

Department of Civil and Environmental Engineering
Massachusetts Institute of Technology

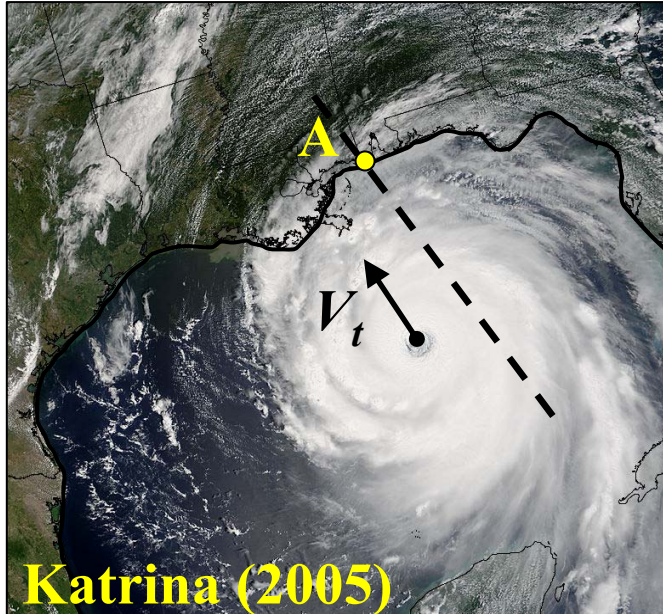
*Extreme Rainfall Intensities and
Long-term Rainfall Risk from
Tropical Cyclones*

By
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Objective

Long-term rainfall risk from TCs at a given location A:



$\lambda_D(i)$: rate at which $I_{max}(D)$ exceeds i at location A (events/year)

$I_{max}(D)$: maximum rainfall intensity at location A for averaging duration D

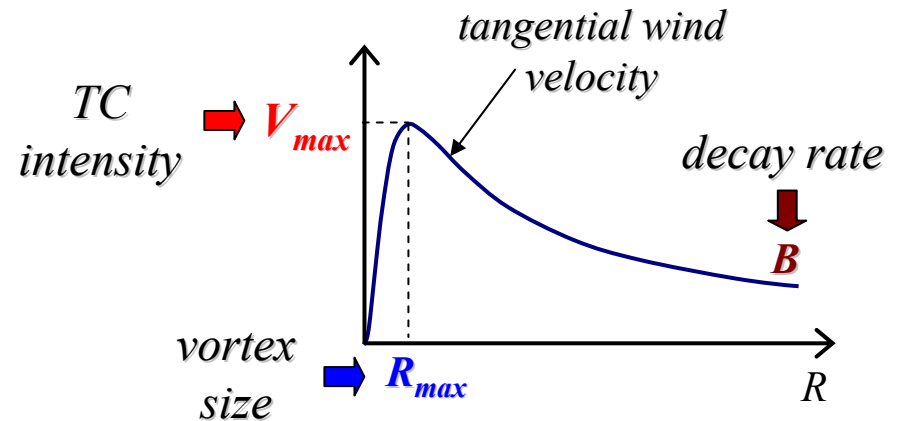
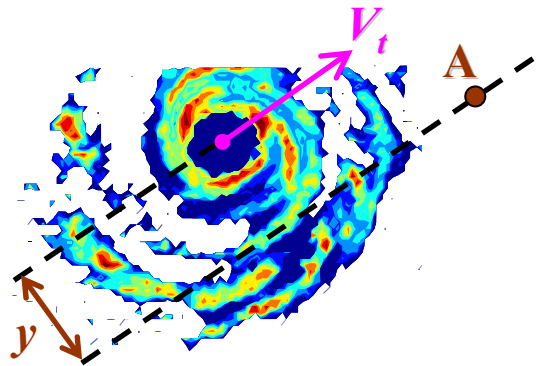
Risk analysis $\Rightarrow \lambda_D(i) = \lambda \int_{\text{all } \omega} P[I_{max}(D) > i | \omega] P[\omega] d\omega$

λ : TC arrival rate [events/yr]
 $P[I_{max}(D) > i | \omega]$: probability that $I_{max}(D)$ exceeds i in a TC with characteristics ω
 $P[\omega]$: probability density of TC characteristics ω

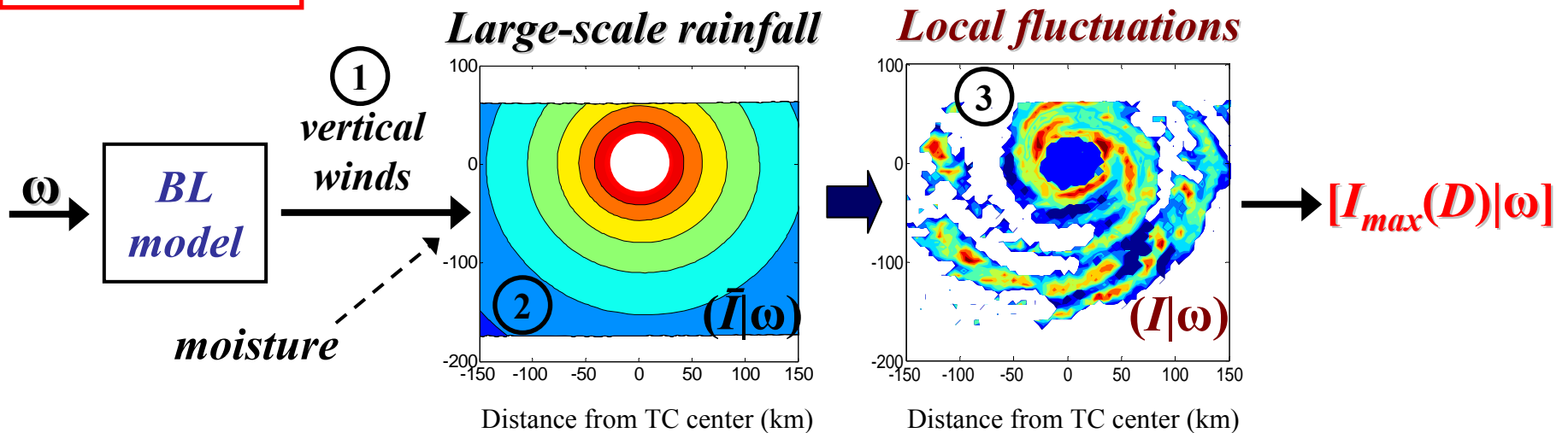
focus (bracketed over $P[I_{max}(D) > i | \omega]$)
local recurrence (literature) (bracketed over $P[\omega]$)

Approach to long-term risk modeling

➤ parameters $\omega = [V_{max}, R_{max}, B, V_t, y]^T$



$$[I_{max}(D)|\omega]$$



Outline

Part 1: BL winds

- Existing BL models \Rightarrow *limitations*
- **MS** model for the BL
- Wind fields: comparison with **MM5**

Part 2: Rainfall from Winds:

- **MS** + *moisture* \Rightarrow **MSR** model
- Validation using **MM5**
- Calibration using **PR** data
- Rainfall **asymmetry** due to motion

Part 3: Rainfall fluctuations

- **Storm-to-storm** variability of rainfall
(*large scales*)
 - **Sub-storm** variability
(*small scales*)
- } 2 alternative approaches

Part 4: Application to New Orleans

- **Recurrence** model for ω
- Theoretical IDF curves for TC rainfall
- Comparison with empirical IDF results on all rainstorms (TCs and non-TCs)
- Design storms for New Orleans

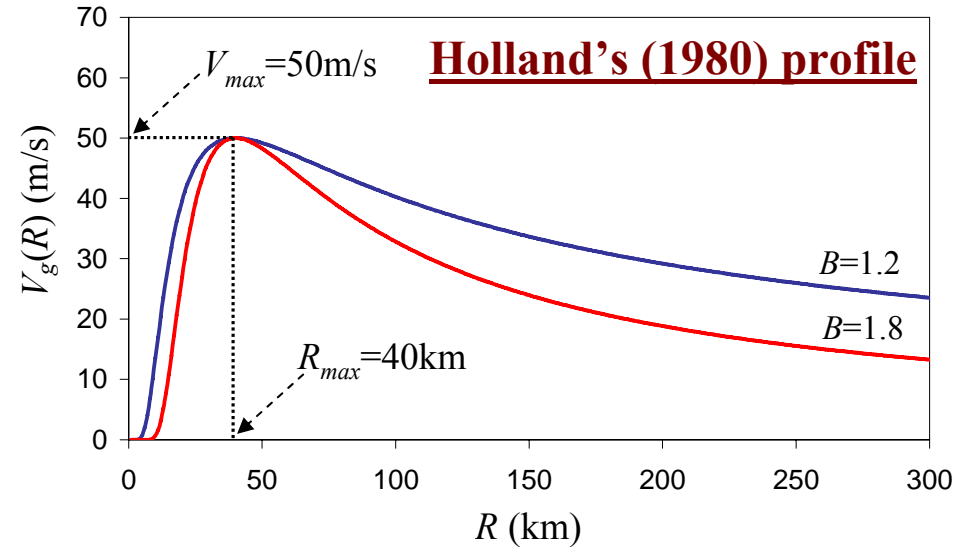
➤ **Conclusions and future directions**

1. Solving the Boundary Layer (BL)...

Conditions at BL top

$$U=0 \quad V=V_g \quad \rightarrow$$

$$\frac{\partial W}{\partial Z} = \frac{\partial U}{\partial Z} = \frac{\partial V}{\partial Z} = 0$$



