Climate change, Hurst phenomenon, and hydrologic statistics

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Abstract. The intensive research of the recent years on climate change has led to the strong conclusion that climate has ever, through the planet history, changed irregularly on all time scales. Climate changes are closely related to the Hurst phenomenon, which has been detected in many long hydroclimatic time series and is stochastically equivalent with a simple scaling behaviour of climate variability over timescale. The climate variability, anthropogenic or natural, increases the uncertainty of the hydrologic processes. It is shown that hydrologic statistics, the branch of hydrology that deals with uncertainty, in its current state is not consistent with the varying character of climate. Typical statistics used in hydrology such as means, variances, cross- and auto-correlations and Hurst coefficients, and the variability thereof, are revisited under the hypothesis of a varying climate following a simple scaling law, and new estimators are studied which in many cases differ dramatically from the classic ones. The new statistical framework is applied to real-world examples for typical tasks such as estimation and hypothesis testing where again the results depart significantly from those of the classic statistics.

Key words Climate change; Hurst phenomenon; hydrologic persistence; hydrologic statistics; hydrologic estimation; hydrologic prediction; statistical testing; uncertainty.

Changement climatique, phénomène Hurst, et statistiqe hydrologique

Résumé. La recherche intensive des années recentes sur le changement climatique a conduit à la conclusion sure que le climat a toujours changé dans l`histoire de cette planète, et ceci de manière irrégulière dans toutes les échelles de temps. Les changements climatiques sont étroitement liés au phénomène Hurst, qui a été détecté dans des nombreuses longues séries temporelles en hydroclimatologie et est stochastiquement équivalent à un comportement

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simple de facteurs d'échelle de la variabilité climatique sur l'échelle de temps. La variabilité climatique, soit anthropogène ou naturelle, fait monter l'incertitude des processus hydrologiques. Il est démontré que les caractéristiques statistiques en hydrologie, le sous-domaine en hydrologie qui s'occupe de l'incertitude, dans son état actuel, n'est pas consistant avec le caractère de variation de climat. Caractéristiques statistiques typiques utilisés en hydrologie comme la moyenne, la variance, les correlations sérielles et croisées et le Hurst coéfficient, et la variabilité des ceux-ci, sont revisés sous les hypothèses d'un climat variable suivant une loi d'échelle simple, et d'estimateurs nouveaux sont étudiés qui, dans beaucoup de cas, diffèrent dramatiquement des classiques. Le nouveau cadre statistique est appliqué à des exemples du monde réel pour des travaux typiques comme l'estimation, les tests d'hypothèses, où, une fois encore, les résultats diffèrent significativement de ceux des statistiques classiques.

Mots clefs changement climatique; phénomène Hurst; persistence; caractéristiques statistiques; estimation hydrologique; prédiction hydrologique; tests statistiques; incertitude.

INTRODUCTION

In the last two decades, climate change has been the subject of intensive scientific research, focusing on the understanding of factors, mechanisms and processes related to climate, and on modelling the climate at the global scale using the so called climatic, or general circulation models. Climatic models describe some of the mechanisms of climate variability that are well understood, such as ice-albedo feedback, CO_2 cycles and greenhouse effects, ocean deep-water circulation, ocean-atmosphere interactions, land-atmosphere interactions, etc. They are capable to reproduce the large-scale distributions of pressure, temperature, precipitation and ocean surface heat flux, and resemble sea surface temperature anomalies related to the El Niño-Southern Oscillation phenomena (e.g., *Ledley et al.*, 1999). Another important field of recent research is the detection and attribution of changes in the past climate. This has been also the subject of scientific debate on whether existing climatic records indicate a significant change of the present climate versus the past, and on whether detected changes are attributed

to natural or anthropogenic forcings.

Thus, there is a number of studies detecting global warming in the past two decades and attributing them to anthropogenic forcings, such as the emissions of CO_2. To refer to a recent example, *Stott et al.* (2000) comparing observations with simulations of a coupled ocean-atmosphere general circulation model conclude that both natural and anthropogenic factors have contributed significantly to 20th century temperature changes (the latter especially for the last 35 years).

On the other side, to invoke another recent study, *Przybylak* (2000) detects no global warming in the recent years. Specifically, he studies mean monthly temperatures of Arctic and sub-Arctic areas, which should play a very important role in the detection of global changes as they are the most sensitive and warming and cooling epochs should be seen most clearly here and should also occur earlier than in other parts of the world. The analyses show that in the Arctic, since the mid-1970s, the annual temperature shows no clear trend and the level of temperature in Greenland in the last 10-20 years is similar to that observed in the 19th century. The author concludes, "the temperature predictions produced by numerical climate models significantly differ from those actually observed."

The issue of climate modelling capability is also examined in the comprehensive study by *Barnett et al.* (1999) who state that, at present, it is not possible to distinguish the relative contributions of specific natural and anthropogenic forcings to observed climate change. The authors also state that "there has been to date no completely convincing demonstration that the anthropogenic effects predicted by advanced climate models have been unambiguously detected in observations" and conclude that "the current state of affairs is not satisfactory." If the current state of affairs is not satisfactory when dealing with the past and present climate, things are even worse when attempting to produce predictions of the future. (After all, it is not so difficult to fit models with a number of adjustable parameters to historical records, but in predicting the future the situation is different.) In another comprehensive study by 36 climate modellers (*Davey et al.*, 2002), who tested the ability of 23 dynamical ocean-atmosphere models to correctly simulate fields of tropical sea surface temperature, surface wind stress,

and upper ocean vertically-averaged temperature, several model deficiencies are found, for example the facts that "interannual variability is commonly too weak in the models" and "no single model is consistent with observed behaviour in the tropical ocean regions"

Such deficiencies are related to the fact that climatic models are necessarily simplified representations of the complex climate system as they do not describe completely the dynamics of all involved processes. Rather, they use simplified representations, known as parameterizations for a large number of processes including mixing, convection and clouds (e.g. *von Storch et al.*, 2001). Besides, climate is influenced by several factors with opposing effects (e.g., *Ledley et al.*, 1999), either anthropogenic (greenhouse gases, aerosols) or natural (solar irradiance, clouds, volcanic activity, etc.). Some of these factors are extremely difficult or even impossible to incorporate in models and to predict. Overall, as *von Storch et al.* (2001) put it, "climate must be considered as a stochastic system, and our climate simulation models as random number generators". Even merely the natural forcings are enough to result in a perpetually changing climate. Indeed, it is well known that climate "changes irregularly, for unknown reasons, on all timescales" (*National Research Council*, 1991, p. 21).

The uncertainty or unpredictability becomes higher when moving from general climatic variables, such as temperature (which is the key variable for most of the studies mentioned above) to hydrologic variables such as rainfall and runoff, and from the coarse spatial scale of climatic models to the finer spatial scale of hydrologic models. In parallel, the importance of these hydrologic variables is higher, when dealing with engineering and management issues such as design and operation of hydrosystems.

In the last two decades, hydrologists have developed different strategies to deal with climate change (*Lettenmaier et al.*, 1996). One of these consists of the so-called downscaling of climatic models' results into the area and timescale of interest. Another option is the adoption of prescribed climate change scenarios with plausible shifts in the average hydrologic regime of the area of interest (e.g. changes 0°C to $+4^{\circ}$C in temperature and 0% to 25% in precipitation; *Lettenmaier et al.*, 1996). The first method suffers from accumulation of uncertainties and errors in the chain of models used, which potentially lead to poor

representation of reality. For example, in a recent study by *Carpenter & Georgakakos* (2001) the large-scale climatic model used explains less than 20% of the observed precipitation variance and, even worse, results in significant scale bias (model precipitation even 5-25 times smaller than the actual one depending on the choice of the neighbouring model grid node, as displayed in their Fig. 6). The second method lacks quantification of uncertainty.

Traditionally, however, hydrologic and water resources studies have been based on the quantification of the natural uncertainties and the resulting risk in terms of probability. The discipline of hydrologic statistics (e.g. *Chow*, 1964; *Yevjevich*, 1972; *Kottegoda*, 1980; *Hirsch et al.*, 1993; *Stedinger et al.*, 1993) has been well developed and applied in hydrologic engineering studies. Hydrologic statistics, though, has been based on the implicit assumption of a stable climate. The questions arise then: (1) Is hydrologic statistics, in its present state, consistent with the assumption of a varying climate? (2) If not, what adaptations are needed to achieve this consistency? (3) Can hydrologic statistics be used to quantify the total uncertainty under a varying climate?

These are the main questions studied in this paper. The usefulness of the answers to these questions from an engineering and management point of view is almost obvious. However, one may argue that the climate change cannot be predicted at all with statistical means, based on historical records that are almost free of anthropogenic influences. This may be correct if indeed, the contribution of anthropogenic forcings is high, relative to that of the natural forcings, a statement that is still unproven. Even if this is the case, the usefulness of statistical estimators consistent with a naturally varying climate is high for the procedure of detecting climate change. Detection of climate change requires demonstrating that the observed change is larger than would be expected to occur by natural causes alone, and this is clearly a statistical problem. In addition, even when working with purely deterministic climatic models, the usefulness of statistics is undeniable in the phase of evaluating the model results.

