Department of Civil and Environmental Engineering
Massachusetts Institute of Technology

Extreme Rainfall Intensities and Long-term Rainfall Risk from Tropical Cyclones

By
Andreas Langousis

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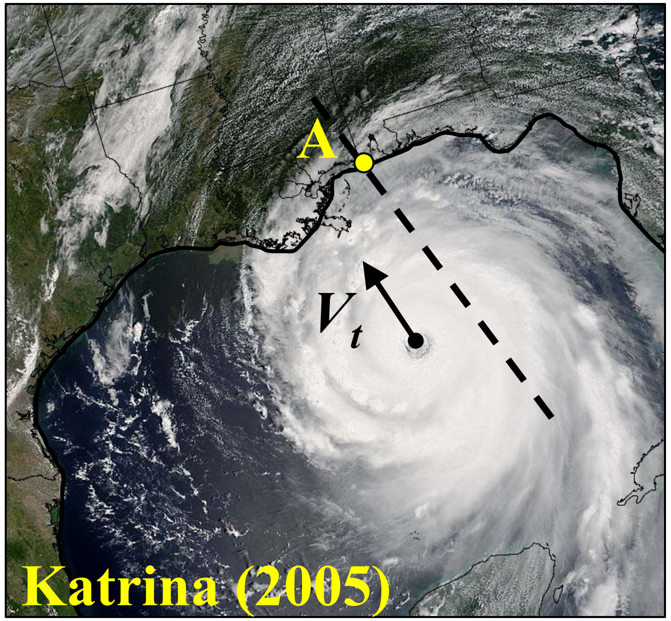
Objective

Long-term rainfall risk from TCs at location A:



$\lambda_D(i)$: rate at which $I_{max}(D)$ exceeds i at location A (events/year)

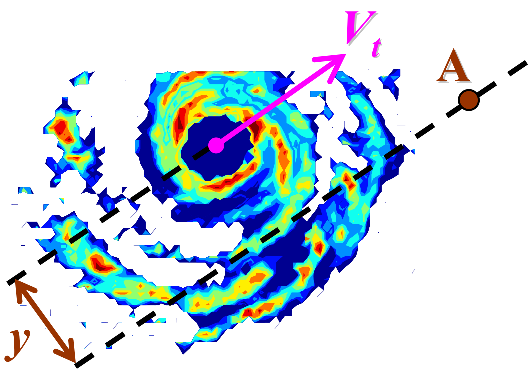
$I_{max}(D)$: maximum rainfall intensity at location A for averaging duration D

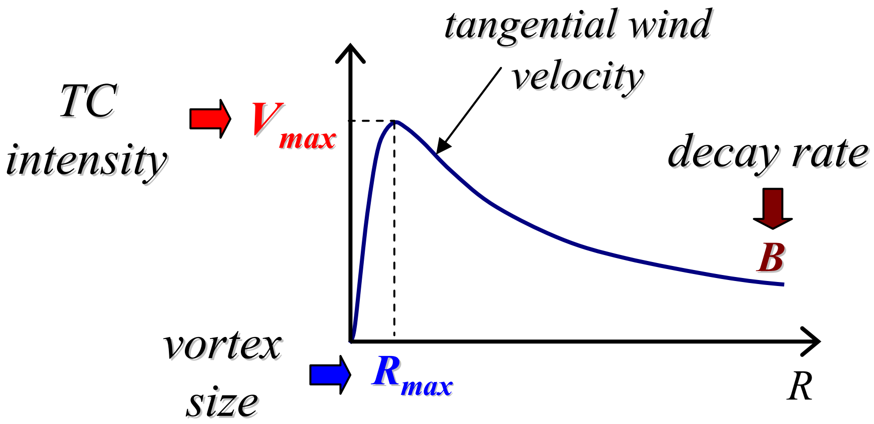
Risk analysis ⇨ $$\lambda_D(i) = \lambda \int_{\text{all } \omega} \underbrace{P[I_{max}(D) > i|\omega]}_{\textit{focus}} \; \underbrace{P[\omega]}_{\textit{local recurrence (literature)}} d\omega$$

- λ: *TC arrival rate* [events/yr]
- $P[\omega]$: *TC characteristics*

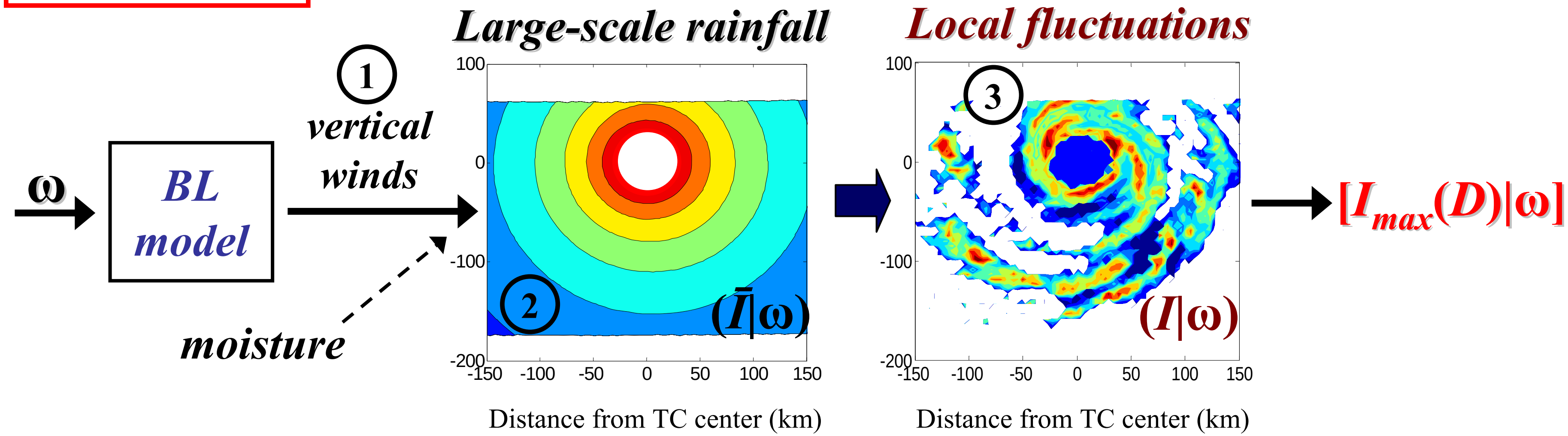
Approach to long-term risk modeling

- **parameters** $\omega = [V_{max}, R_{max}, B, V_t, y]^T$





$[I_{max}(D)|\omega]$



Outline

Part 1: BL winds

- Existing BL models $\Rightarrow$ *limitations*
- **MS** model for the BL
- Wind fields: comparison with **MM5**

Part 2: Rainfall from Winds:

- **MS** + *moisture* $\Rightarrow$ **MSR** model
- Validation using **MM5**
- Calibration using **PR** data
- Rainfall **asymmetry** due to motion

Part 3: Rainfall fluctuations

- **Storm-to-storm** variability of rainfall (*large scales*)
- **Sub-storm** variability (*small scales*) } 2 alternative approaches

Part 4: Application to New Orleans

- ***Recurrence*** model for ω
- Theoretical IDF curves for TC rainfall
- Comparison with empirical IDF results on all rainstorms (TCs and non-TCs)
- Design storms for New Orleans

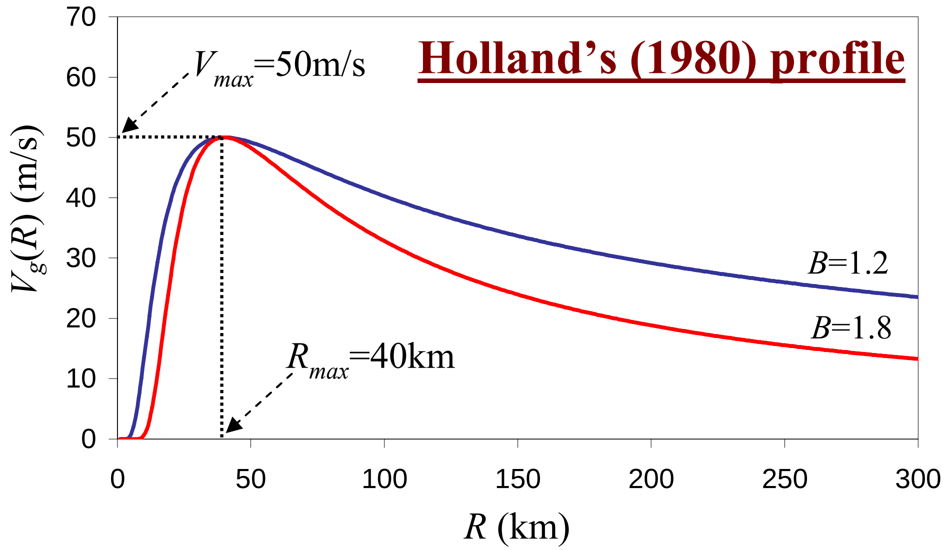
➢ **Conclusions and future directions**

1. Solving the Boundary Layer (BL)…

Conditions at BL top

$$U=0 \quad V=V_g$$

$$\frac{\partial W}{\partial Z}=\frac{\partial U}{\partial Z}=\frac{\partial V}{\partial Z}=0$$



W_H

Main vortex

BL top (Z=H)

BL equations

$$U\frac{\partial U}{\partial R}+\frac{V}{R}\frac{\partial U}{\partial \theta}+W\frac{\partial U}{\partial Z}+\frac{V_g^2-V^2}{R}+f(V_g-V)=K\frac{\partial^2 U}{\partial Z^2}$$

$$U\frac{\partial V}{\partial R}+\frac{V}{R}\frac{\partial V}{\partial \theta}+W\frac{\partial V}{\partial Z}+\frac{UV}{R}+fU=K\frac{\partial^2 V}{\partial Z^2}$$

$$\frac{1}{R}\frac{\partial (RU)}{\partial R}+\frac{1}{R}\frac{\partial V}{\partial \theta}+\frac{\partial W}{\partial Z}=0$$

