An attempt for stochastic forecasting of rainfall

(using weather types and the scaling model)

Presentation at the 4th meeting of the AFORISM project

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Parts of the presentation

1. Data presentation
2. Classification and analysis of intense rainfall events by weather type
3. Fitting of the scaling model to historical data
4. Application of the model for conditional simulation of future rainfall depths
5. Concluding remarks - Future study

Data presentation

Station

Rain recording station at Krikello, Evinos River Basin (at the border)
20 years of continuous operation (with few gaps)

Selection of intense rainfall events

Daily depth ≥ 25 mm or Hourly depth ≥ 7 mm

Data set

Total number of events: 293
At rainy season (Oct-Apr): 200
At dry season (May-Sep): 93

Classification and analysis of intense rainfall events by weather type

Definition and classification of weather types in Greece (by Maheras)

Classification by weather type (continued)

Analysis tools

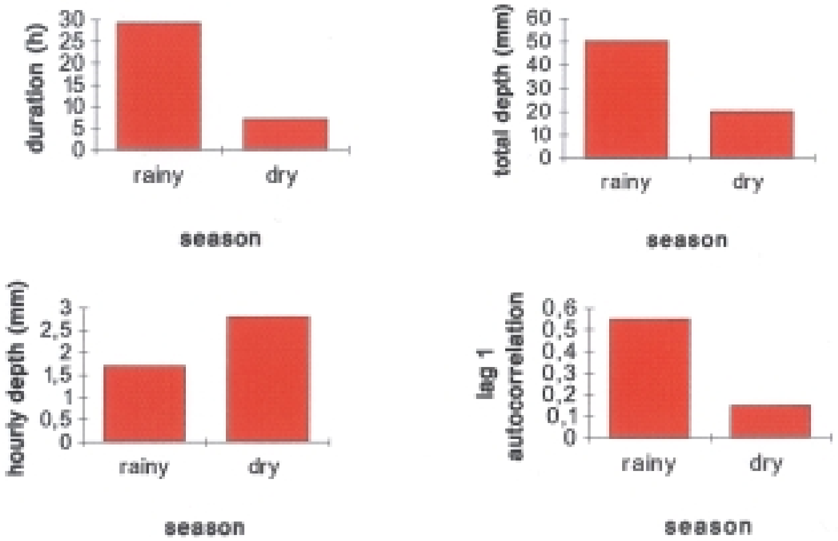
1. Weather maps at surface and 500 mb level, produced by
   a) National Meteorological Service of Greece (1970-1979)
   b) German Meteorological Service (1980-1990)
2. P. Maheras: Personal diary of weather types of Greece in daily basis since 1950 (1992)

Unidentified meteorological characteristics (to be studied in future)

1. Action of fronts (warm or cold)
2. Local meteorological data

Classification by weather type (continued)

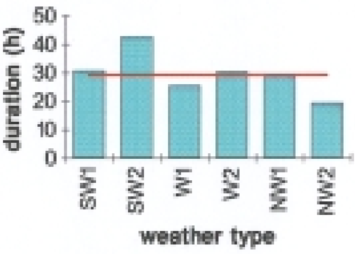
Comparison of rainfall characteristics of the two seasons

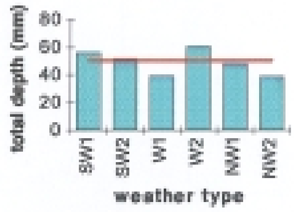


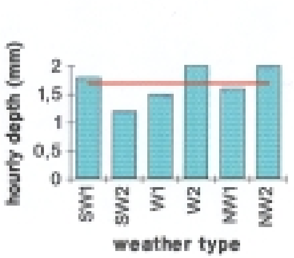
Conclusion: Statistically significant differences between two seasons

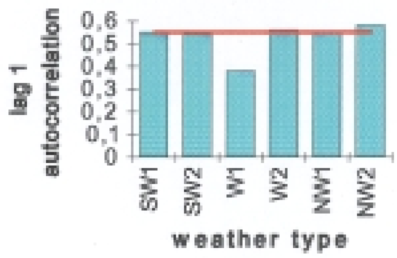
Classification by weather type (continued)

Comparison of rainfall characteristics of each weather type for rainy season





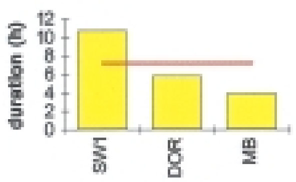


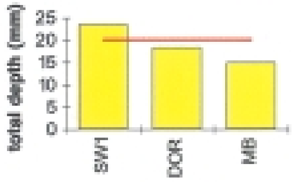


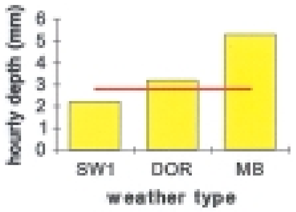
Conclusion: Few significant differences (SW2, NW2) but essentially unimportant.