Main vortex

BL top ($Z=H$)

BL equations

U : radial V : tangential

W : vertical

$$U \frac{\partial U}{\partial R} + \frac{V}{R} \frac{\partial U}{\partial \theta} + W \frac{\partial U}{\partial Z} + \frac{V_g^2 - V^2}{R} + f(V_g - V) = K \frac{\partial^2 U}{\partial Z^2}$$

$$U \frac{\partial V}{\partial R} + \frac{V}{R} \frac{\partial V}{\partial \theta} + W \frac{\partial V}{\partial Z} + \frac{UV}{R} + fU = K \frac{\partial^2 V}{\partial Z^2}$$

$$\frac{1}{R} \frac{\partial(RU)}{\partial R} + \frac{1}{R} \frac{\partial V}{\partial \theta} + \frac{\partial W}{\partial Z} = 0$$

Surface boundary ($Z=0$)

Stress conditions with drag coefficient C_D ($C_D \rightarrow \infty$ for non-slip)

Boundary layer model 1: Kepert (2001)

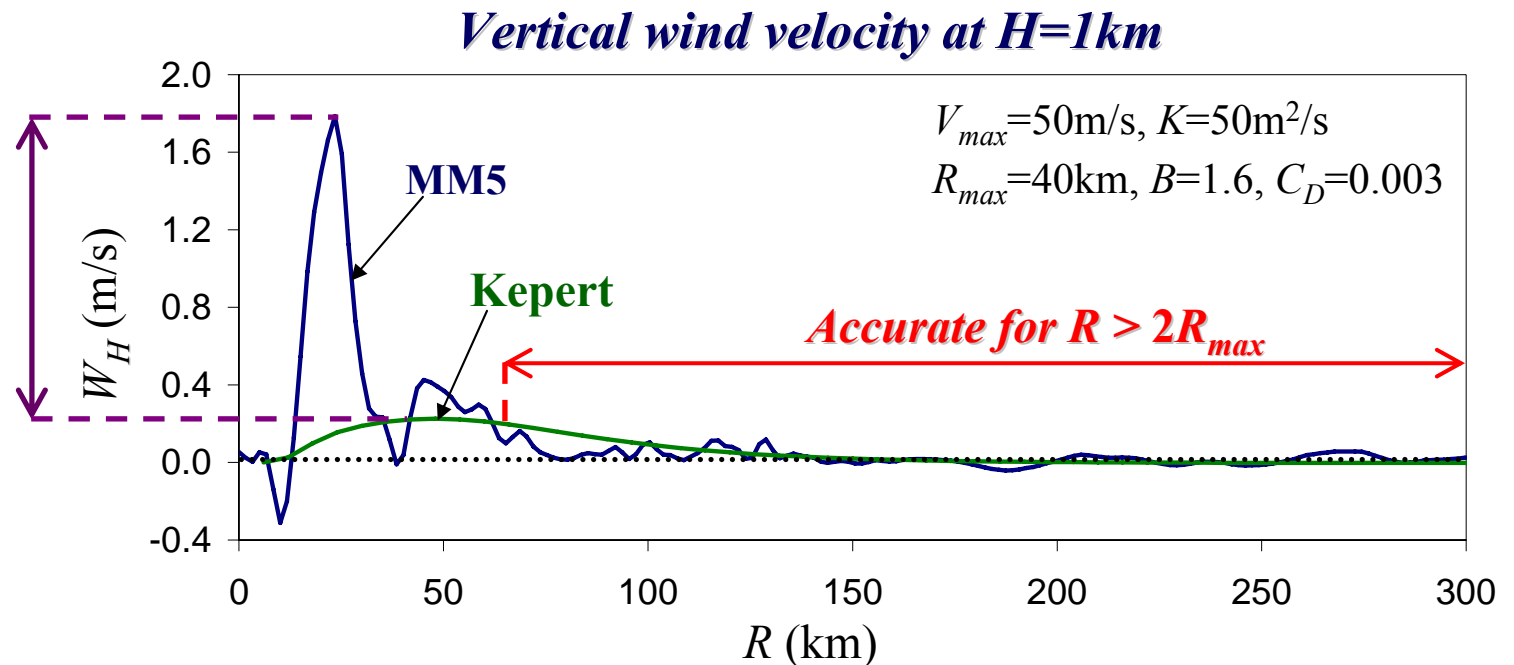
Features:

- ✓ *Analytical and depth resolving*
 - BCs at $Z=0$ and $Z=H \rightarrow \infty$
 - BL scale thickness: $\delta(R, \theta)$
- Accounts for *storm translation*
- ✗ *Linearized version of BL equations*

Model breaks:

- for large horizontal gradients $\Rightarrow R < 2R_{max}$
- for large vertical gradients $\Rightarrow C_D \rightarrow \infty$
- for high translation velocities $\Rightarrow V_t > 5\text{m/s}$
- under inertial neutrality $\Rightarrow B > 1.8$

factor of 6



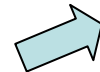
Boundary layer model 2: Shapiro (1983)

Features:

- Vertically averaged
- Accounts for *storm translation*

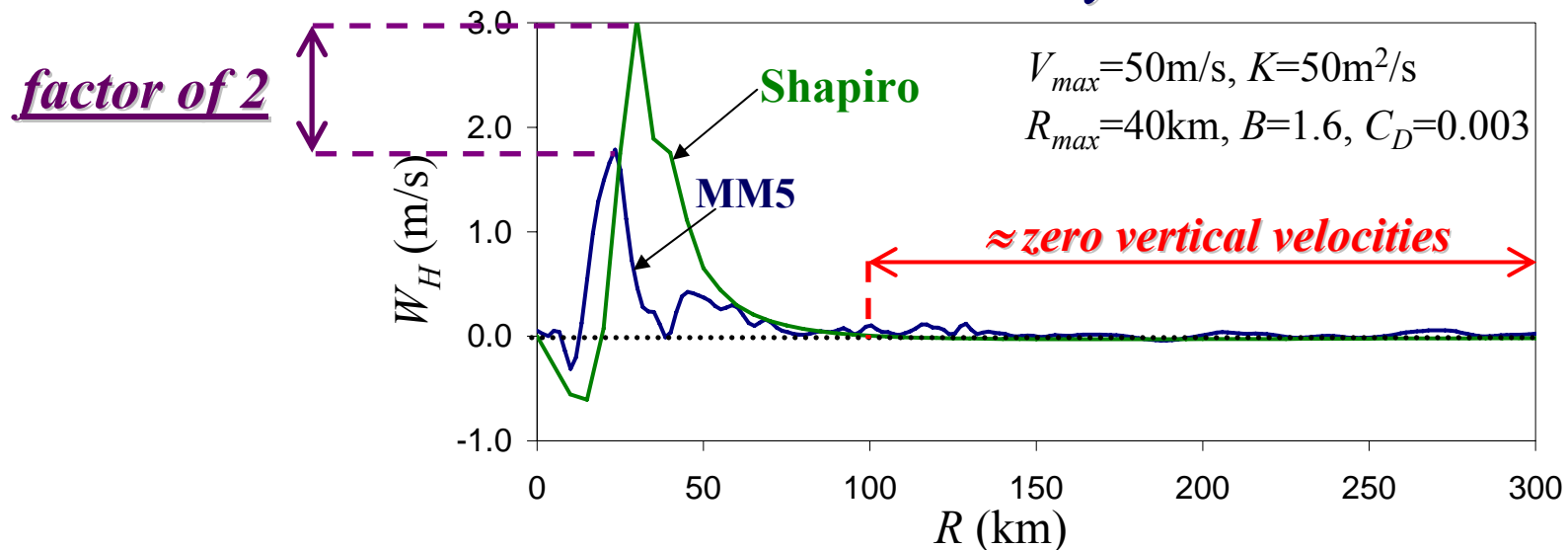
Issues:

- × High horizontal velocities
- × Stability for $R > R_{max}$ requires



constant boundary layer depth $H=1000\text{m}$
vertical diffusion coefficient $\Rightarrow K=50000\text{m}^2/\text{s}$
discretization step $\Rightarrow \Delta R = 5\text{km}$

Vertical wind velocity at $H=1\text{km}$



Boundary layer model 3: Smith (1968)

Karman & Pohlhausen momentum integral method:

- ❖ *Assume* that dependence of V and U on Z is of the *Ekman* type:

$$V(R,Z) = V_g(R) f[Z/\delta(R)]$$

gradient
winds

BL scale
thickness

$$U(R,Z) = E(R) V_g(R) g[Z/\delta(R)]$$

amplitude
coef.