U: radial *V: tangential*

W: vertical

Surface boundary (Z=0)

Stress conditions with drag coefficient C_D ($C_D \rightarrow \infty$ for non-slip)

Boundary layer model 1: Kepert (2001)

Features:

✓ ***Analytical*** *and* ***depth resolving***

- BCs at Z=0 and Z=$H \rightarrow \infty$
- BL scale thickness: $\boldsymbol{\delta(R,\theta)}$

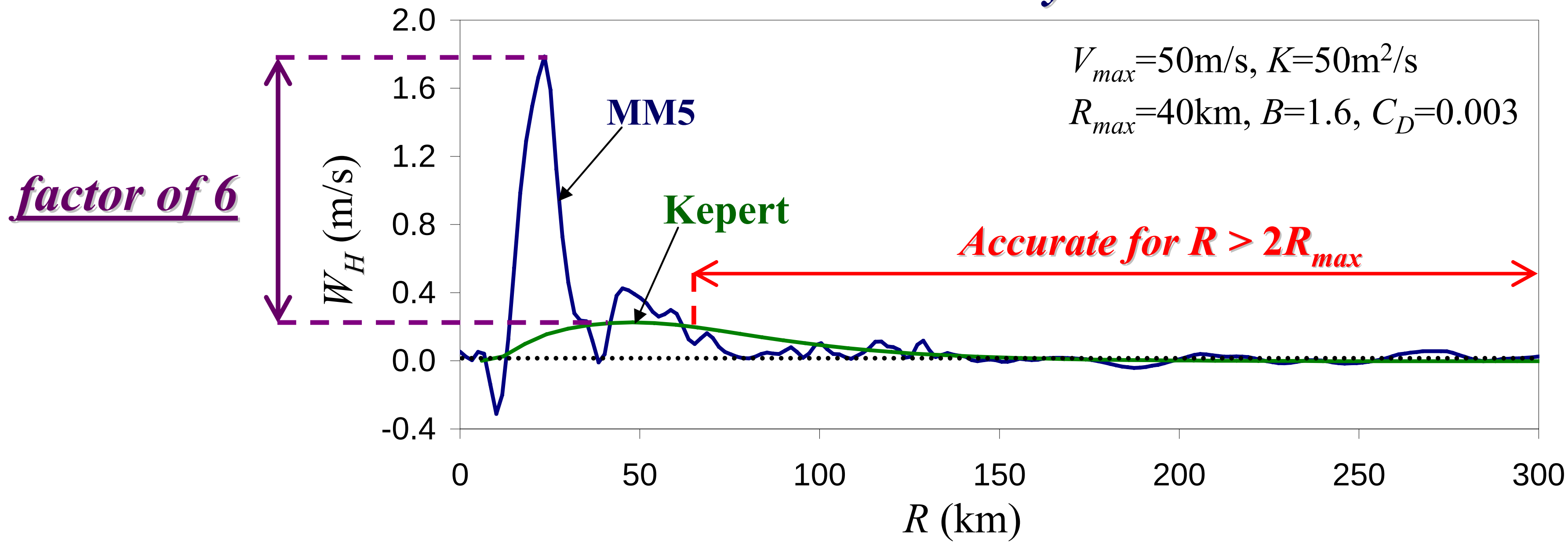
- Accounts for ***storm translation***

× ***Linearized version of BL equations***

Model breaks:

- *for large horizontal gradients* ⇒ $R < 2R_{max}$
- *for large vertical gradients* ⇒ $C_D \rightarrow \infty$
- *for high translation velocities* ⇒ V_t >5m/s
- *under inertial neutrallity* ⇒ B >1.8

Vertical wind velocity at H=1km



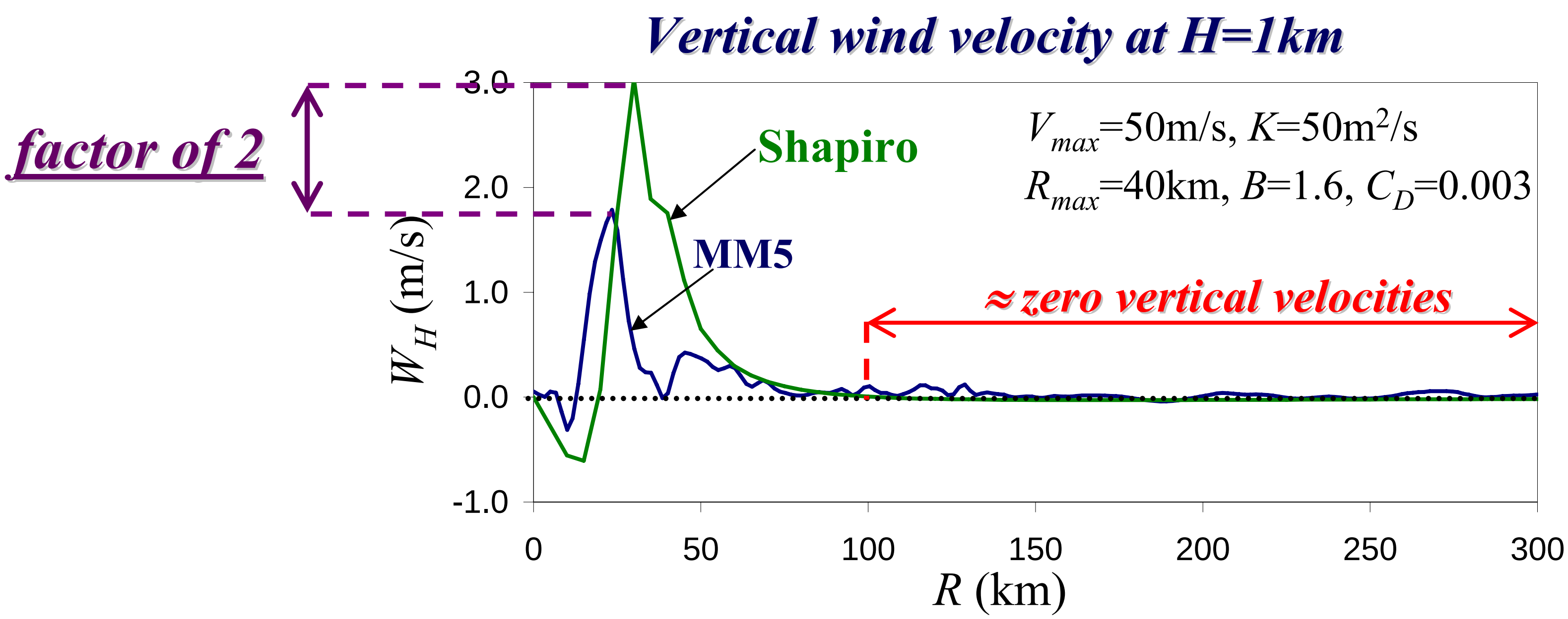
Boundary layer model 2: Shapiro (1983)

Features:

- Vertically averaged
- Accounts for ***storm translation***

Issues:

- *High horizontal velocities*
- ***Stability for $R>R_{max}$ requires***
  - *constant boundary layer depth* $\boldsymbol{H}$**=1000m**
  - *vertical diffusion coefficient* ⇒ $\boldsymbol{K}$**=50000m²/s**
  - *discretization step* ⇒ $\boldsymbol{\Delta R = 5}$***km***



Boundary layer model 3: Smith (1968)

Karman & Pohlhausen momentum integral method:

- ***Assume*** that dependence of V and U on Z is of the ***Ekman*** type:

$$V(R,Z)=V_g(R)\,f[Z/\delta(R)] \qquad U(R,Z)=E(R)\,V_g(R)\,g[Z/\delta(R)]$$

$V_g(R)$: *gradient winds*; $\delta(R)$: *BL scale thickness*; $E(R)$: *amplitude coef.*

Smith (1968): Ekman solutions

$$f(\eta) = -e^{-\eta}\,(a_1 \sin\eta + a_2 \cos\eta)$$
$$g(\eta)=1-e^{-\eta}\,(a_1 \cos\eta + a_2 \sin\eta)$$

- ***Substitute U*** and ***V*** into the BL equations
- ***Integrate*** *in the **vertical** direction* accounting for boundary conditions
- ***Solve*** ordinary differential equations (ODEs) for ***E(R)*** and ***δ(R)***

Limitations:

- ***Stationary hurricanes***
- $a_1, a_2 = const. \Rightarrow$ ***Applies only for non-slip BCs***

Modification of Smith (1968) for a moving storm

MS model

➢ **Wind speeds (relative to the moving vortex):**

$$V(R,\theta,Z)=\Omega\left[R,\theta,\frac{Z}{\delta(R,\theta)}\right]$$

$$U(R,\theta,Z)=E(R,\theta)\ \Psi\left[R,\theta,\frac{Z}{\delta(R,\theta)}\right]$$

$$\Rightarrow\ W_H(R,\theta)=-\frac{1}{R}\int_0^H\frac{\partial(RU)}{\partial R}+\frac{\partial V}{\partial\theta}dZ$$

Ω & Ψ functions:

$$\begin{cases}\Psi(r,\theta,\eta)=g(r,\theta,\eta)\ V_t\cos\theta+f(r,\theta,\eta)\ (V_g-V_t\sin\theta)-V_t\cos\theta\\ \Omega(r,\theta,\eta)=g(r,\theta,\eta)\ (V_g-V_t\sin\theta)-f(r,\theta,\eta)\ V_t\cos\theta+V_t\sin\theta\end{cases}$$