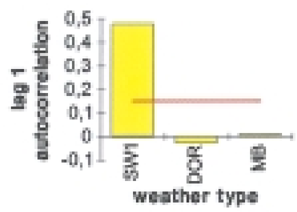
Classification by weather type (continued)

Comparison of rainfall characteristics of each weather type for dry season





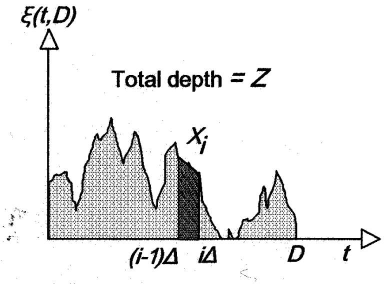




Conclusion: Statistically significant differences. The differences in duration and a scaling assumption can explain the differences in the other characteristics.

Fitting of the scaling model

General structure



$$\{\xi(t,D)\} \overset{d}{=} \{\lambda^{-H}\xi(\lambda t, \lambda D)\}$$

where $\xi()$: instantaneous rainfall intensity
D: duration of the event
t: time ($0 \le t \le D$)
H: scaling exponent

$$E[\xi(t,D)] = c_1 D^H$$
$$Cov[\xi(t,D)\xi(t+\tau,D)] = \left(\varphi(\tau / D) - c_1^2\right)D^{2H}$$
$$\varphi(y) = \tfrac{1}{2}(c_2 + c_1^2)(1-\beta)(2-\beta)\; y^{-\beta}$$

Fitting of the scaling model (continued)

Statistics of incremental and total depths

$$\mathrm{E}[X_i] = c_1 \delta D^{H+1}$$
$$\mathrm{Var}[X_i] = \left[(c_2 + c_1^2)\delta^{-\beta} - c_1^2\right]\delta^2 D^{2(H+1)}$$
$$\mathrm{Cov}\left[X_i, X_j\right] = \left[(c_2 + c_1^2)\delta^{-\beta} f(|j-i|, \beta) - c_1^2\right]\delta^2 D^{2(H+1)}$$
$$\mathrm{E}[Z] = c_1 D^{H+1}$$
$$\mathrm{Var}[Z] = c_2 D^{2(H+1)}$$

where

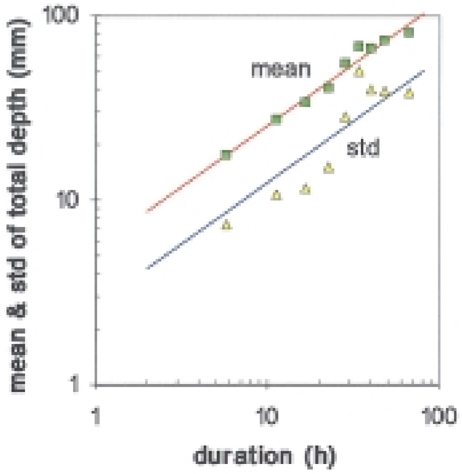
$$\delta = \Delta / D$$
$$f(m,\beta) = \tfrac{1}{2}\left[(m-1)^{2-\beta} + (m+1)^{2-\beta}\right] - m^{2-\beta}, \quad m > 0$$

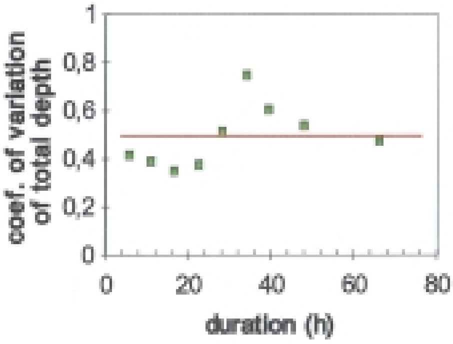
Parameters

c_1:	mean value parameter	rainy season:	5.45	dry season:	9.97
c_2:	variance parameter		7.28		18.26
β:	correlation decay parameter		0.27		0.51
H:	scaling exponent		-0.332		-0.604