Smith (1968): Ekman solutions

$$f(\eta) = -e^{-\eta} (a_1 \sin \eta + a_2 \cos \eta)$$

$$g(\eta) = 1 - e^{-\eta} (a_1 \cos \eta + a_2 \sin \eta)$$

- ❖ *Substitute* U and V into the BL equations
- ❖ *Integrate in the vertical direction* accounting for boundary conditions
- ❖ *Solve* ordinary differential equations (ODEs) for $E(R)$ and $\delta(R)$

Limitations: {

- ❖ *Stationary hurricanes*
- ❖ $a_1, a_2 = \text{const.} \Rightarrow$ *Applies only for non-slip BCs*

Modification of Smith (1968) for a moving storm

MS model

➤ Wind speeds (relative to the moving vortex):

$$\left. \begin{aligned} V(R,\theta,Z) &= \Omega \left[R, \theta, \frac{Z}{\delta(R,\theta)} \right] \\ U(R,\theta,Z) &= E(R,\theta) \Psi \left[R, \theta, \frac{Z}{\delta(R,\theta)} \right] \end{aligned} \right\} \Rightarrow W_H(R,\theta) = -\frac{1}{R} \int_0^H \frac{\partial(RU)}{\partial R} + \frac{\partial V}{\partial \theta} dZ$$

Ω & Ψ functions:
$$\begin{cases} \Psi(r,\theta,\eta) = g(r,\theta,\eta) V_t \cos\theta + f(r,\theta,\eta) (V_g - V_t \sin\theta) - V_t \cos\theta \\ \Omega(r,\theta,\eta) = g(r,\theta,\eta) (V_g - V_t \sin\theta) - f(r,\theta,\eta) V_t \cos\theta + V_t \sin\theta \end{cases}$$

f & g functions:
$$\begin{cases} f(R,\theta,\eta) = -e^{-\eta} [a_1(R,\theta) \sin \eta + a_2(R,\theta) \cos \eta] \\ g(R,\theta,\eta) = 1 - e^{-\eta} [a_1(R,\theta) \cos \eta + a_2(R,\theta) \sin \eta] \end{cases}$$

storm translation speed

surface stresses *solve a linear system for a_1 and a_2*

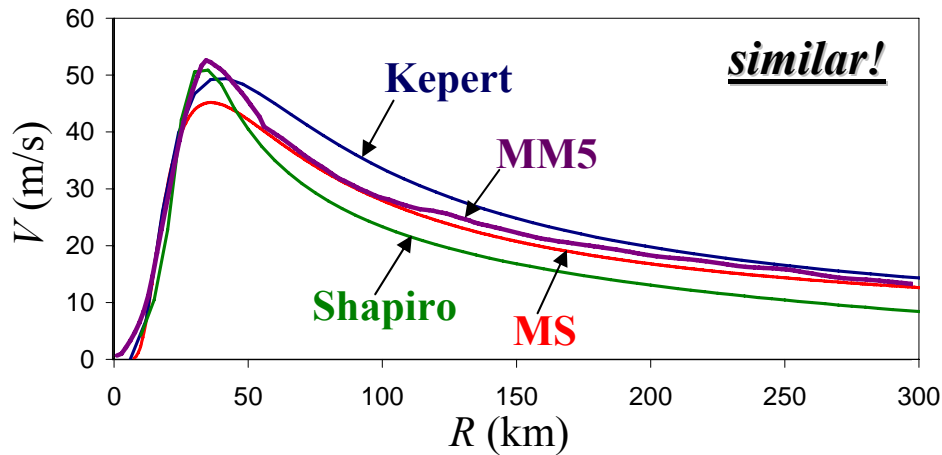
➤ *Solve* a system of non-linear partial DEs for $E(R,\theta)$ and $\delta(R,\theta)$

✓ *Numerically stable and fast formulation*

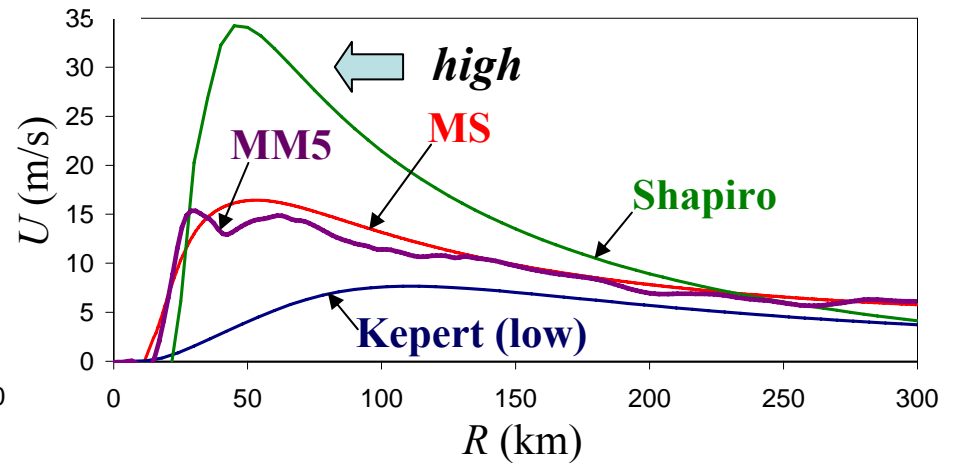
Model comparison: Stationary hurricane

($V_{max}=50\text{m/s}$, $R_{max}=40\text{km}$, $B=1.6$, $K=50\text{m}^2/\text{s}$, $C_D=0.003$)

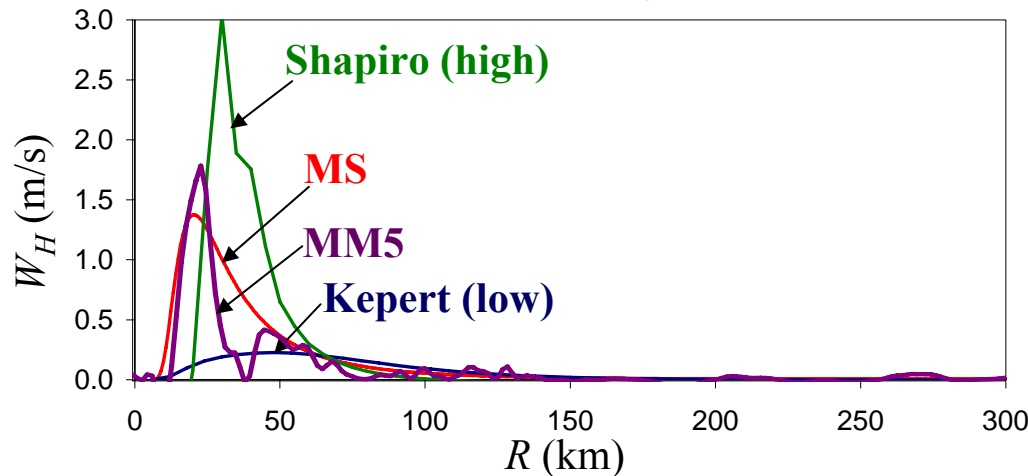
Vertically averaged (1km) tangential winds



Vertically averaged (1km) radial winds



Vertical wind velocity at $H=1\text{km}$



Moving hurricane:
validation in terms
of rain...

2. Rain due to large-scale wind convergence

Assumption:

rainrate = upward water vapor flux at the top of the boundary layer

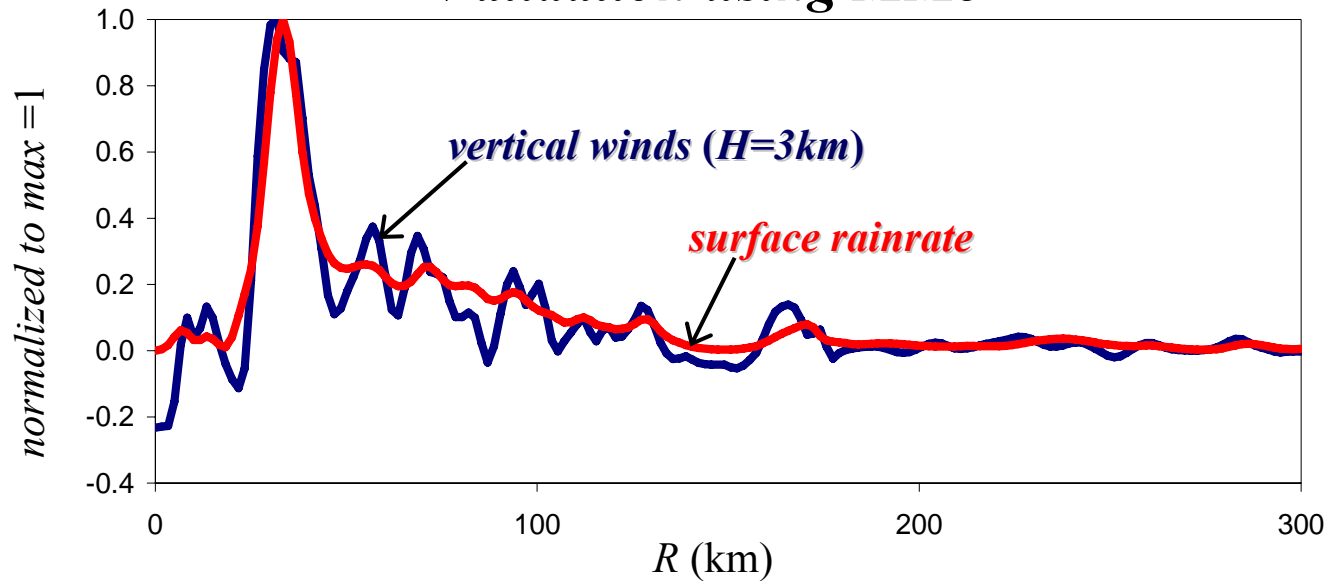
$$\bar{I} \propto W_H$$

large-scale rainfall intensity \nearrow \bar{I}
vertical wind velocity at H \nwarrow W_H
const. = moisture content of air \nearrow

