A CRITICAL REVIEW OF PROBABILITY OF EXTREME RAINFALL: PRINCIPLES AND MODELS

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Abstract

Probabilistic modelling of extreme rainfall has a crucial role in flood risk estimation and consequently in the design and management of flood protection works. This is particularly the case for urban floods, where the plethora of flow control sites and the scarcity of flow measurements make the use of rainfall data indispensable. For half a century, the Gumbel distribution has been the prevailing model of extreme rainfall. Several arguments including theoretical reasons and empirical evidence are supposed to support the appropriateness of the Gumbel distribution, which corresponds to an exponential parent distribution tail. Recently, the applicability of this distribution has been criticized both on theoretical and empirical grounds. Thus, new theoretical arguments based on comparisons of actual and asymptotic extreme value distributions as well as on the principle of maximum entropy indicate that the Extreme Value Type 2 distribution should replace the Gumbel distribution. In addition, several empirical analyses using long rainfall records agree with the new theoretical findings. Furthermore, the empirical analyses show that the Gumbel distribution may significantly underestimate the largest extreme rainfall amounts (albeit its predictions for small return periods of 5-10 years are satisfactory), whereas this distribution would seem as an appropriate model if fewer years of measurements were available (i.e., parts of the long records were used).

1. Introduction

The design and management of flood protection works and measures requires reliable estimation of flood probability and risk. A solid empirical basis for this estimation can be offered by flow observation records with an appropriate length, sufficient to include a sample of representative floods. In practice, however, flow measurements are never enough to support flood modelling. Particularly, in urban floods the control points are numerous and the flow gauge sites scarce or non existing at all (for example in Athens, a city with a history extended over several millennia, traversed by the Kephisos and Ilisos Rivers and other urban streams, no flow gauge with systematic measurements has ever operated). The obvious alternative is the use of hydrological models with rainfall input data and the substitution of rainfall for streamflow empirical information. Notably, even when flow records exist, yet rainfall probability has still a major role in hydrologic practice; for instance in major hydraulic structures, the design floods are generally estimated from appropriately synthesised design storms (e.g. U.S. Department of the Interior, Bureau of Reclamation, 1977, 1987; Sutcliffe, 1978).

However, from the birth time of science, which is typically located in the era of the Ionian philosophers (6th century BC), it is known that the empirical evidence alone never suffices to form a comprehensive and consistent picture of natural phenomena and behaviours. A theory, based on reasoning, is required to interpret empirical observations and draw such a picture. Such a theory has been sought for more than 26 centuries, since the formulation of the first logical explanations of hydrometeorological phenomena by Anaximander (c. 610- c. 547 BC) and Anaximenes (585-525 BC) of Miletus, who studied the formation of clouds, rain and hail (Koutsoyiannis and Xanthopoulos, 1999; Koutsoyiannis et al., 2006). However, still the state of affairs regarding understanding and description of these phenomena and their behaviours may be not satisfactory.

Some of the questions in seeking a fundament for a theory are philosophical questions; for instance the concepts of infinite vs. finite and of determinism vs. indeterminism, including the notions of probability and entropy. It is necessary to briefly discuss these questions because they greatly influence our perception of hydrometeorological phenomena including rainfall and flood.

The history of infinite goes back to the 6th century BC, with Anaximander, who regarded infinite as the cosmological principle, and continues with Zeno of Elea (c. 490- c. 430 BC) and his famous paradoxes, and later with Aristotle (384-328 BC) who introduced the notion of potential infinite, as opposed to the actual or complete infinite. The Aristotelian potential infinite "exists in no other way, but … potentially or by reduction" (Physics, 3.7, 206b16). It is generally claimed that the problem of mathematical infinite was tackled in the late 19th century. According to Bertrand Russell, Zeno's paradoxes "after two thousand years of continual refutation, … made the foundation of a mathematical renaissance (Russell, 1903). Furthermore, "for over two thousand years the human intellect was baffled by the problem [of infinity]… The definite solution to the difficulties is due to Georg Cantor" (Russell, 1926; see also Crossley et al., 1990 and Priest, 2002).

In hydrometeorology, however, the concept of infinity is still not understood and this situation has led to fallacies of upper bounds in precipitation and flood, the well-known concepts of the probable maximum precipitation (PMP) and probable maximum flood (PMF) (World Meteorological Organization, 1986). These contradictory concepts are still in wide use, even though merely the Aristotelian notion of potential infinite would suffice to abandon them. To quote, for example, Dingman (1994, p. 141) "conceptually, we can always imagine that a few more molecules of water could fall beyond any specified limit." This thinking is absolutely consistent with the Aristotelian potential infinite.

Criticisms of the PMP and PMF concepts must have started from the 1970s; among them, one of the neatest was offered by Benson (1973):

> "The 'probable maximum' concept began as 'maximum possible' because it was considered that maximum limits exist for all the elements that act together to produce rainfall, and that these limits could be defined by a study of the natural processes. This was found to be impossible to accomplish – basically because nature is not constrained to limits ... At this point, the concept should have been abandoned and admitted to be a failure. Instead, it was salvaged by the device of renaming it 'probable maximum' instead of 'maximum possible'. This was done, however, at a sacrifice of any meaning or logical consistency that may have existed originally ... The only merit in the value arrived at is that it is a very large one. However, in some instances, maximum probable precipitation or flood values have been exceeded shortly after or before publication, whereas, in some instances, values have been considered by competent scientists to be absurdly high … The method is, therefore, subject to serious criticism on both technical and ethical grounds – technical because of a preponderance of subjective factors in the computation process, and because of a lack of specific or consistent meaning in the result; ethical because of the implication that the design value is virtually free from risk."

More recently, the particular hypotheses and methodologies elements of the different approaches for estimating PMP have been also criticized. The so-called statistical approach to PMP, based on the studies of Hershfield (1961a, 1965) has been revisited recently (Koutsoyiannis, 1999) and it was concluded that the data used by Hershfield do not suggest the existence of an upper limit. To formulate his method, Hershfield compiled a huge and worldwide rainfall data set (a total of 95 000 station-years of annual maximum rainfall belonging to 2645 stations, of which about 90% were in the USA), standardized each record and found the maximum over the 95 000 standardized values, which he asserted PMP. Clearly then, the PMP hypothesis is based on the incorrect interpretation that an observed maximum in precipitation is a physical upper limit; had the sample size been greater, the estimated PMP value would been greater, too.

The situation is perhaps even worse with the so-called moisture maximization approach of PMP estimation (World Meteorological Organization, 1986), which seemingly is more physically based than the statistical approach of Hershfield. This is the most representative and widely used approach to PMP, and is based on the "maximization" of the observed atmospheric moisture content (i.e. to a maximum observed value) and on the assumption that if the moisture content were maximum, then the rainfall depth would be greater than observed by a factor equal to the ratio of the maximum over the observed rainfall depth. Applying this "maximization" procedure for all observed storms, the PMP value is assumed to be the maximum over all maximized depths.

Clearly, then, the approach suffers twice by the incorrect interpretation that an observed maximum is a physical upper limit. This fallacy is used for first time to determine the maximum moisture content (formally, the maximum dew point, assuming that the observed maximum in a record of about 50 years is a physical limit; obviously, had the record length been 100 or 200 years the observed maximum dew point would most likely be higher). This logic is also used for a second time to determine the PMP as the maximum of observed maximized values. Papalexiou and Koutsoyiannis (2006) have demonstrated the arbitrariness of the approach and its enormous sensitivity to the observation records (e.g. a missing rainfall observation could result in 25% reduction of the PMP value). The arbitrary assumptions of the approach extend beyond the confusion of maximum observed quantities with physical limits. For example, the logic of moisture maximization at a particular location is unsupported given that a large storm at this location depends on the convergence of atmospheric moisture from much greater areas.

In conclusion, it is surprising that the contradictory PMP and PMF concepts are regarded by many as concepts more physically based than a probabilistic approach to extreme rainfall and flood. This is particularly the case because the PMP and PMF concepts are greatly based on probabilistic or statistical assumptions, which in addition are rather misrepresentations of physical phenomena and indicate confused interpretation of probability. In turn, as will be discussed in the next section, these very concepts may have also affected probabilistic approaches of hydrological processes, in an attempt to make them consistent with the unsupported assumption of an upper bound.