Fitting of the scaling model (continued)

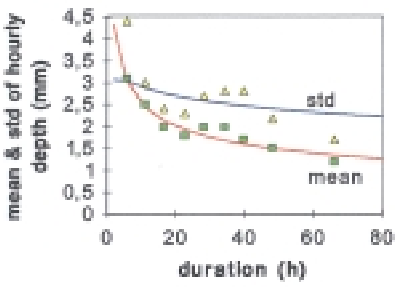
Rainy period - Total rainfall depth

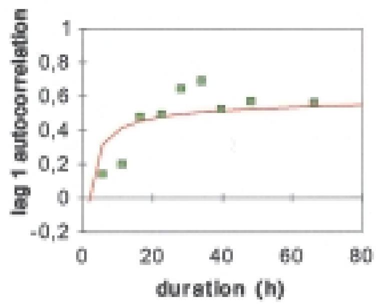


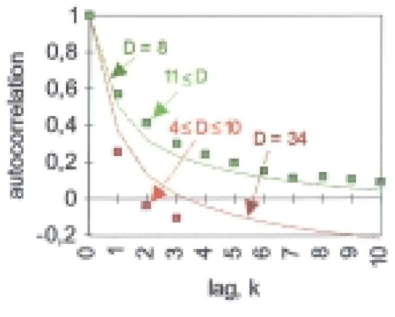


Fitting of the scaling model (continued)

Rainy period - Hourly rainfall depth

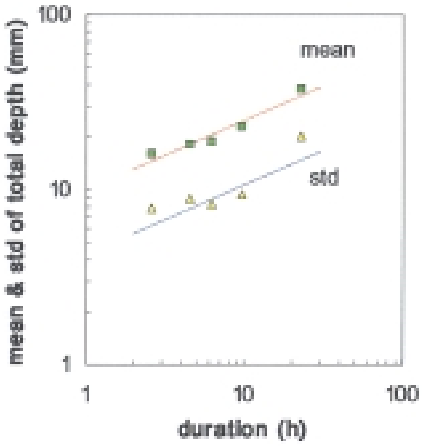


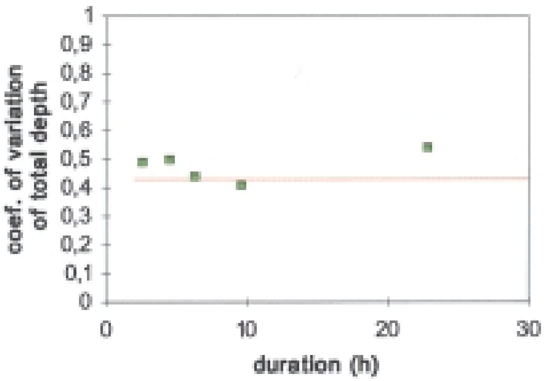




Fitting of the scaling model (continued)

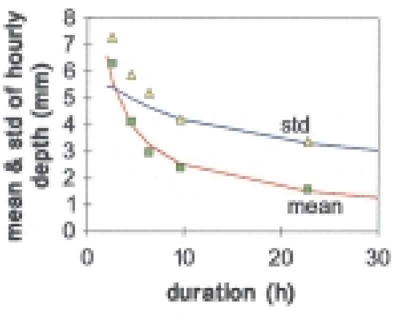
Dry period - Total rainfall depth

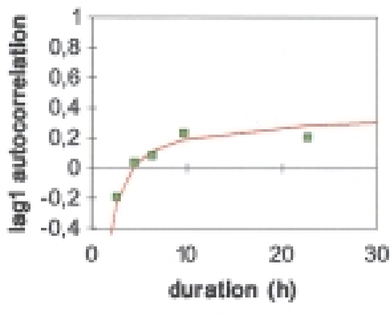


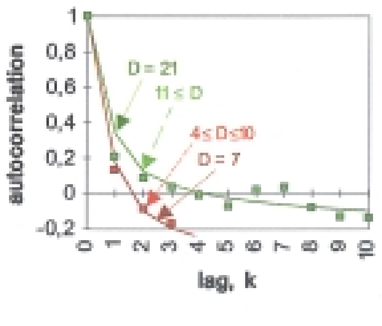


Fitting of the scaling model (continued)

