

## ***Interactive comment on “Just two moments! A cautionary note against use of high-order moments in multifractal models in hydrology” by F. Lombardo et al.***

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### General comments

This paper focuses on heavy-tailed distributions commonly encountered in hydrological practice and highlights problems of high variability and skewness of high-order sample moments associated with such distributions. I share the authors' view that modeling practices and tendencies in hydrology need to be continuously monitored and scrutinized to avoid serious modeling errors that could be made. I find the paper to be well written, well-structured and informative. However, I wish to add some comments

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regarding certain aspects of this study.

### Specific comments

In the text following Eq. (16) of the paper, relative to the Weibull distribution, the authors state that: “the case with (compressed exponential function, i.e. a tail lighter than the exponential one) has less practical importance”. In my view, this assertion is not true because light-tailed distributions are also very common in hydrology, especially for modeling low stream flows, for example, as well as for modeling other types of hydrological variables. The emphasis on the problem of thick-tailed distributions in hydrology should by no means minimize the importance of thin-tailed distributions that are also widely encountered.

After having examined the variability and skewness of sample moments for certain thick-tailed distribution cases, the authors caution against the use of high moment estimators, by characterizing them as “unreliable”. However, the fact that a statistic (e.g., a sample moment) is highly variable and skewed, does not by itself mean that the statistic should be avoided in the modeling. If, for example (as is often the case), the aim is to estimate the quantiles of a hypothesized distribution, then the primary focus should be on the quantile estimates and not on the sample moments that are used to estimate these quantiles. Therefore, questions such as the following should be asked: Are the quantiles of interest situated in the right tail, in the left tail, or in the central part of the distribution? How far in the tail are these quantiles? Does the high variability and skewness of the moment estimators lead to similar characteristics in the quantile estimators? (The answer to this last question may be Yes or No). What is the degree of correlation between the moments' estimators being used, and how does this correlation affect the quantile estimates? One should keep in mind that sample moments of low order which exhibit low variability and skewness but which are highly correlated, are not necessarily better for the modeling than higher order moments that are more variable, but less correlated.

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It is true that the tail type significantly influences the variability and skewness of moment estimates, but questions such as those raised in the previous paragraph (questions that are by no means exhaustive) need to be considered and analyzed attentively before arriving at a decision on what moment orders to use or to avoid, or on whether the method of moments or some other fitting method is best for the modeling at hand. The tail thickness of distributions is a factor to keep in mind, but other important factors also need to be taken into consideration. Consequently, one needs to be careful about the limitations of the following two statements put forward in the Conclusions section of the study:

"Estimators of high moments whose distribution ranges over several orders of magnitude cannot support inference about a natural behaviour nor fitting of models".

"In parameter estimation of three-parameter distributions, it is better to avoid the method of moments and use other fitting methods such as maximum likelihood, L-moments, etc."

Given the wide variety of data that are encountered in hydrology, problems of choice of frequency distributions and/or fitting methods should be handled with great care, as they are highly complex, especially if the aim is to estimate events in the distribution tails. Numerous studies have dealt with fitting thick and thin tailed frequency distributions to hydrological data. In fact, a number of fitting methods either originated or were broadly advocated by researchers working in hydrology (e.g., methods of L-moments, generalized moments, generalized probability weighted moments, etc.). The complexity of fitting distributions to data for the purpose of estimating events in the distribution tails has led to confusion not only among practitioners, but among experienced researchers as well. One observes in fitting frequency distributions to data that some users tend to be overly confident about the use of L-moments, for example; others are not very confident, or overly confident about maximum likelihood estimation; others are confused as to what sample moments to use or not to use in estimations. Recommendations put forward in hydrological studies tend sometimes to be followed blindly by

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practitioners if the limitations of these recommendations are not studied well enough or if they are not clearly drafted. Therefore, the recommendations and conclusions of the present study need to be as clear as possible to avoid any misuse by practitioners. I recommend that the authors express their recommendations in as precise a manner as possible, and specify the limitations of these recommendations as clearly as possible. More specifically, if these recommendations are restricted to heavy-tailed distributions, then this should be well reflected in the title, in the abstract and in the conclusions section of the paper.

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