V_t: *storm translation speed*

f & g functions:

$$\begin{cases}f(R,\theta,\eta)=-e^{-\eta}\,[a_1(R,\theta)\sin\eta+a_2(R,\theta)\cos\eta]\\ g(R,\theta,\eta)=1-e^{-\eta}\,[a_1(R,\theta)\cos\eta+a_2(R,\theta)\sin\eta]\end{cases}$$

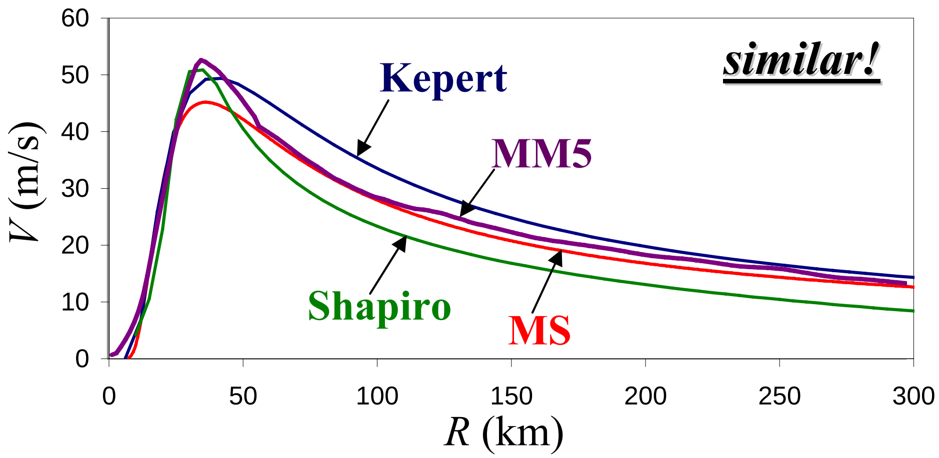
surface stresses — *solve a linear system for* a_1 *and* a_2

➢ ***Solve*** a system of non-linear partial DEs for $E(R,\theta)$ and $\delta(R,\theta)$

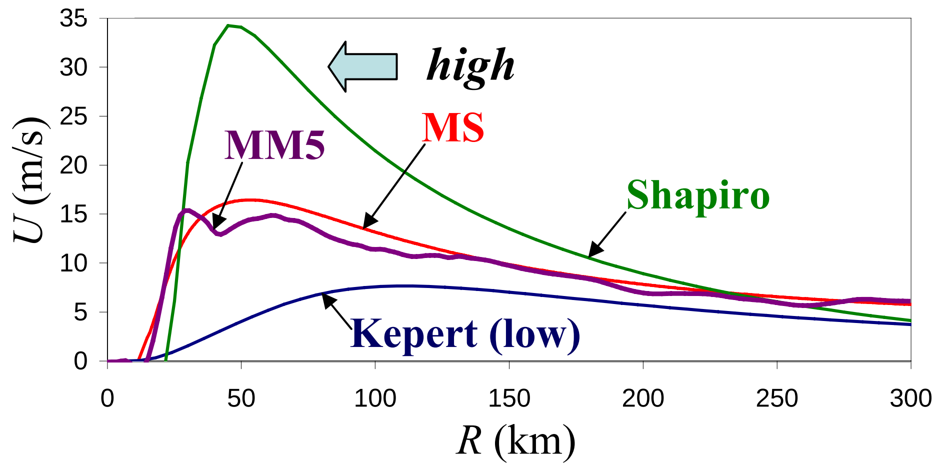
Model comparison: Stationary hurricane

(V_{max}=50m/s, R_{max}=40km, B=1.6, K=50m^2/s, C_D=0.003)

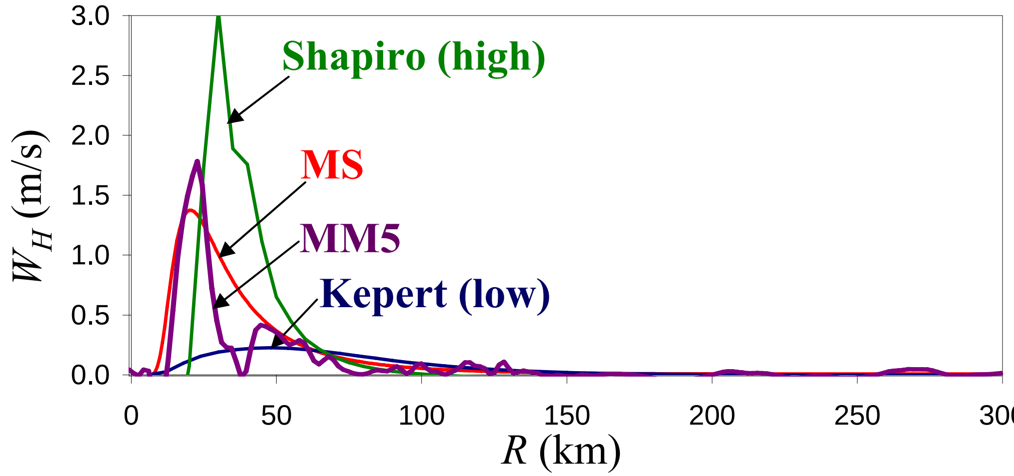
Vertically averaged (1km) ***tangential*** *winds*



Vertically averaged (1km) ***radial*** *winds*



Vertical *wind velocity at H=1km*



Moving hurricane:

validation in terms of rain…

2. Rain due to large-scale wind convergence

Assumption:

rainrate= *upward* ***water vapor flux*** *at the top of the boundary layer*

$$\bar{I} \propto W_H$$

large-scale rainfall intensity ($\bar{I}$)

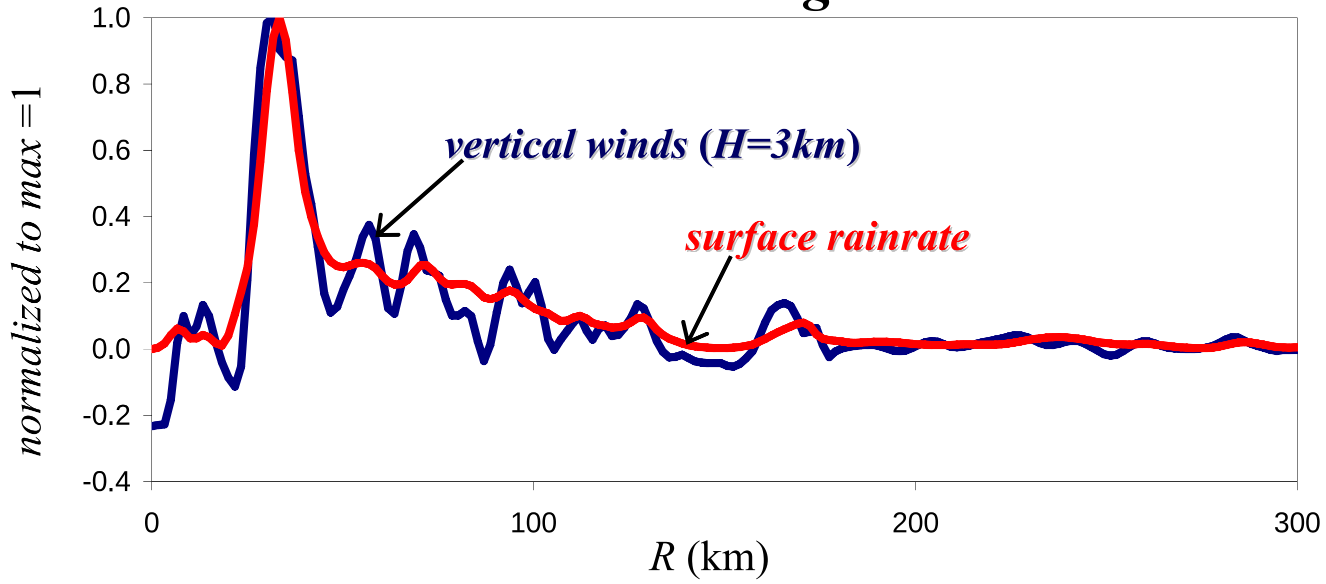
vertical wind velocity at H (W_H)

const.= moisture content of air ($\propto$)

