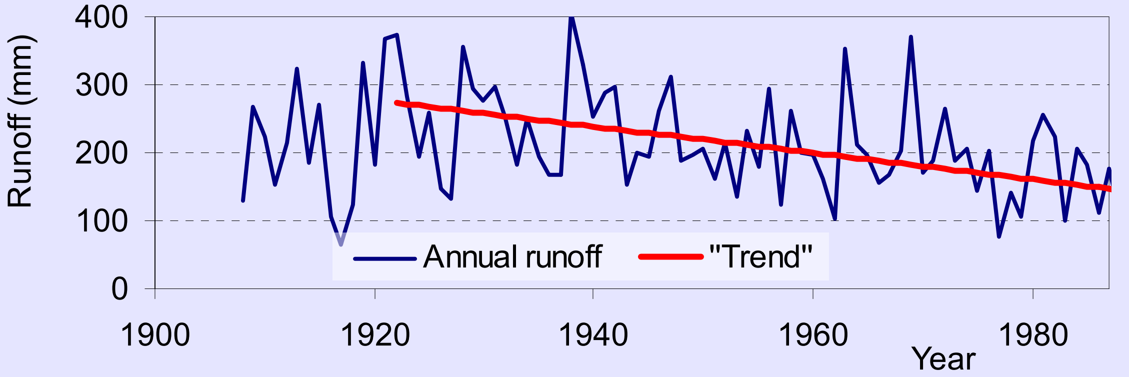
Do rain, do rain Zeus against the earth of Athenians (Ancient Greek prayer)



Back in 1990s – Initial empirical observations

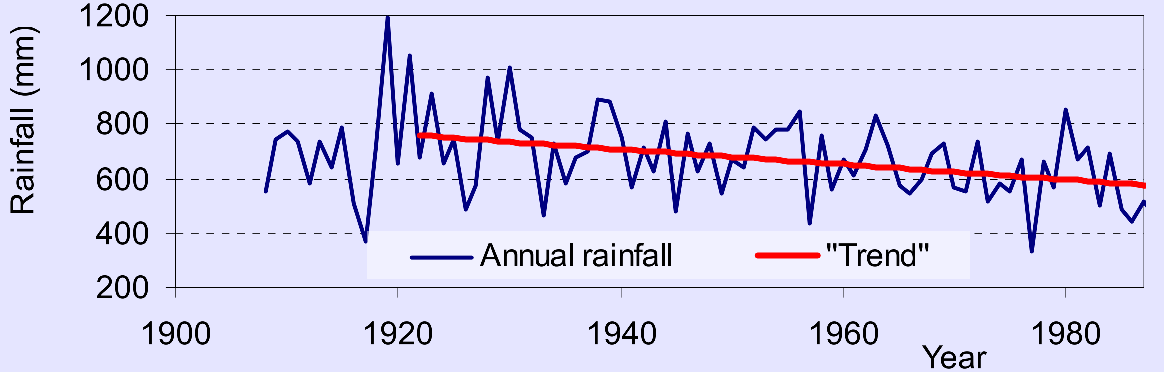
The historical time series of Boeoticos Kephisos runoff (Hydrological years 1907/08-1986/87)

A multi-year «trend» is observed



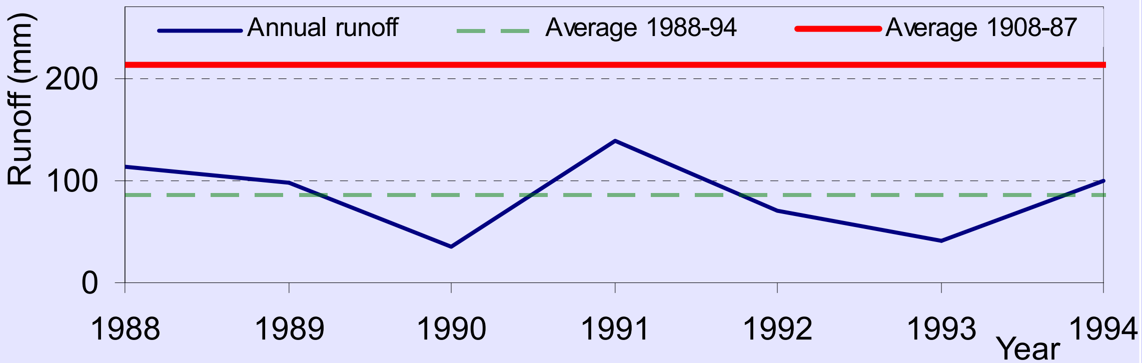
A similar «trend» in the rainfall time series

Explains the «trend» in runoff



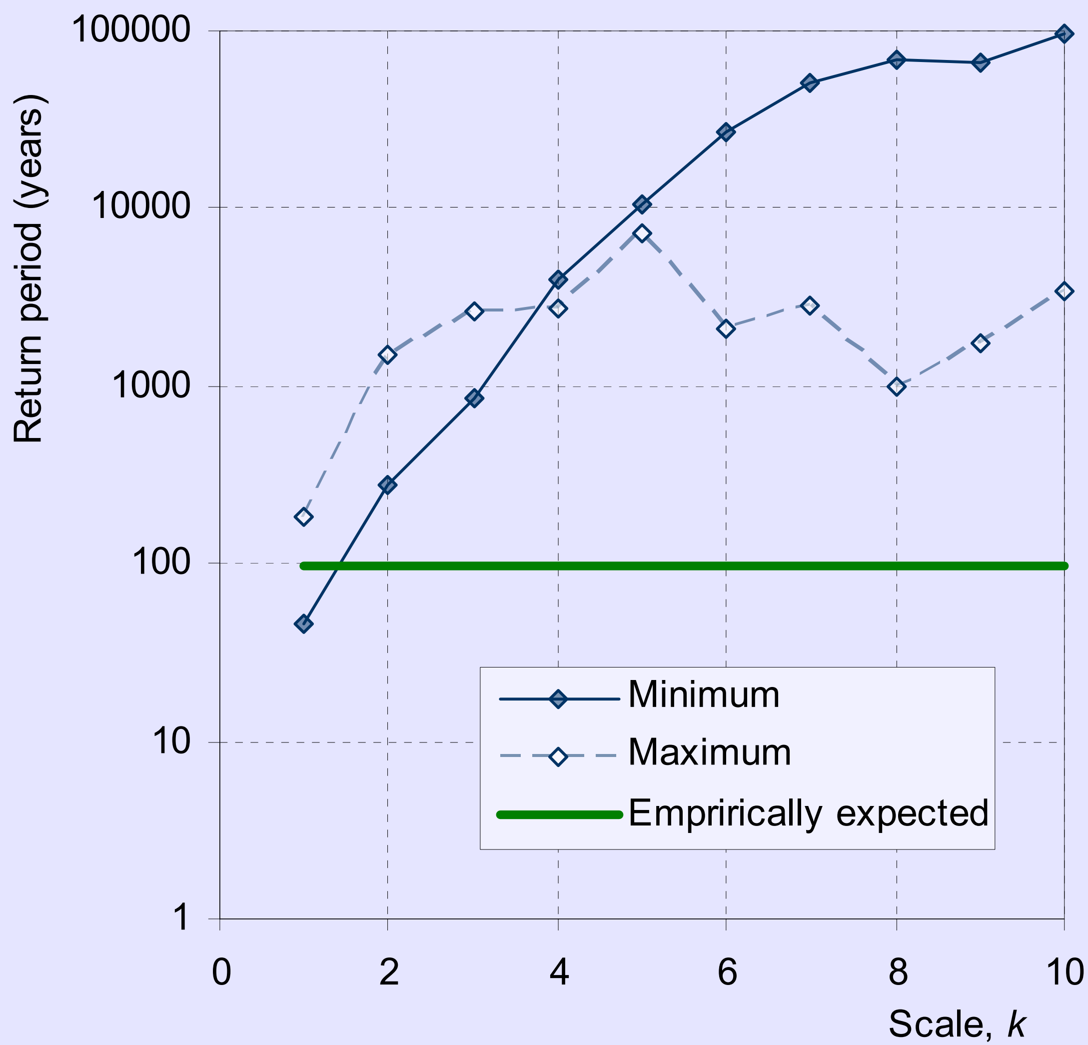
The next years were dry

Intense and persistent drought: Mean flow half that of the historical average, duration 7 years



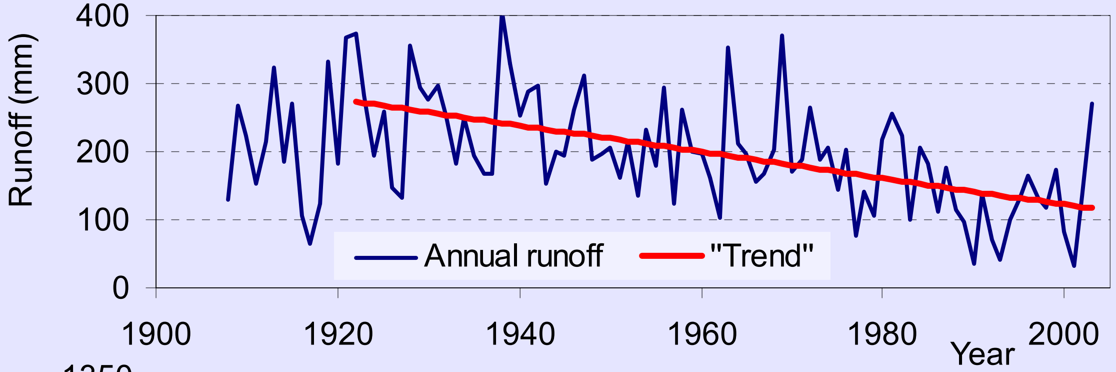
Return period of the persistent drought

- Assessment was done using classical hydrological statistics
- At the annual scale, the drought was a record minimum but with typical magnitude
- Aggregated at larger scales, it appeared something extraordinary
- Similar behaviour was observed for maxima on aggregate scales



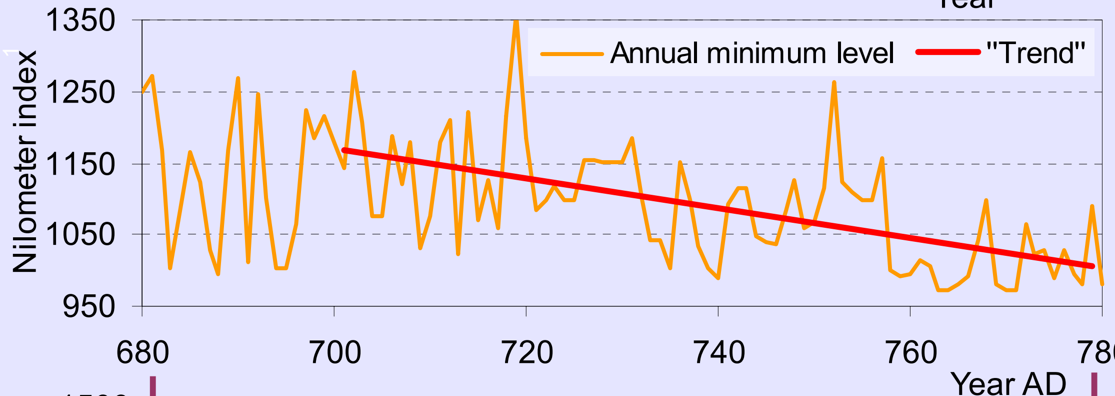
Comparisons with even longer series

The complete historical time series of Boeoticos Kephisos runoff



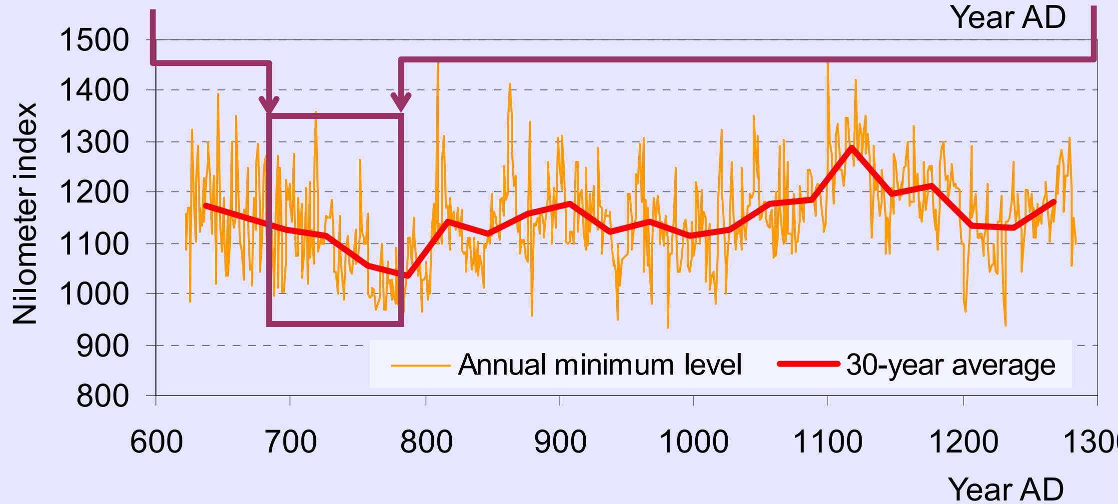
A part of the Nilometer series (the minimum annual water level in the Nile River, in cm)

