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Extreme Rainfall Intensities and Long-term Rainfall Risk from Tropical Cyclones

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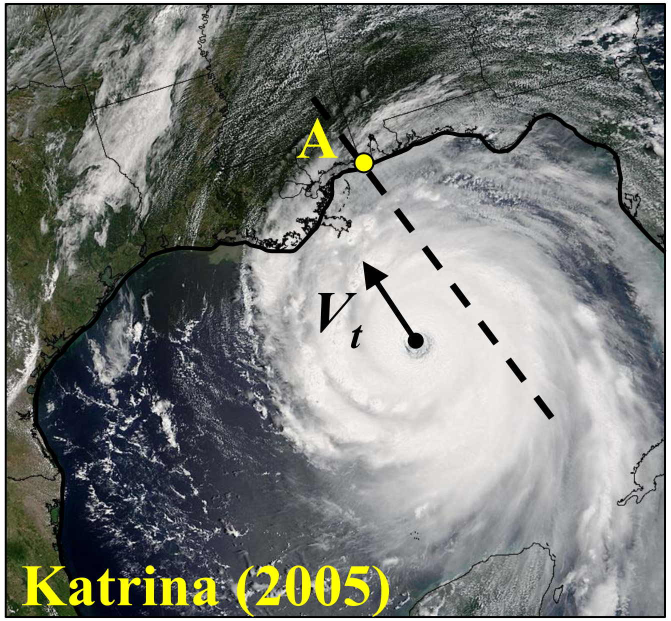
Objective

Long-term rainfall risk from TCs at location A:



$\lambda_D(i)$: rate at which $I_{max}(D)$ exceeds i at location A (events/year)

$I_{max}(D)$: maximum rainfall intensity at location A for averaging duration D

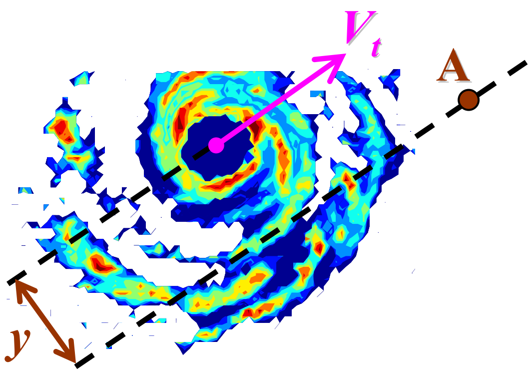
Risk analysis ⇨ $$\lambda_D(i) = \underset{\text{TC arrival rate [events/yr]}}{\lambda} \int_{\text{all } \boldsymbol{\omega}} \underbrace{P[I_{max}(D) > i \mid \boldsymbol{\omega}]}_{\textit{focus}} \underbrace{P[\boldsymbol{\omega}]}_{\textit{local recurrence (literature)}} d\boldsymbol{\omega}$$

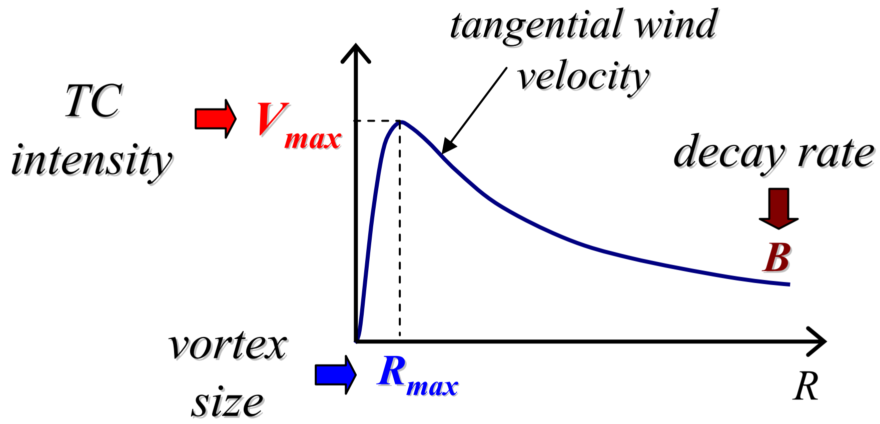
TC arrival rate [events/yr]