This situation harmonizes with a dominant logic in hydrometeorology that probability does not offer a physical insight and is not related to understanding of physical phenomena, but rather it is only an unavoidable modelling tool. In contrast, understanding and insights are regarded as pertinent to deterministic thinking and to mechanistic explanations of phenomena. This logic, however, ought to have been abandoned at the end of the 19th century, after the development of statistical thermophysics and later of the quantum physics which rely upon the concepts of probability and statistics and depart from mechanistic physics. More recently, the study of chaotic dynamical systems and the astonishing results that the evolution of even the simplest nonlinear systems is unpredictable after a short lead time, have demonstrated the ineffectiveness of deterministic thinking. In this respect, even a faithful follower of determinism is inevitably forced to accept probabilistic description of phenomena for practical problems. However, when using probabilistic descriptions the gain may be greater if these descriptions are not regarded as incomprehensible mathematical models but rather as insightful physical descriptions.

The notion of indeterminism is at least as old as Heraclitus (c. 535 - 475 BC) and the notion of probability is the extension (quantifying transformation) of the Aristotelian idea of "potentia" (Popper, 1982, p. 133). The mathematical formalism of probability is much older than the recent notion of chaotic systems albeit its concrete fundament was offered in the mid 20th century by Kolmogorov (1933). The notion of probability may imply indeterminism from the outset (all events are possible, usually with different probabilities, but eventually one occurs) and may differ from the deterministic thinking (only one event is possible but it may be difficult to predict which one).

The notion of probability in synergy with the notion of infinite can remove paradoxical impressions related to upper bounds of physical quantities such as rainfall: The probability that rainfall exceeds any positive number x decreases toward zero as x decreases, becomes inconceivably small for very high x and becomes precisely zero for $x = \infty$. So, there is no need to assume such controversial concepts as PMP. This was explained half a century ago by the famous statistician Feller (1950), using another example, the age of a person:

> "The question then arises as to which numbers can actually represent the life span of a person. Is there a maximal age beyond which life is impossible, or is any age conceivable? We hesitate to admit that man can grow 1000 years old, and yet current actuarial practice admits no bounds to the possible duration of life. According to formulas on which modern mortality tables are based, the proportion of men surviving 1000 years is of the order of magnitude of one in $10^{10^{36}}$ a number with 10^{27} billions of zeros. This statement does not make sense from a biological or sociological point of view, but considered exclusively from a statistical standpoint it certainly does not contradict any experience. There are fewer than 10^{10} people born in a century. To test the contention statistically, more than $10^{10^{35}}$ centuries would be required, which is considerably more than $10^{10^{34/}}$ lifetimes of the earth. Obviously, such extremely small probabilities are compatible with our notion of impossibility. Their use may appear utterly absurd, but it does no harm and is convenient in simplifying many formulas. Moreover, if we were seriously to discard the possibility of living 1000 years, we should have to accept the existence of a maximum age, and the assumption that it should be possible to live x years and impossible to live x years and two seconds is as unappealing as the idea of unlimited life."

In hydrometeorology, the introduction and development of the concepts of probability and statistics have been closely related to the study of extreme rainfall and flood and were greatly determined by the design needs of flood protection works. Empirical ideas similar to the modern probability concepts had been formulated in hydrology about a century ago (for instance, the hydrological frequency curves known as "duration curves"; Hazen, 1914). At about the same time, great mathematicians were developing the theoretical foundation of probability of extreme values (von Bortkiewicz, 1922a, b; von Mises, 1923; Fréchet, 1927; Fisher and Tippet, 1928; Gnedenco, 1941). Around the 1950s the empirical and theoretical approaches converged to form the branch of hydrology now called hydrologic statistics, whose founders were Jenkinson (1955), Gumbel (1958) and later Chow (1964). However, as already stated above, based on the PMP example, the current state of knowledge is not satisfactory and several important questions still wait for answers. For instance, Klemeš (2000) argues that "The distribution models used now, though disguised in rigorous mathematical garb, are no more, and quite likely less, valid for estimating the probabilities of rare events than were the extensions 'by eye' of duration curves employed 50 years ago." Obviously, however, the probabilistic approach to extreme values of hydrological processes signifies a major progress in hydrological science and engineering as it quantifies risk and disputes arbitrary and rather irrational concepts and approaches.

The most important questions that have not received definite answers yet are related in one or another manner to the notion of infinite. These questions concern the asymptotic distribution of maxima, a distribution that assumes a number of events tending to infinity, and are focused on the distribution tails, i.e. the behaviour of the distribution function as the hydrological quantity of interest tends to infinity.

Thus, if one is exempted from the concept of an upper limit to a hydrological quantity and adopts a probabilistic approach, one will accept that the quantity may grow to infinity with decreasing probability of exceedence. In this case, as probability of exceedence tends to zero, there exists a lower limit to the rate of growth which is mathematically proven. This lower limit is represented by the Gumbel distribution, which has the "lightest" possible tail. So, abandoning the PMP concept and adopting the Gumbel distribution can be thought of as a step from a finite upper limit to infinity, but with the slowest possible growth rate towards infinity. Does nature follow the slowest path to infinity? This question is not a philosophical one but has strong engineering implications. If the answer is positive, the design values for flood protection structures or measures will be the smallest possible ones (among those obtained by the probabilistic approach), otherwise they will be higher. These questions are studied in this article with the help of some recent works.

2. Basic concepts of extreme value distributions

It is recalled from probability theory that, given a number n of independent identically distributed random variables, the largest (in the sense of a specific realization) of them (more precisely, the largest order statistic), i.e.:

$$X := \max \{Y_1, Y_2, \ldots, Y_n\} \tag{1}$$

has probability distribution function

$$H_n(x) = [F(x)]^n \tag{2}$$

where $F(x) := P\{Y_i \le x\}$ is the common probability distribution function of each of Y_i. Herein, $F(x)$ will be referred to as parent distribution. If n is not constant but rather can be regarded as a realisation of a random variable with Poisson distribution with mean v, then the distribution of X becomes (e.g. Todorovic and Zelenhasic, 1970; Rossi et al., 1984),

$$H'_v(x) = \exp\{-v[1 - F(x)]\} \tag{3}$$

Since $\ln[F(x)]^n = n \ln \{1 - [1 - F(x)]\} = n \{-[1 - F(x)] - [1 - F(x)]^2 - \ldots\} \approx -n [1 - F(x)]$, it turns out that for large n or large $F(x)$, $H_n(x) \approx H'_n(x)$. Numerical investigation shows that even for relatively small n, the difference between $H_n(x)$ and $H'_n(x)$ is small (e.g., for $n = 10$, the relative error in estimating the exceedence probability $1 - H_n(x)$ from (3) rather than from (2) is about 3% at most).

In hydrological applications concerning the distribution of annual maximum rainfall or flood, it may be assumed that the number of values of Y_i (e.g., the number of storms or floods per year), whose maximum is the variable of interest X (e.g. the maximum rainfall intensity or flood discharge), is not constant. Besides, the Poisson model can be regarded as an acceptable approximation for such applications. Given also the small difference between (3) and (2), it can be concluded that (3) should be regarded as an appropriate model for the practical hydrological applications discussed in this article.

The exact distributions (2) or (3), whose evaluation requires the parent distribution to be known, have rarely been used in hydrological statistics. Instead, hydrological applications have made wide use of asymptotes or limiting extreme value distributions, which are obtained from the exact distributions when n tends to infinity. Gumbel (1958) developed a comprehensive theory of extreme value distributions. According to this, as n tends to infinity $H_n(x)$ converges to one of three possible asymptotes, depending on the mathematical form of $F(x)$. Obviously, the same limiting distributions may also result from $H'_v(x)$ as v tends to infinity. All three asymptotes can be described by a single mathematical expression introduced by Jenkinson (1955, 1969) and become known as the general extreme value (GEV) distribution. This expression is

$$H(x) = \exp\left\{-\left[1 + \kappa\left(\frac{x}{\lambda} - \psi\right)\right]^{-1/\kappa}\right\}, \quad \kappa x \geq \kappa\lambda(\psi - 1/\kappa) \tag{4}$$

where ψ, $\lambda > 0$ and κ are location, scale and shape parameters, respectively; ψ and κ are dimensionless whereas λ has same units as x. (Note that the sign convention of κ in (4) may differ in some hydrological texts). Leadbetter (1974) showed that this holds not only for maxima of independent random variables but for dependent random variables, as well, provided that there is no long-range dependence of high-level exceedences.