A similar «trend»



The complete Nilometer series (622-1284 AD, 663 years)

Upward and downward fluctuations on all scales



The fluctuations on many scales and the "Hurst phenomenon"

- The "weird" (as compared to purely random processes) behaviour of hydrological and other geophysical processes was discovered by the English engineer E. H. Hurst (1950) in the framework of the design of the High Aswan Dam in Nile $\Rightarrow$ **Hurst phenomenon**
- The Polish-French mathematician and engineer B. Mandelbrot (1965-1971) related it to the biblical story of the seven fat and the seven thin cows $\Rightarrow$ **Joseph effect**
- The behaviour has been characterized with several other names $\Rightarrow$ **long-term persistence, long-term memory, long-range dependence, scaling behaviour**
- Most of these names, even though correct, may be misleading for the conceptualization and understanding of the natural behaviour and the causing mechanisms. Probably a better name $\Rightarrow$ **multi-scale fluctuation**
- The behaviour was verified to be omnipresent, not only in geophysical processes (hydrological, climatic), but also in biological (e.g. tree rings), technological (e.g. computer networks), social and economical (e.g. stock market)
- In water resources design and management, it has unfavorable effects (increase of uncertainty)

Easy detection and main effect of Hurst phenomenon

- Fundamental law of classical statistics $\mathrm{StD}[\overline{X}_n] = \dfrac{\sigma}{\sqrt{n}}$

  $\overline{X}_n$ = average of n variables
  σ = standard deviation of each variable
  n = aggregation scale (or sample size)

- Modified law in natural processes $\mathrm{StD}[\overline{X}_n] = \dfrac{\sigma}{n^{1-H}}$, $H > 0.5$

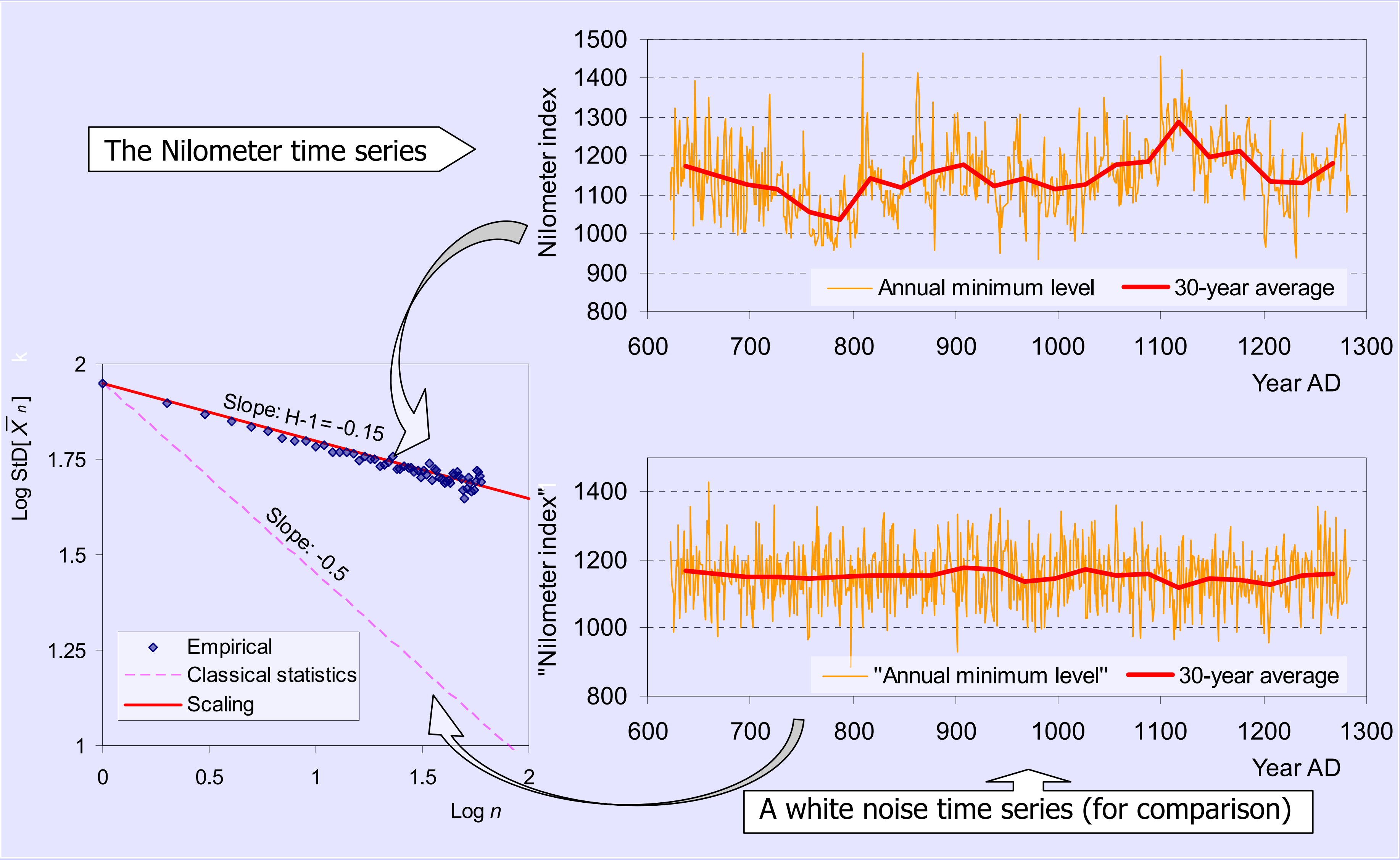
- Example

  To have $\mathrm{StD}[\overline{X}_n] / \sigma = 10\%$

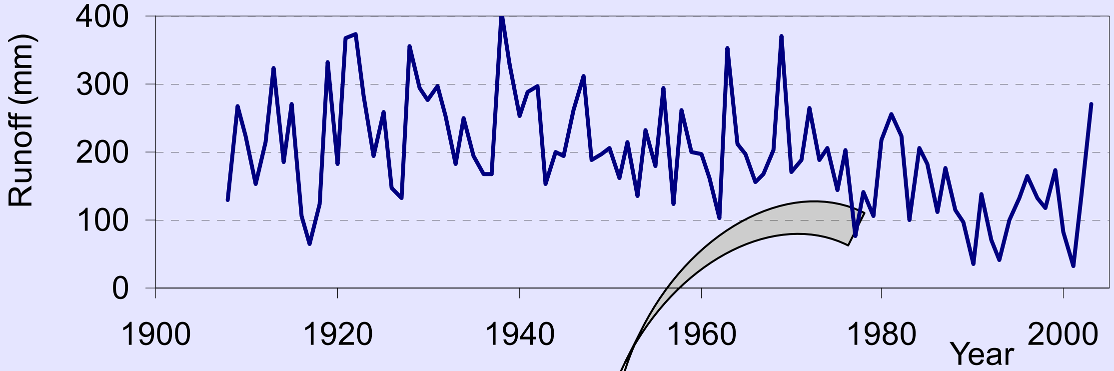
  - n = 30 in classical statistics
  - n = 5 000 for the modified law with H = 0.8

Bomb in the foundation of climatology

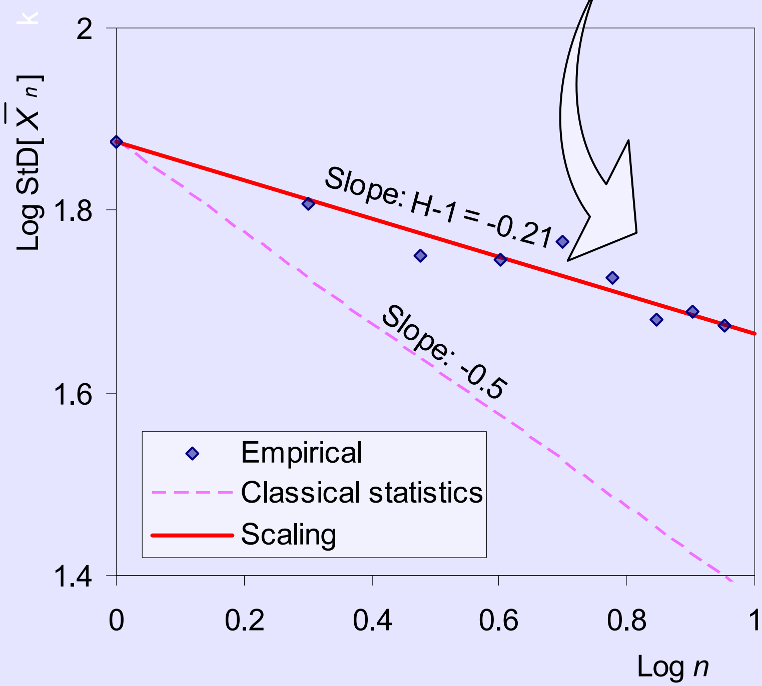
Incongruity of natural processes with typical random processes : (a) The Nilometer series



Incongruity of natural processes with typical random processes : (b) The Boeoticos Kephisos time series



The time series of the Boeoticos Kephisos runoff



Statistical characteristics of all processes			
Sample statistic	Runoff (mm)	Rainfall (mm)	Temperature (^{o}C)
n	96	96	96
m (mm)	197.6	658.4	17.0
s (mm)	87.6	158.9	0.72
C_s	0.36	0.44	0.34
r_1	0.34	0.10	0.31
H	0.79	0.64	0.72

Mathematical description of the Hurst phenomenon

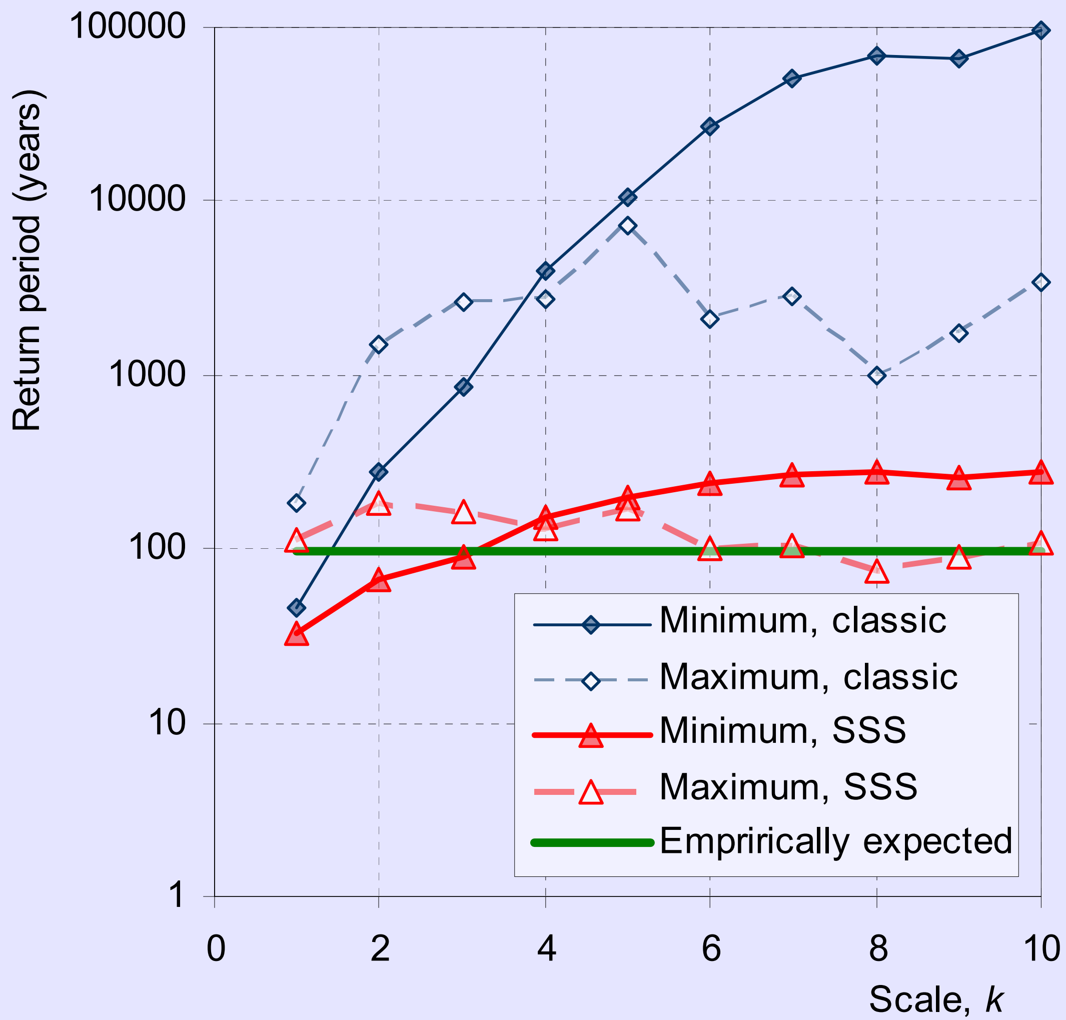
- The mathematical description of the Hurst phenomenon is done on grounds of the probability theory and particularly the theory of stochastic process
- The simple relationship