TC characteristics

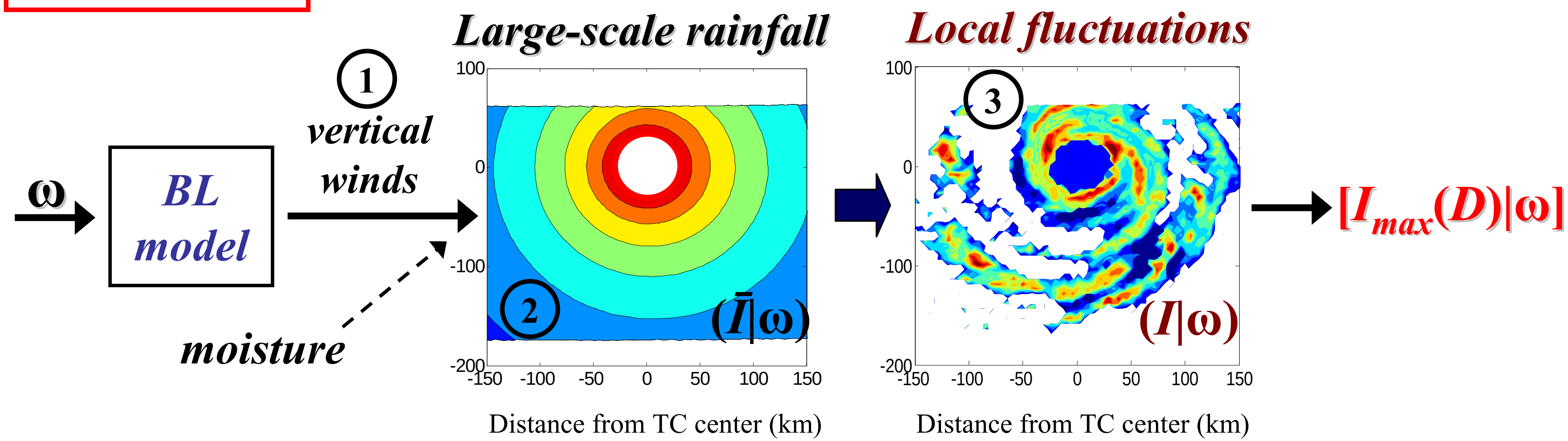
Approach to long-term risk modeling

➢ **parameters** $\omega = [V_{max}, R_{max}, B, V_t, y]^T$





$[I_{max}(D)|\omega]$



Outline

Part 1: BL winds

- Existing BL models $\Rightarrow$ *limitations*
- **MS** model for the BL
- Wind fields: comparison with **MM5**

Part 2: Rainfall from Winds

- **MS** + *moisture* $\Rightarrow$ **MSR** model
- Calibration using **PR** data

Part 3: Rainfall fluctuations

- **Storm-to-storm** variability of rainfall (*large scales*)
- **Sub-storm** variability (*small scales*)

Part 4: Application to New Orleans

- ***Recurrence*** model for **ω**
- Theoretical IDF curves for TC rainfall
- Comparison with empirical IDF results on all rainstorms (TCs and non-TCs)

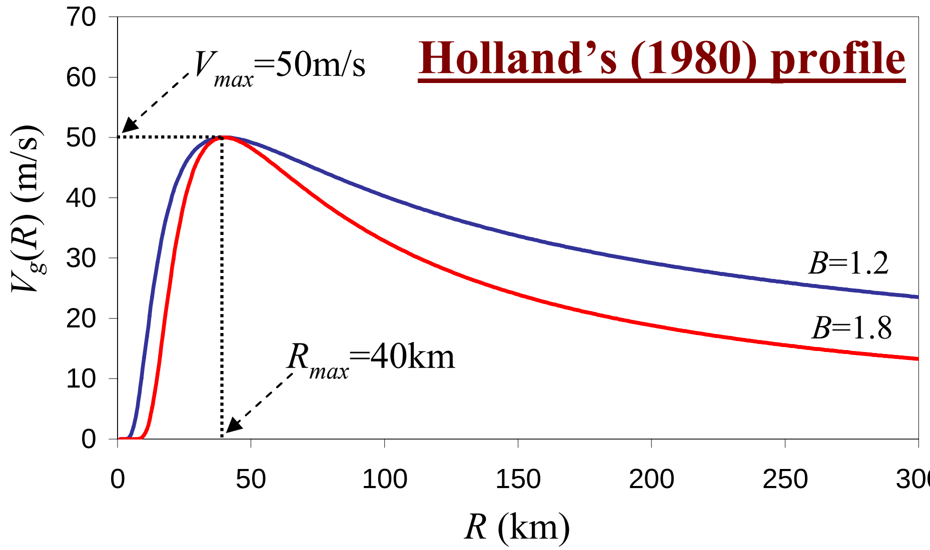
➢ **Conclusions and future directions**

1. Solving the Boundary Layer (BL)…

Conditions at BL top

$$U=0 \quad V=V_g$$

$$\frac{\partial W}{\partial Z}=\frac{\partial U}{\partial Z}=\frac{\partial V}{\partial Z}=0$$



W_H

Main vortex

BL top (Z=H)

BL equations

$$U\frac{\partial U}{\partial R}+\frac{V}{R}\frac{\partial U}{\partial \theta}+W\frac{\partial U}{\partial Z}+\frac{V_g^2-V^2}{R}+f(V_g-V)=K\frac{\partial^2 U}{\partial Z^2}$$

$$U\frac{\partial V}{\partial R}+\frac{V}{R}\frac{\partial V}{\partial \theta}+W\frac{\partial V}{\partial Z}+\frac{UV}{R}+fU=K\frac{\partial^2 V}{\partial Z^2}$$

$$\frac{1}{R}\frac{\partial (RU)}{\partial R}+\frac{1}{R}\frac{\partial V}{\partial \theta}+\frac{\partial W}{\partial Z}=0$$

U: radial   *V: tangential*

W: vertical

Surface boundary (Z=0)

Stress conditions with drag coefficient C_D ($C_D \to \infty$ for non-slip)

Boundary layer model 1: Kepert (2001)

Features:

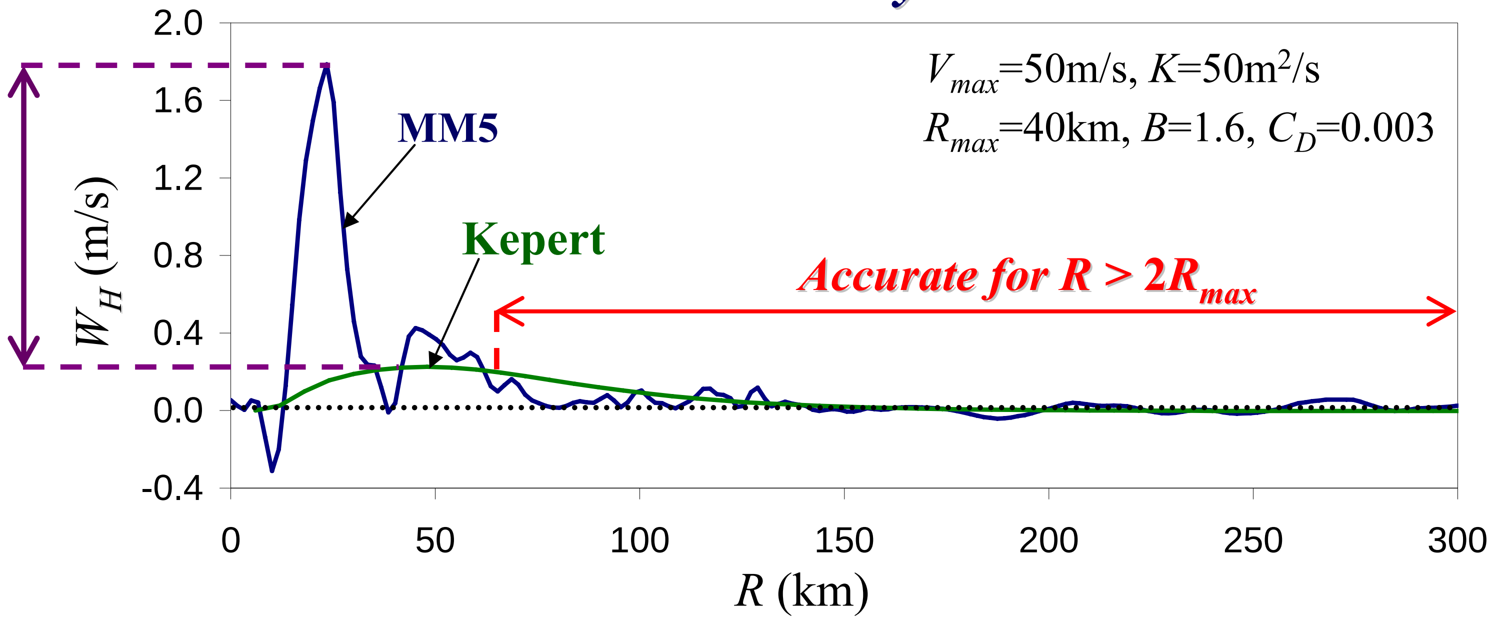
- ✓ ***Analytical*** *and* ***depth resolving***
  - BCs at Z=0 and $Z=H\rightarrow\infty$
  - BL scale thickness: $\boldsymbol{\delta(R,\theta)}$
- Accounts for ***storm translation***
- × ***Linearized version of BL equations***

Model breaks:

- *for large horizontal gradients* $\Rightarrow$ $\boldsymbol{R< 2R_{max}}$
- *for large vertical gradients* $\Rightarrow$ $\boldsymbol{C_D\rightarrow\infty}$
- *for high translation velocities* $\Rightarrow$ $\boldsymbol{V_t}$ **>5m/s**
- *under inertial neutrallity* $\Rightarrow$ $\boldsymbol{B}$ **>1.8**

Vertical wind velocity at H=1km

factor of 6



Boundary layer model 2: Shapiro (1983)

Features:

- Vertically averaged
- Accounts for ***storm translation***

Issues:

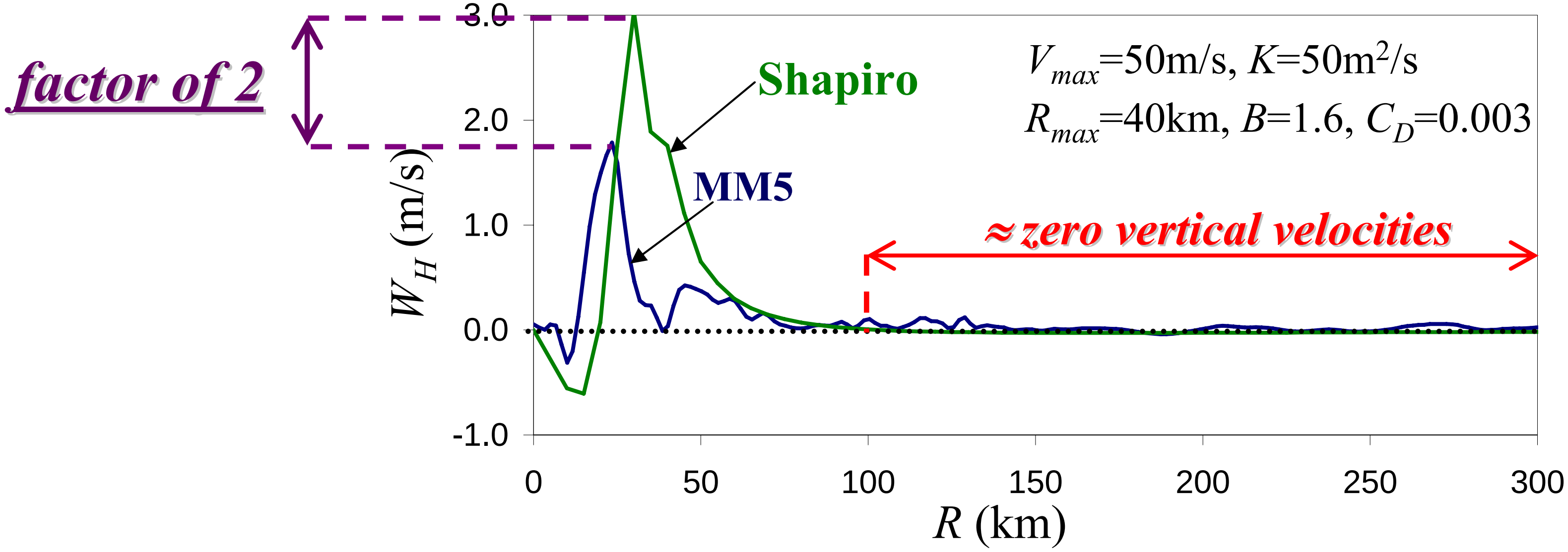
- *High horizontal velocities*
- ***Stability for $R > R_{max}$ requires***

constant boundary layer depth **H=1000m**

vertical diffusion coefficient ⇒ **K=50000m²/s**

discretization step ⇒ ***ΔR = 5km***

Vertical wind velocity at H=1km



Boundary layer model 3: Smith (1968)

Karman & Pohlhausen momentum integral method:

- ***Assume*** that dependence of V and U on Z is of the ***Ekman*** type:

$$V(R,Z)=\underbrace{V_g(R)}_{\text{gradient winds}}\, f[Z/\underbrace{\delta(R)}_{\text{BL scale thickness}}] \qquad U(R,Z)=\underbrace{E(R)}_{\text{amplitude coef.}}\, V_g(R)\, g[Z/\delta(R)]$$

Smith (1968): Ekman solutions

$$f(\eta) = -e^{-\eta}\,(a_1 \sin\eta + a_2 \cos\eta)$$
$$g(\eta)=1-e^{-\eta}\,(a_1 \cos\eta + a_2 \sin\eta)$$

- ***Substitute U*** and ***V*** into the BL equations
- ***Integrate*** *in the* ***vertical*** *direction* accounting for boundary conditions
- ***Solve*** ordinary differential equations (ODEs) for $E(R)$ and $\delta(R)$

Limitations:
- ***Stationary hurricanes***
- a_1, a_2 = *const.* $\Rightarrow$ ***Applies only for non-slip BCs***

Modification of Smith (1968) for a moving storm

MS model

- **Wind speeds (relative to the moving vortex):**

$$\left.\begin{aligned} V(R,\theta,Z) &= \Omega\left[R,\theta,\frac{Z}{\delta(R,\theta)}\right] \\ U(R,\theta,Z) &= E(R,\theta)\ \Psi\left[R,\theta,\frac{Z}{\delta(R,\theta)}\right] \end{aligned}\right\} \Rightarrow W_H(R,\theta) = -\frac{1}{R}\int_0^H \frac{\partial(RU)}{\partial R} + \frac{\partial V}{\partial \theta}\, dZ$$

Ω & Ψ functions:

$$\begin{cases} \Psi(r,\theta,\eta) = g(r,\theta,\eta)\ V_t \cos\theta + f(r,\theta,\eta)\ (V_g - V_t \sin\theta) - V_t \cos\theta \\ \Omega(r,\theta,\eta) = g(r,\theta,\eta)\ (V_g - V_t \sin\theta) - f(r,\theta,\eta)\ V_t \cos\theta + V_t \sin\theta \end{cases}$$

storm translation speed

f & g functions:

$$\begin{cases} f(R,\theta,\eta) = -e^{-\eta}\ [a_1(R,\theta) \sin\eta + a_2(R,\theta) \cos\eta] \\ g(R,\theta,\eta) = 1 - e^{-\eta}\ [a_1(R,\theta) \cos\eta + a_2(R,\theta) \sin\eta] \end{cases}$$