When $\kappa = 0$, the type I distribution of maxima (EV1 or Gumbel distribution) is obtained. Using simple calculus it is found that in this case, (4) takes the form

$$H(x) = \exp[-\exp(-x/\lambda + \psi)] \tag{5}$$

which is unbounded from both from above and below ($-\infty < x < +\infty$).

When $\kappa > 0$, $H(x)$ represents the extreme value distribution of maxima of type II (EV2). In this case the variable is bounded from below and unbounded from above ($\lambda\psi - \lambda/\kappa \leq x < +\infty$). A special case is obtained when the left bound becomes zero ($\psi = 1/\kappa$). This special two-parameter distribution is

$$H(x) = \exp\left\{-\left(\frac{\lambda}{\kappa x}\right)^{1/\kappa}\right\}, \quad x \geq 0 \tag{6}$$

In some texts, (6) is referred to as the EV2 distribution. Here, as in Gumbel (1958), the name EV2 distribution is used for the complete three-parameter form (equation (4)) with $\kappa > 0$. Distribution (6) is referred to as the Fréchet distribution.

When $\kappa < 0$, $H(x)$ represents the type III (EV3) distribution of maxima. This, however, is of no practical interest in hydrology as it refers to random variables bounded from above ($-\infty < x \leq \lambda\psi - \lambda/\kappa$). As discussed in the introduction, many regard an upper bound in hydrological quantities as reasonable. Even Jenkinson (1955) regards the EV3 distribution as “the most frequently found in nature, since it is reasonable to expect the maximum values to have an upper bound”. However, he leaves out rainfall from this conjecture saying “to a considerable extent rainfall amounts are ‘uncontrolled’ and high falls may be recorded”. In fact, he proposes the EV2 distribution for rainfall (note that he uses a different convention, referring to EV2 as type I). In a recent study, Sisson *et al*. (2006), even though detecting EV2 behaviour of rainfall maxima, attempt to incorporate the idea of a PMP upper bound within an EV2 modelling framework (see also Francés, this volume).

The simplicity of the above mathematical expressions is remarkable. This extends to the inverse function $x(H) \equiv x_H$ that is used to estimate a distribution quantile for a given non-exceedence probability H. This is

$$x_H = (\lambda/\kappa)\,[\exp(\kappa\, z_H) - 1] + \lambda\psi \tag{7}$$

where z_H is the so called Gumbel reduced variate, defined as

$$z_H := -\ln(-\ln H) \tag{8}$$

For the Gumbel distribution, (7) takes the special form

$$x_H = \lambda(z_H + \psi) \tag{9}$$

which implies a linear plot of x_H versus z_H (a plot known as the Gumbel probability plot). For the Fréchet distribution, (7) takes the form

$$x_H = \lambda\psi \exp(\kappa\, z_H) \tag{10}$$

which implies a linear plot of $\ln x_H$ versus z_H (a plot referred to as the Fréchet probability plot).

The close relationship between the distribution of maxima $H(x)$ and the tail of the parent distribution $F(x)$ allows for the determination of the latter if the former is known. The tail of $F(x)$ can be represented by the distribution of x conditional on being greater than a certain threshold ξ, i.e. $G_\xi(x) := F(x|x > \xi)$, for which:

$$1 - G_\xi(x) = \frac{1 - F(x)}{1 - F(\xi)}\,,\ x \geq \xi \tag{11}$$

If one chooses ξ so that the exceedence probability $1 - F(\xi)$ equals $1/v$, the reciprocal of the mean number of events in a year (this is implied when the partial duration series is formed from a time series of measurements, by choosing a number of events equal to the number of years of record), and denote $G(x)$ the conditional distribution for this specific value, then:

$$1 - G(x) = v[1 - F(x)] \tag{12}$$

Combining equation (12) with equation (3) it is obtained that:

$$G(x) = 1 + \ln H'_v(x) \tag{13}$$

If $H'_v(x)$ is given by the limit distribution $H(x)$ in equation (4), then it is concluded that for $\kappa > 0$:

$$G(x) = 1 - \left[1 + \kappa\left(\frac{x}{\lambda} - \psi\right)\right]^{-1/\kappa},\quad x \geq \lambda\psi \tag{14}$$

which is the generalized Pareto distribution. Similarly, for $\kappa = 0$:

$$G(x) = 1 - \exp(-x/\lambda + \psi), \quad x \geq \lambda\psi \tag{15}$$

which is the exponential distribution. For the special case $\psi = 1/\kappa$:

$$G(x) = 1 - \left(\frac{\lambda}{\kappa x}\right)^{1/\kappa}, \quad x \geq \lambda/\kappa \tag{16}$$

In this way, a one to one correspondence between the type of the extreme value distribution and the type of the tail of the parent distribution is established. The EV1 distribution ($\kappa = 0$, equation (5)) corresponds to an *exponential* parent distribution *tail* (equation (15)), else known as *short tail*, or *light tail*. The EV2 distribution ($\kappa > 0$, equation (4) including the special case (6)) corresponds to an *over-exponential* parent distribution *tail* (equation (14) including the special case (16)), else known as *hyper-exponential tail*, *Pareto tail*, *power-law tail*, *algebraic tail*, *long tail*, *heavy tail* and *fat tail*.

From the distribution functions $H(x)$ and $G(x)$, two return periods can be defined as follows:

$$T := \delta \,/\, [1 - G(x)], \quad T' := \delta \,/\, [1 - H(x)] \tag{17}$$

where δ is the mean interarrival time of an event that is represented by the variable X. In both cases X represents annual values, so $\delta = 1$ year; δ is most commonly omitted but here we kept it for dimensional consistency, given that the return period has units of time, typically expressed in years.

Equation (16) is precisely a power law relationship between the distribution quantile x and the return period T:

$$x = (\lambda/\kappa)(T/\delta)^{\kappa} \tag{18}$$

In the generalized Pareto case (equation (14)), the corresponding relationship is

$$x = (\lambda/\kappa)[(T/\delta)^{\kappa} - 1 + \kappa\psi] \tag{19}$$

whereas in the exponential case the corresponding relationship is

$$x = \lambda\, [\ln(T/\delta) + \psi)] \tag{20}$$

3. The dominance of the Gumbel distribution

Due to their simplicity and generality, the limiting extreme value distributions have become very widespread in hydrology. In particular, EV1 has been by far the most popular model. In hydrological education is so prevailing that most textbooks contain the EV1 distribution only, omitting EV2. In hydrological engineering studies, especially those analysing rainfall maxima, the use of EV1 has become so common that its adoption is almost automatic, without any reasoning or comparison with other possible models. Sometimes, it is also suggested, or even required, by the guidelines or regulations of several organizations, institutes and country services. Historically, several reasons have been contributed to the prevailing of the Gumbel distribution:

Theoretical reasons. Most types of parent distributions functions that are used in hydrology, such as exponential, gamma, Weibull, normal, lognormal, and the EV1 itself (e.g. Kottegoda and Rosso, 1997) belong to the domain of attraction of the Gumbel distribution. In contrast, the domain of attraction of the EV2 distribution includes parent distributions such as Pareto, Cauchy, log-gamma (also called log-Pearson type 3), and the EV2, which traditionally are not in very common use in hydrology, particularly in rainfall modelling.

Simplicity. The mathematical handling of the two-parameter EV1 is simpler than that of the three-parameter EV2.

Accuracy of estimated parameters. Obviously, two parameters are more accurately estimated than three. For the former case, mean and standard deviation (or second L-moment) suffice, whereas in the latter case the skewness is also required and its estimation is extremely uncertain for typical small-size hydrological samples.