Dry period - Hourly rainfall depth







Application of the model for conditional simulation of future rainfall depths

Overview of the generating scheme

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_k \end{bmatrix} = \begin{bmatrix} \omega_{11} & 0 & \cdots & 0 \\ \omega_{21} & \omega_{22} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{k1} & \omega_{k2} & \cdots & \omega_{kk} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_k \end{bmatrix} \text{ or } \mathbf{X} = \mathbf{\Omega V} \; (V_i \text{ independent, appr. 3-par. gamma})$$

Parameter estimation

$\boldsymbol{\Omega\Omega}^{\mathrm{T}} = \mathrm{Cov}[\boldsymbol{X}, \boldsymbol{X}] \Rightarrow \boldsymbol{\Omega}$ by decomposition(lower triangular)

$$\omega_{ii}\, \mathrm{E}[V_i] = \mathrm{E}[X_i] - \sum_{l=1}^{i-1} \omega_{il}\, \mathrm{E}[V_l]$$

$$\mathrm{Var}[V_i] = 1$$

$$\omega_{ii}^3\, \mu_3[V_i] = \mu_3[X_i] - \sum_{l=1}^{i-1} \omega_{il}^3\, \mu_3[V_l]$$

Application of the model (continued)

Link to the rainfall model

Inputs to the simulation model: $\mathrm{E}[\boldsymbol{X}]$, $\mathrm{Cov}[\boldsymbol{X}, \boldsymbol{X}]$, $\mu_3[\boldsymbol{X}]$

Link to the past of the simulated hyetograph

At time step k: $X_1 = x_1$, $X_2 = x_2$, ..., $X_k = x_k$

Generation steps

1. Generation of total duration D (from the conditional distribution)
2. Generation of sequential hourly depths X_j for $j = k + 1$, ..., D.

Examined cases

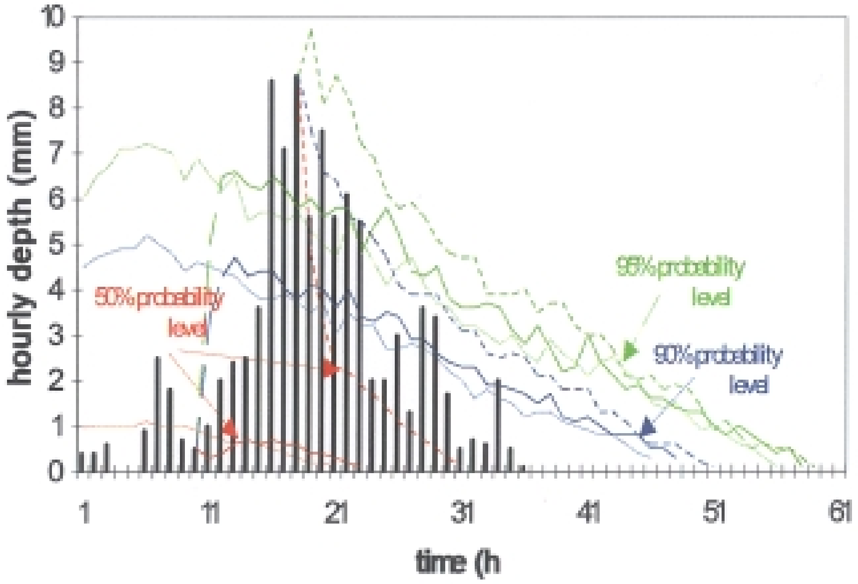
1. Condition for duration: $D > k$.
2. An estimation of the total duration is assumed, i.e. $D_l \leq D \leq D_h$
3. Adaptive simulation (one hour forecasting). At each time step it is assumed that the list of known hourly depths is updated. Duration as in case 1.