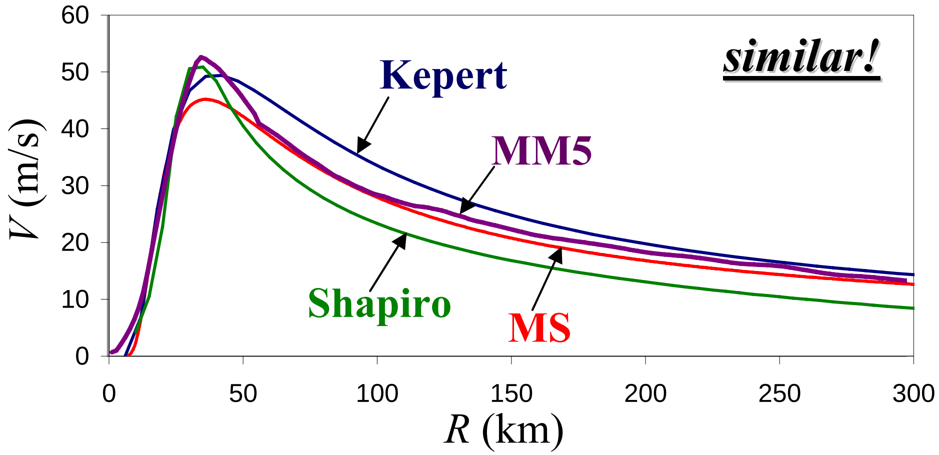
surface stresses *solve a linear system for a_1 and a_2*

- ***Solve*** a system of non-linear partial DEs for $E(R,\theta)$ and $\delta(R,\theta)$

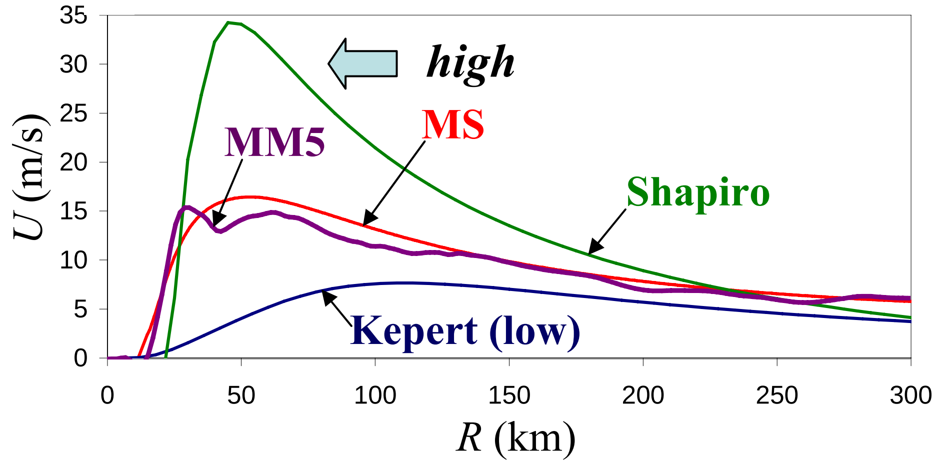
Model comparison: Stationary hurricane

(V_{max}=50m/s, R_{max}=40km, B=1.6, K=50m^2/s, C_D=0.003)

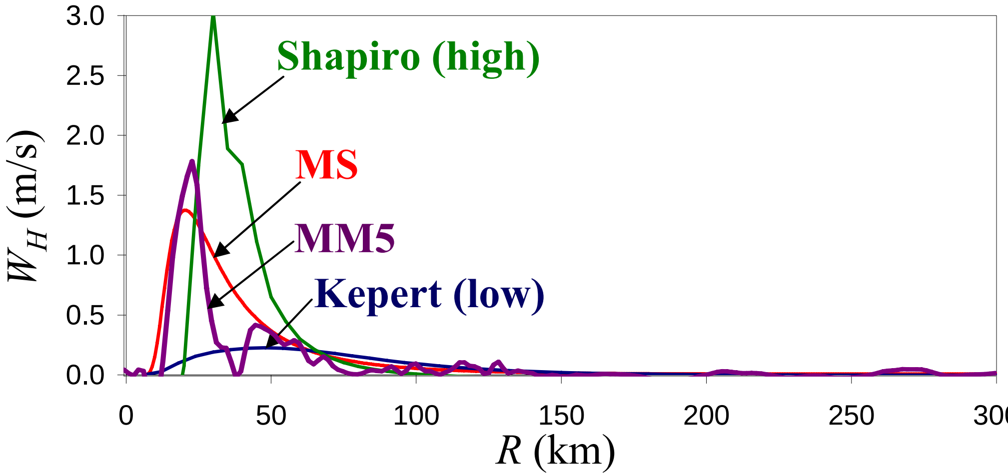
Vertically averaged (1km) ***tangential*** *winds*



Vertically averaged (1km) ***radial*** *winds*



Vertical *wind velocity at H=1km*



...similar results for moving hurricanes

2. Rain due to large-scale wind convergence

Assumption:

rainrate*= upward* ***water vapor flux*** *at the top of the boundary layer*

$$\bar{I} \propto W_H$$

large-scale rainfall intensity

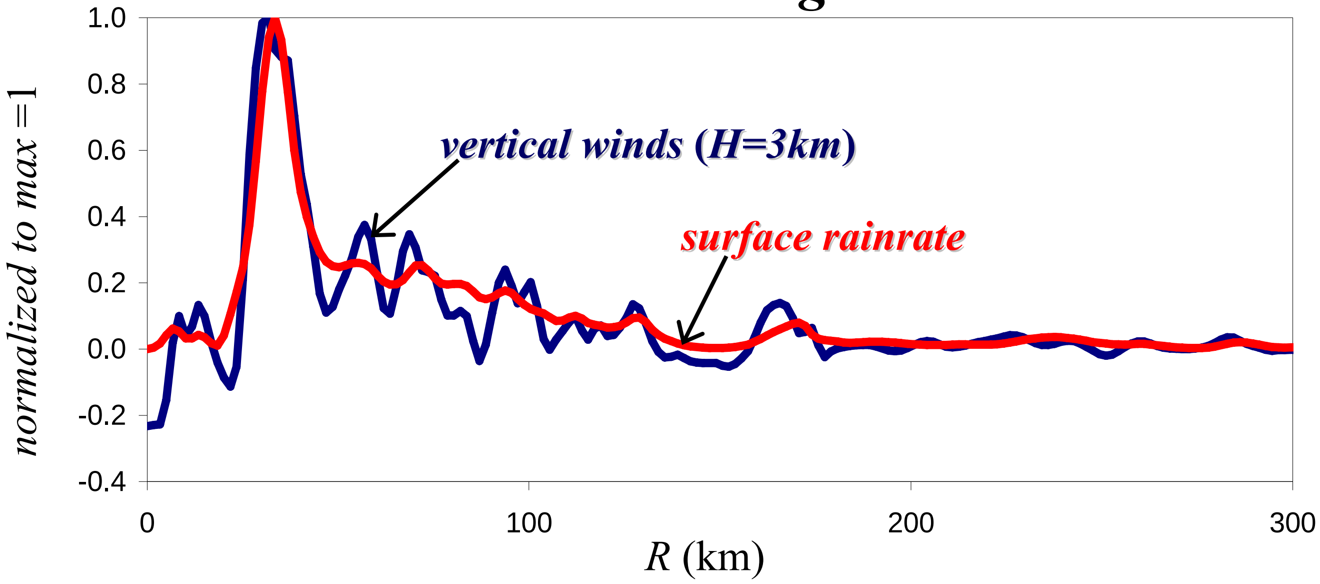
vertical wind velocity at H

const.= moisture content of air

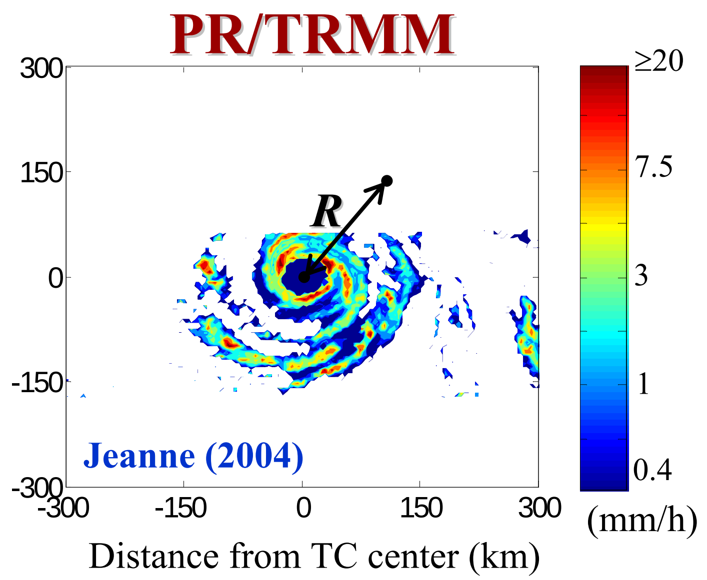
MSR model

...use MS model to calculate W_H

Validation using MM5



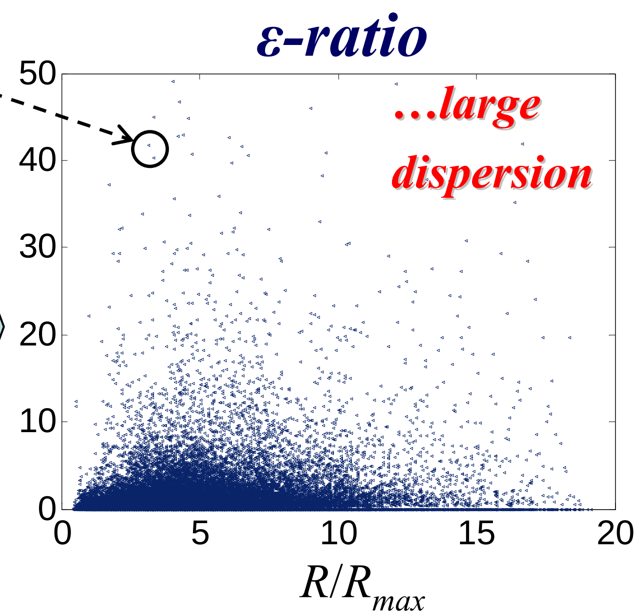
Calibration using PR/TRMM data



$$\varepsilon = \frac{I_{PR}}{I_{MSR}}$$