MSR model

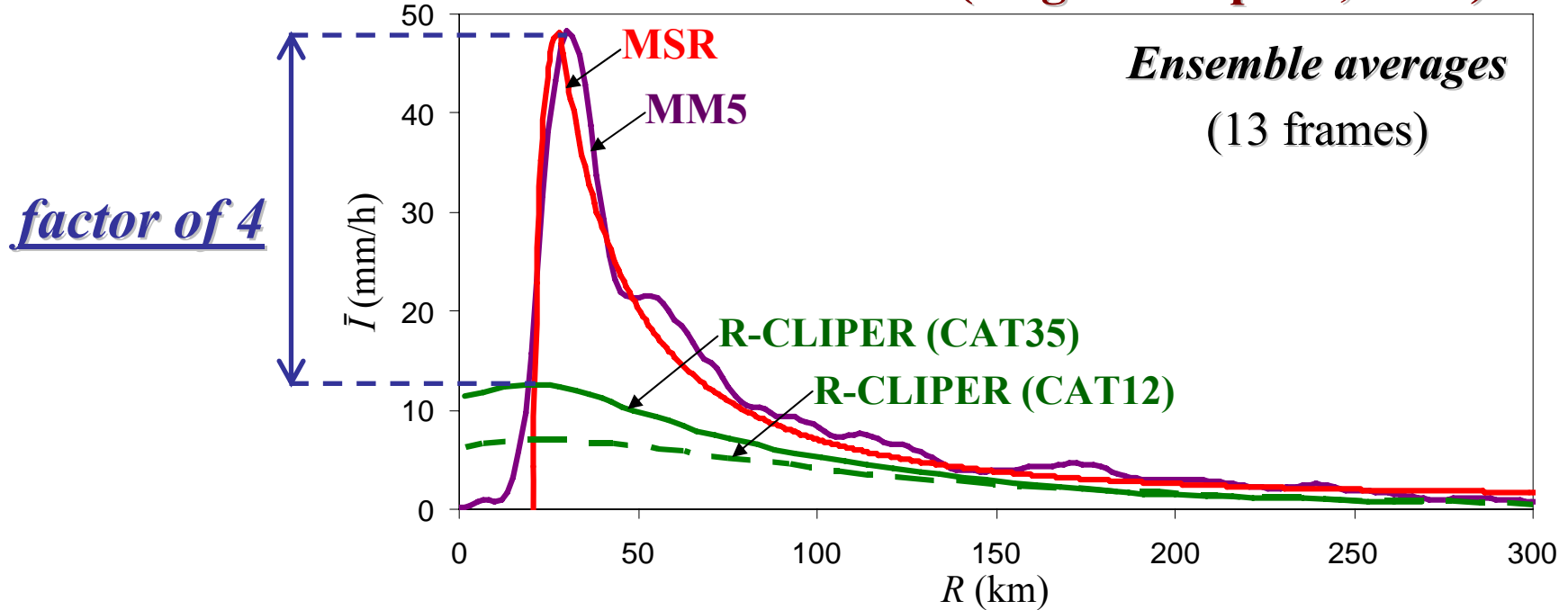
...use *MS* model to calculate W_H

Validation using MM5



Validation: (a) case study using MM5

Hurricane Frances (Aug. 29- Sep. 01, 2004)



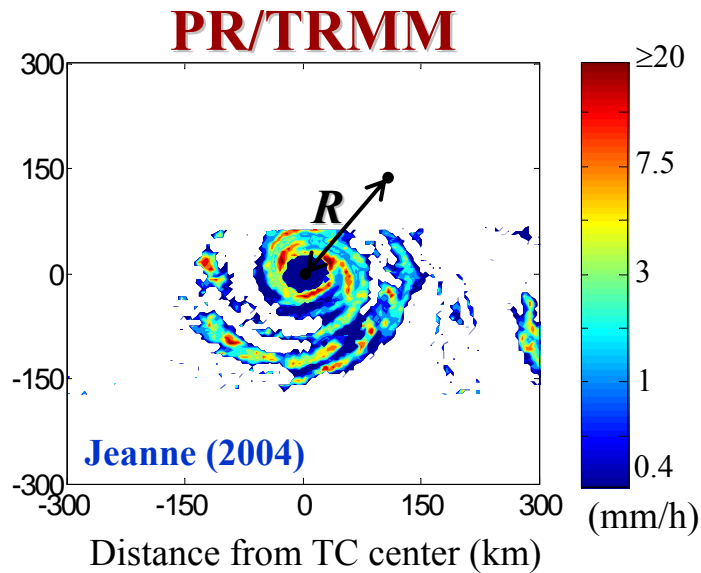
R-CLIPER

• *TMI data limitations* \Rightarrow **biases**

➤ *averaging over storms
with considerably $\neq R_{max}$*

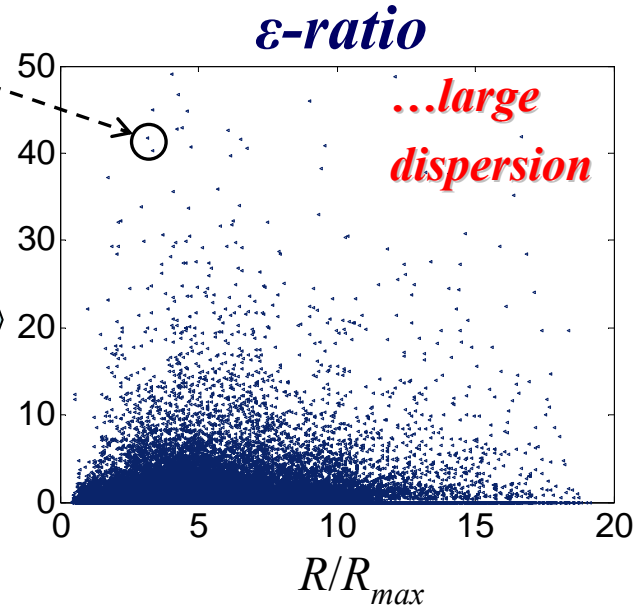
*smearing of high
intensities close to
the core*

Calibration: (b) using PR/TRMM data



$$\varepsilon = \frac{I_{PR}}{I_{MSR}}$$

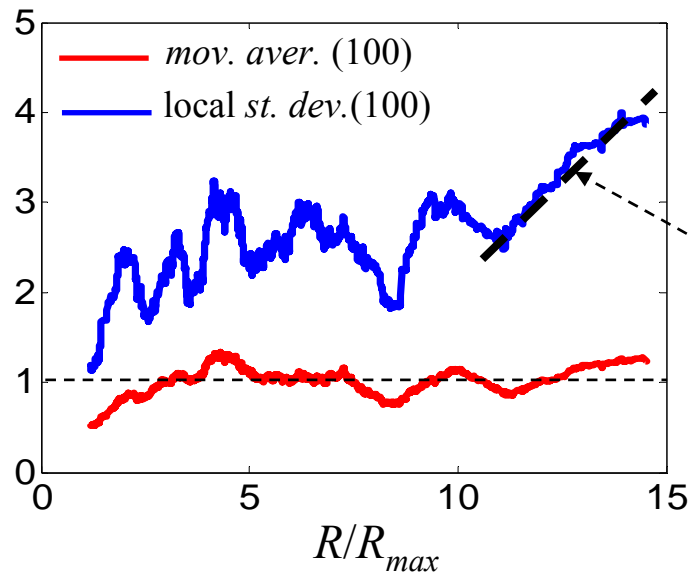
38 storms
48483 points



$B=1$

↓

...almost unbiased estimation



Rainfall asymmetry due to motion

Rainfall asymmetry

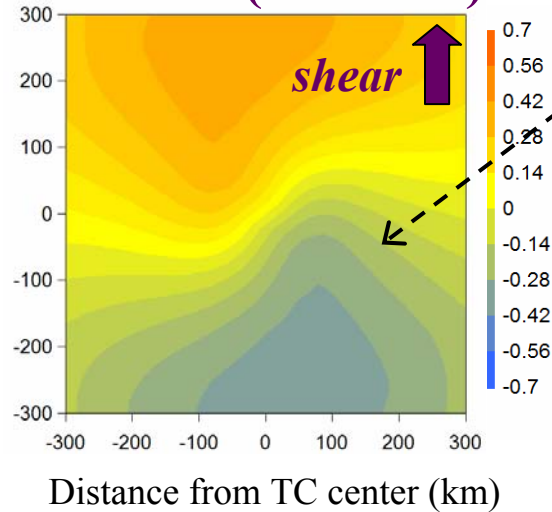
$$A(R, \theta) = \frac{\bar{I}(R, \theta) - \bar{I}_s(R)}{\bar{I}_s(R)}$$

rainfall intensity at (R, θ) azimuthal average

- **Motion** \Rightarrow MSR + cyclonic redistribution of rain
- **Shear**: the difference between the 200 ($\approx 10\text{km}$) and 800-hPa ($\approx 3\text{km}$) wind velocities in the annular region between 200 and 800km from the TC center

➤ On average, shear points to the **right** of motion... ($\approx 75^\circ$)