$$\mathrm{StD}[\overline{X}_n] = \frac{\sigma}{n^{1-H}}$$

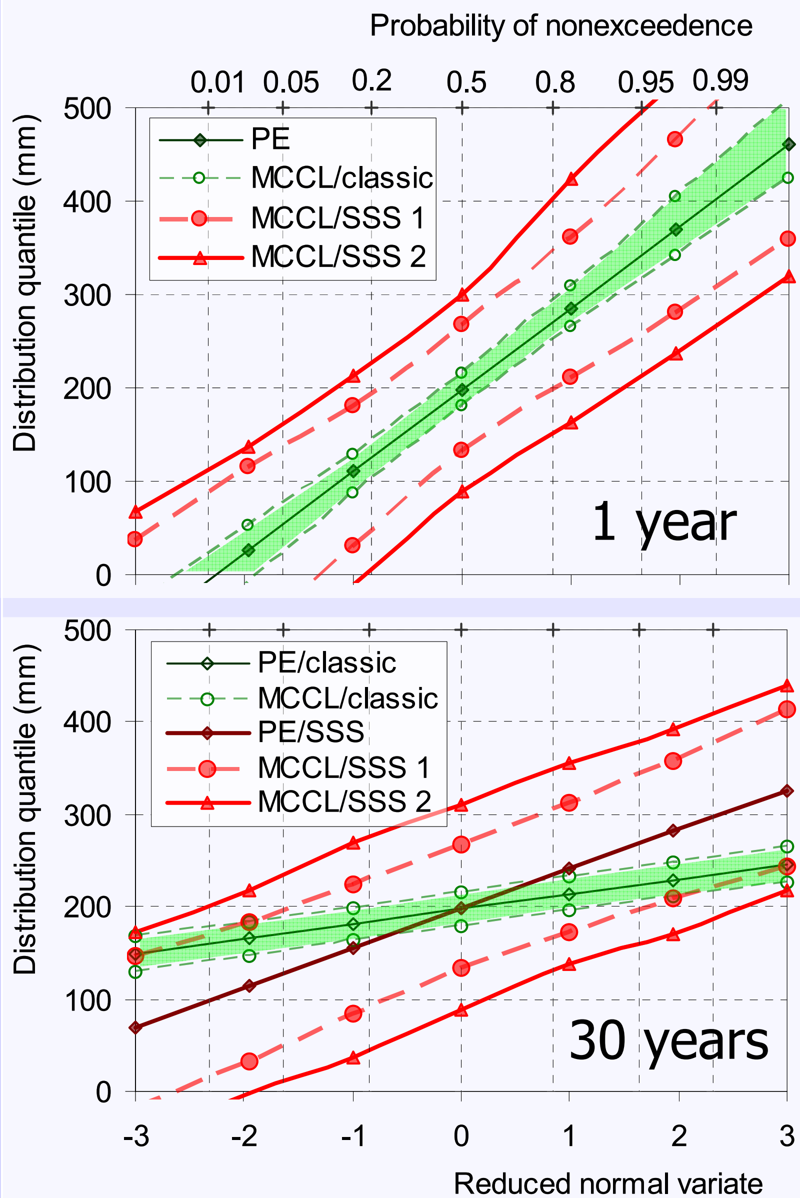
entails a definition (good for our purposes) of a model (stochastic process) reproducing the Hurst phenomenon; n is meant as a scale of aggregation (rather than sample size)
- (Hurst used a different formalism, in terms of the so called rescaled range, which is complicated and probably misleading)
- Today the stochastic process with the above property is called **S**tationary intervals of a **S**elf-**S**imilar process or a **S**imple **S**caling **S**tochastic process (abbreviated as an **SSS** process)
- The SSS process was introduced by the Russian mathematician A. Kolmogorov (1940) who called it Wiener Spiral
- A significant contribution on the SSS process is due to the American mathematician J. Lamperti (1962) who called it a Semi-Stable Process
- The link of the SSS process with the Hurst phenomenon is due to B. Mandelbrot (1965), who called it Fractional Gaussian Noise
- Other given names: Brown noise, Red noise

Back to Boeoticos Kephisos – Adoption of the SSS process

- The trend is a natural and usual behaviour
- The persistent drought is not extraordinary; it is a natural and expected behaviour



Implications on uncertainty: Boeoticos Kephisos runoff



Statistical model	Total uncertainty in runoff (due to variability and parameter estimation) % of average	
	Annual scale	30-year scale
Classical	200	50
SSS	270	200

Classical model
Climate is what you expect
Weather is what you get

SSS model
Weather is what you get … immediately
Climate is what you get
… if you keep expecting a long time

2. Explanation

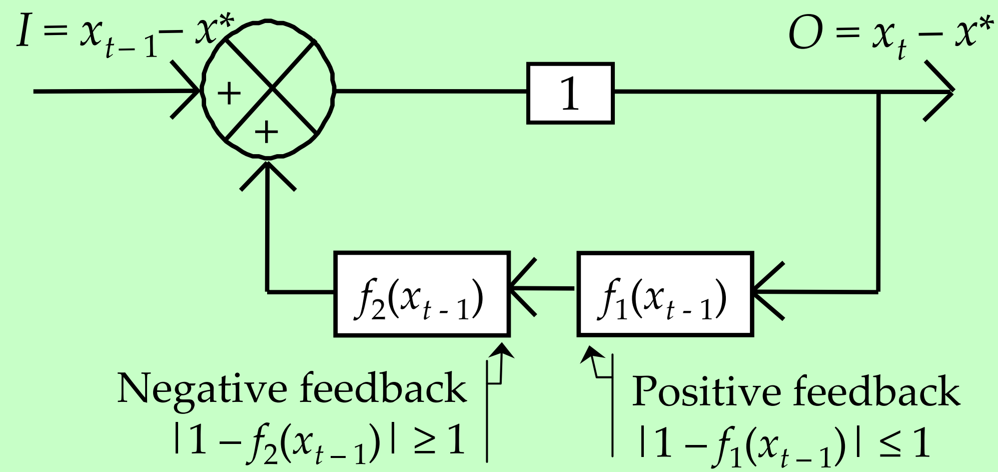
A climatic toy model: A simple system with nonlinear dynamics may produce the Hurst phenomenon

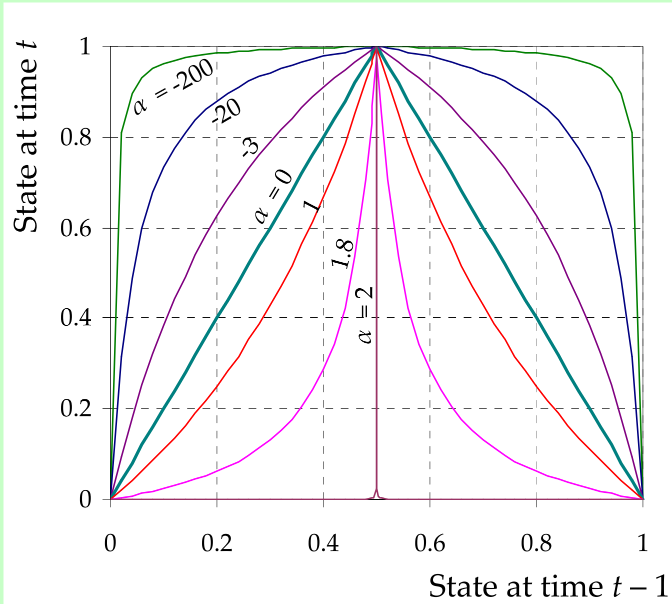
- A simplified climatic system is represented as a circuit with two feedback mechanisms, a positive (amplifying the departure from a stationary state x^*) and a negative (reducing this departure)
- The combined action of the two mechanisms could be represented by a generalized tent transform:

$$x_t = \frac{(2-\alpha)\min(x_{t-1}, 1-x_{t-1})}{1-\alpha\min(x_{t-1}, 1-x_{t-1})}$$

where $0 \le x_t \le 1$, $\alpha < 2$

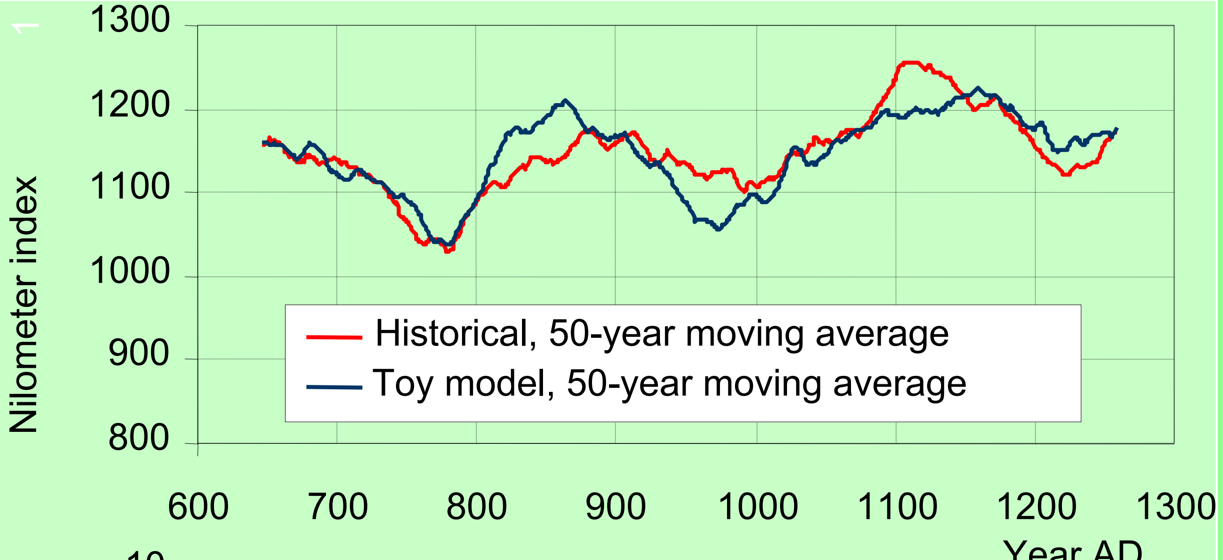
- The parameter α could be assumed to vary in time, following the same tent transform with a constant parameter β