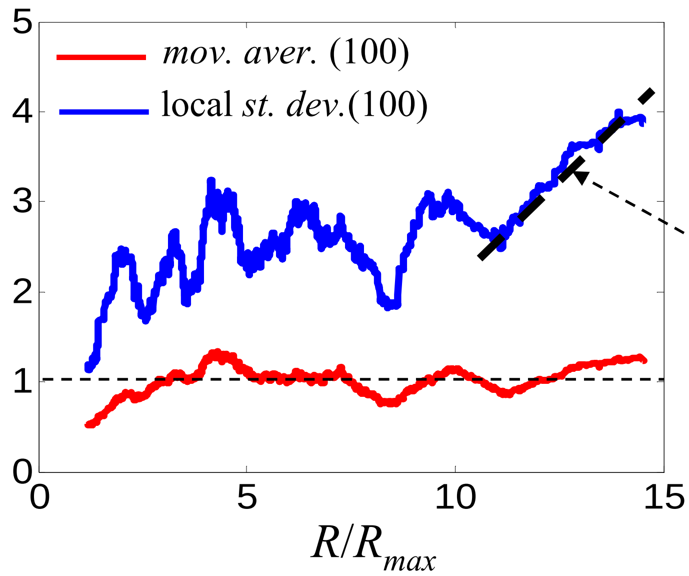
38 frames

48483 points



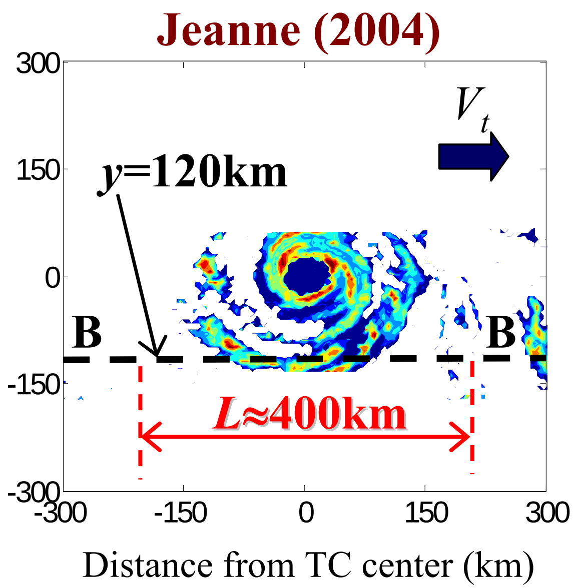
$$\boxed{B=1}$$

...almost ***unbiased*** *estimation*



larger variability in the outer TC

3. Statistical model of rainfall fluctuations

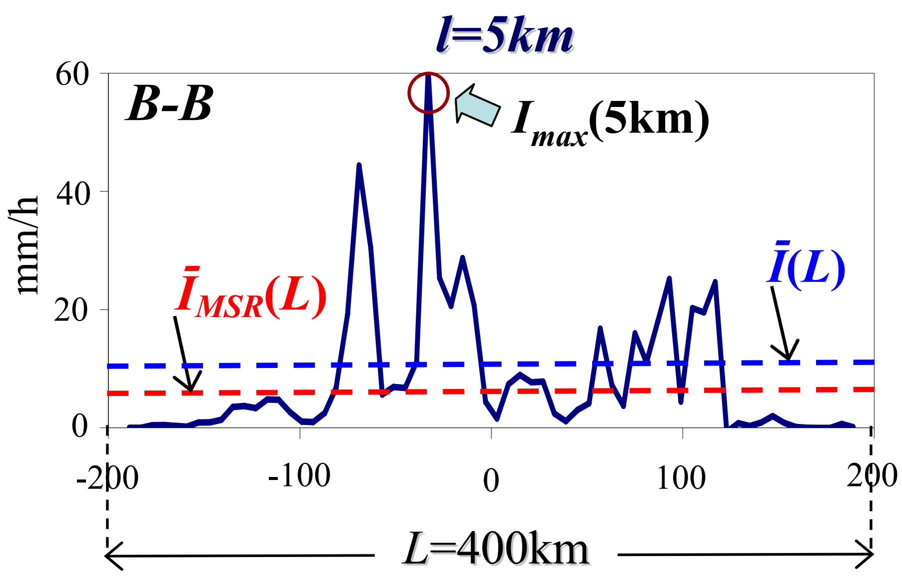


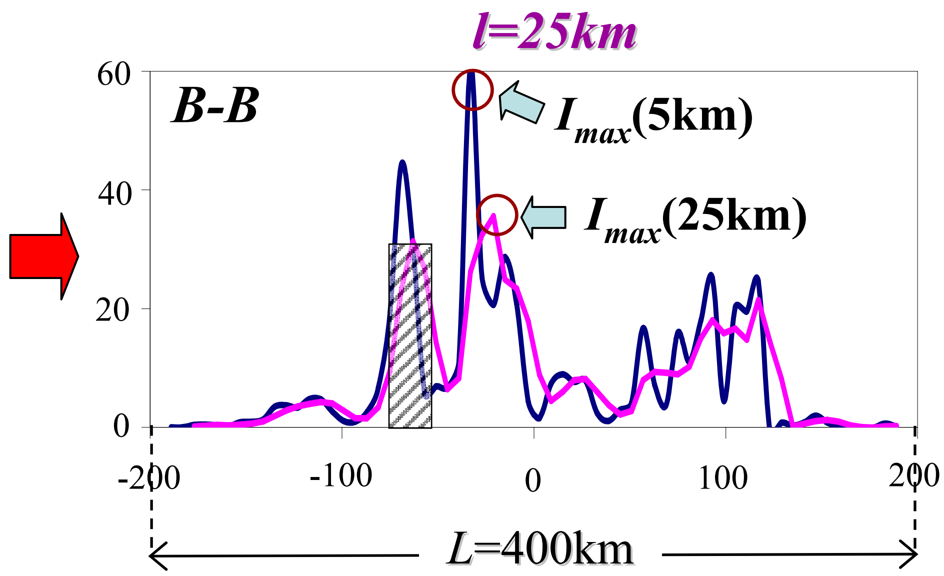
$$I_{max}(l) = \overbrace{\bar{I}_{MSR}(L)\ \beta}^{\text{(large-scales)}}\ \overbrace{\gamma_{max}(l)}^{\text{(small-scales)}}$$