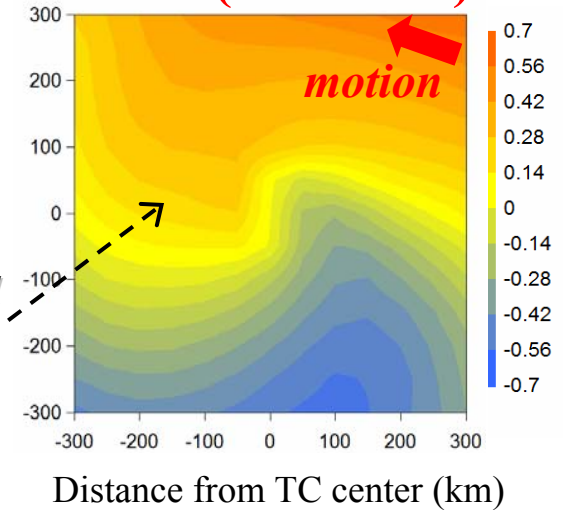
TRMM (observed)



Ensemble average over all TC intensities and shear magnitudes

Ensemble average over all TC intensities and translation speeds

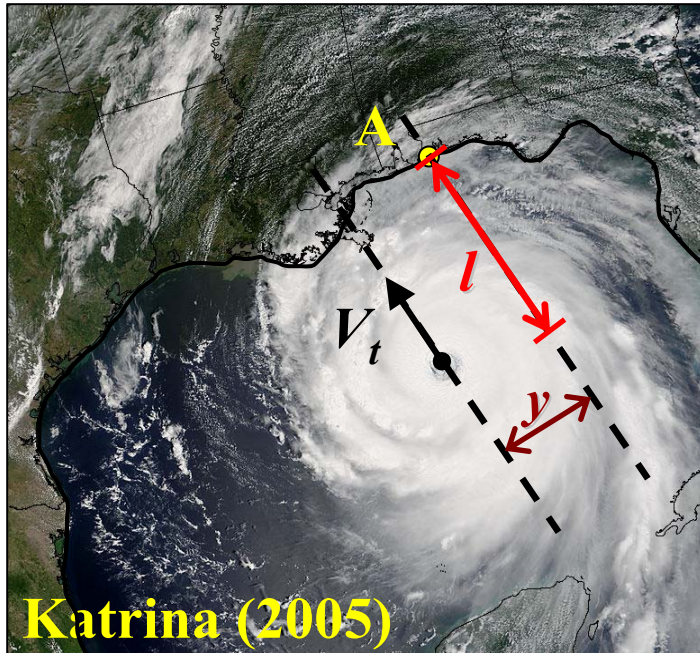
MSR (simulated)



➤ A motion based parameterization of rainfall asymmetry suffices for risk analysis

3. Statistical model of rainfall fluctuations

An observer-type approach:



- ❖ Interested in $I_{max}(D)$, the maximum rainfall intensity at location A for averaging duration D



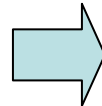
...TRMM products are rainfall snapshots



$$I_{max}(D) = I_{max}(l)$$

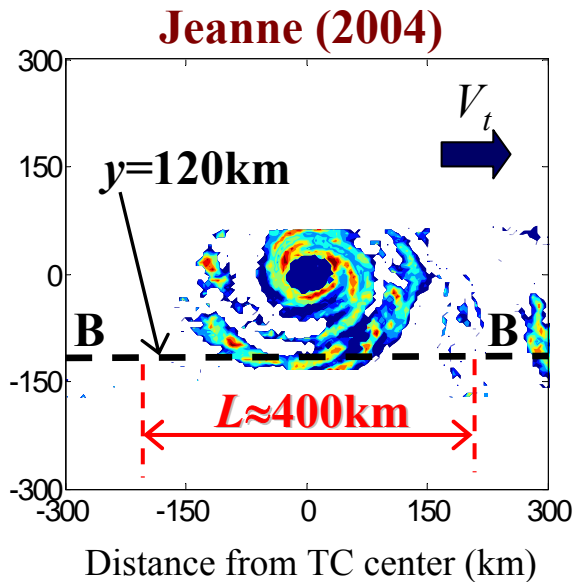
$$l = DV_t$$

Frozen field assumption



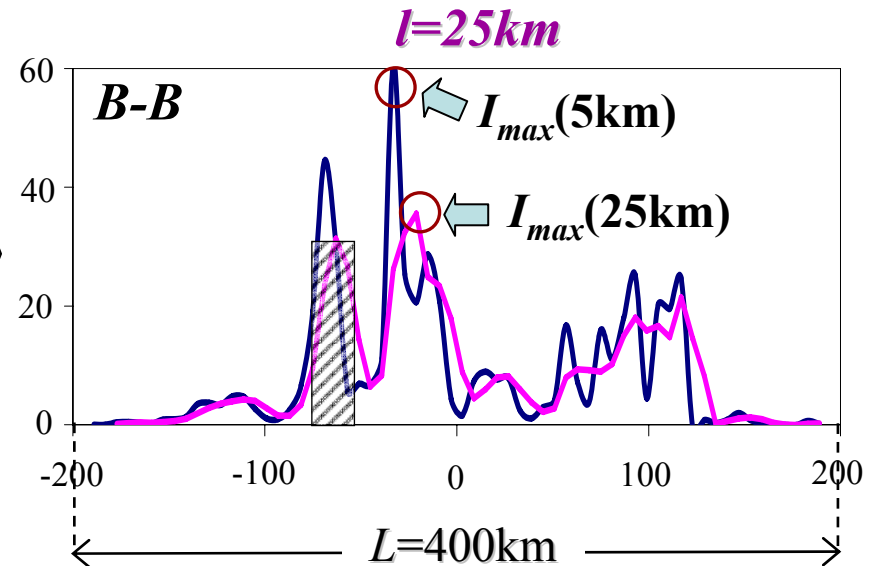
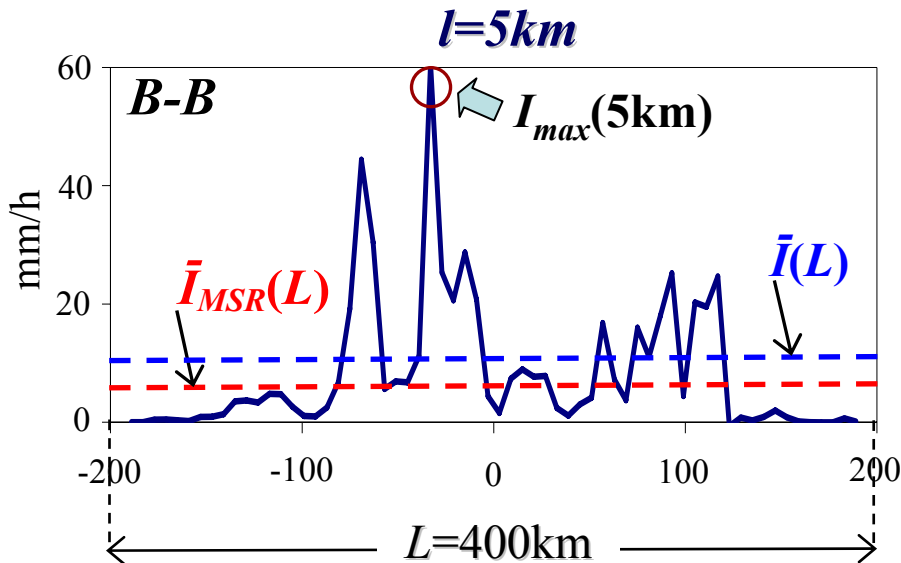
maximum spatially averaged rainfall intensity for a continuously sliding window of length l

Statistical model of $[I_{max}(l)|\omega]$



$$I_{max}(l) = \overbrace{\bar{I}_{MSR}(L)}^{(large-scales)} \underbrace{\beta}_{(small-scales)} \gamma_{max}(l)$$

$\bar{I}_{MSR}(L)$: MSR estimate for the mean rainfall intensity inside L
 β : corrects the model mean relative to the empirical mean
 $\gamma_{max}(l)$: amplification factor for the maximum inside l



Statistical model of $[\beta|\omega]$

Parsimonious parameterization:

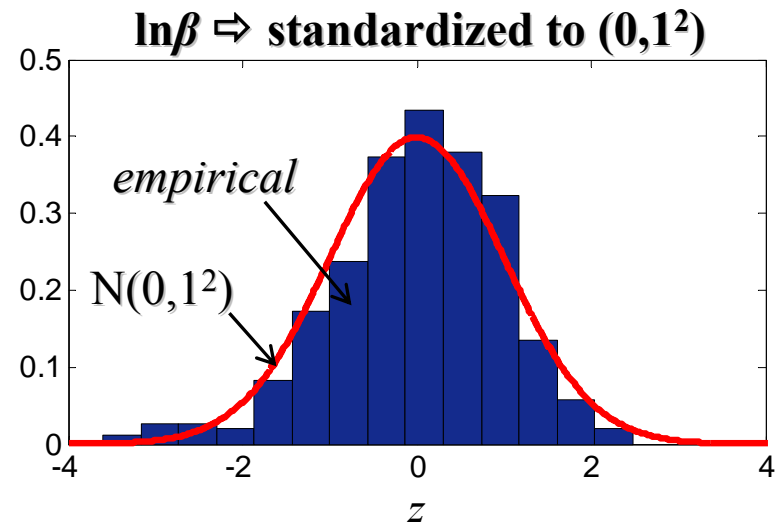
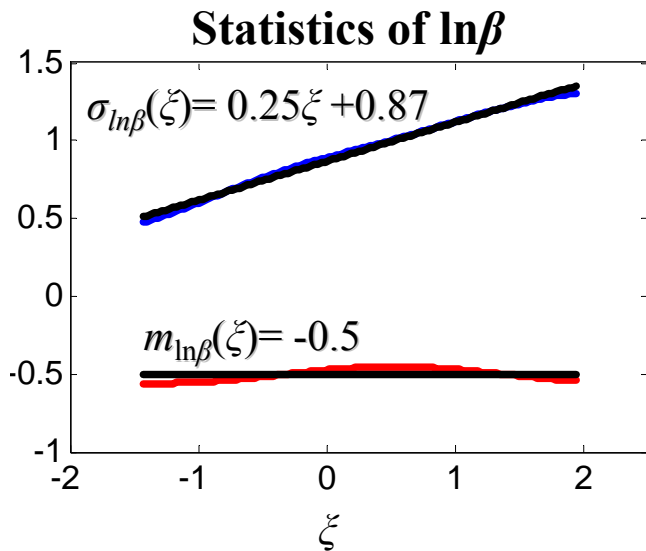
$$\xi = \ln(y') - 0.4 \ln(\bar{I}_{MSR})$$