If one revisits the classic statistical estimators (e.g. for the mean, variance, etc.) and glances at the mathematical proofs of their validity, one will see that they are based on the assumption that statistical samples consist of independent, identically distributed variables.

Obviously, this assumption is not consistent with the nature of hydrometeorological time series. A more consistent representation, which relaxes, to some degree, the assumption of stable climate, is studied in the next section with the help of real world examples. In the subsequent sections typical statistical tasks such as estimation, prediction and hypothesis testing are revisited both theoretically and by means of case studies.

STOCHASTIC REPRESENTATION OF HYDROMETEOROLOGICAL PROCESSES

Varying climate and the Hurst phenomenon

To establish a pragmatic representation of hydrometeorological processes it is very useful to examine observed long time series. In this paper three example time series have been chosen as case studies, which are depicted in Figures 1-3. The first example (Figure 1(a)) is a long time series (992 years) representing the North Hemisphere temperature anomalies in ^{o}C versus 1961-90 mean (from ftp.ngdc.noaa.gov/paleo/contributions_by_author/jones1998/). This series was constructed by *Jones et al.* (1998) using temperature sensitive paleoclimatic multi-proxy data from 10 sites worldwide that include tree rings, ice cores, corals, and historical documents. The second example (Figure 2) is one of the longest series of instrumental meteorological observations, the series of mean annual temperature at Paris/Le Bourget extending through 1764-1995. This is a typical example of a set of similar series of temperatures of European countries that go back to the 18th century (from ftp.cru.uea.ac.uk). The third example (Figure 3) is the longest available streamflow record in Greece, the Boeoticos Kephisos river runoff. The river is located to the north of Athens and the time series length is 91 years (hydrologic years 1907-08 to 1997-98).

A common characteristic in all three examples is that a local multi-year average (plotted in the figures as 5- and 25-year average) is not stable but, rather, it exhibits significant variability. For example, in the first series (Figure 1(a)) during the 16th century there is a falling trend of the local average, which is inverted during *ca.* 1650-1750, becomes again falling during ca. 1750-1850, and becomes rising thereafter. A visual assessment of the magnitude of the overyear variability and trends can be done by comparing the actual time

series with a series of white noise (independent, identically distributed variates). Such a synthetic series of white noise with length and marginal statistical characteristics equal to those of the first original series has been plotted in Figure 1(b). The difference of the original series from white noise is clearly visible; the variability of the latter in the aggregation level of 25 years is much lower than that of the former.

Similar behaviour has been reported in several long time series. For example *Beran* (1994, p. 41), examined a number of geophysical and other long time series and observed that at short time periods, there seem to be local trends but looking at the whole series, these become cycles of (almost) all frequencies, superimposed and in random sequence.

The falling and rising local trends can be regarded as climate changes or variations, considered by many as deterministic components in climatic time series. In all cases, however, these changes are irregular and, in the absence of an accurate deterministic model that could explain them and predict their future, are better modelled as stochastic fluctuations on many timescales. Equivalently, these fluctuations can be regarded as a manifestation of the Hurst phenomenon discovered by *Hurst* (1951). This phenomenon is quantified through the so-called Hurst exponent (or coefficient), the mathematics of which will be discussed later. In a white noise series, such as the one plotted in Figure 1(b), H takes the value 0.5 whereas in real-world time series H is usually greater. H provides a direct means to demonstrate the close relationship of large-scale variability with the Hurst phenomenon: if H is calculated using such a time series the result will be higher than 0.5 but, if the same calculation is repeated for the "detrended" series (i.e., the series after "subtracting" the *a posteriori* identified trends), H will be around 0.5.

In a recent paper (*Koutsoyiannis*, 2002), this relation was more thoroughly explored and it was demonstrated that the large-scale variability of a time series can explain the Hurst phenomenon. This explanation is consistent with observations by *Mesa & Poveda* (1993), who regarded Hurst phenomenon as "probably the result of a mixture of scales" and by *Vanmarcke* (1983, p. 225), who studied a composite random process with two different scales of fluctuation.

Having hypothesized that the Hurst phenomenon is a manifestation of irregular climatic fluctuations on several scales, it can be conjectured that identifying the Hurst phenomenon on a specific series provides some indication of climatic fluctuations. A number of studies have identified the Hurst phenomenon in several environmental quantities such as (to mention a few of the more recent) wind power (*Haslet & Raftery*, 1989); global mean temperatures (*Bloomfield*, 1992); flows of the Nile (*Eltahir*, 1996; *Koutsoyiannis*, 2002); flows of the river Warta, Poland (*Radziejewski & Kundzewicz*, 1997); inflows of Lake Maggiore, Italy (*Montanari et al.*, 1997); indexes of North Atlantic Oscillation (*Stephenson et al.*, 2000); and tree-ring widths, which are indicators of past climate (*Koutsoyiannis*, 2002).

Conclusively, a stochastic representation of hydrometeorological time series that respects the Hurst phenomenon is conjectured to be consistent with the varying climate hypothesis.

Basic assumptions and notation

Let X_i denote a hydrometeorological process with $i = 1, 2, \ldots$, denoting discrete time. In the context of this paper it is assumed that X_i is not periodic, which means that time scale is at least annual. Rather, it will be assumed that the process is stationary, a property that does not hinder to exhibit multiple scale variability. Further, let its mean be denoted as $\mu := E[X_i]$, its autocovariance $\gamma_j := \mathrm{Cov}[X_i, X_{i+j}]$, its autocorrelation $\rho_j := \mathrm{Corr}[X_i, X_{i+j}] = \gamma_j / \gamma_0$ ($j = 0, \pm 1, \pm 2, \ldots$), and its standard deviation $\sigma := \sqrt{\gamma_0}$.

Let k be a positive integer that represents a timescale larger than the basic timescale of the process X_i. The aggregated stochastic process on that timescale is denoted as

$$Z_i^{(k)} := \sum_{l=(i-1)k+1}^{ik} X_l \qquad (1)$$

The statistical characteristics of $Z_i^{(k)}$ for any timescale k can be derived from those of X_i. For example, the mean is

$$E[Z_i^{(k)}] = k\,\mu \qquad (2)$$

whilst the variance and autocovariance (or autocorrelation) depends on the specific structure of γ_j (or ρ_j). The hypothesis is set forward that hydrometeorological processes exhibit scale

invariant properties at any scale longer than annual, i.e.,

$$(Z^{(k)}_i - k\,\mu) \overset{\mathrm{d}}{=} \left(\frac{k}{l}\right)^H (Z^{(l)}_j - l\,\mu) \tag{3}$$

where the symbol $\overset{\mathrm{d}}{=}$ stands for equality in (finite dimensional joint) distribution and H is the Hurst exponent. Equation (3) is valid for any integer i and j (that is, the process is stationary) and any timescales k and l (≥ 1). If X_i is assumed Gaussian, this equation defines in discrete time the process known as fractional Gaussian noise (FGN), which was introduced by *Mandelbrot* (1965). (In fact, the FGN process is typically defined in continuous time; this however is not the scope here). It is easily shown (e.g., *Bras & Rodriguez-Iturbe*, 1985, p. 221) that the process defined by (3) reproduces the Hurst phenomenon. In the paper's scope, the standard name 'fractional Gaussian noise' will be avoided for several reasons: The first term, fractional, is not easily understandable, unless combined with fractals, which is not necessary, when dealing with hydrologic statistics. The second term, Gaussian, may be not appropriate for several hydrologic processes that are asymmetric (non-Gaussian); there is no need to restrict the analysis to Gaussian processes. (Techniques to produce synthetic time series respecting (3) that are not Gaussian have been studied elsewhere – *Koutsoyiannis*, 2000, 2002). The third term, noise, usually describes a random and unstructured process, which is not the case in hydrologic processes that are structured. An alternative name for the process (3) is 'stationary increments of self-similar process' (*Beran*, 1994, p. 50), which is more general but rather complicated. Here it was preferred to use the name simple scaling stochastic process or simple scaling signal (SSS).

As a consequence of (3), for $i = j = l = 1$ one obtains that the variance of the aggregated process is

$$\gamma^{(k)}_0 := \mathrm{Var}[Z^{(k)}_i] = k^{2H}\,\gamma_0 \tag{4}$$

Thus, the standard deviation $\sigma^{(k)} := (\gamma^{(k)}_0)^{1/2}$ is a power law of the scale or level of aggregation k with exponent H. The autocorrelation function, for any aggregated timescale k, is independent of k, and given by

$$\rho^{(k)}_j = \rho_j = (1/2)\,[(j+1)^{2H} + (j-1)^{2H}] - j^{2H} \approx H\,(2H-1)\,j^{2H-2}, \quad j > 0 \tag{5}$$

STATISTICAL ESTIMATION AND PREDICTION UNDER THE SSS REPRESENTATION

In this section, estimations for the most common statistics that are used in hydrologic estimation, prediction and testing will be studied under the hypothesis that the process of interest is SSS. It will be assumed that the sample is a time series of length n whose items correspond to consequent time instances, i.e., $X_1, \ldots, X_n$.