Practical reasons. Probability plots are the most common tools used by practitioners, engineers and hydrologists, to choose an appropriate distribution function. As explained earlier, EV1 offers a linear Gumbel probability plot of observed x_H versus observed z_H (which is estimated in terms of plotting positions, i.e. sample estimates of probability of non-exceedence). In contrast, a linear probability plot for the three-parameter EV2 is not possible to construct (unless the shape parameter κ is fixed). This may be regarded as a primary reason of choosing EV1 against the three-parameter EV2 in practice. For the two parameter EV2 (Fréchet) distribution, a linear plot ($\ln x_H$ versus z_H) is possible as discussed earlier. However, empirical evidence shows that, in most cases, plots of x_H versus z_H give more straight-line arrangements than plots of $\ln x_H$ versus z_H.

From a practical point of view, the choice of an EV1 over an EV2 distribution may be immaterial if small return periods T are considered. For instance, in typical storm sewer networks, designed on the basis on return periods of about 5-10 years, the difference of the two distributions is negligible; besides, in such return periods even interpolation from the empirical distribution would suffice. However, for large T (> 50 years), for which extrapolation is required, EV1 results in probability of exceedence of a certain value significantly lower than EV2. That is, for large rainfall depths, EV1 yields the lowest possible probability of exceedence (the highest possible T) in comparison to those of EV2 for any value of κ. For $T > 1000$, the return period estimated by EV1 could be orders of magnitude higher than that of EV2 (see Figure 3 and its discussion in section 5).

This should be regarded as a strong disadvantage of EV1 from the engineering point of view. Normally, this would be a sufficient reason to avoid the use of EV1 in engineering studies. Obviously, this disadvantage of EV1 would be counterbalanced only by strong empirical evidence and theoretical reasoning. In practice, the small size of common hydrological records (e.g. a few tens of years) cannot provide sufficient empirical evidence for preferring EV1 over EV2. This will be discussed further in section 5. In addition, the theoretical reasons, exhibited above, are not strong enough to justify the adoption of the Gumbel distribution. This will be discussed in section 4.

4. Theoretical justification of the distribution type

As discussed above, the rainfall process at fine time scales (hourly, daily) has been modelled by distributions belonging to the domain of attraction of EV1 such as gamma or Weibull. However, the adoption of these distributions is rather empirical, not based on theoretical reasoning. Thus, the above theoretical justification of the EV1 distribution is inconsistent. In contrast, recently three arguments have been formulated that favour the EV2 over the EV1 distribution, which are summarized below.

Argument 1: Asymptotic vs. actual distribution. What matters in hydrological applications, is the actual distribution of maxima, i.e. $H_n(x)$ or $H'_n(x)$ as given in (2) or (3), respectively. The asymptotic distribution $H(x)$ for $n \to \infty$ provides a useful indication of the behaviour in the tails but not necessarily a model for practical use. It has been observed (Koutsoyiannis, 2004a) that the convergence of $H_n(x)$ to $H(x)$ may be enormously slow. This is demonstrated in Figure 1, which depicts Gumbel probability plots of the exact distribution functions of maxima $H_n(x)$ for $n = 10^3$ and 10^6 for a parent distribution function that is Weibull ($F(y) = 1 - \exp(-y^k)$) with shape parameter $k = 0.5$. The parent distribution belongs to the domain of attraction of the Gumbel limiting distribution, so the Gumbel probability plot tends to a straight line as $n \to \infty$. However, even for n as high as 10^6 the curvature of the distribution function is apparent. Obviously, in hydrological applications, such a high number of events within, say, a year, is not realistic (it can be expected that the number of storms or floods in a location will not exceed the order of 10-10^2). Thus, the limiting distribution for $n \to \infty$ may be not useful. The slow convergence in this case should be contrasted with fast convergence in other limiting situations; for example the distribution of the sum of a number of variables to the normal distribution, according to the central limit theorem, is very fast, so that about 10-30 events suffice to obtain an almost perfect approximation to the normal distribution.

Let us assume that the Weibull distribution (which belongs to the domain of attraction of EV1) with shape parameter smaller than 1 (e.g. $k = 0.5$ as in the example of Figure 1) can be a plausible parent distribution of storms and floods at a fine time scale, which is known to be positively skewed and with J-shaped density function. Accordingly, as observed in Figure 1, the probability plot of the exact distribution of maxima should be a convex curve, rather than a straight line, which indicates that, for a relatively small n, a three-parameter EV2 distribution may approximate sufficiently the exact distribution. Thus, even if the parent distribution belongs to the domain of attraction of the Gumbel distribution, an EV2 distribution can be a choice better than EV1.

Argument 2: Change of domain of attraction due to parameter changes. In argument 1 it was assumed that the random variables Y_i whose maximum values are studied are independent and identically distributed ones. However, it is more plausible to assume that different Y_i have the same type of distribution function $F_i(y)$ but with different parameters. The statistical characteristics (e.g., averages, standard deviations etc.) and, consequently, the parameters of distribution functions exhibit seasonal variation. In addition, evidence from long geophysical records shows that there exist random fluctuations of the statistical properties on multiple large time scales (e.g., tens of years, hundreds of years, etc.).

In this respect, it has been shown theoretically that a gamma parent distribution, which belongs to the domain of attraction of EV1, switches to the EV2 domain of attraction if its scale parameter varies randomly following another gamma distribution function (Koutsoyiannis, 2004a). This point was also made by Katz et al. (2005) for an exponential parent distribution, which is a special case of the gamma distribution function. In addition, it was demonstrated using Monte Carlo simulations (Koutsoyiannis, 2004a) that a gamma parent distribution function with constant shape parameter and scale parameter shifting between two values, which are sampled at random with specified probabilities, results in an actual (for $n = 5$) extreme value distribution which is closely approximated by an EV2 distribution, whereas the EV1 distribution departs significantly from the simulated actual distribution.

Argument 3: Principle of maximum entropy. The principle of maximum entropy is a well established mathematical and physical principle, defined on grounds of probability theory, that can infer the detailed structure or behaviour of a system from rough (macroscopical) information of the system. For a stochastic system, the principle can determine the distribution function of the system states, from assumed macroscopical constraints (e.g. moments) of the system. The classical definition of entropy φ, known as the Boltzmann-Gibbs-Shannon entropy, is

$$\varphi := E[-\ln f(Y)] = -\int_{-\infty}^{\infty} f(y) \ln f(y) \, \mathrm{d}y \tag{9}$$

where $f(y) := \mathrm{d}F(y)/\mathrm{d}y$ denotes the probability density function of the parent variable and $E[.]$ denotes expectation.

In a recent study, Koutsoyiannis (2005a) has shown that the principle of maximum entropy can predict and explain the distribution functions of hydrological variables using only two "macroscopic" statistical properties of observed time series (equality constraints), the mean μ and the standard deviation σ, as well as the inequality constraint that the variables under study are non-negative quantities. For variables with high variation ($\sigma/\mu > 1$) the classical entropy φ fails to apply with these constraints. In this case, a generalized definition of entropy, due to Tsallis (1988, 2004) should be used instead. This is

$$\varphi_q = \frac{1 - \int_0^{\infty} [f(x)]^q \, \mathrm{d}x}{q - 1} \tag{17}$$

and precisely reproduces φ when $q = 0$. Maximization of φ_q with the aforementioned constraints results in Pareto tail of the parent distribution with shape parameter $\kappa = (1 - q)/q$. Now, there is sufficient empirical evidence that at small time scales rainfall exhibits high variation ($\sigma/\mu > 1$). In this case, maximization of Tsallis entropy yields power-type (Pareto) distribution.

5. Empirical justification of the distribution type of extreme rainfall

In seeking empirical evidence to justify the distribution type, one must be aware of bias in statistical estimations and error probability in statistical tests that emerge from typical hydrological samples. In fact, estimation bias and error probability are very large and this explains why the inappropriateness the EV1 distribution was not understood for so many years. Specifically, typical annual maximum rainfall series with record lengths 20–50 years completely hide the EV2 distribution and display EV1 behaviour. This was initially demonstrated by Koutsoyiannis and Baloutsos (2000) using an annual series of maximum daily rainfall in Athens, Greece, extending through 1860–1995 (136 years). This series was found to follow EV2 distribution, but if smaller parts of the series were analysed, the EV1 distribution seemed to be an appropriate model.