$y' = |y/R_{max}|$ function of ω

$$\beta = \frac{\bar{I}(L)}{\bar{I}_{MSR}(L)}$$

$\bar{I}(L)$ → empirical rainfall mean inside interval L
 $\bar{I}_{MSR}(L)$ MSR rainfall estimate

... $\beta(\xi) \sim \text{lognormal}$



➤ Dependence of $m_{\ln\beta}$ and $\sigma_{\ln\beta}$ on V_{max} , R_{max} and y is small and can be neglected

Distribution of $[\gamma_{max}(l) | \omega]$

❖ Direct approach :

$$\gamma_{max}(l) = \frac{I_{max}(l)}{\bar{I}(L)}$$

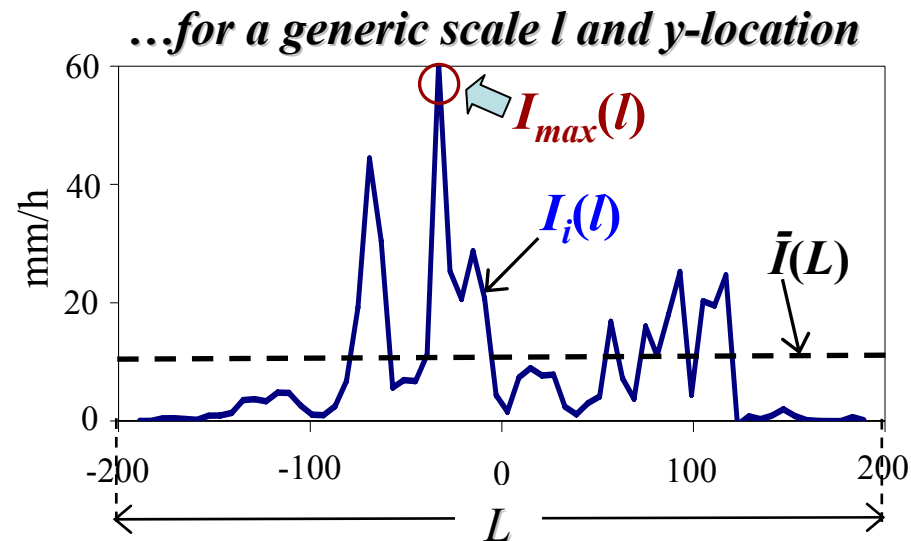
$I_{max}(l)$: maximum rainfall intensity inside a continuously sliding window of length l
 $\bar{I}(L)$: empirical mean inside L

❖ Indirect approach:

$$\gamma_i(l) = \frac{I_i(l)}{\bar{I}(L)}$$

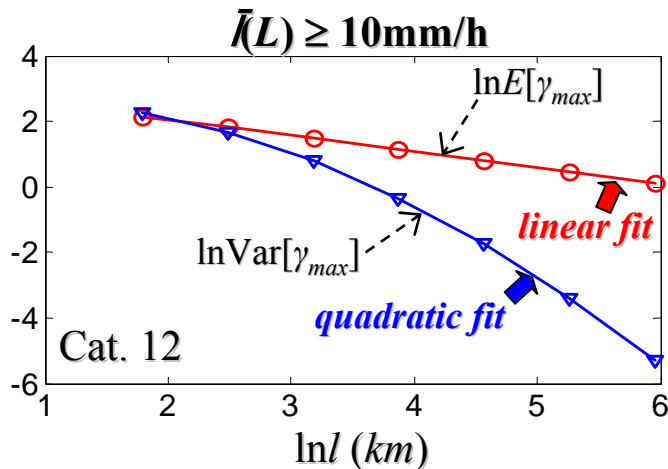
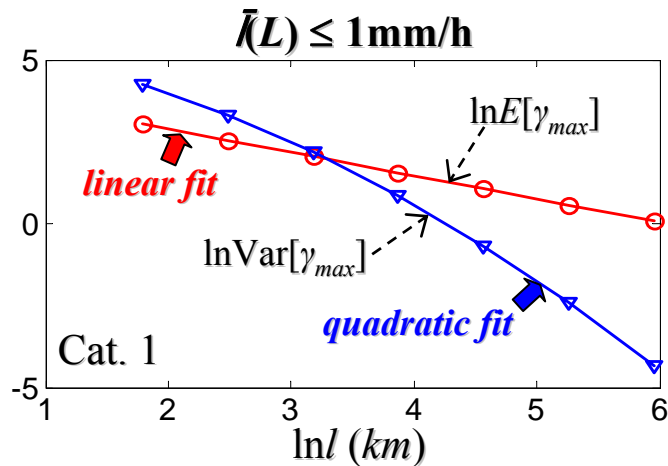
$\gamma_i(l)$: random variable with unit mean
 $\bar{I}(L)$: average rainfall intensity inside l

$$\Rightarrow \gamma_{max} = \max\{\gamma_1, \dots, \gamma_{L/l}\}$$



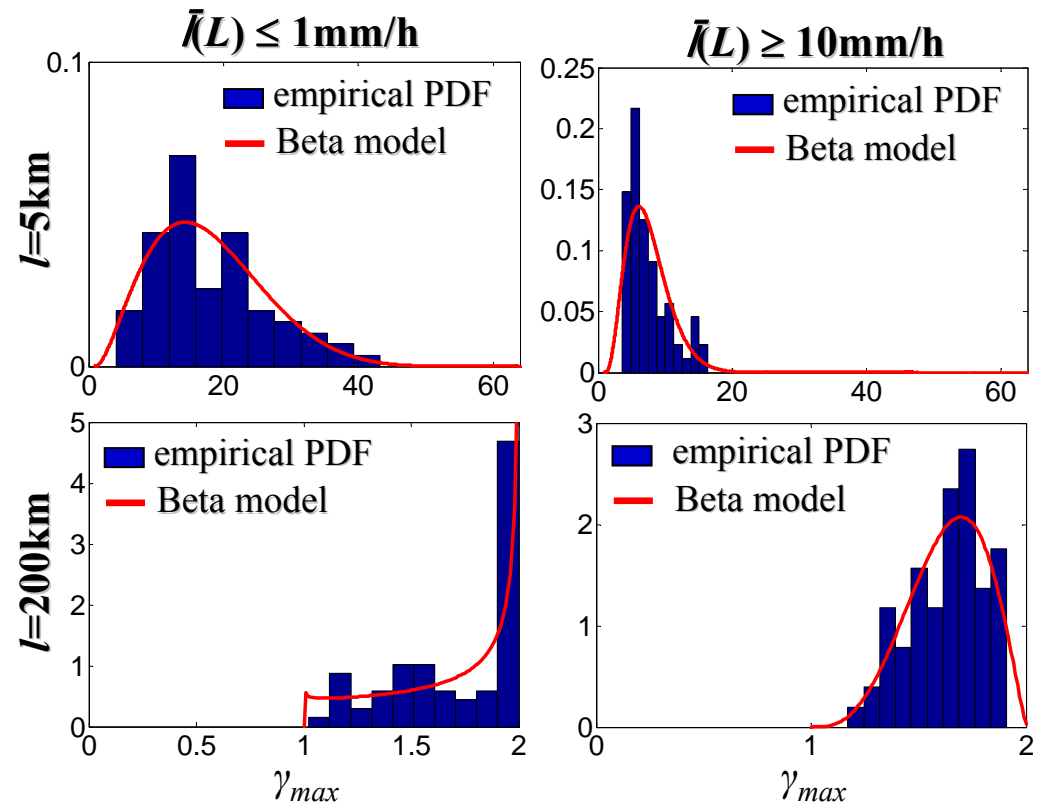
Maxima approach

- Calculate the empirical *mean* and *variance* of γ_{max} for different l and $\bar{\kappa}(L)$



- Develop *parametric expressions* for the dependence of the mean and variance of γ_{max} on l and $\bar{\kappa}(L)$

