



Fig. 10 Plots of the three synthetic time series generated using the statistics of standardised tree rings at Mammoth Creek, Utah, and implementing: (up) the multiple timescale fluctuation approach; (middle) the disaggregation approach; (down) the symmetric moving average approach.



**Fig. 11** Standard deviation of the aggregated processes  $Z_i^{(k)}$  vs. timescale k (logarithmic plots) for the three synthetic time series generated using: (up) the multiple timescale fluctuation approach; (middle) the disaggregation approach; (down) the symmetric moving average approach. For comparison we have also plotted the theoretical curves of the white noise and FGN models.



**Fig. 12** Autocorrelation functions of the three synthetic time series at the basic (annual) scale generated using: (up) the multiple timescale fluctuation approach; (middle) the disaggregation approach; (down) the symmetric moving average approach. For comparison we have also plotted the theoretical curves of the AR(1) and FGN models and empirical functions of three additional series with large length (64 000) generated using the same three methods.



**Fig. 14** Mean rescaled range  $E[R_k^{**}]$  vs. time length k (logarithmic plots) for the three synthetic time series generated using: (up) the multiple timescale fluctuation approach; (middle) the disaggregation approach; (down) the symmetric moving average approach. For comparison we have also plotted approximate theoretical curves for the white noise and FGN models.

## **APPENDIX 2: Derivation of (16)**

We observe that

$$Z_1^{(kj+k)} = Z_1^{(kj)} + Z_{j+1}^{(k)}$$
(B1)

and consequently

$$\operatorname{Var}[Z_1^{(kj+k)}] = \operatorname{Var}[Z_1^{(kj)}] + \operatorname{Var}[Z_{j+1}^{(k)}] + 2\operatorname{Cov}[Z_1^{(kj)}, Z_{j+1}^{(k)}]$$
(B2)

From (15) we get

$$\operatorname{Var}[Z_{1}^{(kj+k)}] = \left(\frac{kj+k}{k}\right)^{H} \operatorname{Var}[Z_{1}^{(k)}], \quad \operatorname{Var}[Z_{1}^{(kj)}] = \left(\frac{kj}{k}\right)^{H} \operatorname{Var}[Z_{1}^{(k)}]$$
(B3)

and we conclude that

$$\operatorname{Cov}[Z_1^{(kj)}, Z_{j+1}] = (\operatorname{Var}[Z_1^{(k)}] / 2) [(j+1)^{2H} - j^{2H} - 1]$$
(B4)

Besides,

$$Z_1^{(kj)} = \sum_{i=1}^j Z_i^{(k)}$$
(B5)

so that

$$\operatorname{Cov}[Z_1^{(k_j)}, Z_{j+1}^{(k)}] = \operatorname{Var}[Z_1^{(k)}] \sum_{i=1}^j \rho_i^{(k)}$$
(B6)

and thus

$$\sum_{i=1}^{j} \rho_i^{(k)} = (1/2) \left[ (j+1)^{2H} - j^{2H} - 1 \right]$$
(B7)

Likewise,

$$\sum_{i=1}^{j-1} \rho_i^{(k)} = (1/2) \left[ j^{2H} - (j-1)^{2H} - 1 \right]$$
(B8)

Subtracting (B8) from (B7) we get (16).