Estimation of mean

The simpler statistic estimated from a time series is the average $\overline{X}$ with standard estimator

$$\overline{X} := \frac{1}{n} \sum_{i=1}^{n} X_i \tag{6}$$

As it can be directly verified, $\overline{X}$ is an unbiased estimator regardless of the type of the process X_i, i.e., $E[\overline{X}] = \mu$. Moreover, it is very close to the best linear unbiased estimator of the process mean for SSS (*Adenstedt*, 1974; *Beran*, 1994, p. 150). In classic statistics, its variance is

$$\mathrm{Var}[\overline{X}] = \frac{\sigma^2}{n} \tag{7}$$

which, however, is not valid in SSS. Instead, observing that $\overline{X} = Z_1^{(n)}/n$ and using (4) one obtains

$$\mathrm{Var}[\overline{X}] = \frac{\sigma^2}{n^{2-2H}} \tag{8}$$

This is a known result for SSS (*Adenstedt*, 1974; *Beran*, 1994, p. 54). It is reminded that the square root of $\mathrm{Var}[\overline{X}]$ is the standard error in estimating the true mean from the observed time series. For $H = 0.5$ both (7) and (8) result in the same standard error, which is inversely proportional to the square root of the length of the time series. For large H, however, the difference of (7) and (8) becomes very significant. For example, in a time series of $n = 100$ years observations, according to the classic statistics, the standard estimation error is $\sigma/10$. However, if $H = 0.8$ the correct standard error, as given by (8), is $\sigma/2.5$, i.e. 4 times larger. To have an estimation error equal to $\sigma/10$, the required length of the time series would be 100 000 years! Obviously, this dramatic difference should induce substantial differences in other

common statistics, as well, as it will be seen in the following sections.

Estimation of variance and standard deviation for known Hurst coefficient

The classic variance estimator

$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2 \tag{9}$$

is no longer an unbiased estimator for SSS. It has been shown (*Beran*, 1994, p. 156) that a consistent SSS estimator, which becomes unbiased for known H, is,

$$\widetilde{S}^2 := \frac{n-1}{n-n^{2H-1}} S^2 = \frac{1}{n-n^{2H-1}} \sum_{i=1}^{n} (X_i - \bar{X})^2 \tag{10}$$

When $H = 0.5$, (10) becomes identical to (9). This expression has some similarity with earlier expressions that corrected the variance estimator in terms of the lag-1 autocorrelation (*Salas*, 1993).

The statistic $\widetilde{S}$ (the square root of $\widetilde{S}^2$) can then be considered as an estimator of the standard deviation σ. However, since the square root (as any nonlinear transformation) does not preserve unbiasedness, $\widetilde{S}$ is biased. A more consistent estimator (approximately unbiased for known H and for normal distribution of X_i) is

$$\widetilde{\widetilde{S}} := \sqrt{\frac{n-1/2}{n-n^{2H-1}}} \, S = \sqrt{\frac{n-1/2}{(n-1)\,(n-n^{2H-1})}} \, \sqrt{\sum_{i=1}^{n} (X_i - \bar{X})^2} \tag{11}$$

This was based on a systematic Monte Carlo study. It can be easily verified that, when $H = 0.5$, $\widetilde{\widetilde{S}} = [(n - ½) / (n - 1)]^{0.5} S$. This is a very good approximation of the true unbiased estimator, which for independent Gaussian X_i is $\widetilde{\widetilde{S}} = \{[(n-1)/2]^{0.5} \, \Gamma[(n-1)/2] / \Gamma(n/2)\} \, S$.

The variance of S for a normal distribution of X_i in the classic statistics is

$$\mathrm{Var}[S] \approx \frac{\sigma^2}{2(n-c)} \tag{12}$$

where c is taken typically 0 or 1 but, more accurately, can be taken 0.75 (the exact variance is $\{1 - [2/(n-1)]\,\Gamma^2(n/2)/\Gamma^2[(n-1)/2]\}\,\sigma^2$. In the SSS case, with the same Monte Carlo study it was found that

$$\mathrm{Var}[\widetilde{\widetilde{S}}] \approx \frac{(0.1\,n + 0.8)^{\lambda(H)}\,\sigma^2}{2\,(n-1)}, \text{ with } \lambda(H) := 0.088\,(4\,H^2 - 1)^2 \tag{13}$$

It can be verified that (13) results in higher variance than the one given by (12) (apart from

the case $H = 0.5$, when the two equations become almost identical). It is underscored that both SSS results (11) and (13) refer to normal distribution of X_i. Preliminary investigation showed that (11) can be also used for gamma distributed X_i but the variability in this case is higher than the one given by (13).

A demonstration of the consequences of using the inappropriate classic estimators of variance and standard deviation is given in *Koutsoyiannis* (2003). In brief, (9) results in underestimation of variance, which for small timescales is not serious but increases for larger scales. The increasingly underestimated standard deviation with the increase of scale k, when the classic estimator is used, has another important consequence: it obscures the presence of the Hurst phenomenon for small samples. Specifically, in a logarithmic plot of standard deviation versus scale, the slope of the curve of classic estimate is not constant, as implied by the scaling law (4), but decreases with the increase of scale k. Not only does it result in underestimation of the Hurst coefficient itself, but it may also lead to a conclusion that for large scales this slope tends to 0.5, a value indicating absence of the Hurst phenomenon.

Simultaneous estimation of variance and Hurst coefficient

When the Hurst exponent is unknown, which is the most usual case when dealing with an observed time series, (10)-(11) cannot be applied as they contain the unknown H. Therefore, a new algorithm, consistent with the SSS statistics, is proposed. This algorithm is based on classic sample estimates $s^{(k)}$ of standard deviations $\sigma^{(k)}$ for timescales k ranging from 1 to a maximum value $k' := [n / 10]$. This maximum value was chosen so that $s^{(k)}$ can be estimated from at least 10 data values.

Combining (11) and (4) and assuming $E[\tilde{\tilde{S}}] = \sigma$ one obtains

$$s^{(k)} \approx c_k(H)\, k^H \sigma \quad \text{with } c_k(H) := \sqrt{\frac{n/k - (n/k)^{2H-1}}{n/k - 1/2}} \tag{14}$$

Ignoring $c_k(H)$ in (14), one can estimate H and σ by linear regression of $\ln s^{(k)}$ on $\ln k$. A graphical depiction of this regression is always very important, as it provides visual evidence of the appropriateness of the SSS model for the time series of interest. This regression is a known algorithm (e.g. *Montanari et al.*, 1997; *Koutsoyiannis*, 2002), which, however,

introduces negative bias for both H and σ when H is high. To reduce bias, in a second trial, the estimate of H from the regression can be used to determine $c_k(H)$ and then perform another regression of $\ln s^{(k)}$ on $\ln[c_k(H)\,k]$. This procedure can be continued until convergence.

A more systematic approach can be formulated in terms of minimizing a fitting error, i.e.,

$$e^2(\sigma, H) := \sum_{k=1}^{k'} \frac{[\ln \sigma^{(k)} - \ln s^{(k)}]^2}{k^p} = \sum_{k=1}^{k'} \frac{[\ln \sigma + H \ln k + \ln c_k(H) - \ln s^{(k)}]^2}{k^p} \qquad (15)$$

where a weight equal to $1/k^p$ was assigned to the partial error of each scale k. For $p = 0$ the weights are equal whereas for $p = 1, 2, \ldots$, decreasing weights are assigned to increasing scales; this is reasonable because at larger scales the sample size is smaller and thus uncertainty larger. Using Monte Carlo experiments it was found that, although differences in estimates caused by different values of p in the range 0 to 2 are not so important, $p = 2$ results in slightly more efficient estimates (i.e., with smaller variation) and thus is preferable. From (14) and (15) it can be observed that, when σ tends to zero or infinity or when H tends to 1, $e^2(\sigma, H)$ tends to infinity. This guarantees that a global minimum exits, in which $\sigma > 0$ and $H < 1$. However, an analytical procedure to locate it is very difficult to set up. Therefore, minimization of $e^2(\sigma, H)$ should be done numerically and several numerical procedures can be devised for this purpose. A detailed iterative procedure is given in *Koutsoyiannis* (2003).