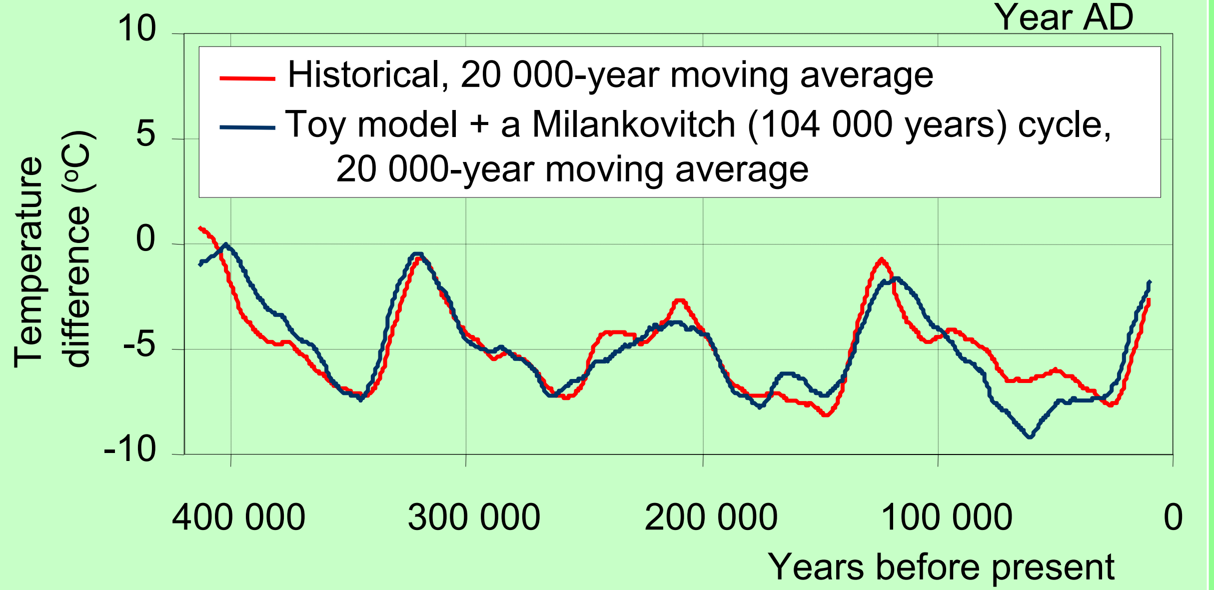


Demonstration: Toy model fitted to two long time series

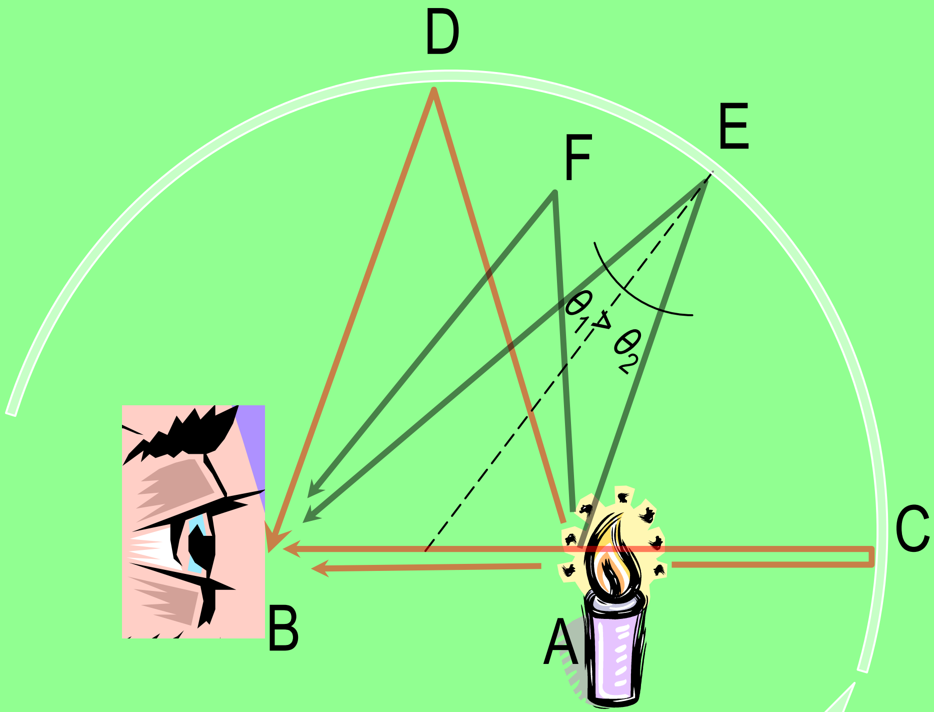
The Nilometer time series



The Vostok (Antarctica) ice core deuterium data set going back to 422 766 years before present
Reconstructed temperature difference with reference to the mean recent time value



Towards a more general explanation: Nature loves extremes ...



A semi-cylindrical mirror

Why light follows the red paths from A to B (AB, ACB, ADB) and not other (the black) ones (e.g. AEB, AFB)?

- The red paths are those that (a) reach the mirror and (b) form an angle of incidence equal to the angle of reflection

  (True for most cases; not true for AB; not general or informative)

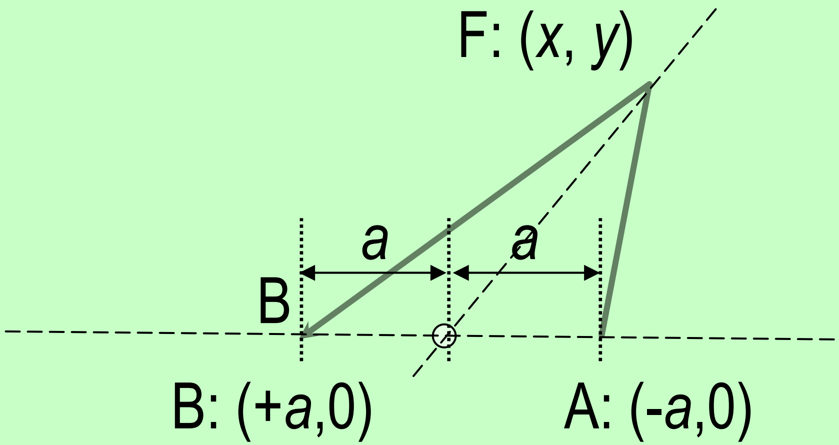
- The red paths have minimum travel time (or length)

  (Fermat's principle – Not true for ADB)

- The red paths have extreme (stationary, i.e. minimum or maximum) travel time (or length)

  (True)

The light example – no mirror

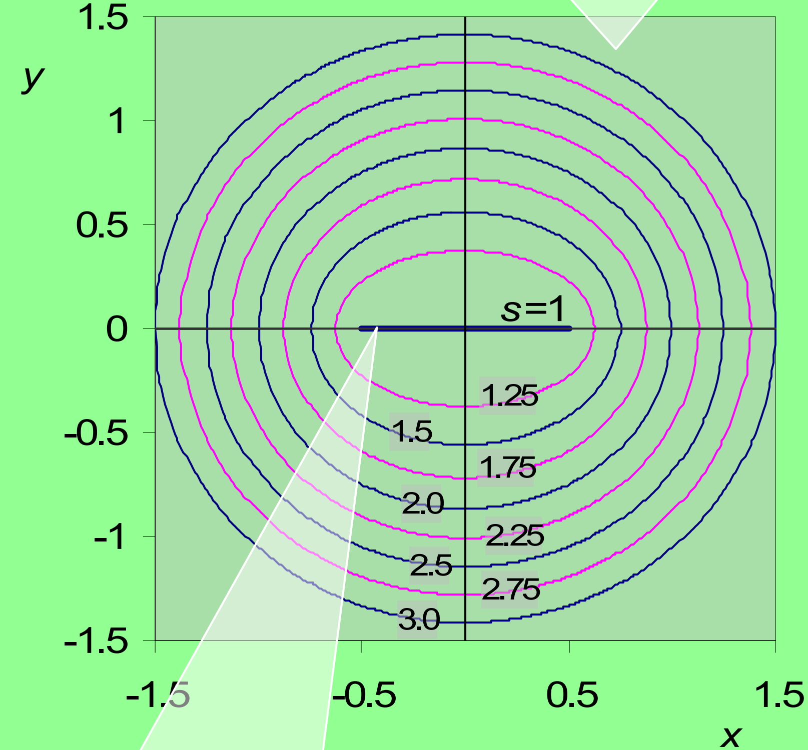


Assume that light can travel from A to B along a broken line with a break point F with coordinates (x, y).
(This is not restrictive: later we can add a second, third, … break points)
The travel distance is $s(x, y)$ = AF + FB where

$$AF = \sqrt{(x - a)^2 + y^2}$$

$$FB = \sqrt{(x + a)^2 + y^2}$$

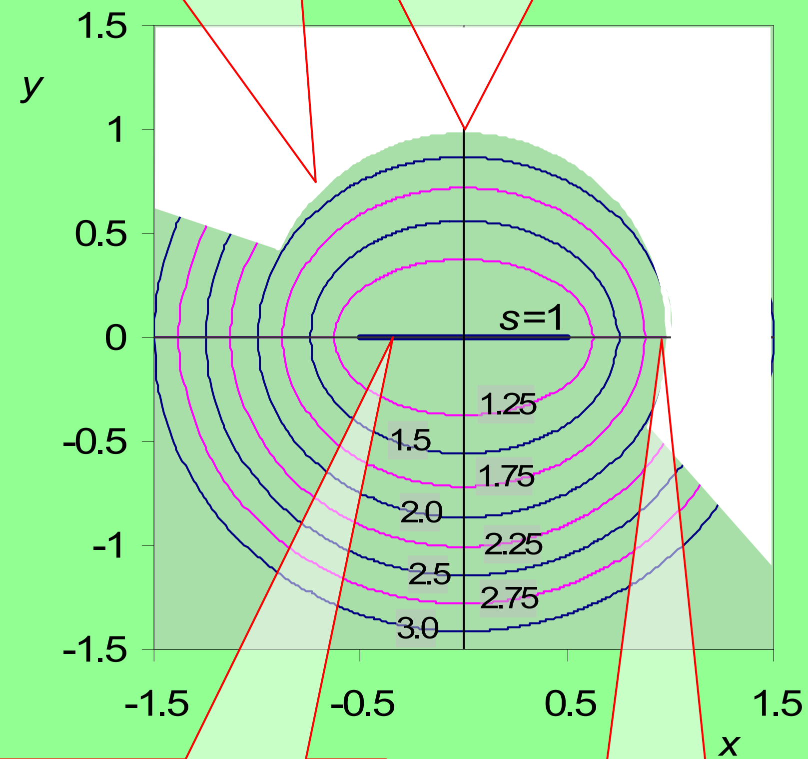
Contours of the distance $s(x, y)$ assuming a = 0.5

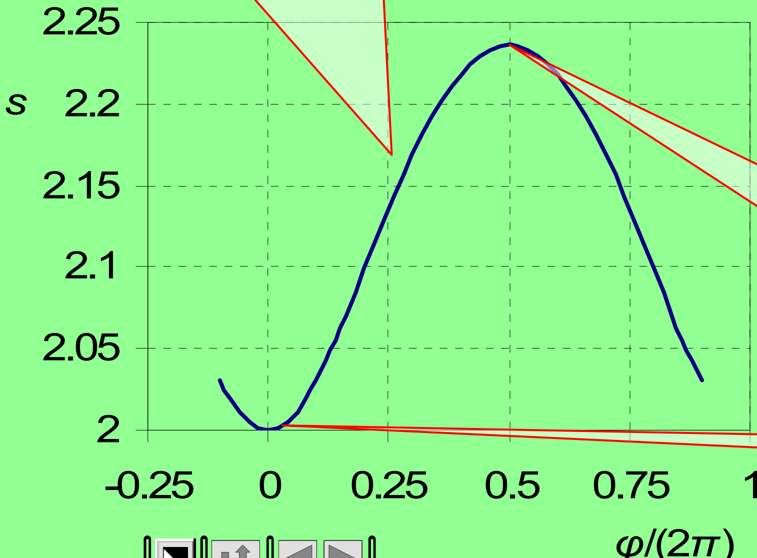


Line of minimum distance $s(x, y)$ = 1
Infinite points F essentially describing the same path

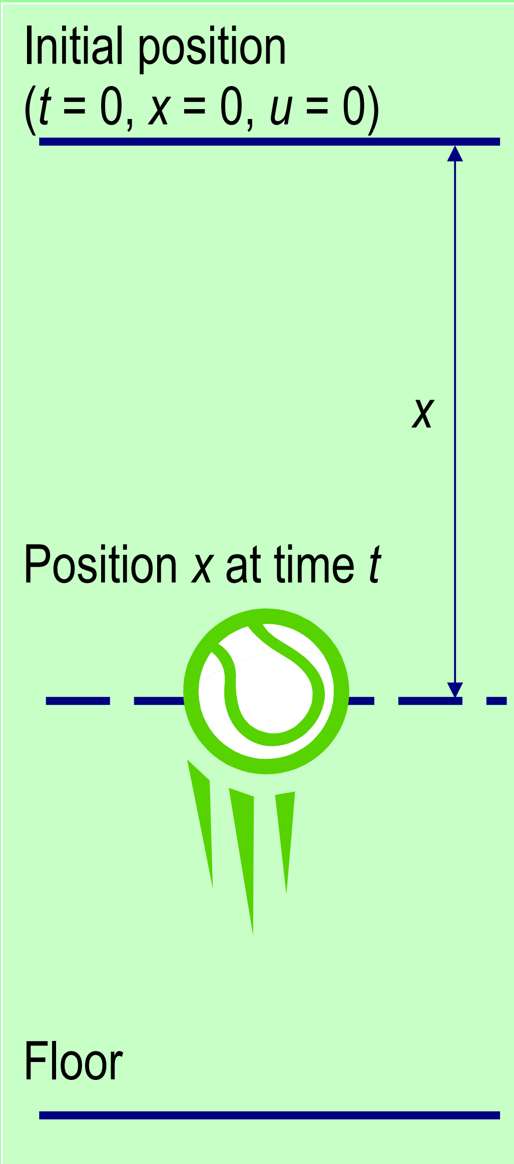
The light example with mirror

- The mirror introduces an inequality constraint in the optimization: the point F should not be behind the mirror
- Two points of local optima emerge on the mirror surface (the curve where the constraint is binding)





A second example: a falling weight



Quantities involved

- Potential energy: $V = -m\, g\, x$
- Kinetic energy: $T = (1/2)\, m\, u^2 = (1/2)\, m\, (dx/dt)^2$
- Total energy: $E = T + V$
- Lagrangian: $L = T - V$
- Action: $S = \int L\, dt$

Alternative methodologies to find equations for the movement

1. Directly by integrating $d^2x/dt^2 = g$
2. From conservation of total energy
3. From minimization of action (more difficult)

- All methodologies result in same solution ($x = g\, t^2/2$, $u = dx/dt = g\, t$)

Principle of least action (Hamilton's principle – applicable both in classical and in quantum physics)

- From all possible motions between two points, the true motion has least action
- More correct to substitute "extreme" (or "stationary") for "least"

How nature works? (a hypothesis ...)