A systematic Monte Carlo simulation study to address this problem has been done in Koutsoyiannis (2004a). Some of the results, concerning the estimation bias, are depicted in Figure 2. A negative bias, defined as estimated κ minus true κ, is apparent, for both the moments and L-moments estimators. It can be observed that for true κ = 0.15 (a value that is typical for extreme rainfall, as will be discussed later) and for a record length of 20 years the bias of the method of moments is –0.15, which means that the estimated κ will be zero! Even for a record length of 50 years the negative bias is high (b = –0.12), so that κ will be estimated at 0.03, a value that will not give good reason for preferring EV2 to EV1.

The situation is improved if L-moments estimators are used as the resulting bias is much lower. However the method of L-moments is relatively new (Hosking et al., 1985; Hosking, 1990) and its use has not been very common so far. In addition, even the method of L-moments is susceptible to type II error (no rejection of the null false hypothesis of an EV1 distribution against the true alternative hypothesis of EV2 distribution) with a high probability. As demonstrated in Koutsoyiannis (2004a) for κ = 0.15 and record length 20 years the frequency of not rejecting the EV1 distribution is 80%. Even for record length 50 years this frequency is high: 62%.

The results of this analysis show that (a) only long records (e.g. 100 years or more) could provide evidence of the distribution type of extreme rainfall, and (b) even with these records, the estimation of the shape parameter κ of the GEV distribution is highly uncertain, and an ensemble of many records should be used to obtain a reliable estimate.

In this respect, Koutsoyiannis (2004b) compiled an ensemble of annual maximum daily rainfall series from 169 stations of the Northern Hemishpere (28 from Europe and 141 from the USA) roughly belonging to six major climatic zones. All series had lengths from 100 to 154 years, the top three (in terms of length) being Florence, Genoa and Athens, with record lengths 154, 148 and 143 years respectively. The empirical distribution of one of the stations (Athens, Greece) is shown in Figure 3, on Gumbel probability plot, along with the theoretical EV2 and EV1 distributions fitted by several methods. The plot clearly shows that (a) the EV2 distribution fits the empirical one better than the EV1 distribution; for the highest observed daily rainfall (~150 mm), EV2 and EV1 assign return periods of ~200 and ~1000 years (differing by a factor of 5), respectively; for a rainfall depth of ~220 mm, EV2 and EV1 assign return periods of ~1000 and ~100 000 years (differing by two orders of magnitude), respectively. These observations demonstrate how important the correct choice of the theoretical model is and how much the EV1 distribution underestimates the return period of extreme rainfall.

In addition, a PMP value, estimated by Hershfield's method is also plotted in Figure 3. As discussed above, this value should not be regarded as an upper bound of rainfall but just as a value with high return period. It turns out from Figure 3 that the return period of this PMP values is around 50 000 years. It may be useful to mention that the aforementioned critical revisit (Koutsoyiannis, 1999) of Hershfield's data set, on which his method was based, revealed that Hershfield's PMP should be regarded as a rainfall value with return period of about 65 000 years.

These findings are representative of a general behaviour of all 169 rainfall records. In fact, in more than 90% of the records the estimated κ by the methods of maximum likelihood and L-moments were positive. The small percentage of non-positive κ in the remaining records is fully explained as a statistical sampling effect. This provides sufficient support for a general applicability of the EV2 distribution worldwide. Furthermore, the ensemble of all samples were analysed in combination and it was found that several dimensionless statistics, including the coefficient of variation of the annual maximum series, are virtually constant worldwide, except for an error that can be attributed to a pure statistical sampling effect. This enabled the formation of a compound series of annual maxima, after standardization by mean, for all 169 stations. The empirical distribution of the compound series is shown in Figure 4, on Gumbel probability plot, along with the theoretical EV2 and EV1 distributions fitted by several methods. The plot clearly shows that the EV2 distribution fits the empirical one whereas the EV1 distribution is totally inappropriate. The compound series also supported the estimation of a unique κ for all stations, which was found to be 0.15.

The same data set was revisited in Koutsoyiannis (2005a) in a framework investigating the applicability of the maximum entropy principle in hydrology. In this case, instead of series of annual maxima, the series-above-threshold were constructed for 168 out of 169 records (in the Athens case only the annual maximum values were available, and thus the construction of a series-above-threshold was not possible). All series were standardized by their mean and merged in one sample with length 17 922 station-years. The empirical distribution of this sample is depicted in Figure 5 (double logarithmic plot), where values lower than 0.79 are not shown, as this number is the lowest value of the merged series-above-threshold. In addition, several theoretical distribution functions are also plotted. Among these, the Pareto distribution is obtained by the maximum entropy principle for coefficient of variation $\sigma/\mu = 1.19$. The agreement of the Pareto distribution with the empirical one is remarkable. The Pareto distribution is precisely consistent with the EV2 distribution of the annual maximum, as justified in section 2. The shape parameter of the Pareto distribution, as obtained by the maximum entropy principle, is 0.15, the same value with the one obtained by fitting the EV2 distribution in the compound series of annual maximum rainfall.

Additional empirical evidence with same conclusions is provided by the aforementioned Hershfield's (1961a) data set. Koutsoyiannis (1999) showed that this is consistent with the EV2 distribution with $\kappa = 0.13$. The plot of Figure 6 (EV2 probability plot with fixed $\kappa = 0.15$, which is further explained in section 7) indicates that the value $\kappa = 0.15$ can be acceptable for that data set too. This enhances the trust that an EV2 distribution with $\kappa = 0.15$ can be thought of as a generalized model appropriate for mid latitude areas of the north hemisphere.

Additional empirical evidence with same orientation was provided by Chaouche (2001) and Chaouche et al. (2002). Chaouche (2001) exploited a data base of 200 rainfall series of various time steps (month, day, hour, minute) from the five continents, each including more than 100 years of data. Using multifractal analyses he showed that (a) an EV2/Pareto type law describes the rainfall amounts for large return periods; (b) the exponent of this law is scale invariant over scales greater than an hour; and (c) this exponent is almost space invariant.

Other studies have also expressed scepticism for the appropriateness of the Gumbel distribution for the case of rainfall extremes and suggested hyper-exponential tail behaviour. Thus, Wilks (1993), who investigated empirically several distributions which are potentially suitable for describing extreme rainfall, using rainfall records of 13 stations in the USA with lengths ranging from 39 to 91 years, noted that EV1 often underestimates the largest extreme rainfall amounts and suggested an update and revision of the Technical Paper 40 (Hershfield, 1961b), a widely used climatological atlas of United States that was compiled fitting EV1 distributions to annual extreme rainfall data. Coles et al. (2003) and Coles and Pericchi (2003) concluded that inference based on the Gumbel model to annual maxima may result in unrealistically high return periods for certain observed events and suggested a number of modifications to standard methods, among which is the replacement of the Gumbel model with the GEV model. Mora et al. (2005) confirmed that rainfall in Marseille (a raingauge included in the study by Koutsoyiannis, 2004b) shows hyper-exponential tail behaviour. They also provided two regional studies in the Languedoc-Roussillon region (south of France) with 15 and 23 gauges, for which they found that a similar distribution with hyper-exponential tail could be fitted; this, when compared with previous estimations, leads to a significant increase in the depth of rare rainfall. On the same lines, Bacro and Chaouche (2006) showed that the distribution of extreme daily rainfall at Marseille is not in the Gumbel law domain. Sisson et al. (2006) highlighted the fact that standard Gumbel analyses routinely assign near-zero probability to subsequently observed disasters, and that for San Juan, Puerto Rico, standard 100-year predicted rainfall estimates may be routinely underestimated by a factor of two. Schaefer et al. (2006) using the methodology by Hosking and Wallis (1997) for regional precipitation-frequency analysis and spatial mapping for 24-hour and 2-hour durations for the Wahington State, USA, found that the distribution of rainfall maxima in this State generally follows the EV2 distribution type.

6. The distribution tails in other hydrological processes

The theoretical arguments presented in section 4 that support the EV2 over the EV1 distribution are not related merely to rainfall but rather to any process with high variability. Thus, it could be expected that other processes should also exhibit a similar behaviour.