To demonstrate the performance of the algorithm, a Monte Carlo experiment has been done using an ensemble of 100 samples with $n = 50$ and 100, generated for $H = 0.80$, $\mu = 2$ and $\sigma = 0.50$. For comparison, similar series of white noise ($H = 0.50$) were also used. The resulting values of H and σ using the above algorithm are shown in Figure 4 by means of box plots. The figure shows a good performance of the algorithm for estimation of both H and σ. For comparison, the algorithm based on the regression method mentioned above, which essentially is based on the classic estimator of σ, has been also applied. As shown in Figure 4, this apparently results in underestimation of both H and σ even for $n = 100$. Finally, another comparison is performed using the traditional Hurst's algorithm based on the rescaled range for the ensemble with $n = 100$ and $H = 0.8$. Clearly, Figure 4 shows that this algorithm is

inappropriate. It exhibits a negative bias and, more importantly, the dispersion of the results is more than double that of the proposed algorithm. Notably, in a non-ignorable percentage of samples, this algorithm resulted in H greater than 1, which is mathematically inconsistent, as well as in H smaller than 0.5, which is physically inconsistent (although mathematically acceptable). On the contrary, the proposed algorithm, as discussed earlier, results in $H < 1$; in the simulation, it never resulted in H smaller than 0.5 when the true H is 0.8.

Estimation of distribution quantiles

The problem of estimation of quantiles of hydrologic variables, for certain values of probability of non-exceedance, is central to hydrologic statistics. Clearly, the above results lead to dramatic differences of these quantiles in comparison with their classic estimations.

To demonstrate this, the case of normally distributed X_i is considered, where the classic estimator of the u-quantile (the value of the variable for probability of non-exceedance u) is

$$\hat{X}_u = \bar{X} + \zeta_u S \tag{16}$$

where ζ_u is the u-quantile of the standard normal distribution. Its classic confidence limits for confidence coefficient γ are given by (e.g., *Stedinger et al.*, 1993)

$$\hat{x}_{u_{1,2}} = \hat{x}_u \pm \zeta_{(1+\gamma/2)}\, \varepsilon_u, \text{ with } \varepsilon_u = \frac{s}{\sqrt{n}}\sqrt{1 + \frac{\zeta_u^2}{2}} \tag{17}$$

For the SSS case, assuming a known H, the estimator of the u-quantile for any scale k becomes

$$\hat{Z}_u^{(k)} = k\,\bar{X} + \zeta_u\, k^H\, \widetilde{\widetilde{S}} \tag{18}$$

The confidence limits can be estimated using (8) and (13) and also assuming that, for normally distributed X_i, $\bar{X}$ and $\widetilde{\widetilde{S}}$ are independent (as in the classic case), a fact verified by Monte Carlo simulations. After algebraic manipulations, these confidence limits become

$$\hat{z}_{u_{1,2}}^{(k)} = \hat{z}_u^{(k)} \pm \zeta_{(1+\gamma/2)}\, \varepsilon_u^{(k)}, \text{ with } \varepsilon_u^{(k)} = k\frac{\widetilde{\widetilde{s}}}{n^{1-H}}\sqrt{1 + \frac{\zeta_u^2\,(0.1\,n + 0.8)^{\lambda(H)}}{2\,(k/n)^{2-2H}\,(n-1)}} \tag{19}$$

To demonstrate the difference of the classic and SSS estimators, the estimates of quantiles

from the first 10 synthetic samples of the ensemble already described in the previous subsection for $n = 100$ have been plotted (on Gauss probability paper) in Figure 5 for both cases, classic and SSS (panels (a) and (b) respectively). This figure is for the basic timescale ($k = 1$). The theoretical distribution function is also plotted along with 95% confidence limits around it that correspond to the true mean and standard deviation equal to the largest of the 10 sample standard deviations (0.59 and 0.77 for the classic and the SSS case, respectively). It is observed in Figure 5 that the confidence interval determined by the SSS estimator is 3 to 5 times wider than that of the classic estimator. Even though the largest of the 10 sample standard deviations was used to determine the confidence limits, more than half of the 10 empirical quantile curves lie outside of the 95% classic confidence limits for the probability domain shown in the graph. On the contrary, all curves lie within the 95% SSS confidence limits. This demonstrates the inappropriateness of classic estimators and the appropriateness of the SSS ones.

When both the standard deviation and the Hurst exponent are unknown, apparently the confidence intervals will be even wider. Their theoretical determination, however, is very difficult and therefore Monte Carlo methods should be the appropriate choice.

Estimation of cross-covariances and cross-correlations

Assuming that two processes X_i and Y_i are both SSS with common H and mutually correlated, it can be shown (see *Koutsoyiannis*, 2003) that: (a) the typical cross-covariance estimator is biased; (b) a consistent estimator can be formed with an expression analogous to (10); and (c) the classic estimator of the cross-correlation coefficient remains valid also for SSS. These results have been verified by Monte Carlo experiments with bivariate time series generated using the multivariate method by *Koutsoyiannis* (2000).

Estimation of auto-covariances and auto-correlations

It is known that for series with nonzero autocorrelation, the typical estimator of autocovariance is biased downward (*Salas*, 1993). It can be shown that the typical estimator of the lag l autocovariance

$$G_l := \frac{1}{n} \sum_{i=1}^{n-l} (X_i - \overline{X})(X_{i+l} - \overline{X}) \tag{20}$$

is biased for SSS processes and an approximately unbiased estimator of γ_l is

$$\widetilde{G}_l := G_l + \frac{1}{n^{2-2H}} \widetilde{S}^2 = G_l + \frac{n-1}{n^{3-2H}-n} S^2 \tag{21}$$

(a proof can be found in *Koutsoyiannis*, 2003). Consequently, a consistent estimator of the autocorrelation coefficient ρ_l will be

$$\widetilde{R}_l := \frac{\widetilde{G}_l}{\widetilde{S}^2} = R_l \left(1 - \frac{1}{n^{2-2H}} \right) + \frac{1}{n^{2-2H}} \tag{22}$$

where R_l is the classic estimator of the autocorrelation coefficient, i.e.,

$$R_l := \frac{n}{n-1} \frac{G_k}{S^2} \tag{23}$$

Both (21) and (22) are in agreement with the asymptotic results that were determined by *Hosking* (1996) for a normally distributed SSS and for $n \to \infty$.

Clearly, the classic estimate of autocorrelation is highly biased for large H. This is demonstrated in Figure 6 by means of the Monte Carlo experiment already discussed earlier. Not only are the classic estimates of autocorrelation coefficient significantly lower than the true values but they also vanish off for lags 5-10 thus obscuring the long-term persistence of the process. This may have dramatic consequences as the process may be taken as a short-memory one. On the contrary, the SSS estimator captures very well the long-term persistence of the process and agrees well with the theoretical autocorrelation function. Apart from the point estimates of autocorrelations, 90% confidence limits, estimated from simulation, have been also plotted in Figure 6. It can be observed that, when H is regarded as known ($H = 0.80$) the SSS confidence zone is as wide as the classic one but significantly shifted upward. However, when H is regarded as unknown (with different value in each sample estimated using the proposed algorithm) the SSS confidence zone becomes significantly wider. For instance, for lag 5 and for $n = 100$, the 90% confidence zone extends between 0 and 0.55. This indicates the vastly high uncertainty in estimating autocorrelograph, even for sample size as high as 100.

CASE STUDIES

In this section the three real-world time series examined before are discussed further. In Figure 7, the standard deviation of the *Jones's* (1998) proxy time series of temperature anomalies has been plotted versus timescale. Clearly, the standard deviation of the aggregated process is a power function of timescale and this is apparent even using the classic estimator of standard deviation. The exclusion of the data of the last century, which can be suspect for anthropogenic influence, does not alter the shape of the curve of standard deviation versus slope, as also shown in Figure 7. The slope of this curve in the log-log plot is much higher than 0.5. The estimate of Hurst coefficient is 0.86 using the regression method and 0.88 using the proposed method.

In Figure 8(a) the lag-1 and lag-2 autocorrelation coefficient versus scale have been plotted. The SSS model implies that these autocorrelations are independent of scale (equation (5)). The classic empirical estimates of autocorrelation verify this theoretical expectation, but not fully for very large scales (> 20), where autocorrelation decreases somehow. However, the SSS estimates agree well with the model, as they are almost constant even for scales as large as 50 years. In Figure 8(b) the autocorrelation function for the basic scale ($k = 1$) has been plotted versus lag. Due to the large length of the time series, the long-term persistence of the time series is obvious here even when the classic autocorrelation estimator is used. However, in this case the empirical autocorrelation departs from theoretical ones for lags > 40, even when the value $H = 0.86$ is used. When the SSS estimate is used along with the value $H = 0.88$, the model fits satisfactorily to the empirical autocorrelation function for lags up to 200.