Property

- She preserves a few quantities (mass, momentum energy, ….)
- She optimizes a single quantity (Dependent on the specific system - Difficult to find what this quantity is)
- She disallows some states (Dependent on the specific system – Maybe difficult to find)

Mathematical formulation

- One equation per preserved quantity:

$$g_i(\mathbf{s}) = c_i, \quad i = 1, \ldots, k$$

where c_i constants; **s** the size n vector of state variables ($n \geq k$, sometimes $n = \infty$)

- A single "optimation":

optimize $f(\mathbf{s})$

[i.e. maximize/minimize $f(\mathbf{s})$] **This is equivalent to many equations** (as many as required to determine **s**)
Conversely, many equations can be combined into an "optimation"

- Inequality constraints:

$$h_j(\mathbf{s}) \geq 0, \quad j = 1, \ldots, m$$

- In conclusion, we may find how nature works solving the problem:

$$\begin{aligned} &\text{optimize } f(\mathbf{s}) \\ &\text{s.t. } g_i(\mathbf{s}) = c_i, \quad i = 1, \ldots, k \\ &\phantom{\text{s.t. }} h_j(\mathbf{s}) \geq 0, \quad j = 1, \ldots, m \end{aligned}$$

The typical “optimizable” quantity in complex systems ...

- … is entropy – entropie – Entropie – entropia – entropía – entropi – entrópia – entroopia – entropija – энтропия – ентропія – 熵 – エントロピー – سمقياس – אנטרופיה– εντροπία
- The word is ancient Greek (εντροπία, a feminine noun meaning: turning into; turning towards someone’s position; turning round and round)
- The scientific term is due to Clausius (1850)
- The entropy concept was fundamental to formulate the second law of thermodynamics
- Boltzmann (1877), then complemented by Gibbs (1948), gave it a statistical mechanical content, showing that entropy of a macroscopical stationary state is proportional to the logarithm of the number *w* of possible microscopical states that correspond to this macroscopical state
- Shannon (1948) generalized the mathematical form of entropy and also explored it further. At the same time, Kolmogorov (1957) founded the concept on more mathematical grounds on the basis of the measure theory

What is entropy?

- Entropy is defined on grounds of probability theory
- For a discrete random variable X taking values x_j with probability mass function $p_j \equiv p(x_j)$, j = 1,…,w, the Boltzmann-Gibbs-Shannon (or extensive) entropy is defined as

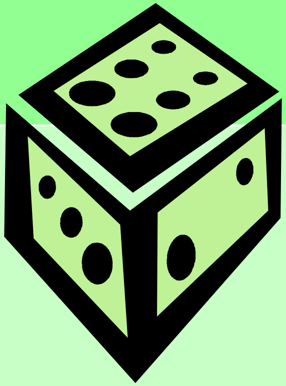
$$\varphi := E[-\ln p(X)] = -\sum_{j=1}^{w} p_j \ln p_j\,, \qquad \text{where } \sum_{j=1}^{w} p_j = 1$$

- For a continuous random variable X with probability density function $f(x)$, the entropy is defined as

$$\varphi := E[-\ln f(X)] = -\int_{-\infty}^{\infty} f(x) \ln f(x)\, dx, \qquad \text{where } \int_{-\infty}^{\infty} f(x)\, dx = 1$$

- In both cases the entropy φ is a measure of **uncertainty** about X and equals the **information** gained when X is observed.
- In other disciplines (statistical mechanics, thermodynamics, dynamical systems, fluid mechanics), entropy is regarded as a measure of **order** or **disorder** and **complexity**.
- Generalizations of the entropy definition have been introduced more recently (Renyi, Tsallis)

Entropy maximization: The die example



- What is the probability that the outcome of a toss of a die will be i? (i = 1, …, 6)
- The entropy is:

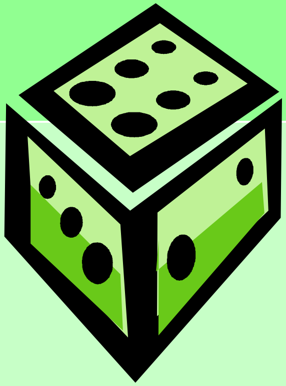
$$\varphi := E[-\ln p(X)] = -p_1 \ln p_1 - p_2 \ln p_2 - \ldots - p_6 \ln p_6$$

- The equality constraint (mass preservation) is

$$p_1 + p_2 + \ldots + p_6 = 1$$

- The inequality constraint is $p_i \geq 0$
- Solution of the optimization problem (e.g. by the Lagrange method) yields a single maximum: $p_1 = p_2 = \ldots = p_6 = 1/6$
- This method, the application of the Maximum Entropy Principle (mathematically, an "optimation" form) is equivalent to the Principle of Insufficient Reason (Bernoulli-Laplace; mathematically, an "equation" form)

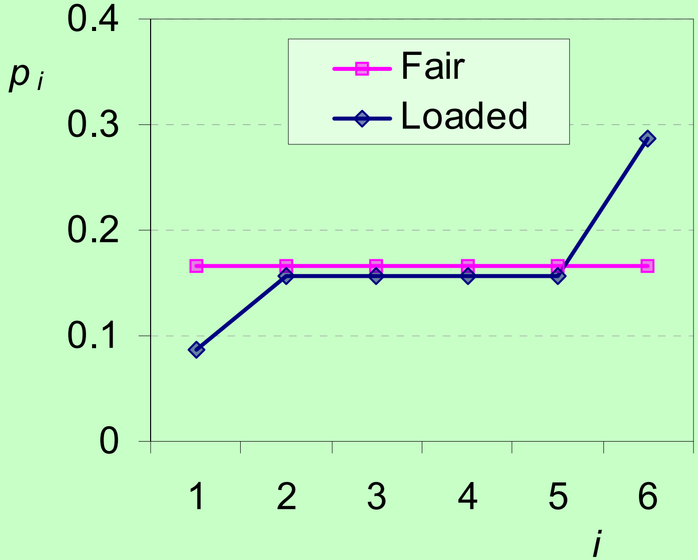
Entropy maximization: The loaded die example



- What is the probability that the outcome of a toss of a die will be i (i = 1, …, 6) if we know that it is loaded, so that $p_6 - p_1 = 0.2$?
- The IS principle does not work in this case
- The ME principle works. We simply pose an additional constraint:

$$p_6 - p_1 = 0.2$$

- The solution of the optimization problem (e.g. by the Lagrange method) is a single maximum:



Entropy maximization: The temperature example

- What will be the temperature in my house (T_H), compared to that of the environment (T_E)? (Assume an open window and no heating equipment)
- Take a space of environment (E) in contact to the house (H) with volume equal to that of the house

- Partition the continuous range of kinetic energy of molecules into several classes i = 1 (coldest), 2, …, k (hottest)
- Denote p_i the probability that a molecule belongs to class i, and partition it to $p_{\mathrm{H}i}$ and $p_{\mathrm{E}i}$, if the molecule is in the house or the environment, respectively
- Form the entropy in terms of $p_{\mathrm{H}i}$ and $p_{\mathrm{E}i}$
- Maximize entropy conditional on $p_{\mathrm{H}i} + p_{\mathrm{E}i} = p_i$
- The result is $p_{\mathrm{H}i} = p_{\mathrm{E}i}$
- Equal number of molecules of each class are in the house and the environment, so $T_H = T_E$
- This could be obtained also from the IR principle

Formalization of the principle of maximum entropy

- In a probabilistic context, the principle of ME was introduced by Janes (1957)
- In a probabilistic context, the principle of ME is used to infer unknown probabilities from known information
- In a physical context, it determines thermodynamical states
- The principle postulates that the entropy of a random variable should be at maximum, under some conditions, formulated as constraints, which incorporate the information that is given about this variable
- Typical constraints used in a probabilistic or physical context are:

Mass; Mean/Momentum; Non-negativity

$$\underbrace{\int_{-\infty}^{\infty} f(x)\, dx = 1}_{\text{Mass}}, \quad \underbrace{E[X] = \int_{-\infty}^{\infty} x\, f(x)\, dx = \mu}_{\text{Mean/Momentum}} \qquad \underbrace{x \geq 0}_{\text{Non-negativity}}$$

Variance/Energy; Dependence/Stress

$$\underbrace{E[X^2] = \int_{-\infty}^{\infty} x^2 f(x)\, dx = \sigma^2 + \mu^2}_{\text{Variance/Energy}}, \quad \underbrace{E[X_i X_{i+1}] = \int_{-\infty}^{\infty} x_i\, x_{i+1}\, f(x_i, x_{i+1})\, dx_i\, dx_{i+1} = \rho\, \sigma^2 + \mu^2}_{\text{Dependence/Stress}}$$

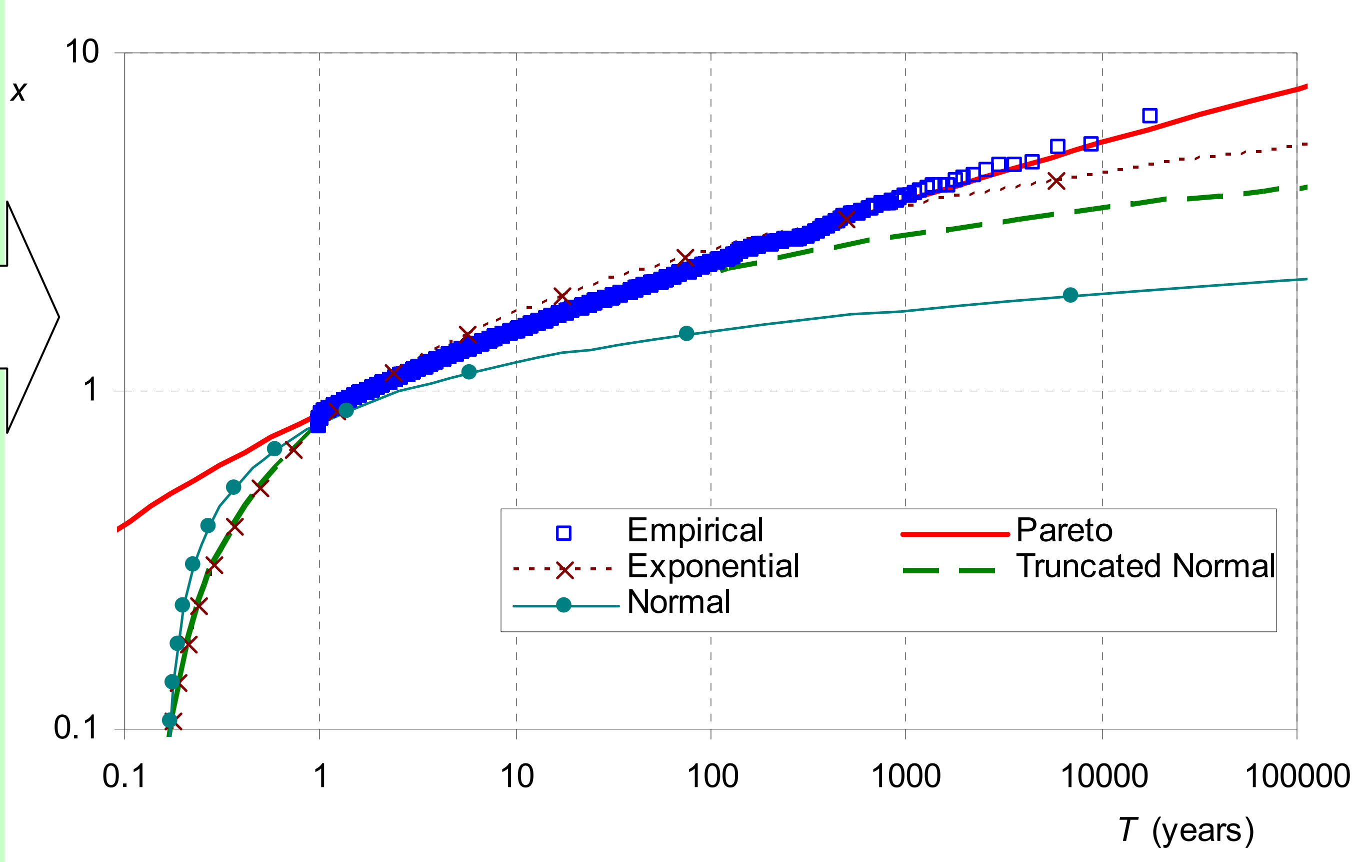
Some results of ME interesting to hydrology