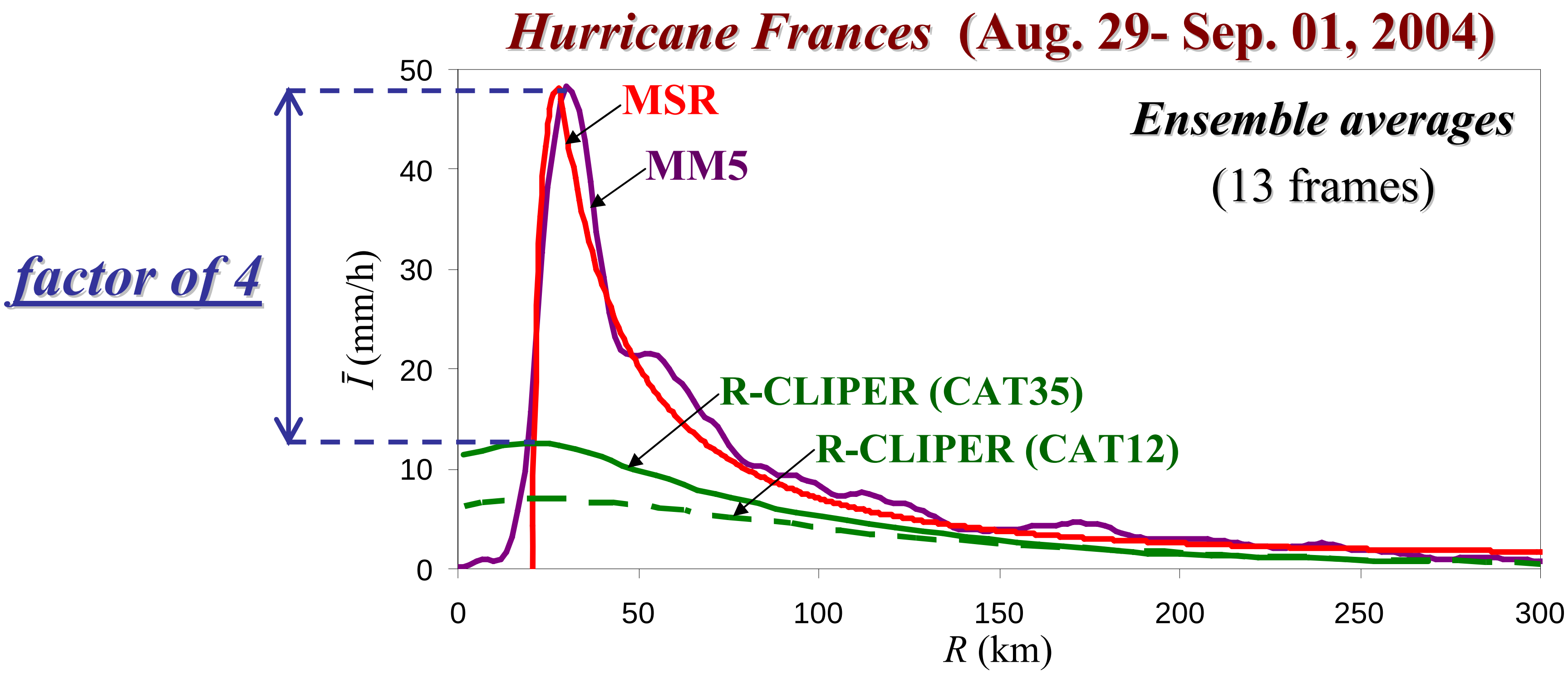
MSR model

...use MS model to calculate W_H

Validation using MM5



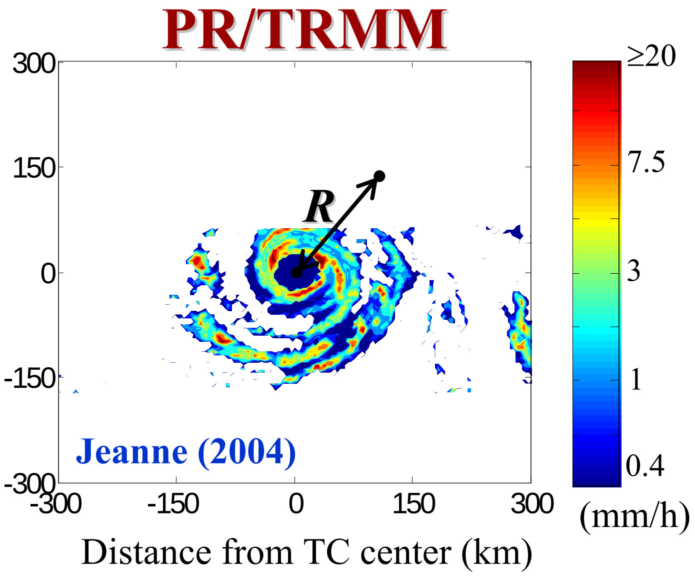
Validation: (a) case study using MM5



R-CLIPER

- ***TMI** data limitations* ⇨ **biases**
- ***averaging** over storms with considerably ≠ R_{max}* ⇨ ***smearing of high intensities close to the core***

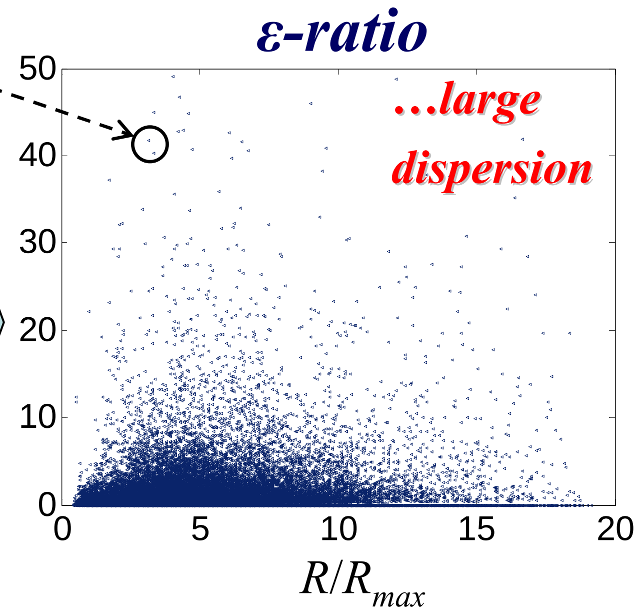
Calibration: (b) using PR/TRMM data



$$\varepsilon = \frac{I_{PR}}{I_{MSR}}$$

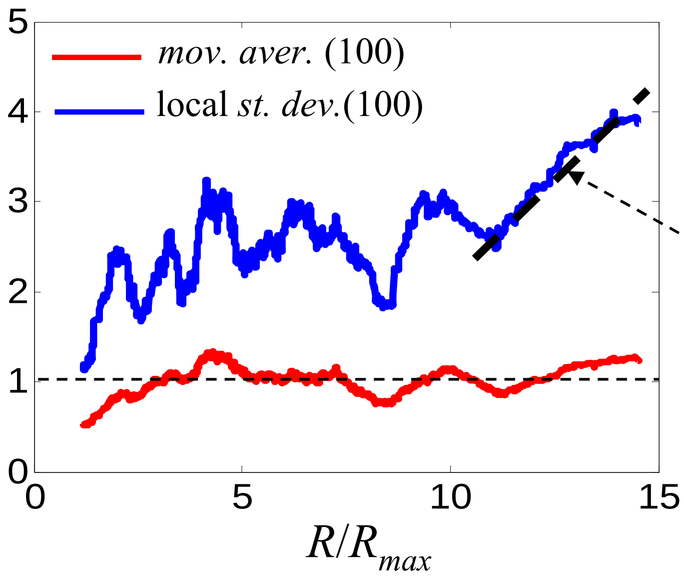
38 frames

48483 points



$B = 1$

...*almost* ***unbiased*** *estimation*



larger variability in the outer TC

Rainfall asymmetry due to motion

Rainfall asymmetry

$$A(R,\theta) = \frac{\bar{I}(R,\theta) - \bar{I}_s(R)}{\bar{I}_s(R)}$$

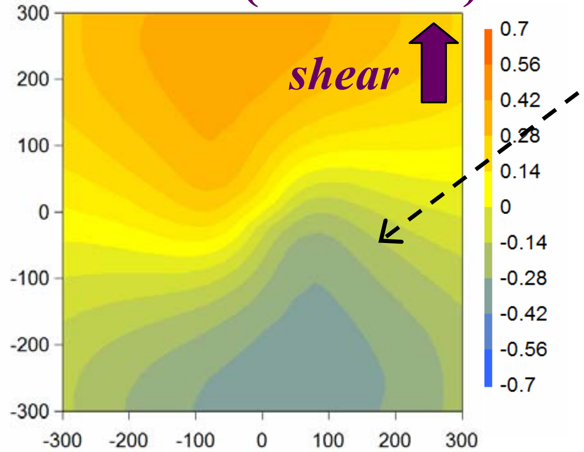
$\bar{I}(R,\theta)$: rainfall intensity at (R,θ)

$\bar{I}_s(R)$: azimuthal average

- ***Motion*** ⇨ ***MSR + cyclonic redistribution of rain***
- ***Shear:*** *the difference between the 200 (≈10km) and 800-hPa (≈3km) wind velocities in the annular region between 200 and 800km from the TC center*

➢ *On average,* ***shear*** *points to the* ***right*** *of* ***motion…*** (≈ 75°)

TRMM (observed)

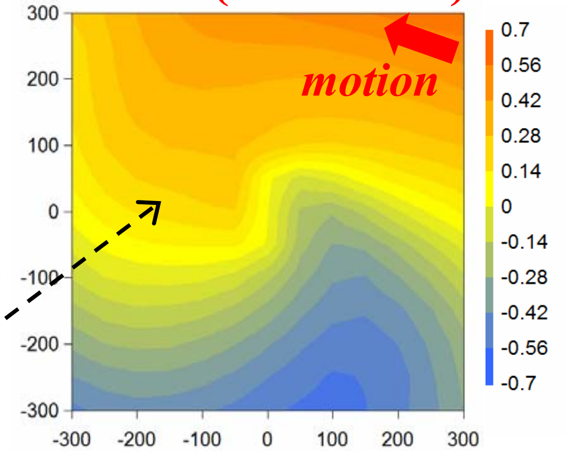


Distance from TC center (km)

Ensemble average over all TC ***intensities*** *and* ***shear*** *magnitudes*

Ensemble average over all TC ***intensities*** *and* ***translation speeds***

MSR (simulated)