In Figure 9, the point estimates and the 99% confidence limits of the quantiles of the temperature anomalies have been plotted for probability of non-exceedance u ranging from 1% to 99%, assuming a normal distribution as verified from the time series. This is done for two timescales, the basic ($k = 1$) that represents the annual variation of temperature anomaly, and the 30-year timescale, which typically is assumed to be sufficient to smooth out the annual variations and provide values representative of the climate. (For the latter, the averaged rather than aggregated time series, i.e., $z_i^{(30)}/30$, has been used). It is observed in

Figure 9 that the variation of the 30-year average is only slightly lower than that of the annual values. For example, the point estimate of the 99%-quantile of annual temperature anomaly is 0.6^{o}C above average, whereas that of the thirty-year average is 0.4^{o}C above average (which over all 992 years is not zero but rather –0.30^{o}C as in this time series the temperature anomalies are expressed as differences from the 1961-90 mean). If one considers the upper 99% confidence limits of quantiles, these values become about 0.9^{o}C and 0.7^{o}C above average, respectively. These figures indicate that an increase of the thirty-year average temperature by 0.5^{o}C, does not provide strong statistical evidence of an unusual change of climate. (It is noted that the observed temperature increase since 1850 is around 0.5^{o}C).

The results related to this example, however, contain a high degree of uncertainty due to the proxy character of the time series. This problem however, does not emerge in the next example, the Paris temperature time series. In Figure 10(a), the standard deviation of Paris temperature has been plotted versus timescale. Again, the standard deviation of the aggregated process is a power function of timescale and this is apparent even using the classic estimator of standard deviation. The Hurst coefficient, estimated using the proposed method, is 0.79. The autocorrelation coefficients also verify the presence of long-term persistence (a plot for this time series similar to that of Figure 8 can be found in *Koutsoyiannis*, 2003). Otherwise, as shown in Figure 2, there is no remarkable pattern in this time series that would require further statistical analysis.

An interesting pattern exists in the third example time series, the Boeoticos Kephisos runoff, shown in Figure 3. This is the falling trend since 1920, lasting 78 out of a total of 91 years. The characteristic plot of standard deviation versus time scale, shown in Figure 10(b), again verifies the presence of long-term persistence. As in the previous examples, the relation of standard deviation versus scale is a power law; the Hurst exponent is 0.79.

Typically, in hydrologic statistics a trend is detected using the Kendall's τ statistic (e.g. *Kottegoda*, 1980, p. 32) defined as

$$\tau := \frac{4\,p}{n\,(n-1)} - 1 \qquad (24)$$

where p is the number pairs of observations (x_j, x_i; $j > i$) in which $x_j < x_i$. In a random series τ has mean 0, variance $2(2n + 5)/9n(n − 1)$, and distribution converging rapidly to normal. In this example, the application of the test results in $\tau = 0.40$ for $n = 78$. The standard deviation of τ is 0.077, and eventually Kendall's test results in rejection of the null hypothesis that a trend does not exist for an attained significance level as low as 8.8×10^{-8}; this is typically considered as sufficient statistical evidence that a trend really exits. However, this is not correct because the time series is not a series of independent values but it exhibits SSS properties. To find the value of the standard deviation of τ, stochastic simulation can be used. Thus, an ensemble of 100 time series with $n = 78$ and $H = 0.78$ was generated, from which it was found that the standard deviation of τ is 0.173 (more than twice greater than 0.077) and the attained significance level of the Kendall's test becomes now 0.01. Still, this is not absolutely correct because it does not correspond exactly to the formulation of the null and alternative hypotheses. It is recalled that, to formulate the hypotheses to be tested, an exploratory data analysis of the complete 91-year series was done and this trend was located at 78 out of the 91 years. On the other hand, it is known that the validity of the confirmatory tests is based on the assumption that the investigator developed the hypothesis prior to examining the data (*Hirsch et al.*, 1993, p. 17.5). To re-establish the validity of the test, a different stochastic simulation that is consistent with the procedure of formulating the hypotheses was performed. Specifically, an ensemble of 100 time series was generated, each with $n = 91$ and $H = 0.78$, and in each of these series that 78-year period which gave the maximum value of τ (in absolute value) was located. Now the standard deviation of τ over the 100 series is 0.252 (more than three times greater than 0.077) and the attained significance level of the Kendall's test is 0.055. This means that the trend is not statistically significant at the 1% or even the 5% significance level.

The falling trend could be alternatively viewed as a downward jump. This jump, as shown in Figure 3, can be located between 1971 and 1972. The 65-year period before the jump and the 26-year period after the jump have averages 439.3 mm and 276.6 mm, respectively, whereas the entire 91-year average is 392.8 mm. In classic statistics, the difference 493.3 −

276.6 = 162.7 mm would be tested for being statistically significant using the typical statistical test for equality of means. Indeed, this test results in rejection of the hypothesis of equality of the means at an attained significance level as low as 8.2×10^{-6}. This typical test, however, is incorrect because it was based on the classic statistics. To perform a more accurate test, stochastic simulation was performed. In each of the 100 generated 91-year long synthetic series already discussed in the previous paragraph the 26-year period with the minimum average was located and the difference from the average of the entire 91-year series was taken. Then, the probability that this difference exceeds 392.8 – 276.6 = 116.2 mm was determined, which is as high as 22%. This probability is the attained significance level of the test and this means that the hypothesis of equality of means is not rejected at the usual significant levels.

In conclusion no statistically significant trend or jump is detected in the Boeoticos Kephisos runoff time series. The above statistical analysis defeats earlier analyses of the same time series (e.g. *Nalbantis et al.*, 1993), which detected statistically significant trends or jumps using classic statistics.

SUMMARY, CONCLUSIONS AND DISCUSSION

According to the experts of climate modelling, "the current state of affairs is not satisfactory" (*Barnett et al.*, 1999) in terms of prediction capabilities of the climate evolution and quantification of the related uncertainty. Moreover, the unpredictability of future climate in deterministic terms may by a structural characteristic of the climate system (rather than a matter of current weaknesses of models) since, according to *von Storch et al.* (2001), "climate must be considered as a stochastic system".

Therefore, probability-based methods may be good alternatives to quantify uncertainty, even under a varying climate. However, hydrologic statistics, the branch of hydrology that deals with uncertainty, has been based on the implicit assumption of a stable climate. This disagrees with the fact that climate has ever, through the planet history, changed irregularly on all time scales, as witnessed by long hydroclimatic time series. Observed shifts in such

time series were often regarded as deterministic components (trends or jumps) and removed from the time series so that the residual can be processed using classic statistics. This would be an efficient approach if a deterministic model existed, which could explain these components and also predict their future. This, however, is hardly the case, as most typically the trends or shifts are identified only *a posteriori* and expressed mathematically by equations lacking physical meaning (e.g. using linear regression) and thus applicable only to the relevant parts of the time series and not in their future evolution. A more consistent alternative is to approach this behaviour in a stochastic manner. A stochastic basis for dealing with these shifts and trends is offered by simple scaling processes that are consistent with the assumption of hydroclimatic fluctuations on multiple timescales, a behaviour that is none other than the Hurst phenomenon. The Hurst coefficient is the exponent of the power-law relationship between the aggregated standard deviation at any timescale and the scale length.

When the simple scaling hypothesis is employed and the typical statistical descriptors used in hydrologic statistics are revisited under this hypothesis it is found out that:

1. the classic sample average remains an unbiased estimator of the true mean, but its variance, which expresses the uncertainty of its estimation, is dramatically higher than the value given by the classic statistics;

2. the classic estimators of variance and standard deviation are no more unbiased and the variance thereof is larger than that of the classic statistics;

3. the quantiles of a given distribution function differ from those of the classic statistics and their confidence intervals are radically wider than those implied by the classic statistics;

4. the classic estimators of cross-correlations between two variables remain almost unbiased, but those of autocorrelations are highly biased downward; the confidence intervals of the latter are much wider than those implied by the classic statistics.

For all above statistical descriptors, generalized estimators exist which are unbiased or almost unbiased under the simple scaling hypothesis. These estimators depend on the Hurst exponent H, in addition to the other dependencies used in classic statistics. The classic

estimators are derived as special cases of the generalized estimators when $H = 0.5$. The Hurst exponent itself has been considered as another statistic and an algorithm has been developed to estimate it, simultaneously with standard deviation, in an approximately unbiased manner.

The application of the developed statistical framework to three hydrometeorological time series with lengths ranging from 91 to 992 years showed that all three series are consistent with the scaling hypothesis (Hurst phenomenon). In addition, it is shown that several patterns within these times series would be regarded as evident trends or shifts if classic statistical tests were used, but using modified tests, based on the scaling hypothesis, it turns out that they are regular behaviour of the time series.

Apparently, the consistency of geophysical time series with the scaling hypothesis is not exhausted to the three time series analysed in this paper. In several studies, a large number of long geophysical time series have been found to exhibit the Hurst phenomenon. The analyses of this paper show that in time series with short length the classic statistics can hide the scaling behaviour. This concerns the Hurst exponent and the autocorrelation function (Figure 6), whose classic estimate hides a fat tail. Therefore, it can be the case that short time series, classified as random noise without scaling behaviour, in fact exhibit the Hurst phenomenon.