- Assume that a hydrometeorological variable *X* (e.g. temperature, rainfall, runoff) is continuous and positive, has known mean μ and known variation σ/μ. Estimate the distribution function with only this information, applying the ME principle
- The results are:
  - Maximum entropy + Low variation → (Truncated) normal distribution
  - Maximum entropy + High variation → Power-type (Pareto) distribution
  - Maximum entropy + High variation + High return periods → State scaling
- The celebrated state scaling ($x_T \sim T^{\kappa}$,where T is the return period and x_T the corresponding quantile) is only:
  - a consequence of the ME principle,
  - an approximation, good for high return periods and for variables with high variation
- Real world time series (especially long ones) validate the applicability of the ME principle in hydrometeorological processes

ME application to extreme daily rainfall worldwide

Data set: Daily rainfall from 168 stations worldwide each having at least 100 years of measurements; series above threshold, standardized by mean and unified; period 1822-2002; 17922 station-years of data

$\mu = 0.28$
(mean minus threshold)
$\sigma/\mu = 1.19 > 1$
ME distribution: Pareto
$\kappa = 0.15$
$\varphi_q = 1.16$



Conclusion:
Scaling
for $T > \sim 50$ yr

Entropic quantities of a stochastic process

- The *order 1 entropy* (or simply *entropy* or *unconditional entropy*) refers to the marginal distribution of the process X_i:

$$\varphi := E[-\ln f(X_i)] = -\int_{-\infty}^{\infty} f(x) \ln f(x)\, dx, \quad \text{where} \int_{-\infty}^{\infty} f(x)\, dx = 1$$

- The *order n entropy* refers to the joint distribution of the vector of variables $\mathbf{X}_n = (X_1, \ldots, X_n)$ taking values $\mathbf{x}_n = (x_1, \ldots, x_n)$:

$$\varphi_n := E[-\ln f(\mathbf{X}_n)] = -\int_{D_n} f(\mathbf{x}_n) \ln f(\mathbf{x}_n)\, d\mathbf{x}_n$$

- The *order m conditional entropy* refers to the distribution of a future variable (for one time step ahead) conditional on known *m* past and present variables (Papoulis, 1991):

$$\varphi_{c,m} := E[-\ln f(X_1|X_0, \ldots, X_{-m+1})] = \varphi_m - \varphi_{m-1}$$

- The *conditional entropy* refers to the case where the entire past is observed:

$$\varphi_c := \lim_{m \to \infty} \varphi_{c,m}$$

- The *information gain* when present and past are observed is:

$$\psi := \varphi - \varphi_c$$

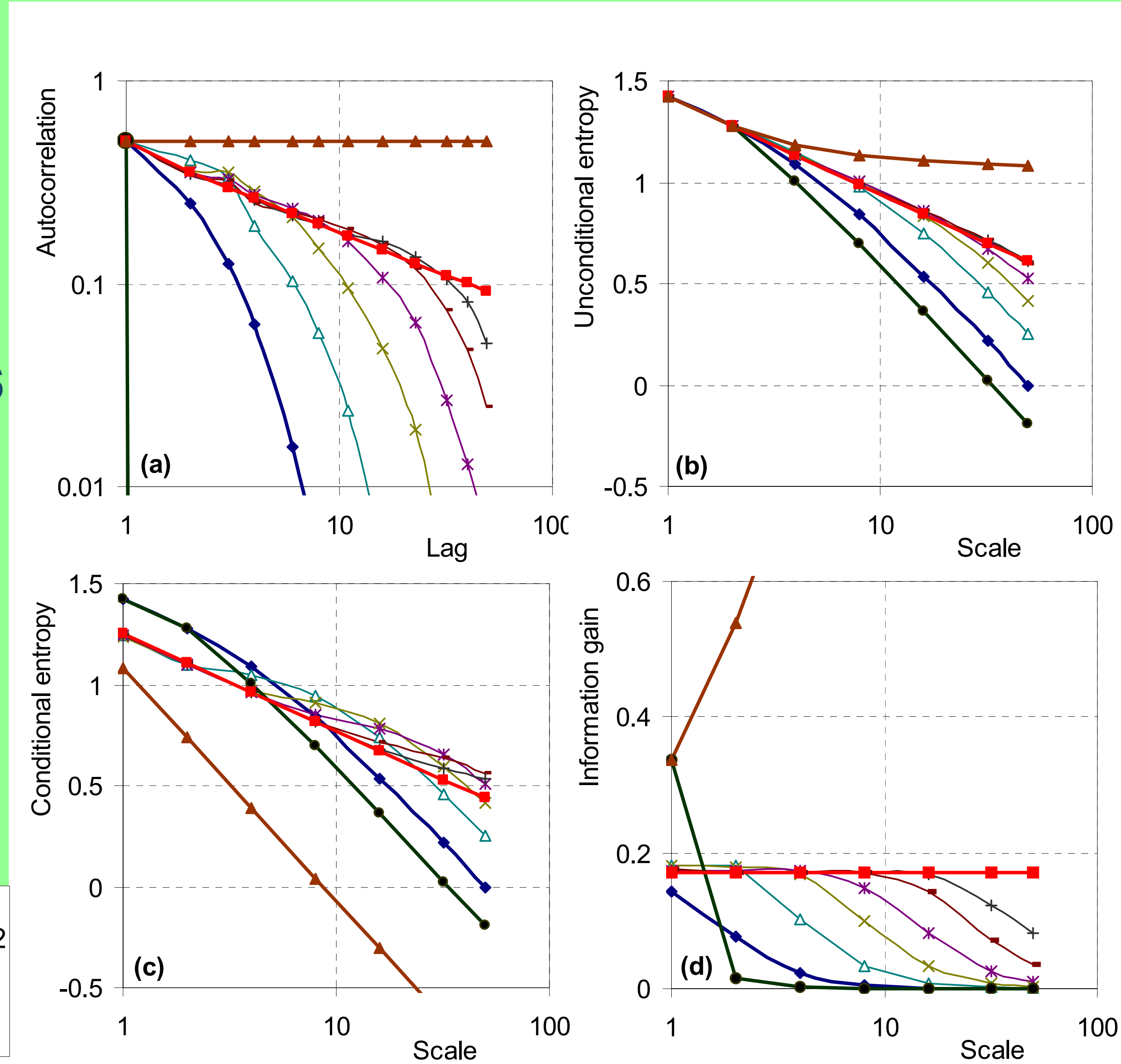
Note: notation assumes stationarity

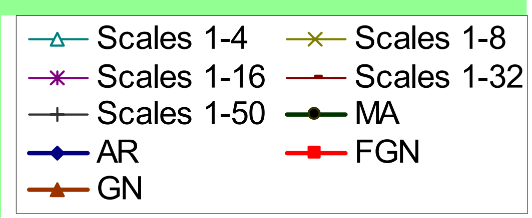
Entropy maximization for a stochastic process

- The purpose is to determine not only marginal probabilities but the dependence structure as well
- All five constrains are used (mass/mean/variance/dependence/non-negativity)
- The lag one autocorrelation (used in the dependence constraint) is determined for the basic (annual) scale but the entropy maximization is done on other scales as well
- The variation is low ($\sigma/\mu << 1$) and thus the process is virtually Gaussian (intermediate result). This is valid for annual and over-annual time scales
- For a Gaussian process the nth order entropy is given as $\varphi_n = \ln\sqrt{(2\,\pi\,e)^n\,\delta_n}$ where δ_n is the determinant of the autocovariance matrix $c_n :=$ Cov[$\mathbf{X}_n$, $\mathbf{X}_n$].
- The autocovariance function is assumed unknown to be determined by application of the ME principle. Additional constraints for this are:
  - Mathematical feasibility, i.e. positive definiteness of c_n (positive δ_n)
  - Physical feasibility, i.e. autocorrelation function (a) positive and (b) non increasing with lag and time scale
    (Note: periodicity that may result in negative autocorrelations is not considered here due to annual and over-annual time scales)

Demonstration: Maximization of unconditional entropy averaged over ranges of scales

Conclusion:
As the range of time scales widens, the dependence tends to SSS





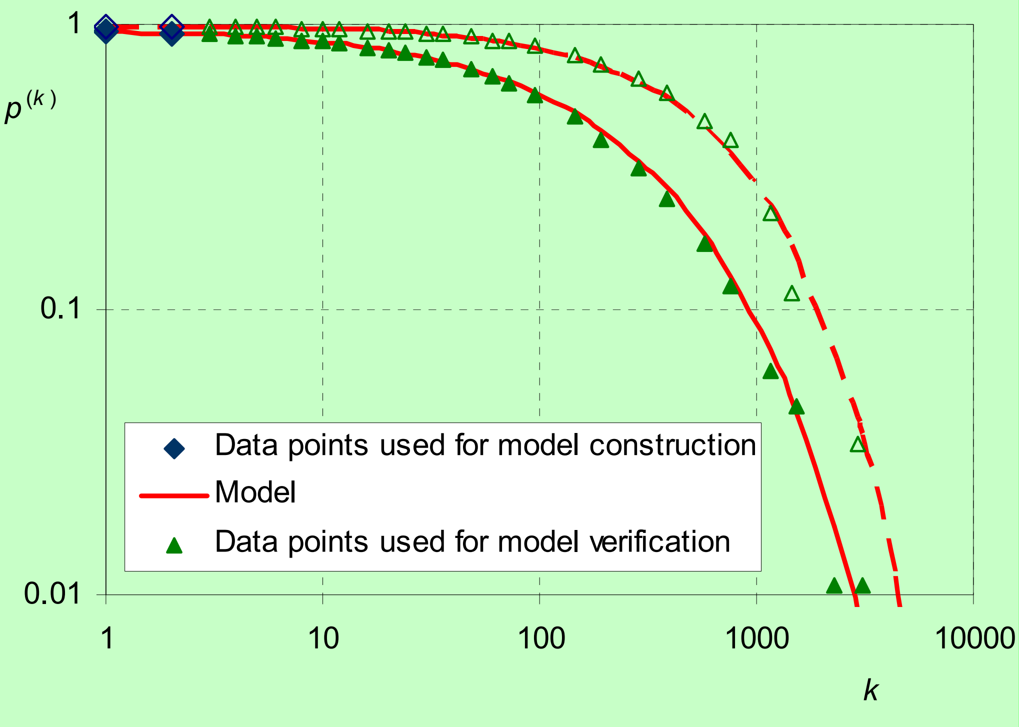
Results of the ME principle in stochastic processes

- Maximum entropy + Low variation + Dominance of a single time scale → Normal distribution + Time independence
- Maximum entropy + Low variation + Time dependence + Dominance of a single time scale → Normal distribution + Markovian (short-range) time dependence
- Maximum entropy + Low variation + Time dependence + Equal importance of time scales → Normal distribution + Time scaling (long-range dependence / Hurst phenomenon)
- The time scaling behaviour is a result of the principle of maximum entropy
- The omnipresence of time scaling in numerous long hydrological time series, validates the applicability of the ME principle

Another peculiar dependence explained by ME

- Rainfall at small scales is intermittent
- The dependence of the rainfall occurrence process is not Markovian neither scaling but in between; it has been known as clustering or overdispersion
- The models used for the rainfall occurrence process (point processes) are essentially those describing clustering of stars and galaxies
- The ME principle applied with the binary state rainfall process in more or less the same way as in the continuous state process explains this dependence

Probability $p^{(k)}$ that an interval of k hours is dry, as estimated from the Athens rainfall data set and predicted by the model of maximum entropy for the entire year (full triangles and full line) and the dry season (empty triangles and dashed line)



Interpretation of results

- The successful application of the ME principle in nature offers an explanation for of a plethora of phenomena (e.g. thermodynamic) and statistical behaviours including
  - the emergence of normal distribution, in many (but not all) cases
  - the scaling behaviour in state, in other cases
  - the scaling behaviour in time
  - the clustering behaviour in rainfall occurrence
- This can be interpreted as dominance of uncertainty in nature
- It harmonizes with the Socratic view: «Ἓν οἶδα, ὅτι οὐδέν οἶδα» (One I know, that I know nothing)
- This view was not a confession of modesty – Socrates regarded the knowledge of ignorance as a matter of supremacy
- In this respect, the knowledge of the dominance of uncertainty can assist to safer design and management of hydrosystems

