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A Simple Theoretical Model for the Mean Rainfall Field of Tropical Cyclones

Andreas Langousis and Daniele Veneziano

(andlag@mit.edu) (venezian@mit.edu)

Department of Civil and Environmental Engineering, M.I.T., Cambridge Mass., U.S.A.

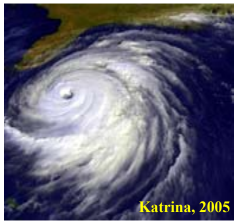
Objectives

Develop a simple theoretical approximation for the mean rainfall field of tropical cyclones (TCs) and study how this mean rainfall field depends on TC :

- Maximum tangential wind velocity V_{max}
- Radius of maximum winds R_{max}
- Holland's B parameter of the tangential wind profile
- Translation velocity **V**

1. Basic structure of tropical cyclones

Tropical cyclones (TCs) are a class of low pressure rotating systems that develop over tropical and subtropical waters. These systems have a non-frontal core, well organized convection and cyclonic surface wind circulation.



Flow regions

I. **Boundary layer:** Surface stresses cause radial influx and low level convergence.

II. **Main vortex:** Tangential winds are approximately in gradient-wind balance => *Negligible radial flux*.

III. **Subsidence region:** The inward directed pressure gradient force decreases with height leading to high-level divergence.

IV. **Eye:** Downward directed flux of relatively dry air originated from region III.

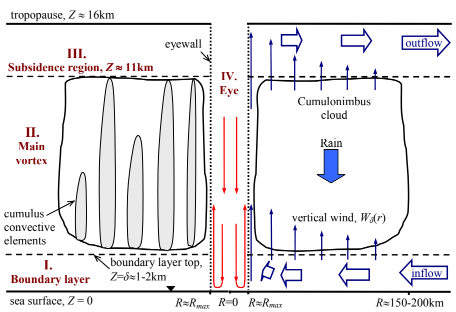


Figure 1: Schematic structure of a mature hurricane.

Rainfall is contributed mainly by elevations between 2 and 6km.

Focus on regions I and II should suffice to model the mean rainfall field.

2. Methodology (axi-symmetric component)

Step 1: Use a *parametric* model for the tangential winds in the main vortex.

$$V(R) = V_{max}\sqrt{(R_{max}/R)^B \exp[1-(R_{max}/R)^B]}$$ **Holland (1980)**

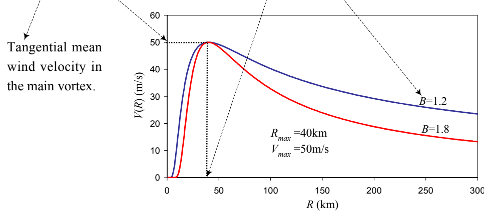


Figure 2: Holland's (1980) model for two different values of the shape parameter *B*.

Step 2: Use Smith's (1968) *theoretical* model to describe the *radial* and *tangential* fluxes inside the boundary layer.

Step 3: Integrate the continuity equation in the vertical direction to obtain the vertical velocity $W_\delta(R)$ at the top of the boundary layer.

Step 4: Assume that the *upward water vapor mass flux* at the top of the boundary layer *equals* the *downward rainwater flux*:

$$i(R) = \frac{\rho_{air}}{\rho_w} q_w(T)\, W_\delta(R) \qquad (1)$$

Rainfall intensity; Dry air density; Liquid water density; Water vapor mixing ratio (≈10-15gr/kgr); Depth averaged temperature (≈20°C); Vertical wind velocity

Validation of step 4 using MM5

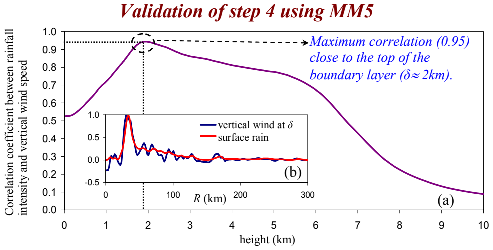


Figure 3: (a) Correlation of vertical wind velocity at different elevations and surface rainfall intensity for hurricane Frances (2004) using MM5. (b) Radial profiles of simulated surface rainfall intensity and vertical wind velocity at the elevation of maximum correlation (2km). Both profiles are normalized to have maximum value equal to 1.