Simulated hyetographs

1000 simulated hyetographs for each event

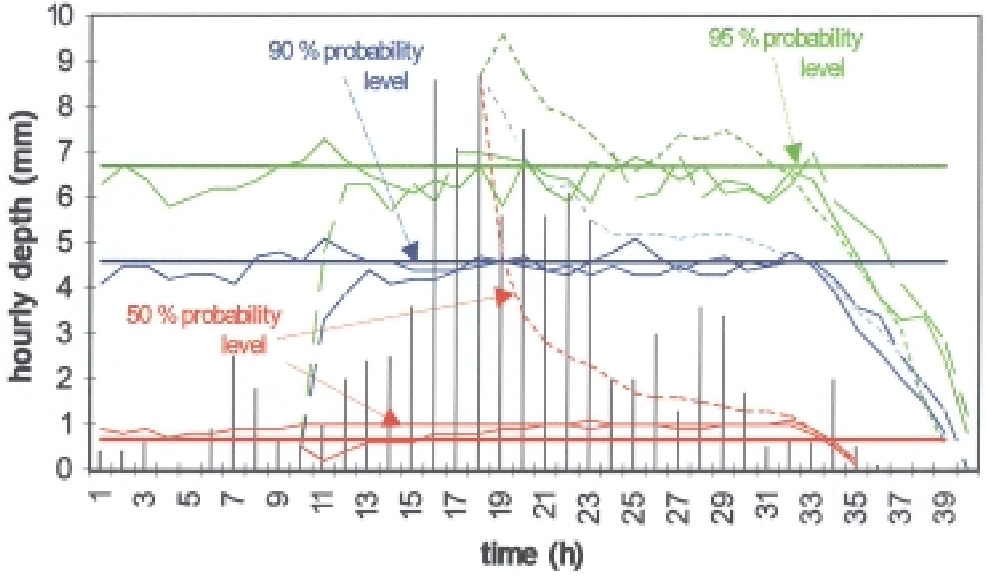
Application of the model (continued)

Case 1 ($D > k$)



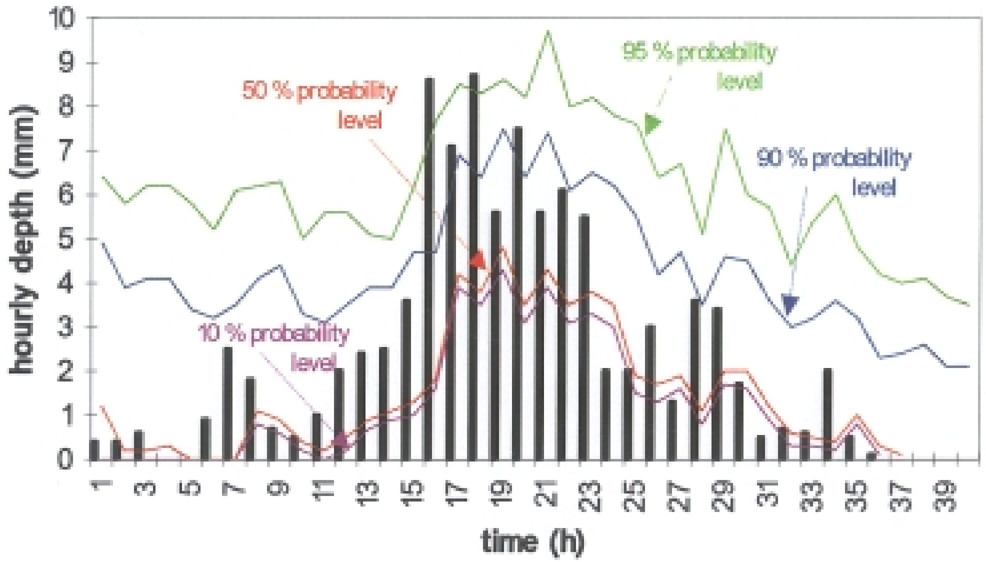
Application of the model (continued)

Case 2 ($D_l \leq D \leq D_h$, where $|D_{l,h} - D| = 0.1\ D$)



Application of the model (continued)

Case 3 (Adaptive simulation)



Concluding remarks - Future study

1. Events of different weather types do not have notable differences in their stochastic structure. Thus, the introduction of weather types does not facilitate the stochastic modelling of rainfall.
2. The main essential difference detected is due to seasonal variability.
3. Future analysis of weather types will include other weather characteristics such as front action and local meteorological data as an attempt to explain a portion of the high variance of the rainfall depths.
4. The scaling rainfall model fits the intense rainfall data in both seasons and thus, can provide a basis for modelling of intense rainfall events.
5. Due to the high coefficient of variation (> 1) and the relevantly low autocorrelation function of hourly depths, the stochastic forecasting of the evolution of the rainfall process is impractical. However, an one-hour forecasting by stochastic methods may be feasible.