This is the case for flood runoff. In fact, as demonstrated by Koutsoyiannis (2005b, c) and Gaume (2006), there are theoretical reasons by which we can conclude that the type of extreme value distribution in rainfall and runoff will be the same. If rainfall follows the EV1 distribution, then it can be shown that runoff will also follow the EV2 distribution. Conversely, if runoff follows the EV2 distribution, then rainfall should necessarily follow the EV2 distribution. Perhaps, the EV2 distribution in flood is easier to verify empirically (due to magnification of variability of extremes) and thus, the EV1 distribution has not been as standard in flood modelling as is in rainfall modelling. Thus, the log-gamma model, which belongs to the domain of attraction of EV2 has more frequently used in flood modelling. For instance, this model is the federally adopted approach to flood frequency in the USA (US Water Resources Council, 1982). But when flood frequency is estimated from rainfall, which is modelled using the EV1 model, then the flood frequency becomes necessarily consistent to the EV1, as explained above. Several more recent studies have also supported a three-parameter GEV over an EV1 distribution for floods (Farquharson *et al.*, 1992; Madsen et al., 1997).

Similar results have been provided by fractal/multifractal analyses. Thus, Turcotte (1994) studied flood peaks over threshold in 1200 stations in the United States and concluded that they follow a fractal law, which essentially is described by equation (18). Pandey et al. (1998) established power-law distributions for daily mean streamflows in 19 river basins in the USA. Similarly, Malamud and Turcotte (2006) examined six river basins from different climatic regions and hydrologic conditions in the USA and concluded in power law distributions using either flood peaks over threshold or all daily mean streamflows, also considering in some cases paleoflood data.

Naturally, other hydrological processes driven by runoff are anticipated to follow long-tail distributions, too. However, it may be more difficult to verify empirically the type of distribution tail in such cases, because instrumental records are typically much shorter. Nevertheless, reconstructions of time series are possible in some other cases, for instance, in sediment yield time series from sediment deposits. Thus, Katz et al. (2005) were able to detect long tail behaviour in the annual sediment yield time series Nicolay Lake on Cornwall Island, Canada. In addition, Katz et al. (2005) provide an excellent review of the tail behaviours of several ecological variables.

7. Practical issues for the application of the EV2 distribution

As discussed in section 3, the simplicity and the two-parameter form of the EV1 distribution are strong points that made it prevail in hydrology. However, if the shape parameter of the EV2 distribution is fixed (in extreme rainfall $\kappa = 0.15$, as discussed in section 5) the general handling of the distribution becomes as simple as that of the EV1 distribution. For example, the estimation of the remaining two parameters becomes similar to that of the EV1 distribution. That is, the scale parameter can be estimated by the method of moments from:

$$\lambda = c_1 \sigma \tag{21}$$

where $c_1 = \kappa / \sqrt{\Gamma(1 - 2\kappa) - \Gamma^2(1 - \kappa)}$ or $c_1 = 0.61$ for $\kappa = 0.15$, while in the EV1 case $c_1 = 0.78$. The relevant estimate for the method of L-moments is:

$$\lambda = c_2 \lambda_2 \tag{22}$$

where λ_2 is the second L-moment and $c_2 = \kappa / [\Gamma(1 - \kappa)(2^\kappa - 1)]$ or $c_2 = 1.23$ for $\kappa = 0.15$, while in the EV1 case $c_2 = 1.443$. The estimate of the location parameter for both the method of moments and L-moments is:

$$\psi = \mu / \lambda - c_3 \tag{23}$$

where $c_3 = [\Gamma(1 - \kappa) - 1] / \kappa$ or $c_3 = 0.75$ for $\kappa = 0.15$, while in the EV1 case $c_3 = 0.577$.

If, in addition to λ and ψ, the shape parameter is to be estimated directly from the sample (which is not advisable but it may be useful for comparisons) the following approximate equations can be used (Koutsoyiannis, 2004b):

$$\kappa = \frac{1}{3} - \frac{1}{0.31 + 0.91C_s + \sqrt{(0.91C_s)^2 + 1.8}} \tag{24}$$

$$\kappa = 8c - 3c^2, \quad c := \frac{\ln 2}{\ln 3} - \frac{2}{3 + \tau_3} \tag{25}$$

where C_s and τ_3 are the regular and L skewness coefficients, respectively. The former corresponds to the method of moments and the resulting error is smaller than ±0.01 for $-1 < \kappa < 1/3$ ($-2 < C_s < \infty$). The latter corresponds to the method of L-moments and the resulting error is smaller than ±0.008 for $-1 < \kappa < 1$ ($-1/3 < \tau_3 < 1$).

The construction of linear probability plots is also easy if κ is fixed. It suffices to replace in the horizontal axis the Gumbel reduced variate $z_H = -\ln(-\ln H)$ (equation (8)) with the GEV reduced variate $z_H = [(-\ln H)^{-\kappa} - 1]/\kappa$. An example of such a plot is depicted in Figure 6.

8. Resulting intensity-duration-frequency curves

The construction of rainfall intensity-duration-frequency (IDF) relationships or curves is one of the most common practical tasks related to the probabilistic description of extreme rainfall. Unfortunately, however, the construction is typically performed by empirical procedures (e.g. Chow et al., 1988). Even the terms “duration” and “frequency” in IDF are misnomers; in fact, “duration” should read “timescale” (in order not to be confused with the duration of a rainfall event) and “frequency” should read “return period”. Thus, the IDF relationships are mathematical expressions of the rainfall intensity $i(d, T)$ averaged over timescale d and exceeded on a return period T.

The recent theoretical advances in the probabilistic description can support a more theoretically based, mathematically consistent, and physically sound approach. A few assumptions are needed to support such an approach, namely:

1. The separability assumption, according to which the influences of return period and timescale are separable (Koutsoyiannis et al., 1998), i.e.,

   $$i(d, T) = a(T) \,/\, b(d) \tag{26}$$

   where $a(T)$ and $b(d)$ are mathematical expressions to be determined.

2. The similarity assumption, according to which the distribution of average rainfall intensity conditional on being wet is statistically similar for all time scales (Koutsoyiannis, 2006).

3. A stochastic description of rainfall intermittency, which, as suggested by Koutsoyiannis (2006) should be a generalization of a Markov chain process that results applying the maximum entropy principle to the rainfall occurrence process.

4. A probabilistic distribution of the rainfall depth at any scale, which as discussed above should be of Pareto/EV2 type.

Based on assumptions 1-3, Koutsoyiannis (2006) showed that the function $b(d)$ can be approximated for relatively short timescales by the expression (here written in slightly different form)

$$b(d) = (1 + d/\theta)^{\eta} \tag{27}$$

where $\theta >$ is a parameter with units same as the timescale d and η is a dimensionless parameter with values in the interval (0, 1). This resembles an expression historically established with empirical considerations. The approximate character of (46) as well as that of assumptions 1 and 2 should be underlined. At the same time, it should be noted that (46) is more accurate than a pure power law of $b(d)$, which has been suggested by modern fractal approaches. Particularly, (46) implies a decrease of rainfall intensity on small timescales, as compared to what is predicted by a power law. This is very important for the design of urban drainage networks that have small concentration times.

Furthermore, assumption 3 combined with (19) results in

$$i(d, T) = (\lambda/\kappa)[(T/\delta)^{\kappa} - 1 + \kappa\psi] \tag{28}$$

By comparison of (28) with (26), we conclude that only the scale parameter λ should be a function of timescale d and particularly that $\lambda \sim (1 + d/\theta)^{-\eta}$. We easily then deduce that the final form of the IDF will be

$$i(d, T) = \lambda' \frac{(T/\delta)^{\kappa} - \psi'}{(1 + d/\theta)^{\eta}} \tag{29}$$

where $\psi' := 1 - \kappa\psi$ and $\lambda' := (\lambda/\kappa)\,(1 + d/\theta)^{\eta}$, which should be constant, independent of d, Notice that (29) is dimensionally consistent and that the return period T refers to the parent distribution (and thus it can take values smaller than $\delta = 1$ year, but necessarily greater than $\delta\psi'^{1/\kappa}$). Also, notice that the numerator of (29) differs from a pure power law that has been commonly used in engineering practice. By virtue of (13) and (17), (29) can be easily converted in terms of the return period of the distribution of maxima and takes the form

$$i(d, T) = \lambda' \frac{[-\ln(1 - \delta/T')]^{-\kappa} - \psi'}{(1 + d/\theta)^{\eta}} \tag{30}$$

In the latter case, obviously T' should be greater than $\delta = 1$ year. All parameters are precisely the same in both (29) and (30). Consistent parameter estimation techniques for these relationships have been discussed in Koutsoyiannis et al. (1998).