3. Operational synthesis

Stochastic simulation/forecasting of hydrological processes

- Question: Why simulated series (from stochastic models)?
- Answer:
  - Deterministic forecasts for long horizons (appropriate for the hydrosystem management) are impossible
  - In a stochastic framework, analytical solutions for a hydrosystem as complex as that of Athens are not feasible or would assume oversimplification of the system
  - Of numerical methods, Monte Carlo simulation (stochastic simulation) is the most convenient
  - Detailed inflow and other (rainfall, evaporation) hydrological series are needed at many sites simultaneously and at several time scales for Monte Carlo simulation the hydrosystem
  - The acceptable failure probability level for Athens is of the order of 10^{-2}: one failure in 100 years on the average
  - For a reasonable estimation error in the failure probability we need 1000-10 000 years of data
  - Historical hydrological records are too short

Requirements for stochastic simulation

1. Multivariate model
2. Multiple time scales of operation: annual to monthly or sub-monthly
3. Multiple time scales of preservation: multi-year (reproduction of the Hurst phenomenon) to sub-monthly (reproduction of sub-annual periodicity)
4. Preservation of essential marginal statistics up to third order (skewness)
5. Preservation of joint second order statistics
   - autocorrelations of any type and any lag
   - concurrent cross-correlations
6. Parsimony of parameters
7. Performance in simulation mode (steady state simulations) and in forecast mode, given the current and historical values (terminating simulations)

Models with such features did not exist (particularly, the ARMA type models were not useful)

Stochastic simulation/forecast strategy 1

- Consider the prediction of a single variable W (e.g. the monthly flow one month ahead), conditional on a number s of other variables with known values that compose the vector $\mathbf{Z}$. Use the linear model:

$$W = \mathbf{a}^T \mathbf{Z} + V$$

where $\mathbf{a}$ is a vector of parameters (the superscript T denotes the transpose of a vector or matrix) and V is the prediction error, assumed independent of $\mathbf{Z}$; for simplicity, $\mathbf{Z}$ is assumed normalized and standardized with zero mean and unit variance

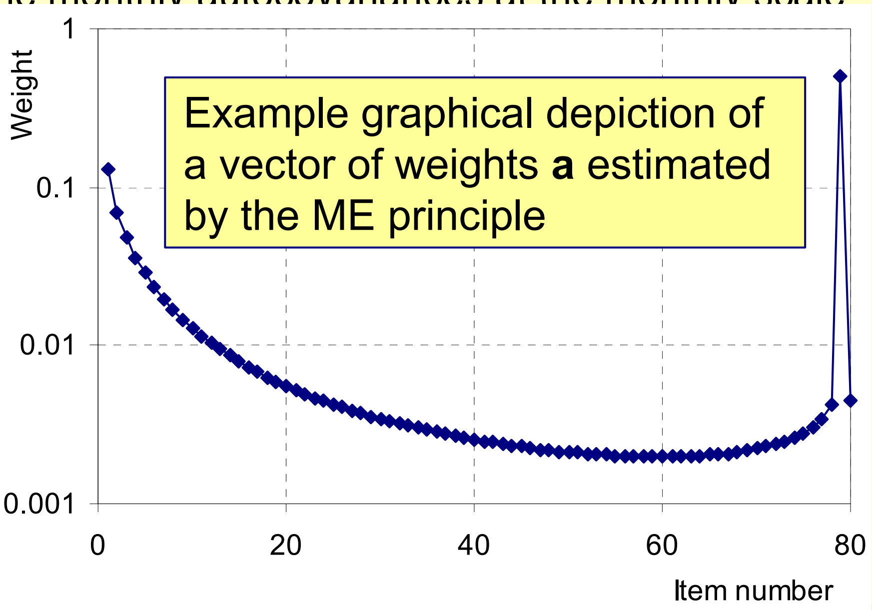
- In forecast mode, $V = 0$ (to obtain the expected value of W conditional on $\mathbf{Z} = \mathbf{z}$); in simulation mode V is generated from the normal distribution independently of **$\mathbf{Z}$;** in multivariate mode, $\mathbf{Z}$ may contain variables of different locations and several models of this type, applied sequentially, are needed
- The vector $\mathbf{Z}$ may contain very many elements; for instance:
  - All available flow measurements of the same month on previous years – to take account of long-range dependence
  - The flows of the some previous months of the same year (2 variables) – to take account of short-range dependence
- The model parameters are estimated from (Koutsoyiannis, 2000)

$$\mathbf{a}^T = \boldsymbol{\eta}^T \mathbf{h}^{-1}, \quad \mathrm{Var}[V] = 1 - \boldsymbol{\eta}^T \mathbf{h}^{-1} \boldsymbol{\eta} = 1 - \mathbf{a}^T \boldsymbol{\eta}$$

where $\boldsymbol{\eta} := \mathrm{Cov}[W, \mathbf{Z}]$ and $\mathbf{h} := \mathrm{Cov}[\mathbf{Z}, \mathbf{Z}]$

Parameter estimation

- Both **a** and Var[V] are estimated from the vector **η** := Cov[W, **Z**] and the matrix **h** := Cov[**Z**, **Z**] that contain numerous items
- For example, assuming 78 years of data and 2 previous months, the number of elements of **η** and **h** will be 80 + 80 × 80 = 6480 for each month; such a number of parameters cannot be estimated from 78 monthly data values
- However, most covariances in **η** and **h** depend on:
  - 2-3 parameters (same for all months) expressing the long-range dependence, as estimated by application on the ME principle on a multi-time scale setting (a stationary component)
  - 2-3 parameters (per month) expressing the monthly autocovariances at the monthly scale (a cyclostationary component)
- All other covariances that cannot be derived from these parameters are left 'unestimated' (in terms of statistics) and are calculated by the ME principle, applied on a single scale
- The entropy maximization in this case has an easy analytical solution that can be formulated as a generalized Cholesky decomposition (assuming that **h** = **b b**^{T})



Stochastic simulation/forecast strategy 2

- Stage 1: Generate annual time series
  - Use a parsimonious model yet capable of describing over-annual scaling
  - No need to describe sub-annual periodicity
- Stage 2: Disaggregate the annual into sub-annual time series
  - Use a parsimonious model structure such as PAR(1)
  - Couple it to the annual model
  - So, no need to describe over-annual scaling explicitly

Annual model: Generalized generating scheme for any covariance structure

Typical (backward) moving average (**BMA**) scheme: $X_i = \ldots + a_1 V_{i-1} + a_0 V_i$ where V_i independent random variables and a_i numerical coefficients

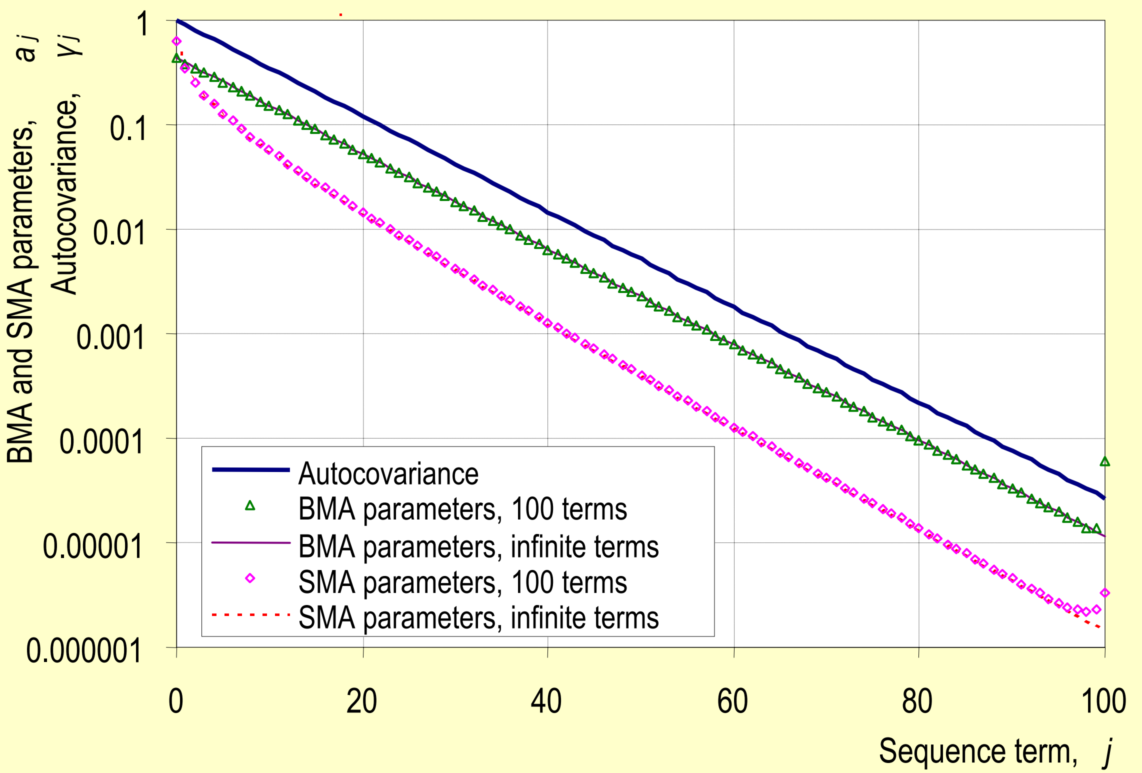
Symmetric moving average (**SMA**) scheme $X_i = \ldots + a_1 V_{i-1} + a_0 V_i + a_1 V_{i+1} + \ldots$

SMA has several advantages over BMA. Among them, it allows a closed solution for a_i:

$s_a(\omega) = [2\, s_\gamma(\omega)]^{1/2}$

where $s_a(\omega)$ and $s_\gamma(\omega)$ the Discrete Fourier Transforms of the series a_j and γ_j, respectively.

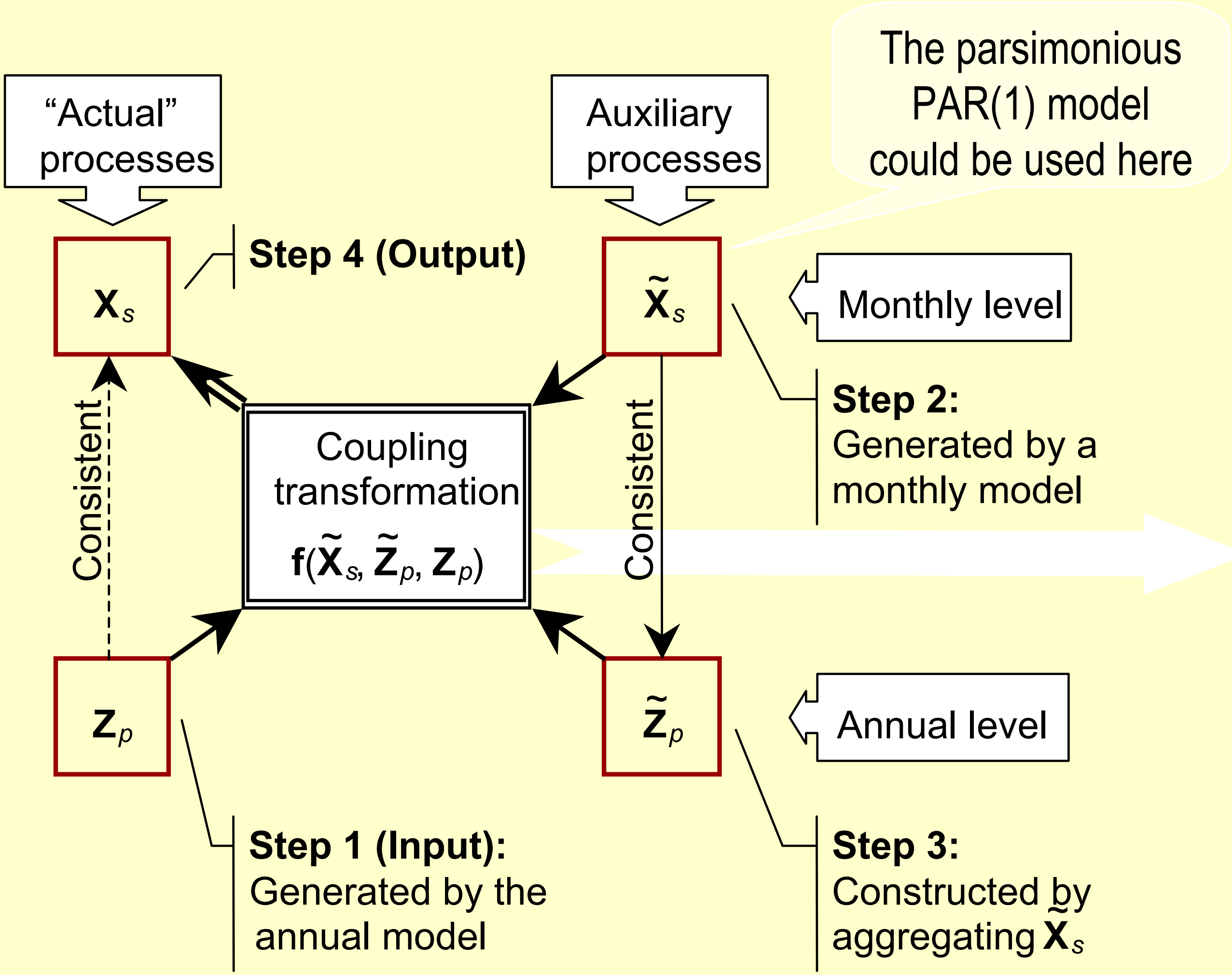
Both schemes are applicable for multivariate problems



Annual model: Stochastic simulation in forecast mode

- In forecast mode, the observed present and past values must condition the hydrological time series of the future
- This is attainable using a two-step algorithm
  1. Generate future time series without reference to the known present and past values
  2. Adjust future time series using the known present and past values and a linear adjusting algorithm
- The linear adjusting algorithm:
  1. is expressed in terms of covariances among variables
  2. preserves exactly means, variances and covariances
  3. is easily implemented

Coupling stochastic models of different time scales



The linear transformation

$$\mathbf{X}_s = \tilde{\mathbf{X}}_s + \mathbf{h}\,(\mathbf{Z}_p - \tilde{\mathbf{Z}}_p)$$

where

$$\mathbf{h} = \mathrm{Cov}[\mathbf{X}_s, \mathbf{Z}_p] \cdot \{\mathrm{Cov}[\mathbf{Z}_p, \mathbf{Z}_p]\}^{-1}$$

preserves the vectors of means, the variance-covariance matrix and any linear relationship that holds among $\mathbf{X}_s$ and $\mathbf{Z}_p$.