In conclusion, the analyses of this paper attempt to cast a warning that the classic hydrologic statistics describes only a portion of the natural uncertainty of hydroclimatic processes, because it is based on the implicit assumption of a stable climate. In addition, its use may characterize a regular behaviour of hydroclimatic processes as unusual phenomenon. Furthermore, the analyses show that it is feasible to adapt the classic hydrologic statistics so as to quantify (but, not to reduce) the total uncertainty under a varying climate. In some cases, the adapted estimators are based on simplifying assumptions, like the assumption of normal distribution of the process of interest. It may be difficult to derive closed analytical estimators appropriate for any type of distribution and for every statistic involved in hydrologic estimation and testing. However, stochastic simulation provides an easy and directly applicable means for problems that cannot be solved analytically, as demonstrated in the examples of hypothesis testing presented here. Obviously, further and more detailed analyses

of several related issues of hydrologic statistics, and investigations of a large number of data sets, are needed before a concrete base of methodologies, appropriate for different types of water resources problems, can be established.

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FIGURES
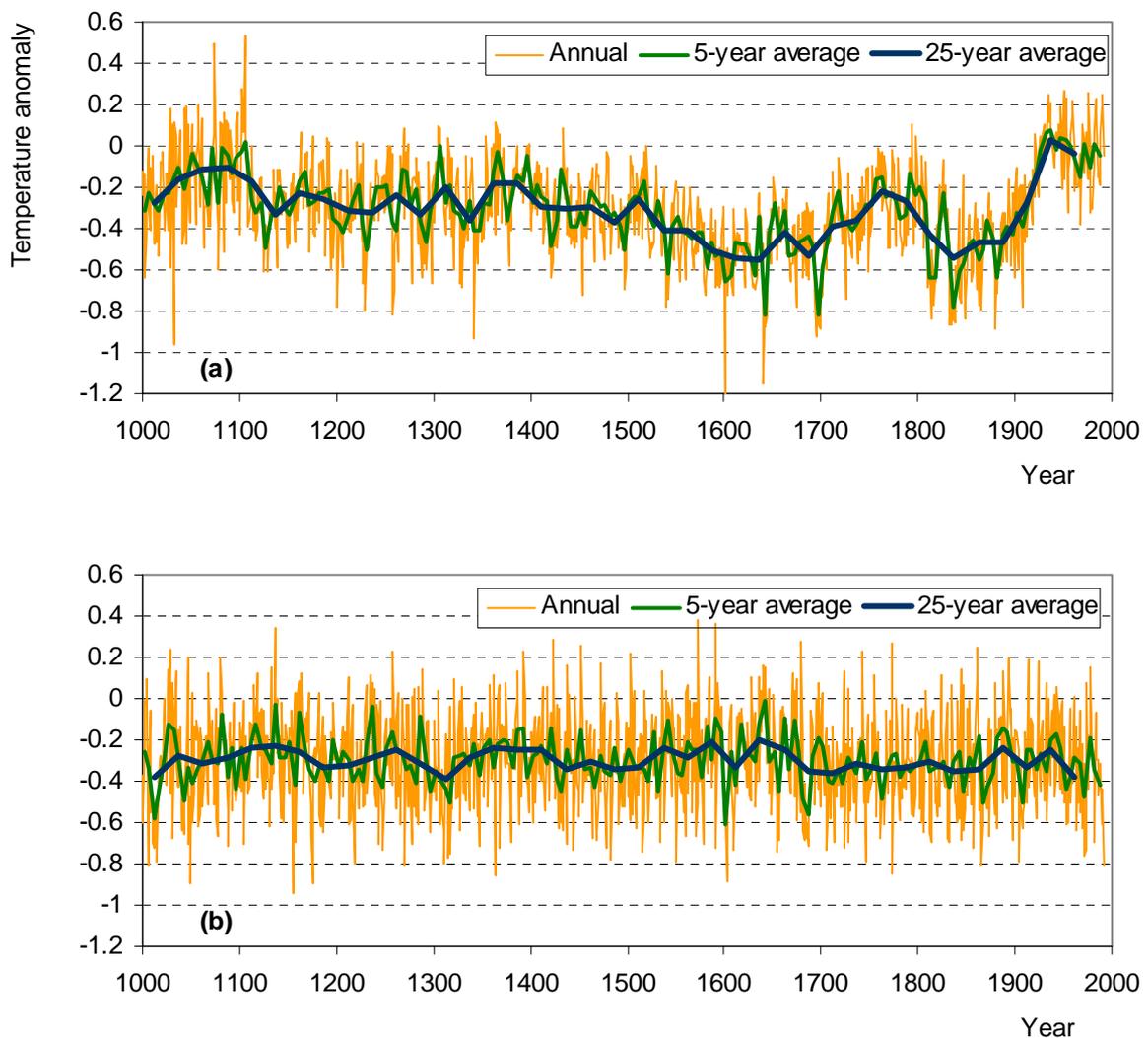