Given $W_\delta(R)$, the proportionality model in equation (1) has accuracy similar to MM5.

3. Sensitivity analysis

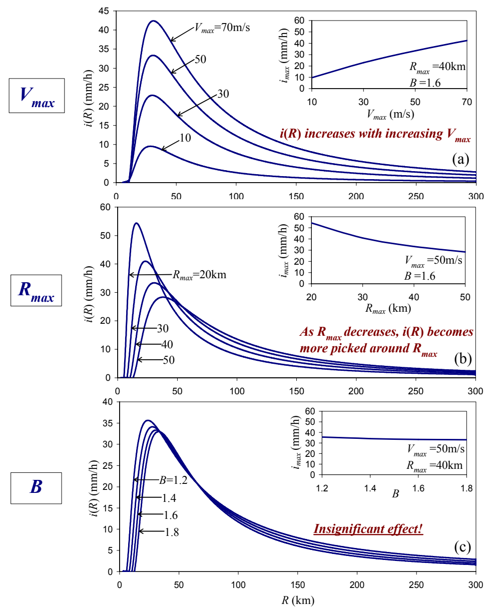


Figure 4: The effect of V_{max}, R_{max} and B on the radial distribution of rainfall intensity $i(R)$.

4. Validation (Hurricane Frances, 2004)

- Fit Holland's model to flight-level tangential wind observations.
- Apply steps 2-4. Use q_w =11gr/kgr (T=20°C, saturation ratio=0.9). See **Figures 5** and **6**.

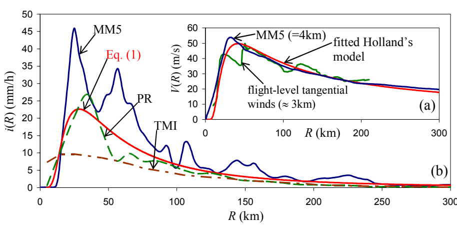


Figure 5: Comparison of model estimates of tangential winds $V(R)$ and surface rainfall intensity $i(R)$ with observed values and MM5 calculations for hurricane Frances (30 August 2004, 18:00 UTC).

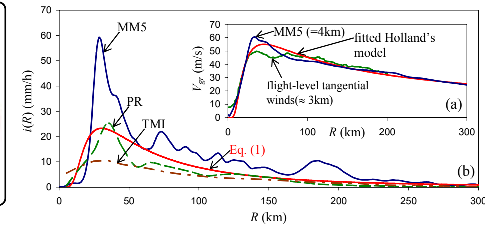


Figure 6: Same as Figure 5 for 02 September 2004, 19:00 UTC.

- Especially close to the TC center, TMI estimates under-predict surface rainfall intensity and should not be considered accurate.
- MM5 overpredicts rainfall (*need for calibration*….).
- The proposed approximation produces estimates of the azimuthal average rainfall intensity $i(R)$ close to the PR data from TRMM.

5. Extensions

Effect of motion on rainfall asymmetry

- Use Kepert's (2001) linearized boundary layer model to estimate the effect of motion on $W_\delta(R,\theta)$, where θ is the azimuth relative to the direction of motion.
- Estimate $i(R,\theta)$ accounting for the effects of *upward spiral motion of moist air*, *water vapor condensation* and *downward spiral motion of rain*.

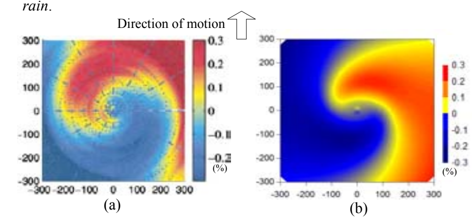


Figure 7: (a) Average rainfall asymmetry in the North Indian Oceanic Basin (Lonfat *et al.*, 2004), (b) simulated rainfall asymetry for V_{max}=50m/s, R_{max} =40km, *B*=1.5, and translation velocity V=4m/s.

Future extensions should account for vertical wind shear and the effect of landfall.

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