- Find a suitable distribution model for γ_{max} , bounded in $[1, L/l]$

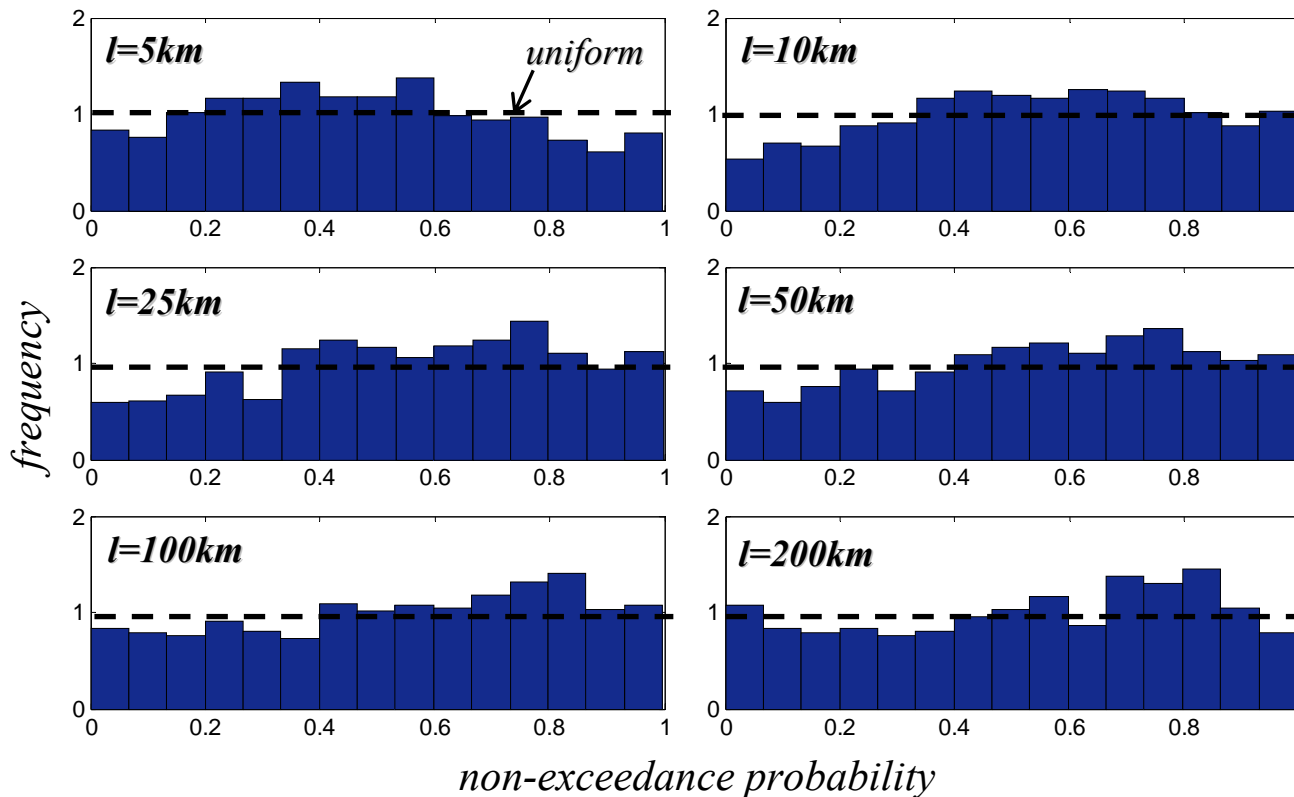


...statistical assessment

➤ For each spatial scale l , use the model to calculate the **theoretical** non-exceedance probability of the **empirical maxima**.

➤ If the model is correct, then $\Rightarrow F_{I_{max}}(I_{max,emp}) \sim U[0,1]$

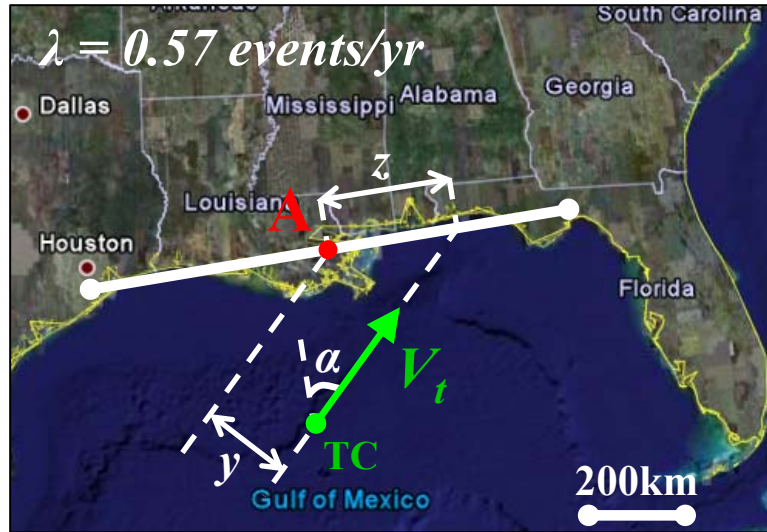
Direct approach



Similar results for
other approaches...

4. Application to New Orleans

➤ Recurrence model for $\omega = [V_{max}, R_{max}, V_t, y]^T$...and $B = 1$



$$[V_{max} | \Delta P] \sim \left\{ \begin{array}{l} \text{lognormal with} \\ m = 4.8 \Delta P^{0.559}, \sigma = 0.15 \text{ m} \\ \text{(Willoughby and Rahn, 2004)} \end{array} \right. \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{(ind.)}$$

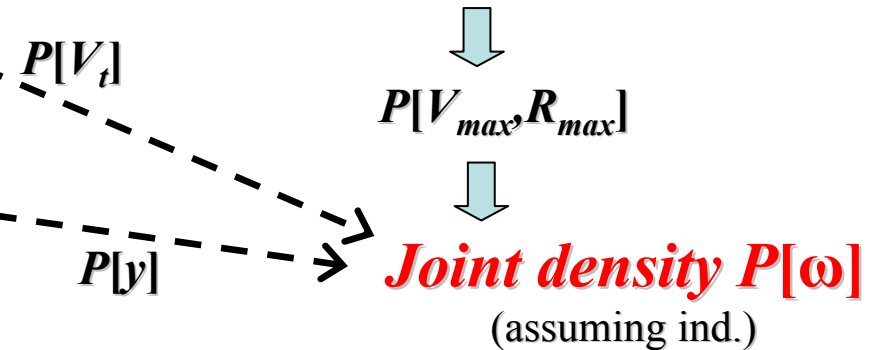
$$[R_{max} | \Delta P] \sim \left\{ \begin{array}{l} \text{lognormal with} \\ m = 3.962 - 0.00567 \Delta P, \sigma = 0.313 \\ \text{(Vickery et al., 2000)} \end{array} \right. \quad \left. \begin{array}{l} \\ \\ \end{array} \right\}$$

$$\Delta P \text{ (mb)} \sim \left\{ \begin{array}{l} \text{shifted lognormal with} \\ m_{\ln \Delta P} = 3.15, \sigma_{\ln \Delta P} = 0.68, \\ \text{Shift par.} = 18 \text{ mb (IPET, 2006)} \end{array} \right.$$

$$V_t \sim \left\{ \begin{array}{l} \text{LN with } m = 6 \text{ m/s \& } \sigma = 2.5 \text{ m/s} \\ \text{(Vickery et al., 2000, Chen et al. 2006)} \end{array} \right.$$

$$\left. \begin{array}{l} z \sim \text{U}[-500 \text{ km}, 500 \text{ km}] \\ \alpha \sim \text{N}[-5.4^\circ, (34.9^\circ)^2] \\ \text{(IPET, 2006)} \end{array} \right\} \text{(ind.)} \Rightarrow$$