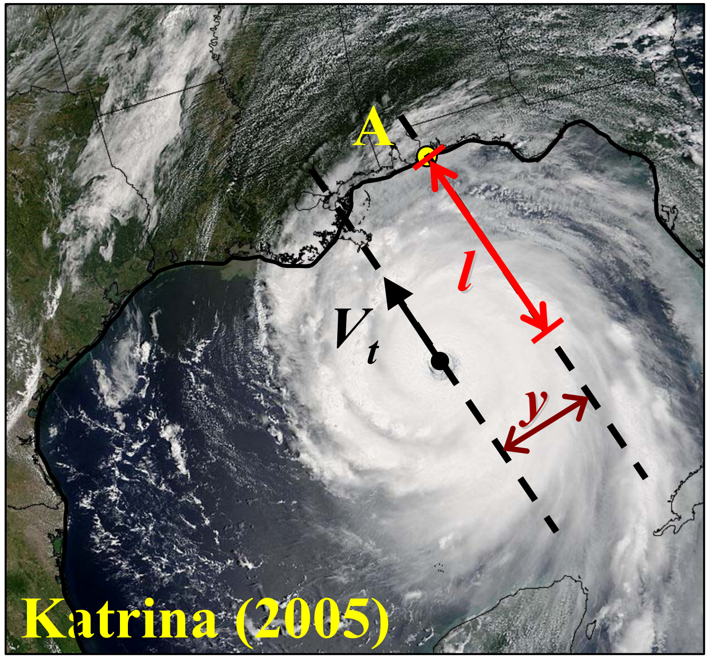


Distance from TC center (km)

➢ ***A motion based parameterization of rainfall asymmetry suffices for risk analysis***

3. Statistical model of rainfall fluctuations

An observer-type approach:



- ***Interested** in $I_{max}(D)$, the **maximum** rainfall **intensity at** location **A** for averaging duration **D***

…TRMM products are rainfall snapshots

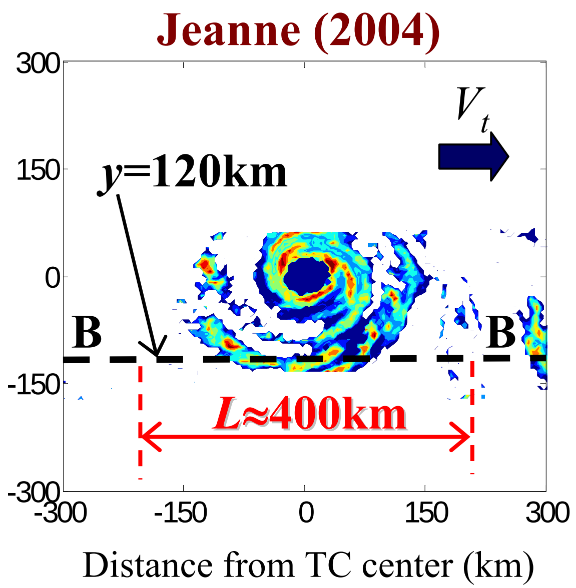
$$I_{max}(D) = I_{max}(l)$$

Frozen field assumption

maximum** spatially averaged rainfall **intensity** for a continuously sliding window of length **l

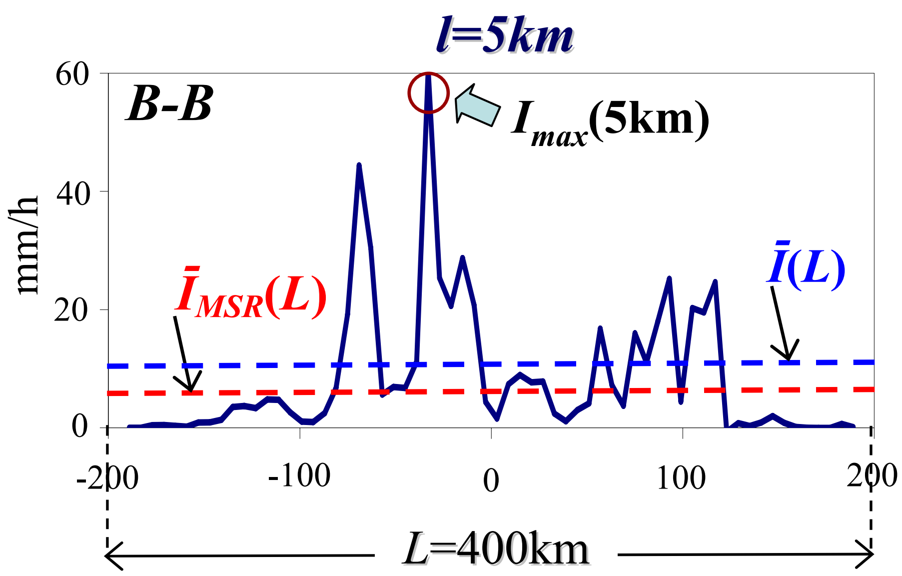
$$l = DV_t$$

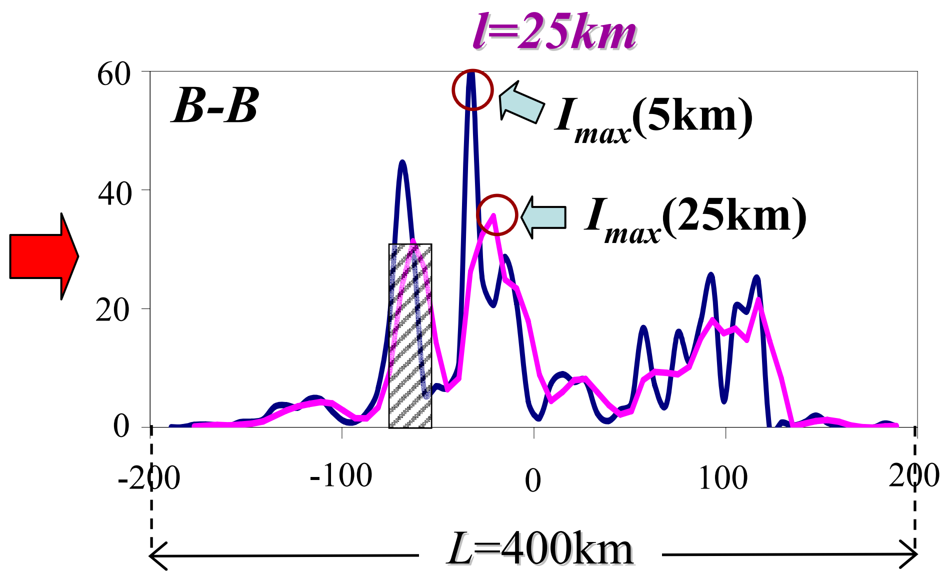
Statistical model of $[I_{max}(l)|\omega]$



$$I_{max}(l) = \underbrace{\bar{I}_{MSR}(L)\ \beta}_{\text{(large-scales)}}\ \underbrace{\gamma_{max}(l)}_{\text{(small-scales)}}$$

- $\bar{I}_{MSR}(L)$: ***MSR** estimate for the mean **rainfall intensity** inside **L***
- β: *corrects the **model** mean relative to the **empirical** mean*
- $\gamma_{max}(l)$: ***amplification factor** for the maximum inside l*





Statistical model of $[\beta|\omega]$

Parsimonious parameterization:

$$\xi = \ln(y') - 0.4\ln(\bar{I}_{MSR})$$

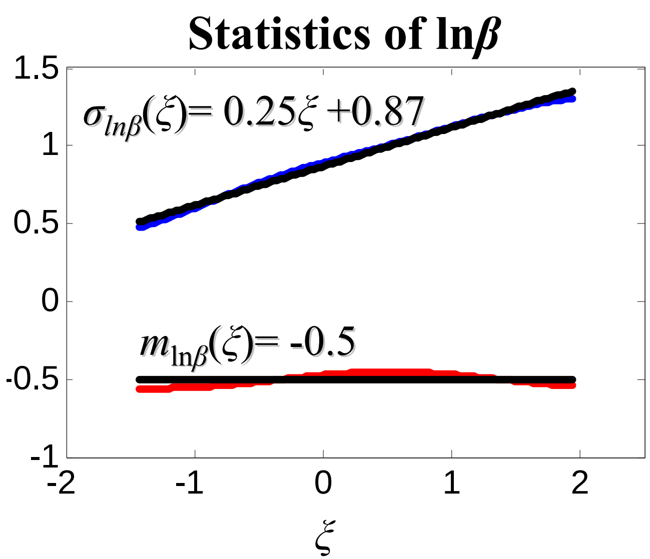
$y' = |y/R_{max}|$ function of ω

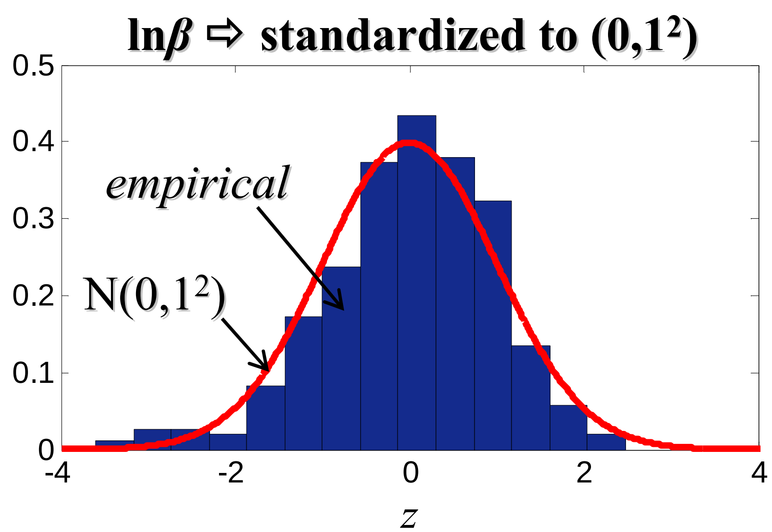
$$\beta = \frac{\bar{I}(L)}{\bar{I}_{MSR}(L)}$$

$\bar{I}(L)$ → ***empirical*** *rainfall mean inside interval L*

$\bar{I}_{MSR}(L)$ → ***MSR*** *rainfall estimate*

…$\beta(\xi)$ ~ lognormal





- *Dependence of $m_{ln\beta}$ and $\sigma_{ln\beta}$ on V_{max}, R_{max} and y is small and can be neglected*