Figure 1 (a) Plot of the North Hemisphere temperature anomalies, reconstructed by *Jones et al.* (1998) using proxy data; (b) for comparison, a series of white noise with mean and standard deviation equal to those of the original series is also plotted.
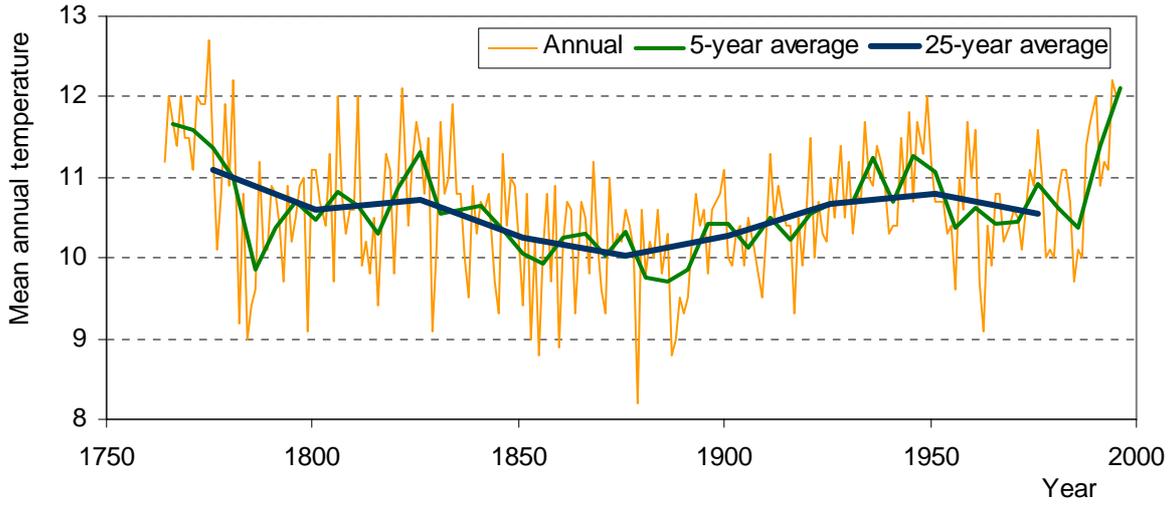
Figure 2 Plot of the time series of mean annual temperature at Paris/Le Bourget.
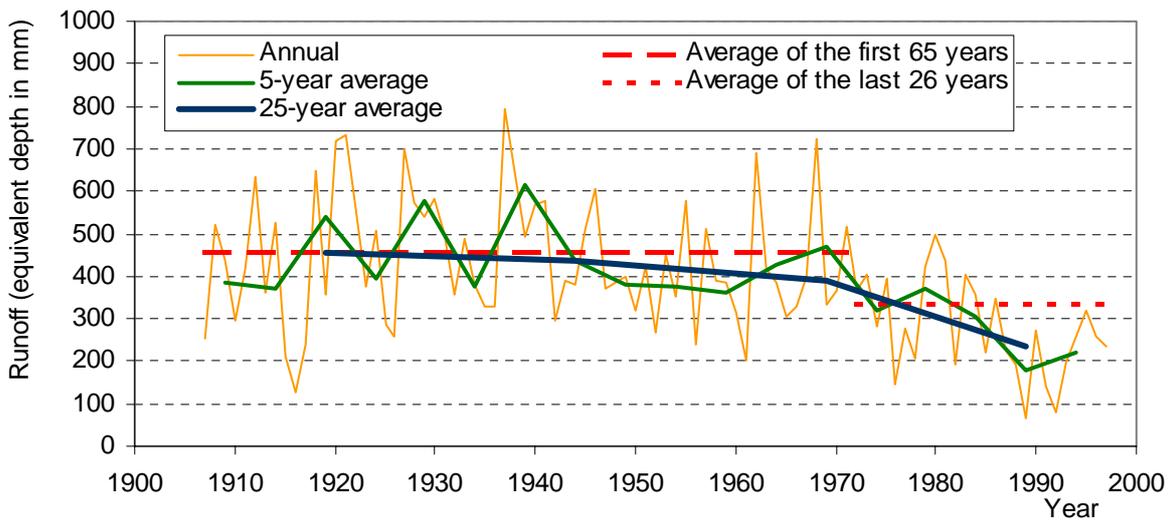
Figure 3 Plot of the time series of the equivalent runoff depth of the Boeotikos Kephisos river basin, Greece.
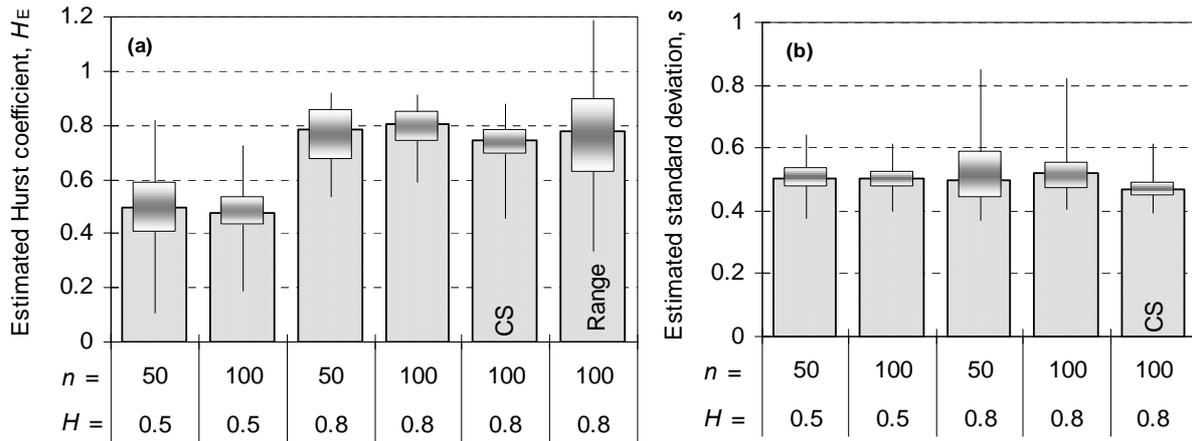
Figure 4 Box plots of the estimated Hurst coefficients (a) and standard deviations (b) from ensembles of synthetic series with theoretical $H = 0.5$ or 0.8 and $\sigma = 0.5$. Bars correspond to the median of 100 estimations (from 100 synthetic series), upper and lower edges of boxes correspond to the 75%- and 25%-quantiles, respectively, and whiskers correspond to the maximum and minimum estimated values. The first four boxes correspond to the proposed method, the fifth boxes (annotated as CS) correspond to estimates using classic statistics, and the sixth box in panel (a) (annotated as Range) corresponds to the estimation using the original Hurst's algorithm based on the rescaled range.
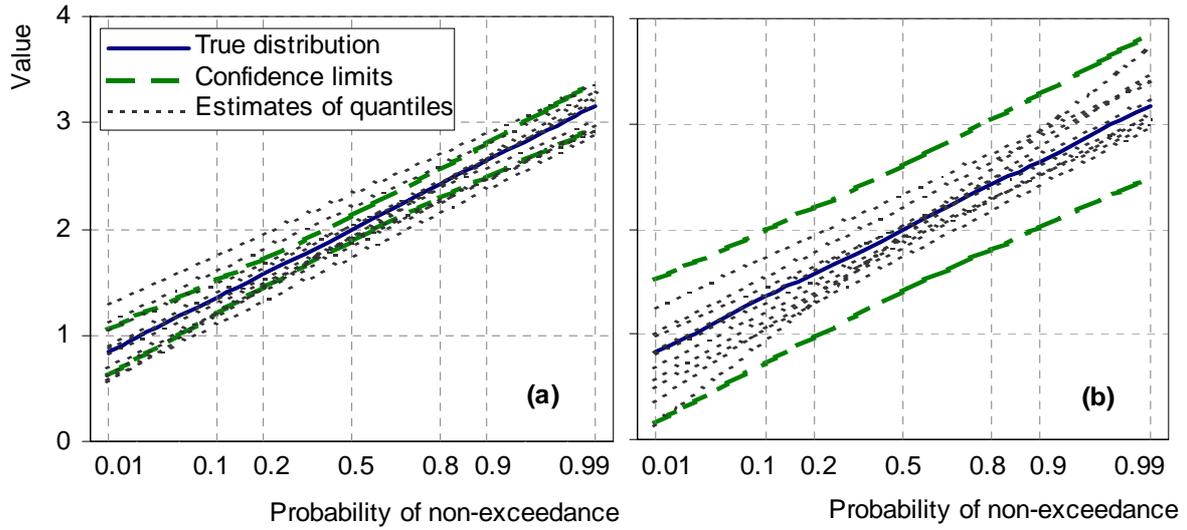
Figure 5 Estimates of quantiles at the basic timescale ($k = 1$) from 10 synthetic time series with length $n = 100$ generated with theoretical parameters $H = 0.8$, $\mu = 2$ and $\sigma = 0.5$, in comparison with the true distribution and 95% confidence limits: (a) using classic statistics; (b) using SSS statistics.