9. Conclusions

Historically, the modelling of rainfall has suffered from several fallacies, such as the existence of an upper bound (PMP), and empirical practices that do not have theoretical support. Rational thinking and fundamental scientific principles, formulated since the birth of science in ancient Greece, can help combat such fallacies.

Probability, statistics and stochastic processes have offered a better alternative in perceiving and modelling of the rainfall process. However, even the probabilistic approaches have suffered from misconceptions and bad practices that have resulted in underestimation of rainfall variability and uncertainty. Among them is the wide application of the Gumbel or EV1 distribution, which has been the prevailing model for rainfall extremes despite the fact that it yields unsafe (the smallest possible) design rainfall values.

More recent studies have provided theoretical arguments and general empirical evidence from many rainfall records worldwide, which suggest a long distribution tail and favour the EV2 distribution of maxima. Simultaneously, they explain that the broad use of the EV1 distribution worldwide is in fact related to statistical biases and errors due to small sample sizes, rather than to the real behaviour of rainfall maxima, which should be better described by the EV2 distribution. Similar behaviours have been also detected in other hydrological processes such as streamflow and sediment transport.

The new methodological framework is more theoretically consistent, and more mathematically and physically sound (justified by the physico-mathematical principle of maximum entropy). Simultaneously, it is very simple so as to allow its easy implementation in typical engineering tasks such as estimation and prediction of design parameters, including the construction of IDF curves. The new framework imposes also some requirements for stochastic models of rainfall, many of which are currently not consistent with the long tail behaviour of the rainfall distribution.

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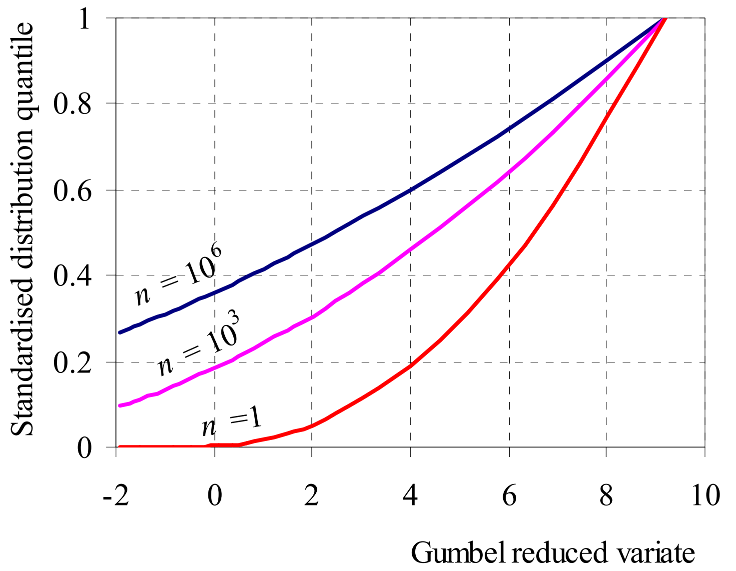

Figure 1: Gumbel probability plots of exact distribution function of maxima $H_n(x)$ for $n = 10^3$ and 10^6, also in comparison with the parent distribution function $F(y) \equiv H_1(y)$, which is Weibull with shape parameter $k = 0.5$. The distribution quantile has been standardised by $x_{0.9999}$ corresponding to $z_H = 9.21$ (from Koutsoyiannis, 2004a).

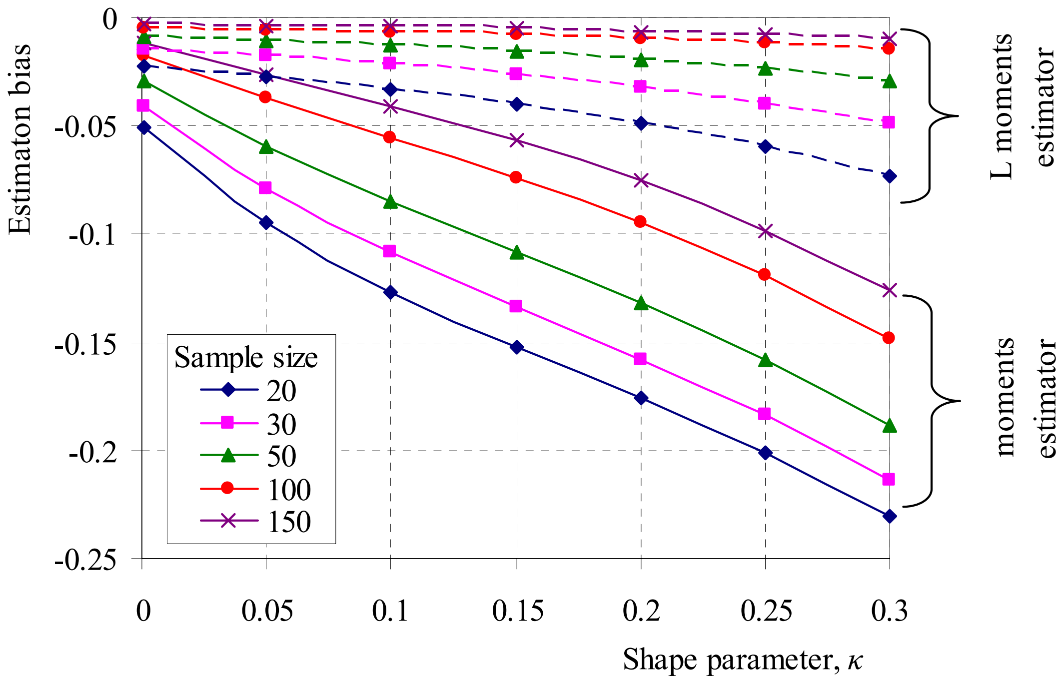

Figure 2: Bias in estimating the shape parameter κ of the GEV distribution using the methods of moments and L-moments (from Koutsoyiannis, 2004a).

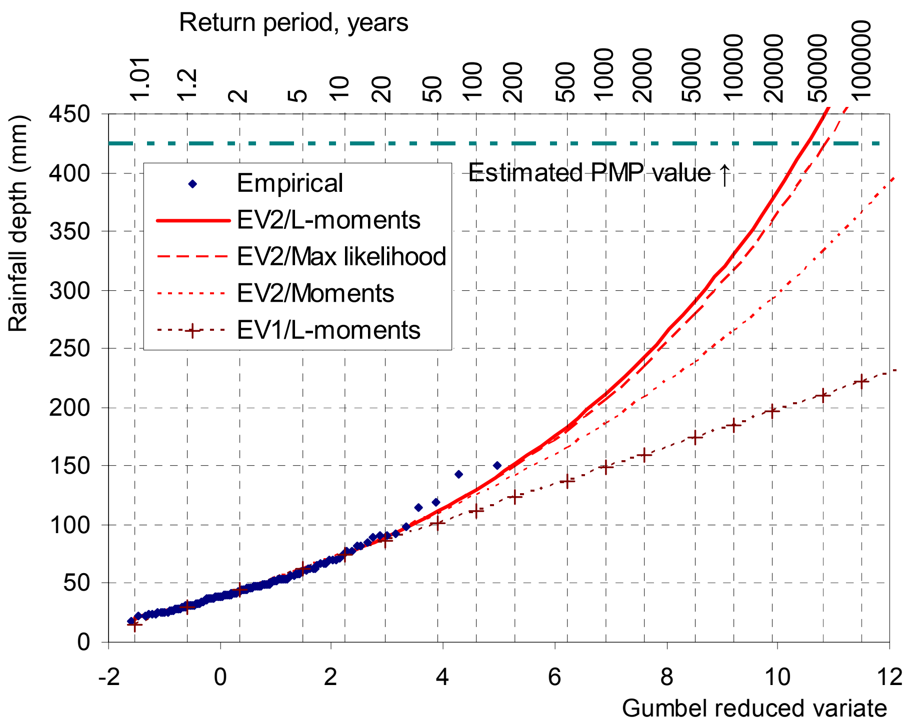

Figure 3: Empirical distribution and theoretical EV2 and EV1 distributions fitted by several methods for the annual maximum daily rainfall series of Athens, National Observatory, Greece (Gumbel probability plot; from Koutsoyiannis, 2004b). The PMP value (424.1 mm) was estimated by Koutsoyiannis and Baloutsos (2000).