Distribution of $[\gamma_{max}(l) | \omega]$

❖ Direct approach :

$$\gamma_{max}(l) = \frac{I_{max}(l)}{\bar{I}(L)}$$

empirical *mean inside L* (for $\bar{I}(L)$)

maximum *rainfall* ***intensity*** *inside a continuously sliding window of length l* (for $I_{max}(l)$)

❖ Indirect approach:

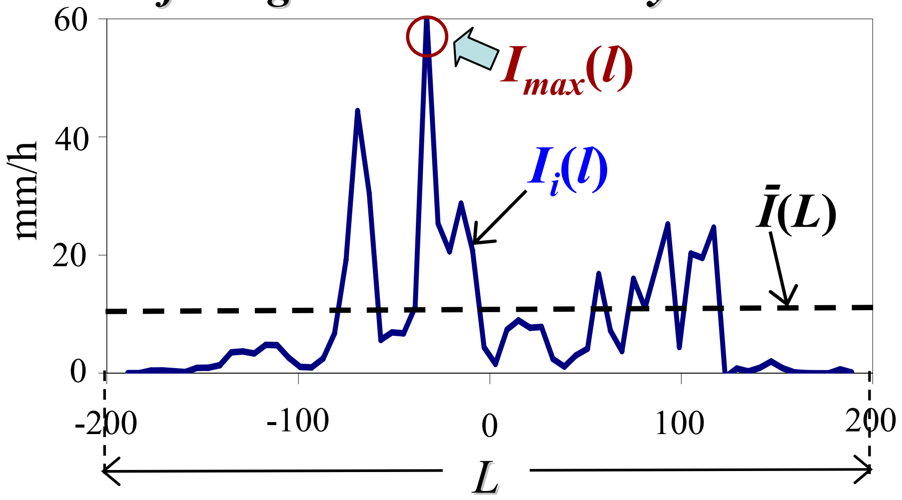
$$\gamma_i(l) = \frac{I_i(l)}{\bar{I}(L)}$$

random variable *with unit mean* (for $\gamma_i(l)$)

average *rainfall* ***intensity*** *inside l* (for $I_i(l)$)

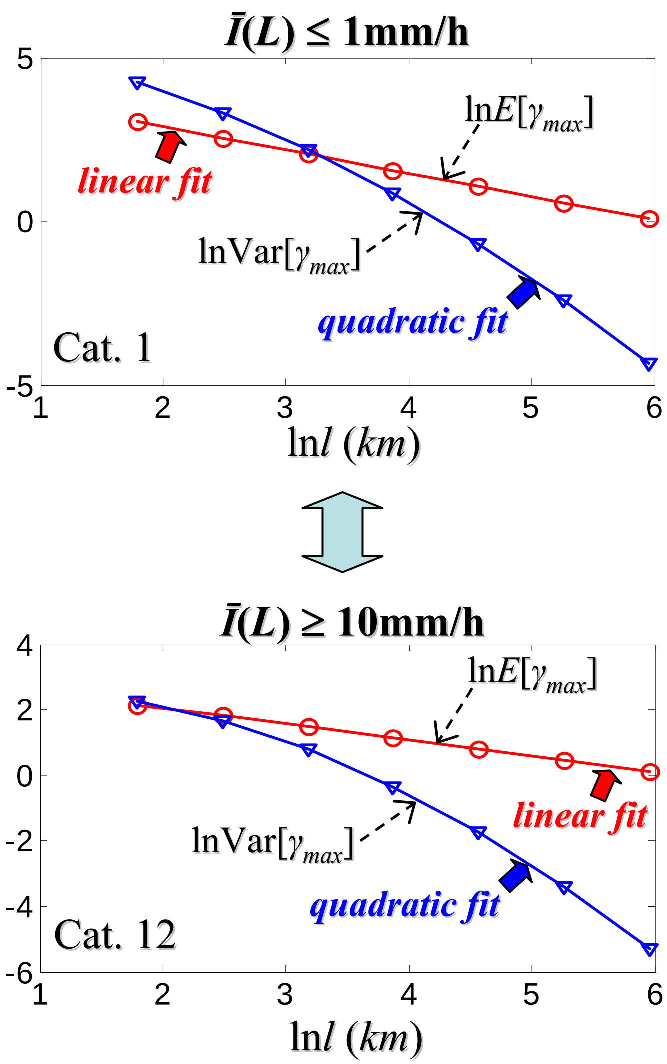
$\Rightarrow \gamma_{max} = max\{\gamma_1, \ldots, \gamma_{L/l}\}$

…for a generic scale l and y-location

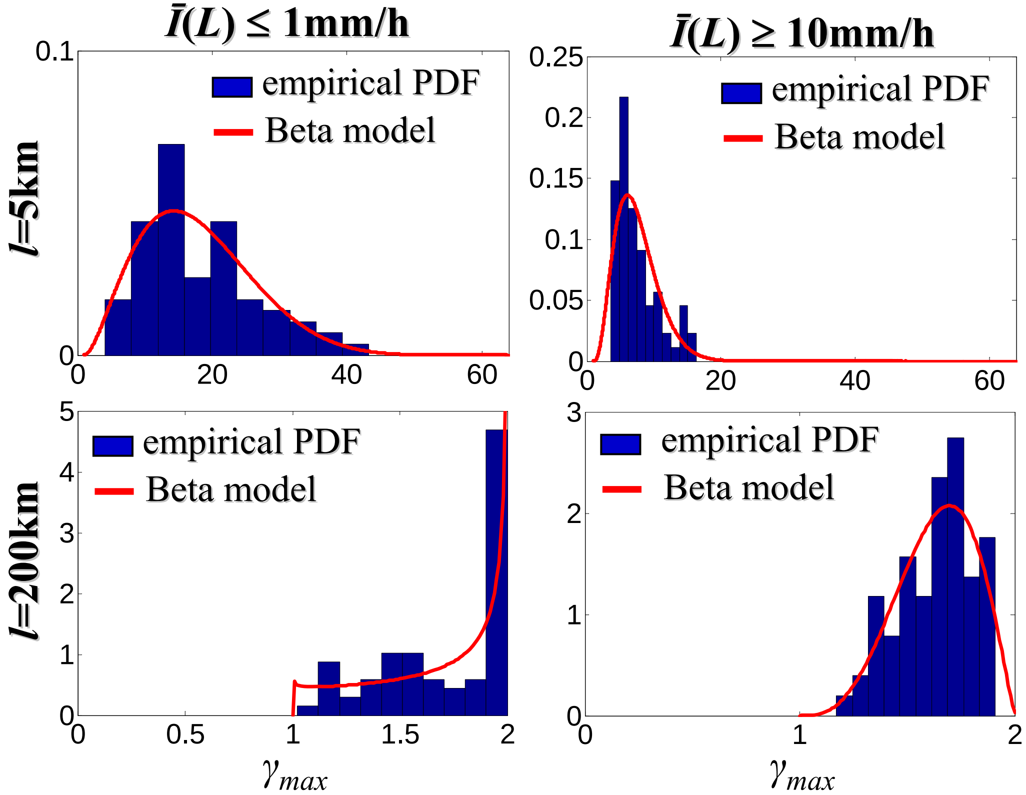


Maxima approach

- Calculate the empirical ***mean*** and ***variance*** of γ_{max} for different l and $\bar{I}(L)$



- Develop ***parametric expressions*** for the dependence of the mean and variance of γ_{max} on l and $\bar{I}(L)$
  - Find a suitable distribution model for γ_{max}, bounded in [1,L/l]

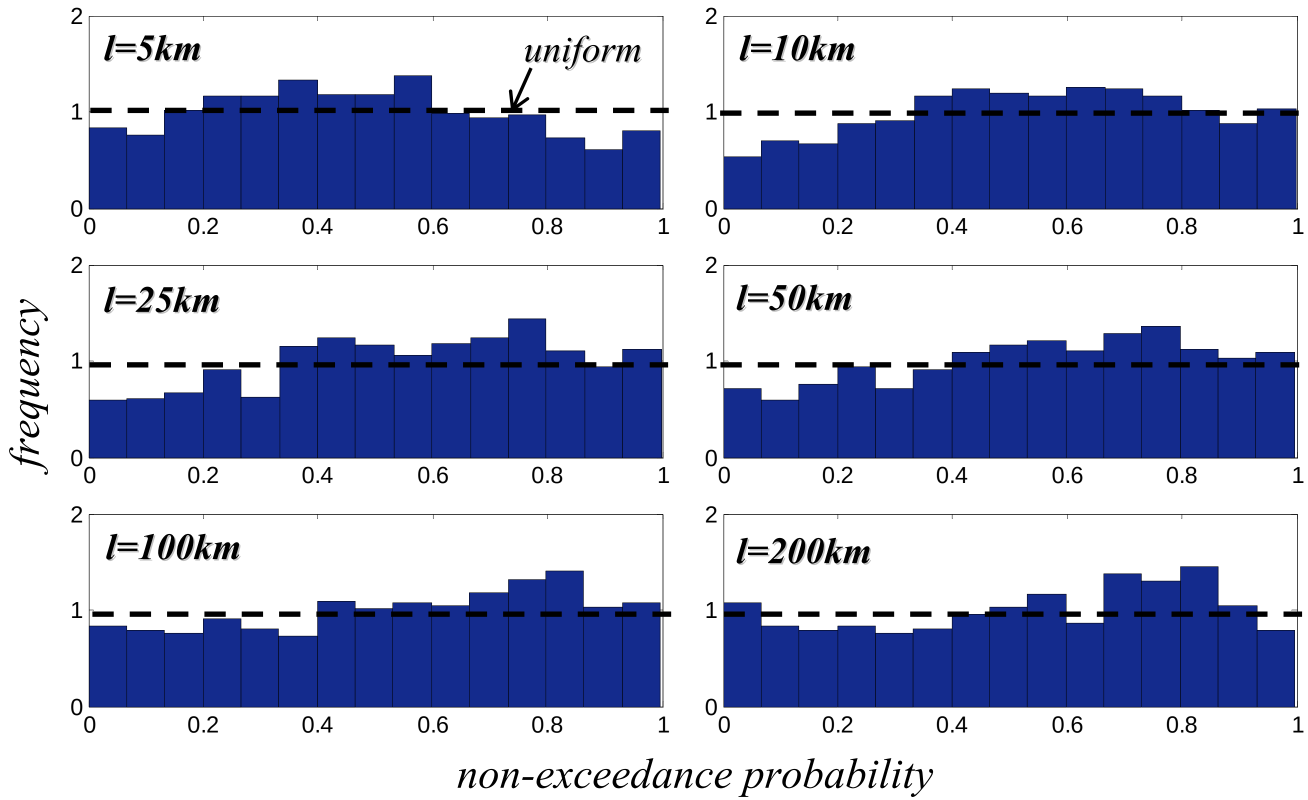


…statistical assessment

- *For each spatial scale **l**, use the model to calculate the **theoretical** non-exceedance **probability** of the **empirical maxima**.*