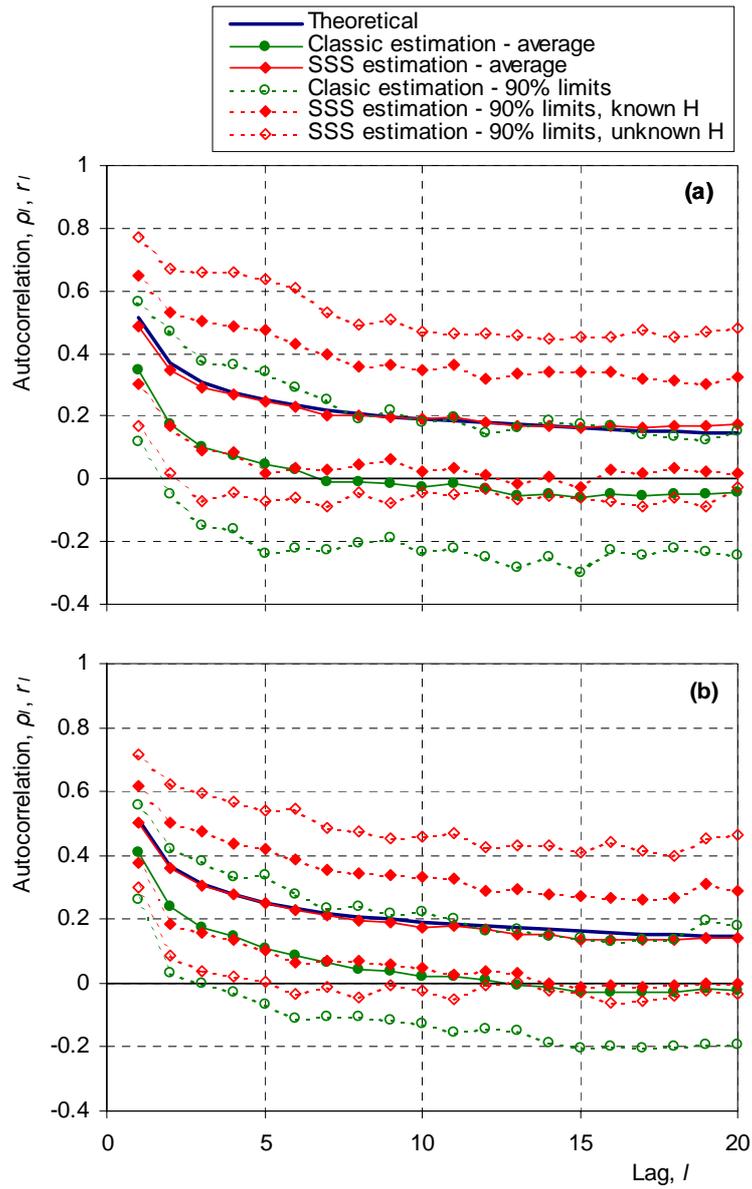
Figure 6 Comparison of theoretical and empirical autocorrelation functions at the basic scale (k = 1) for a Monte Carlo experiment with theoretical H = 0.8 and σ = 0.5: (a) n = 50; (a) n = 100.
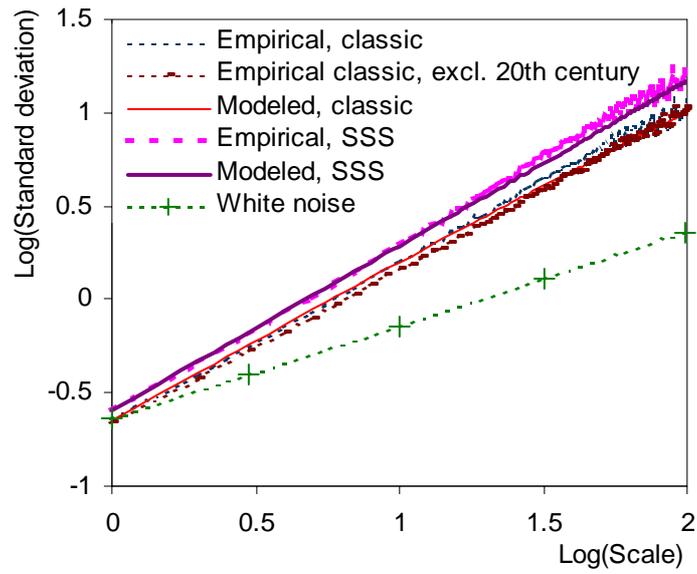
Figure 7 Standard deviation of the aggregated processes versus timescale (logarithmic plot) for the Jones's time series of the North Hemisphere temperature anomalies. For comparison, the theoretical curve of the white noise model is also plotted.
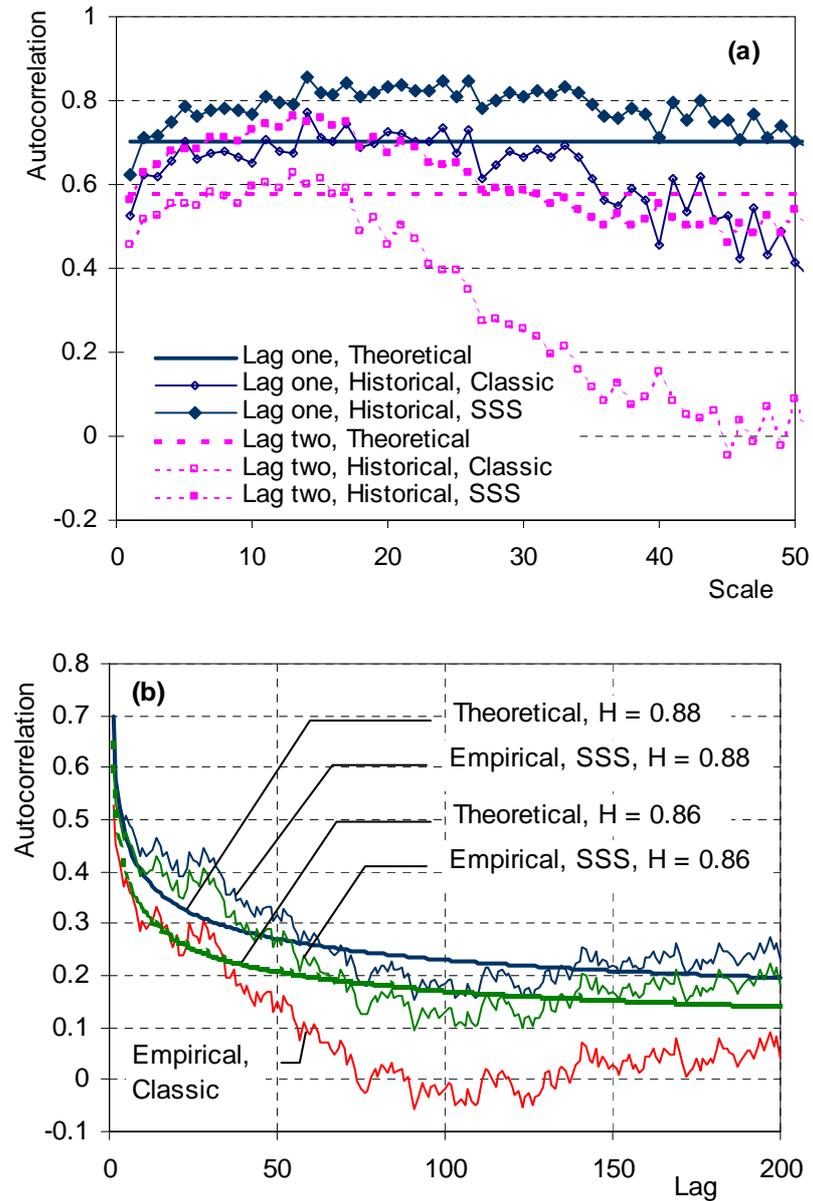
Figure 8 Autocorrelation coefficients of the Jones's time series of the North Hemisphere temperature anomalies: (a) lag 1 and lag 2 autocorrelations of the aggregated process versus timescale, k; (b) autocorrelation versus lag for the basic timescale, $k = 1$.
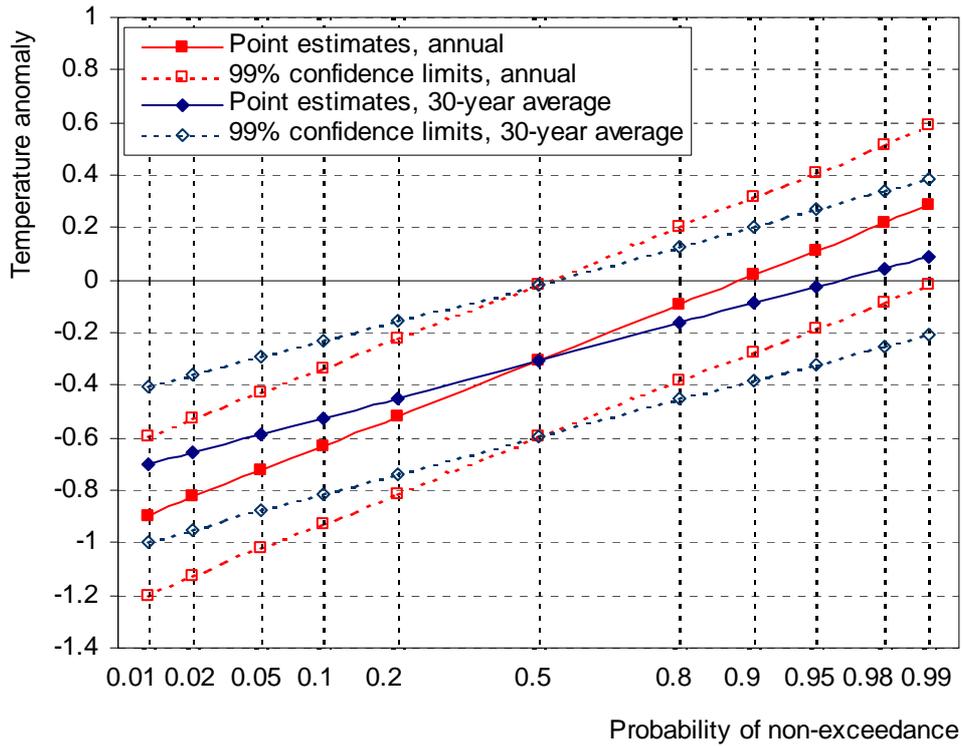
Figure 9 Point estimates of quantiles at the basic timescale (annual values, $k = 1$) and the 30-year timescale (30-year averages, $k = 30$), and 99% confidence limits thereof for the Jones's time series of the North Hemisphere temperature anomalies.
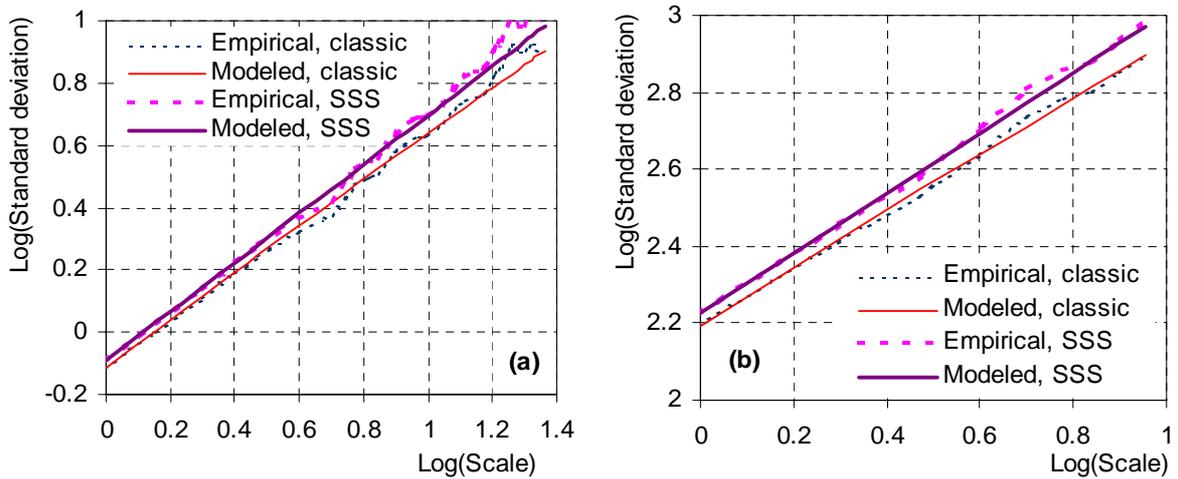
Figure 10 Standard deviation of the aggregated processes versus timescale (logarithmic plot) for the time series of (a) mean annual temperature at Paris/Le Bourget; (b) runoff time series of the Boeotikos Kephisos river basin.