$$y = -z \cos(\alpha)$$

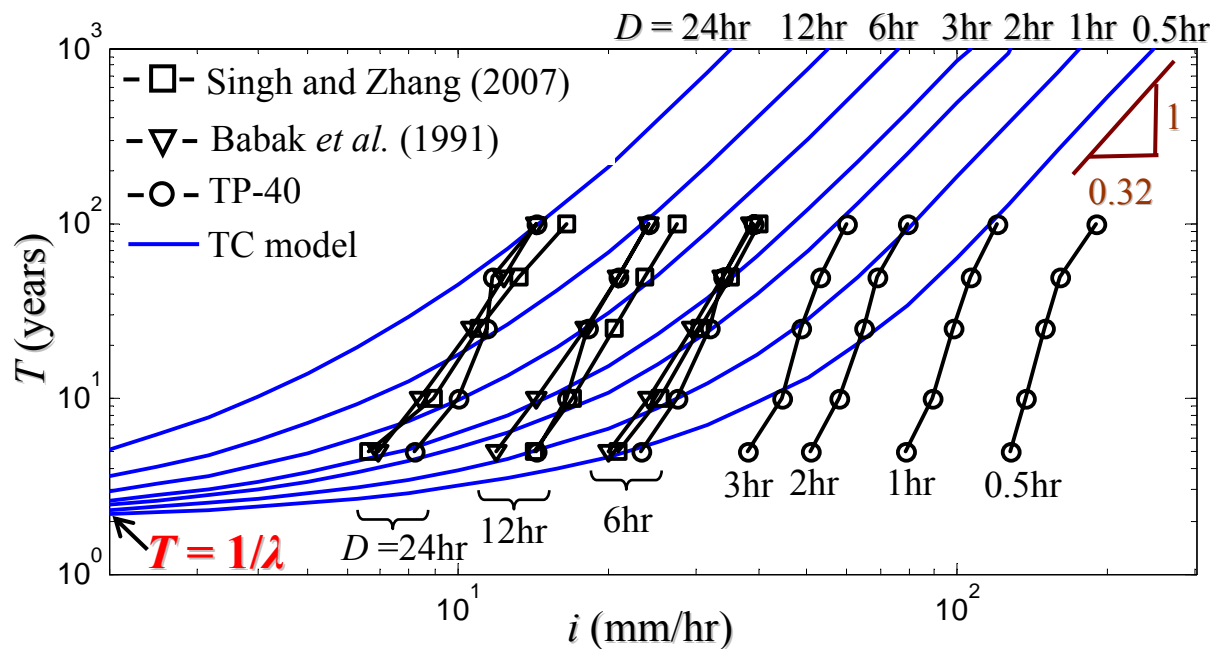
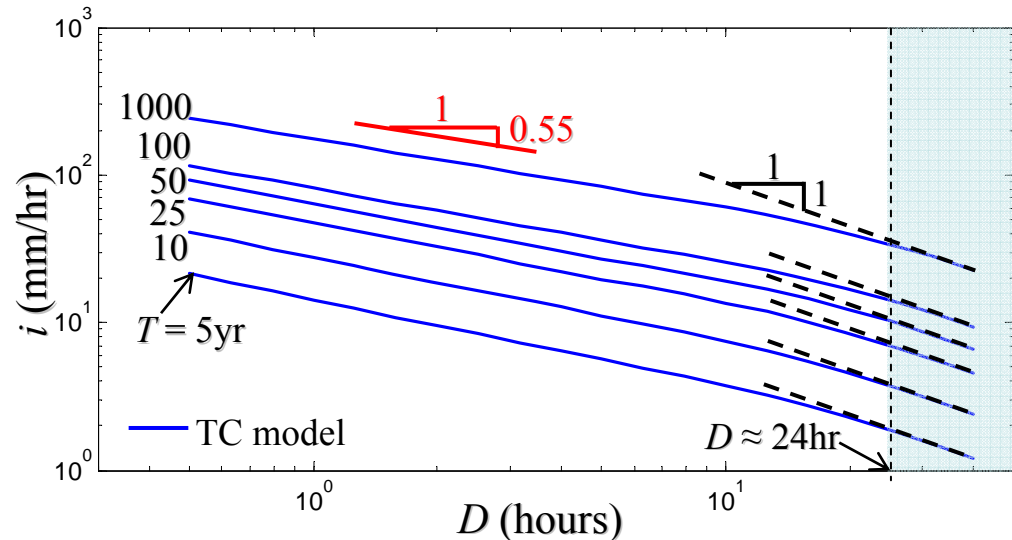


Application to New Orleans: IDF curves

Rainfall Risk and IDF curves:

$$\lambda_D(i) = \lambda \int_{\text{all } \omega} P[I_{\max}(D) > i | \omega] P[\omega] d\omega$$

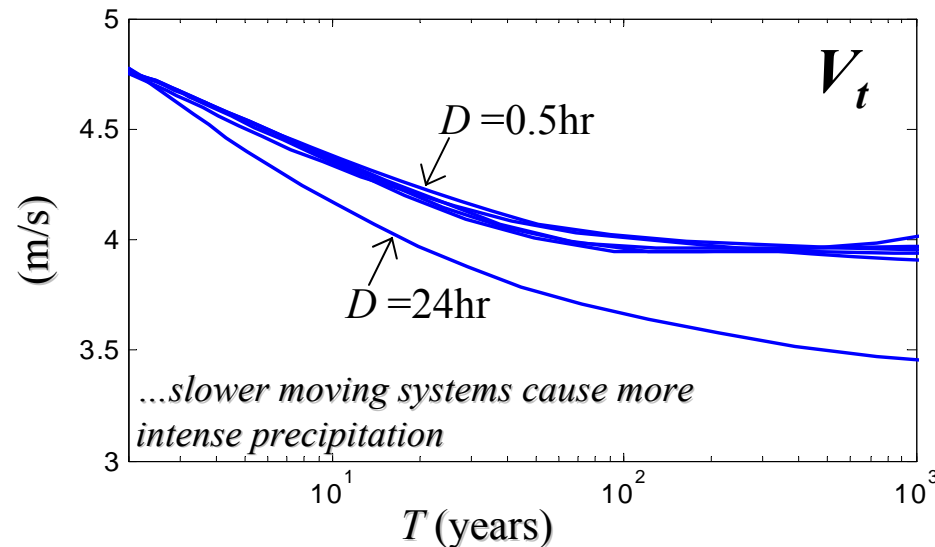
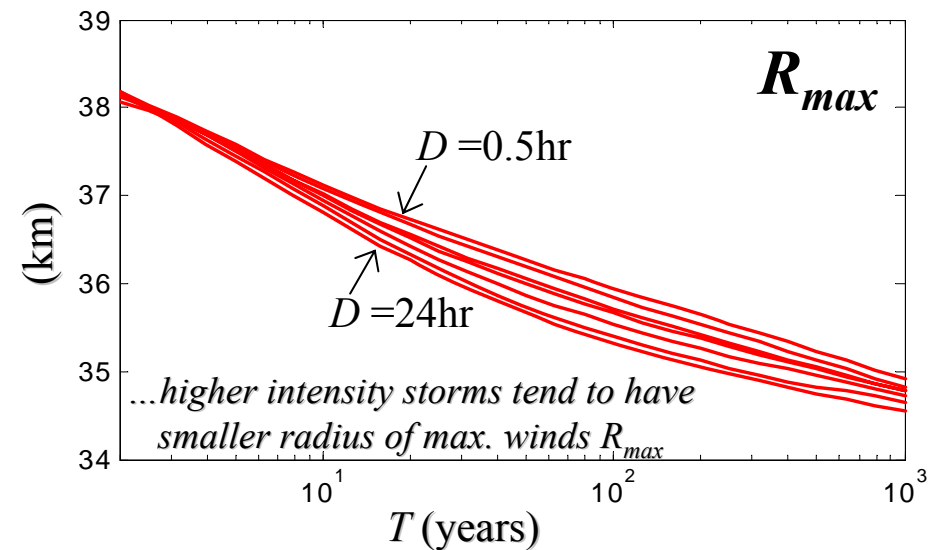
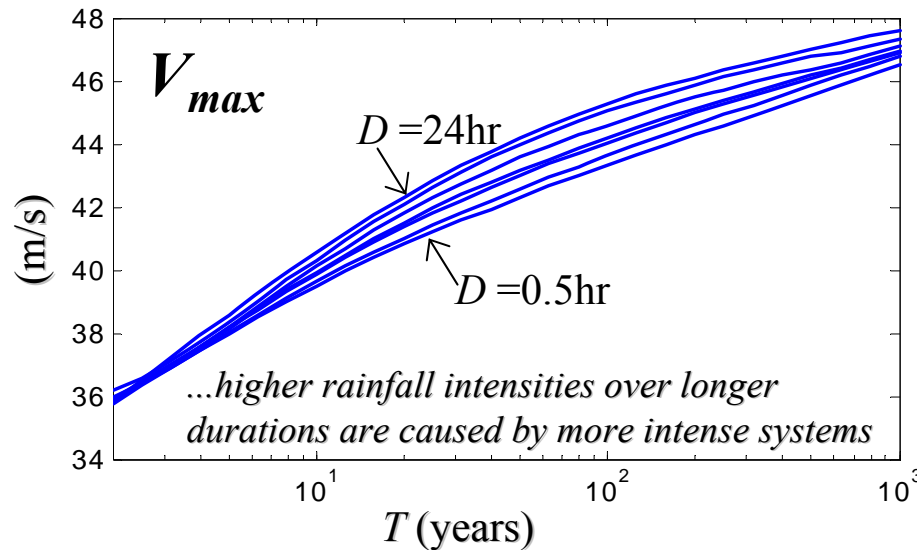
IDFs: plots of i against D and $T = 1/\lambda_D(i)$ (years)



- For **large D** and **T** TCs dominate risk.
- For **small D** applies the rule: “convection is convection”

Design storms for New Orleans

Modal values of $[\omega|D,T]$:



$$y \approx R_{max}$$

MSR model maximum

Conclusions (1)

➤ *Developed a Model of peak rainfall intensities from TCs with the following characteristics:*

- *Explicit parameterization of the hurricane: $\omega = [V_{max}, R_{max}, V_t, y]^T$*
- *Physical model (MSR) to obtain large-scale rainfall given ω*
- *Statistical model for large-scale (storm-to-storm) rainfall fluctuations: β*
- *Statistical model for small-scale variability on rainfall maxima: $\gamma_{max}(l)$*
- *Calibration and validation using PR/TRMM data*

Conclusions (2)

Uses of Model:

- ❖ *Mean wind field characterization:* **MS model**
- ❖ *Obtain distribution of maximum rainfall intensity given storm parameters ω :* **MSR + Stat. model**
- ❖ *Obtain design rainfall intensities i for given (D, T)*
- ❖ *Obtain design storm parameters ω for given (D, T)*
- ❖ *Assess relative importance of TCs and other rainstorms*
- ❖ *Complement wind, surge, and wave risk models with a rain model*

Future Directions

- ❖ Develop a simple parameterization for \bar{I}_{MSR}
- ❖ Extend to locations farther inland
- ❖ Short-term rainfall forecasting
- ❖ Apply a similar approach to assess risk from TC winds

Thanks!