- *If the model is correct, then* ⇨ $F_{I_{max}}(I_{max,emp}) \sim U[0,1]$

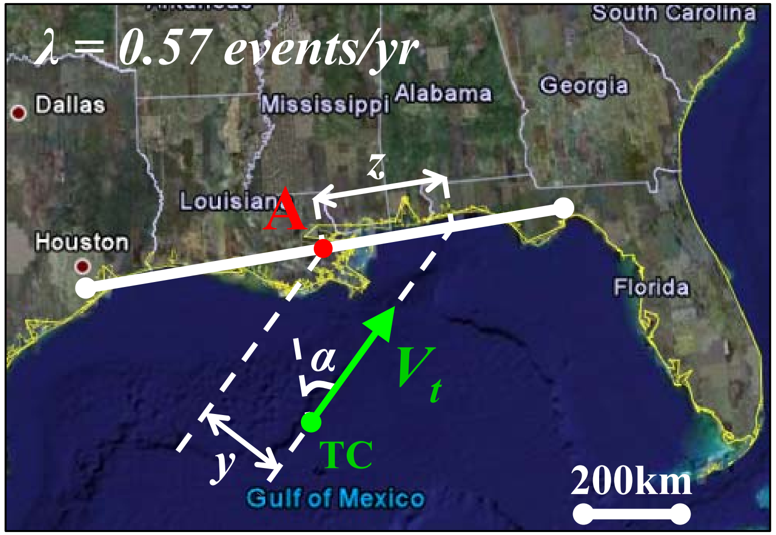
Direct approach



Similar results for other approaches…

4. Application to New Orleans

➢ **Recurrence model for** $\omega = [V_{max}, R_{max}, V_t, y]^T$ ***…and B = 1***



$[V_{max}|\Delta P] \sim$ (m/s) — **lognormal** with $m = 4.8\,\Delta P^{0.559}$, $\sigma = 0.15\,m$ (Willoughby and Rahn, 2004)

$[R_{max}|\Delta P] \sim$ (km) — **lognormal** with $m = 3.962\text{-}0.00567\Delta P$, $\sigma = 0.313$ (Vickery *et al*., 2000)

(ind.)

ΔP (mb) $\sim$ **shifted lognormal** with $m_{ln\Delta P} = 3.15$, $\sigma_{ln\Delta P} = 0.68$, *Shift par.* = 18mb (IPET, 2006)

⇩

$P[V_{max}, R_{max}]$

⇩

$V_t \sim$ **LN with m = 6m/s & σ = 2.5m/s** (Vickery *et al*., 2000, Chen *et al*. 2006)

$P[V_t]$

$z \sim$ **U[-500km, 500km]**

$\alpha \sim$ **N[-5.4°,(34.9°)²]** (IPET, 2006)

(ind) ⇨ $y = -z\cos(\alpha)$

$P[y]$

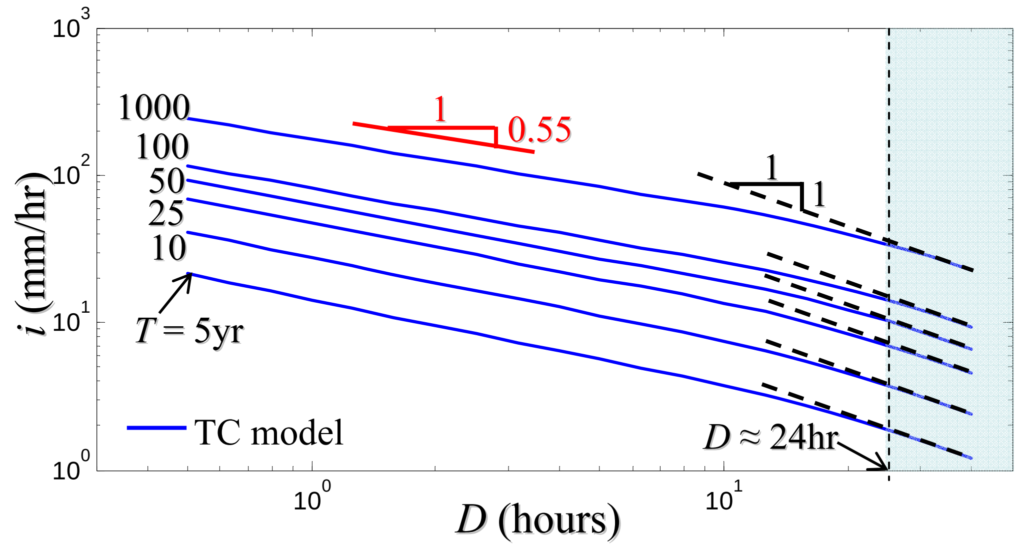
Joint density $P[\omega]$
(assuming ind.)

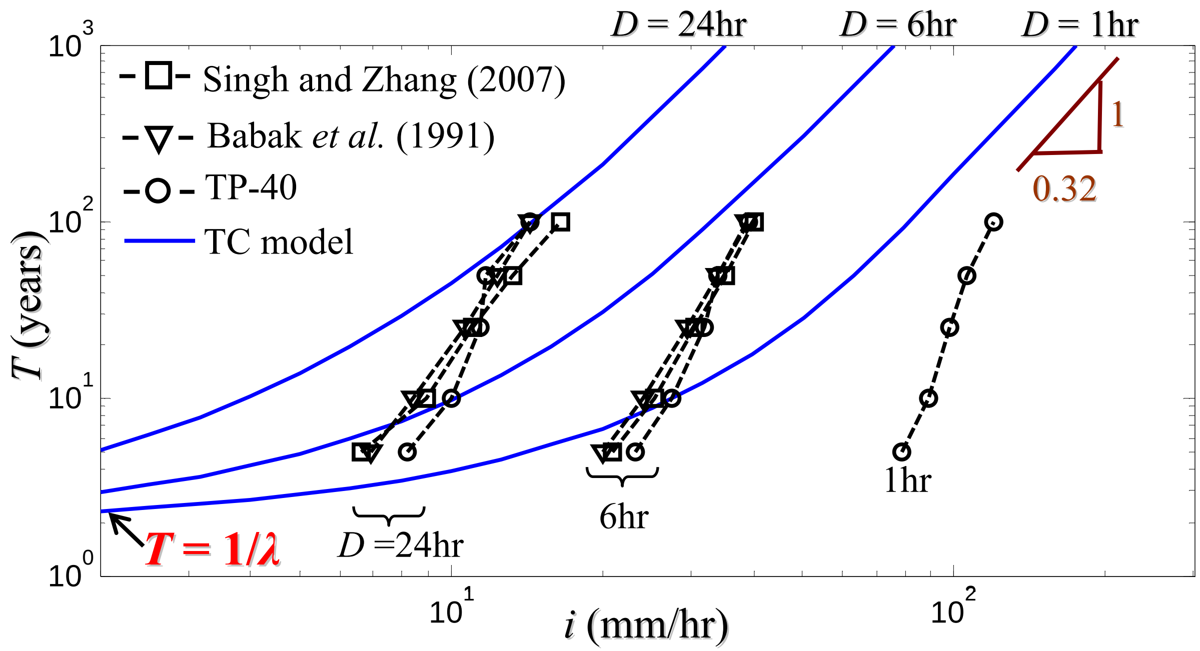
Application to New Orleans: IDF curves

Rainfall Risk and IDF curves:

$$\lambda_D(i) = \lambda \int_{\text{all } \omega} P[I_{max}(D) > i|\omega]\, P[\omega]\, d\omega$$

IDFs: *plots of i against* ***D*** *and* $T = 1/\lambda_D(i)$ (years)