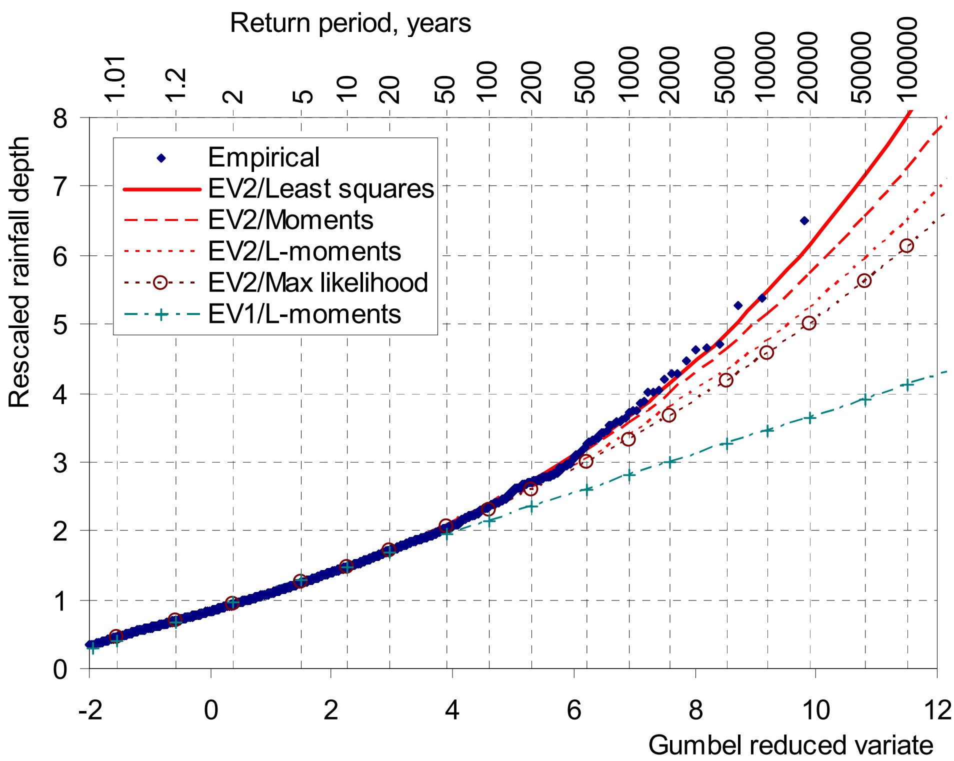

Figure 4: Empirical distribution and theoretical EV2 and EV1 distributions fitted by several methods for the unified record of all 169 annual maximum rescaled daily rainfall series (18 065 station-years; from Koutsoyiannis 2004b).

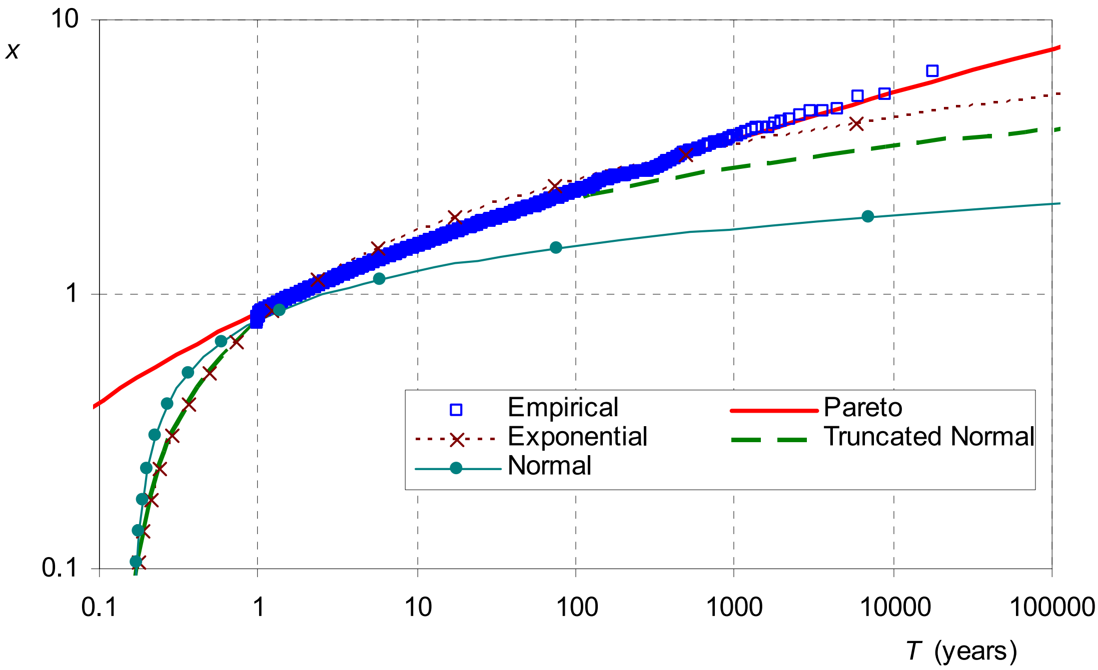

Figure 5: Plot of daily rainfall depth from the unified standardized sample above threshold, formed from data of 168 stations worldwide, *vs* return period, in comparison to Pareto, exponential, truncated normal and normal distributions (adapted from Koutsoyiannis, 2005a).

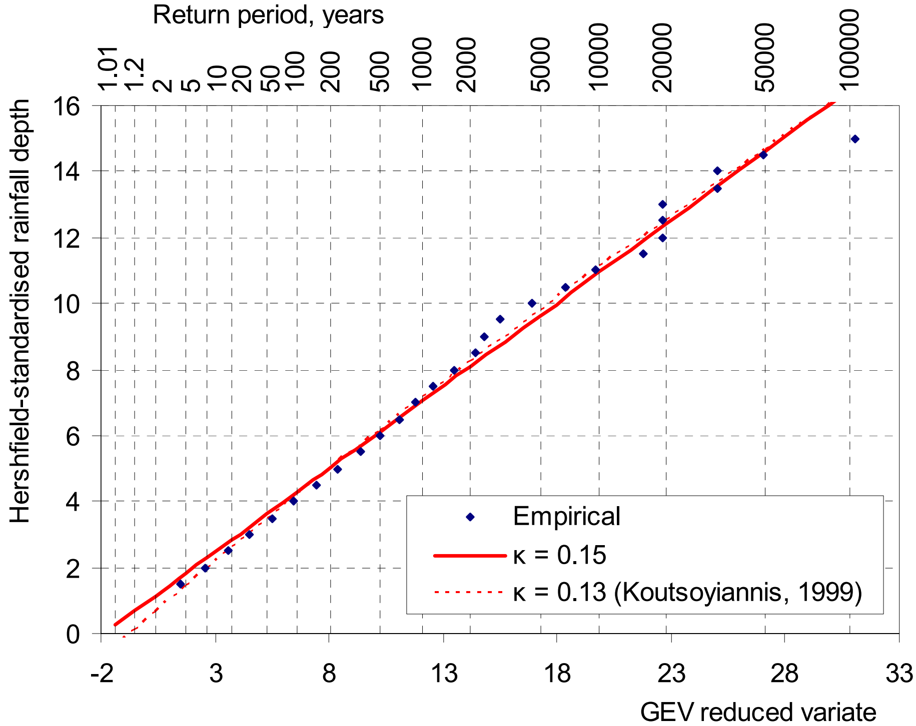

Figure 6: Empirical distribution of standardized rainfall depth $k = (X - \mu)/\sigma$ for Hershfield's (1961a) data set (95 000 station years from 2645 stations), as determined by Koutsoyiannis (1999), and fitted EV2 distributions with κ = 0.13 (Koutsoyiannis, 1999) and κ = 0.15 (Koutsoyiannis, 2004b) (EV2 probability plots with fixed κ = 0.15).