Handling of skewness in multivariate problems: Optimized decomposition of covariance matrices

- Consider any linear multivariate stochastic model of the form

$$\mathbf{Y} = \mathbf{a}\,\mathbf{Z} + \mathbf{b}\,\mathbf{V}$$

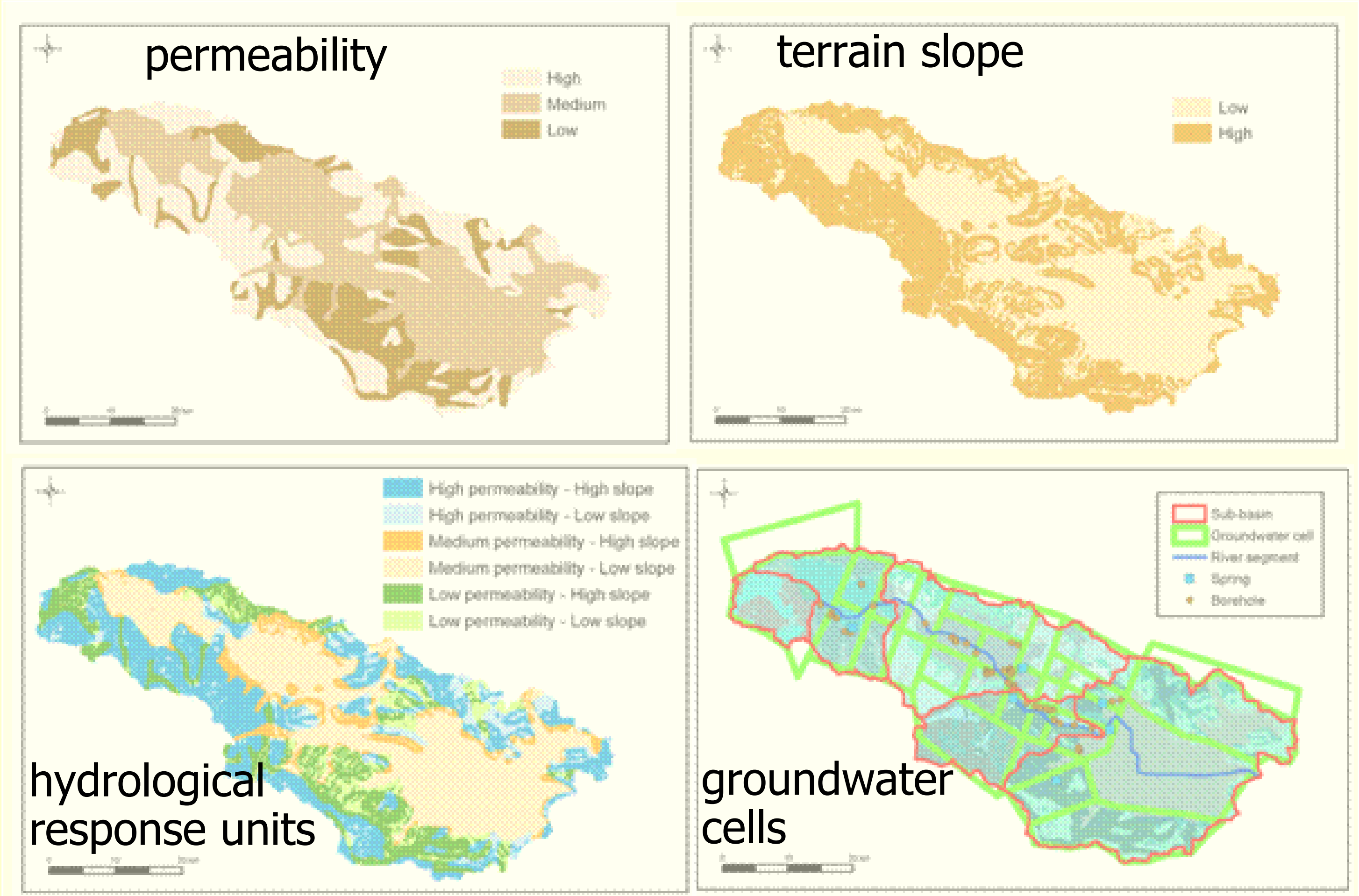
  where **Y**: vector of variables to be generated, **Z**: vector of variables with known values, **V**: vector of innovations, and **a** and **b**: matrices of parameters
- The parameter matrix **b** is related to a covariance matrix **c** by

$$\mathbf{b}\,\mathbf{b}^T = \mathbf{c}$$

- This equation may have infinite solutions or no solution (if **c** is not positive definite)
- The skewness coefficients **ξ** of innovations **V** depend on **b**
- The smaller the values of **ξ**, the more attainable the preservation of the skewness coefficients of the actual variables **Y**
- Therefore, the problem of determination of **b** can be seen as an optimization problem that combines
  - minimization of skewness **ξ**, and
  - minimization of the error $||\mathbf{b}\,\mathbf{b}^T - \mathbf{c}||$
- A fast optimisation algorithm has been developed for this problem
- The algorithm works even for **c** that are not positive definite

Models developed are not only stochastic …

In the Boeoticos Kephisos River basin a hydrological model of the entire hydrological cycle had to be developed, which was demanding due to the extended karstic activity and the intensive withdrawals for irrigation



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Early stage

Restoration of the Hadrianean aqueduct (19th century)

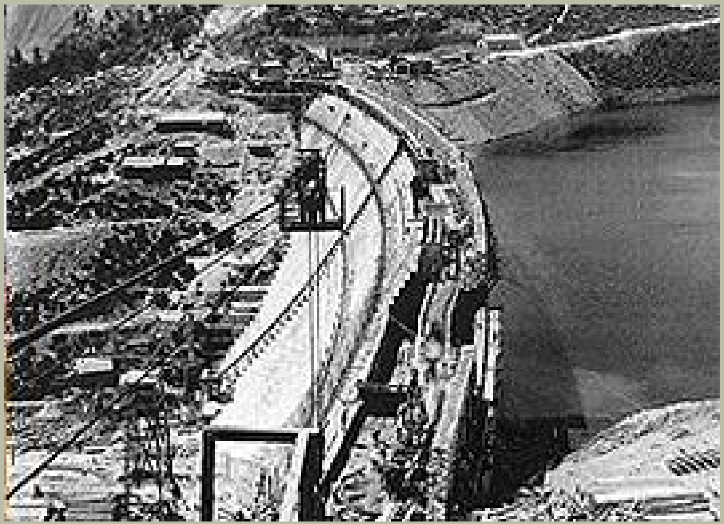
Part of the Peisistratian aqueduct

Supplementary water collection and distribution in Athens (early 20th century until 1930s)

Milestones

Marathon dam

Today

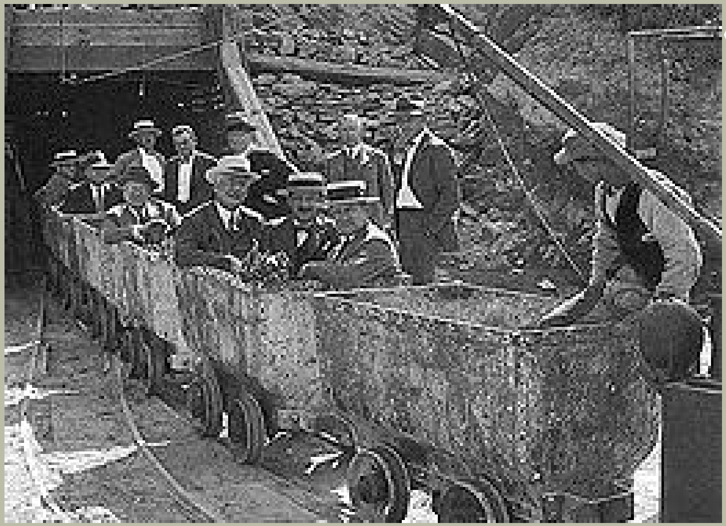


Construction of dam, 1928

Construction of spillway, 1928

Marathon dam (2)

Devastating flood, 1926



Inauguration of Boyati tunnel, 1928

Marathon spillway in action, 1941

Hydrosystem

Previous pictures

Hylike lake and pumping stations

Hylike lake

Hylike, floating pumping stations

Hylike, main pumping station

Kiourka pumping station

Mornos reservoir and aqueduct

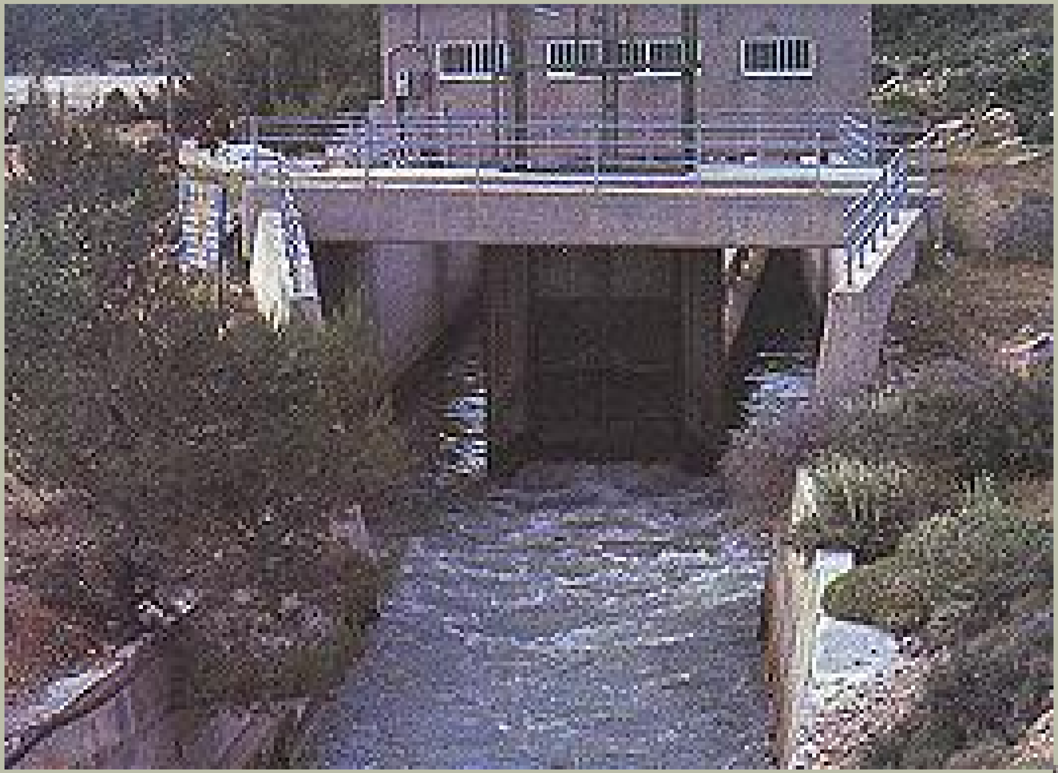
Mornos canal at Delphi

Mornos reservoir

Mornos canal at Thebes plain

Siphon at Distomo

Control of Mornos aqueduct



Canal flow control construction

Aqueduct supervizing & control centre

Evinos dam and tunnel

Evinos dam during construction

Construction of the Evinos-Mornos connection tunnel

Treatment plants

Perissos water treatment plant

Aspropyrgos water treatment plant