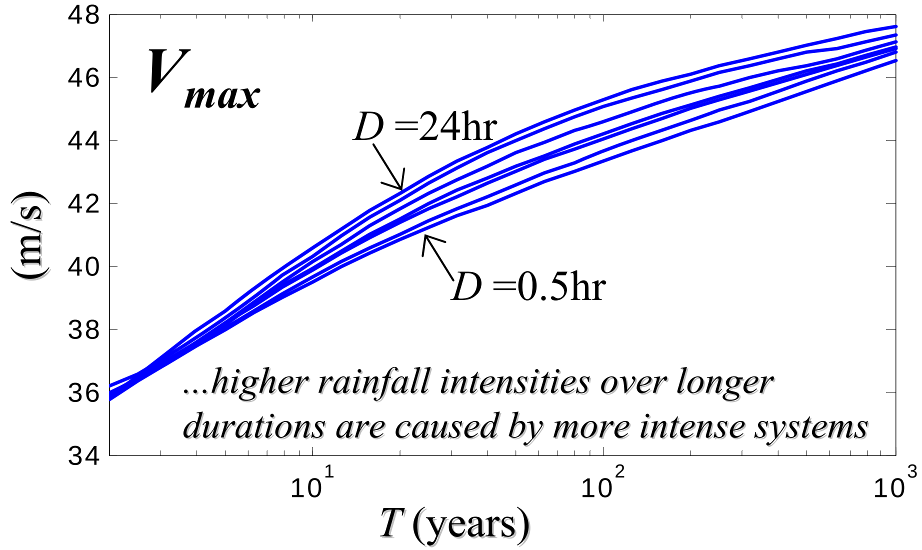


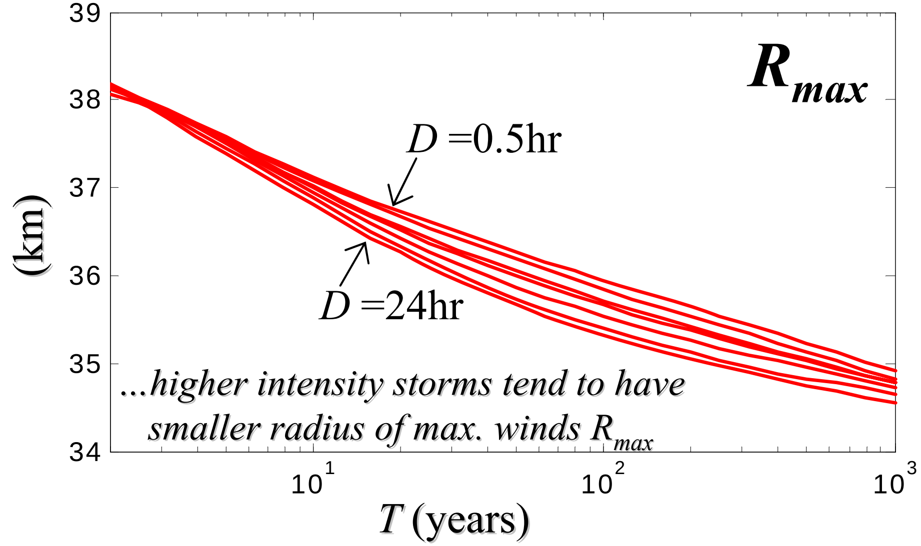


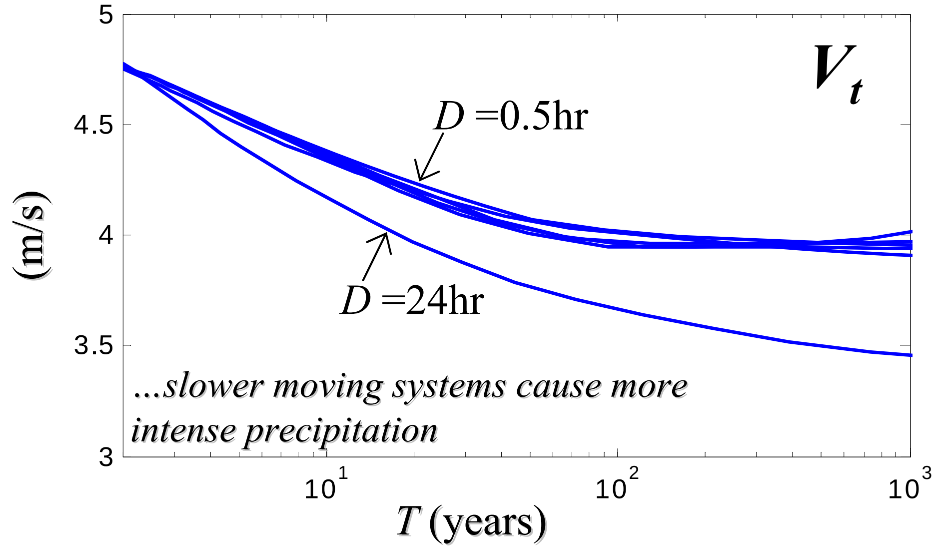
- For ***large D*** and ***T*** TCs dominate risk.
- For ***small D*** applies the rule: *"convection is convection"*

Design storms for New Orleans

<u>*Modal values of* [ω|*D*,*T*]:</u>







$y \approx R_{max}$

MSR *model maximum*

Conclusions (1)

- *Developed a model of* ***peak rainfall intensities*** *from TCs with the following characteristics:*

  - *Explicit parameterization of the hurricane:* $\boldsymbol{\omega}=[V_{max}, R_{max}, V_t, y]^T$
  - *Physical model* (MSR) *to obtain* ***large-scale rainfall*** *given* $\boldsymbol{\omega}$
  - *Statistical model for* ***large-scale*** *(storm-to-storm) rainfall fluctuations:* $\boldsymbol{\beta}$
  - *Statistical model for* ***small-scale*** *variability on rainfall maxima:* $\gamma_{max}(l)$
  - ***Calibration*** *and* ***validation*** *using PR/TRMM data*

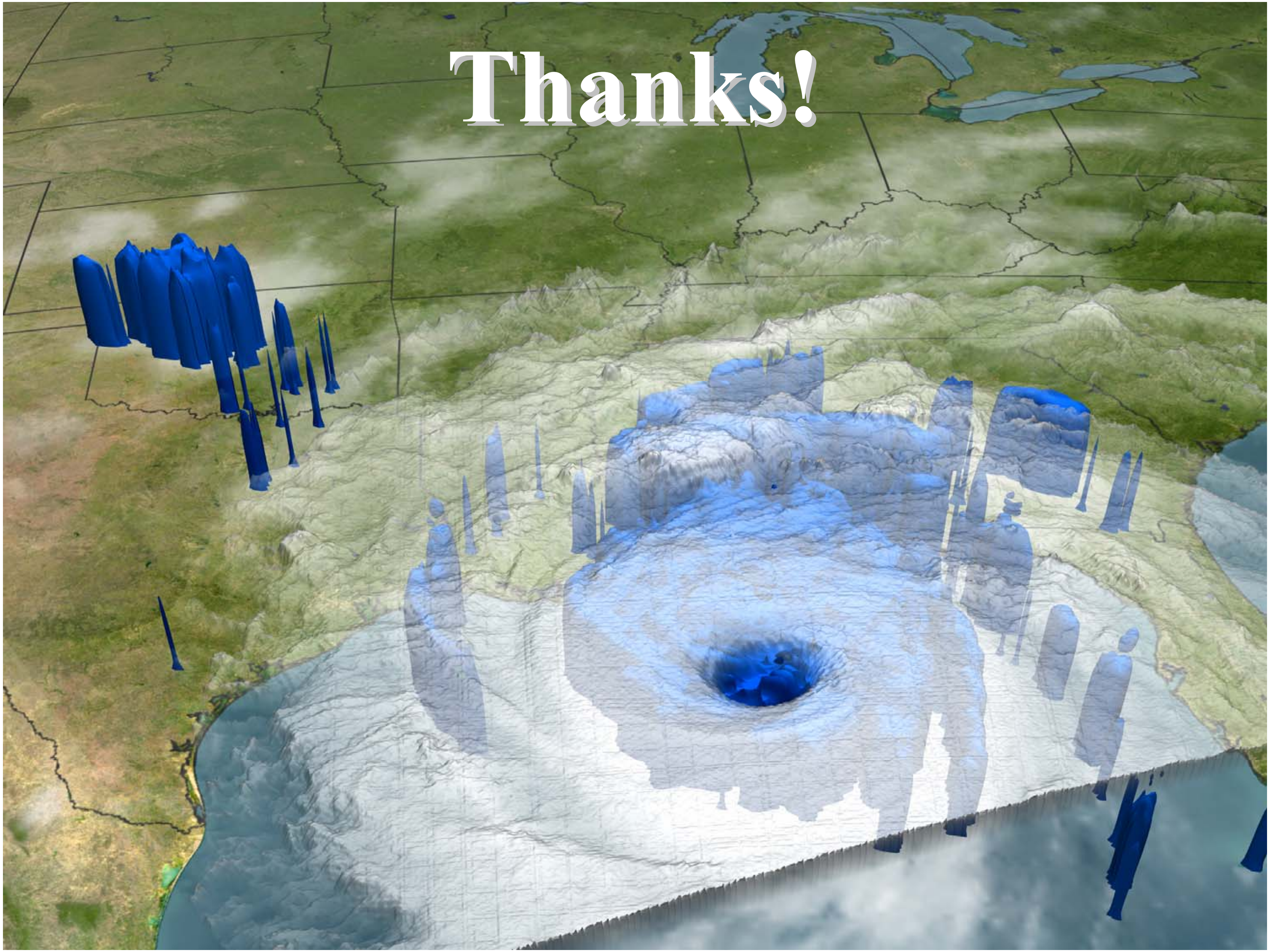
Conclusions (2)

Uses of Model:

- *Mean* ***wind*** *field characterization:* **MS model**
- *Obtain distribution of* ***maximum rainfall*** *intensity given storm parameters* **ω**: **MSR** + **Stat. model**
- *Obtain* ***design*** *rainfall* ***intensities*** *i for given* (D,T)
- *Obtain* ***design*** *storm parameters* **ω** *for given* (D,T)
- ***Assess*** *relative importance of TCs and other rainstorms*
- ***Complement*** *wind, surge, and wave risk models with a rain model*

Future Directions

- Develop a simple parameterization for $\bar{I}_{MSR}$
- Extend to locations farther inland
- Short-term rainfall forecasting
- Apply a similar approach to assess risk from TC winds


Thanks!