MSR** estimate for the mean **rainfall intensity** inside **L

*corrects the **model** mean relative to the **empirical** mean*

***amplification factor** for the maximum inside l*





Statistical models for $[\beta|\omega]$ and $[\gamma_{max}(l)|\omega]$

Model for $[\beta\,|\omega]$

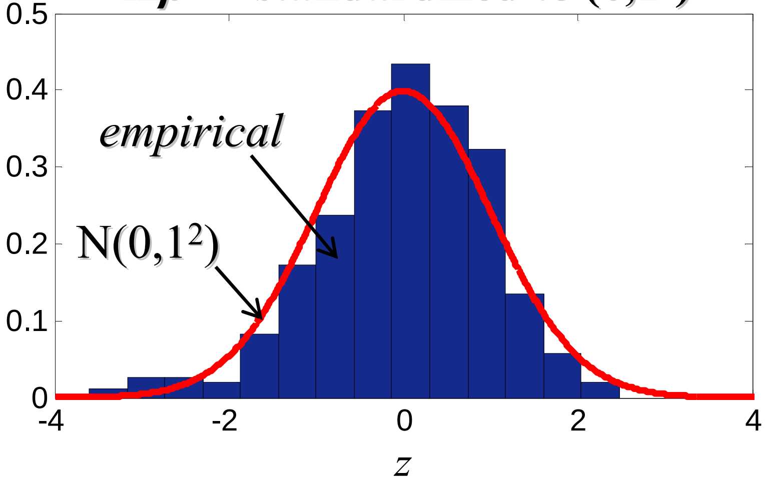
$$\beta = \frac{\bar{I}(L)}{\bar{I}_{MSR}(L)}$$

empirical *mean inside L*

model *estimate*

… $\beta\,(y, \bar{I}_{MSR})$ ~ lognormal

lnβ ⇨ standardized to $(0,1^2)$

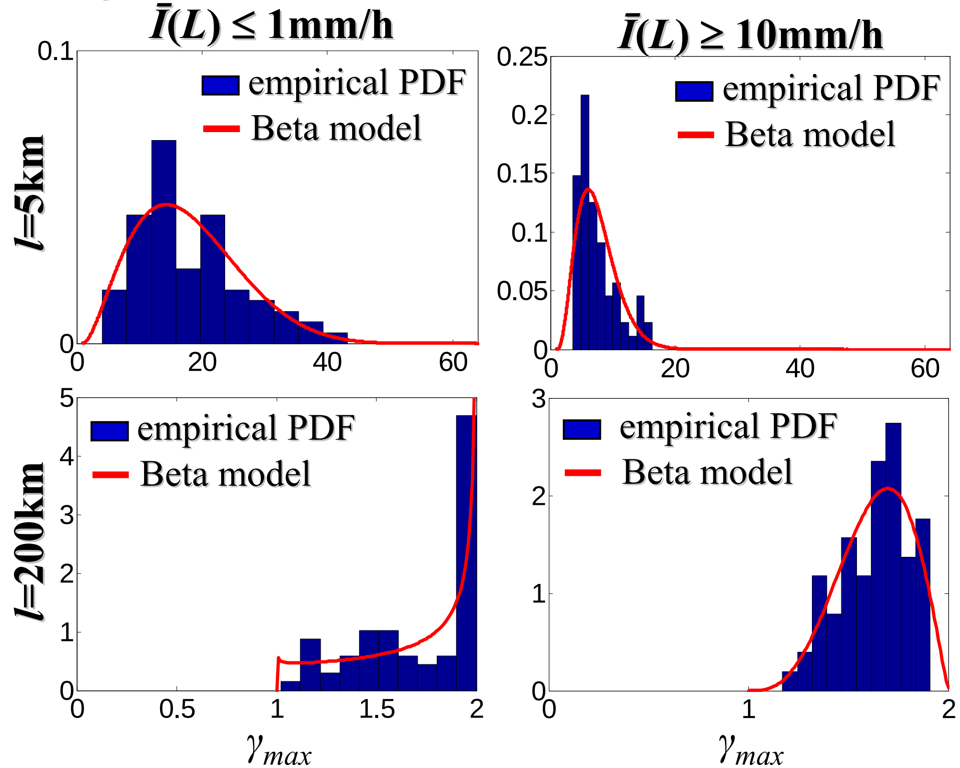


Model for $[\gamma_{max}(l)|\omega]$

$$\gamma_{max}(l) = \frac{I_{max}(l)}{\bar{I}(L)}$$

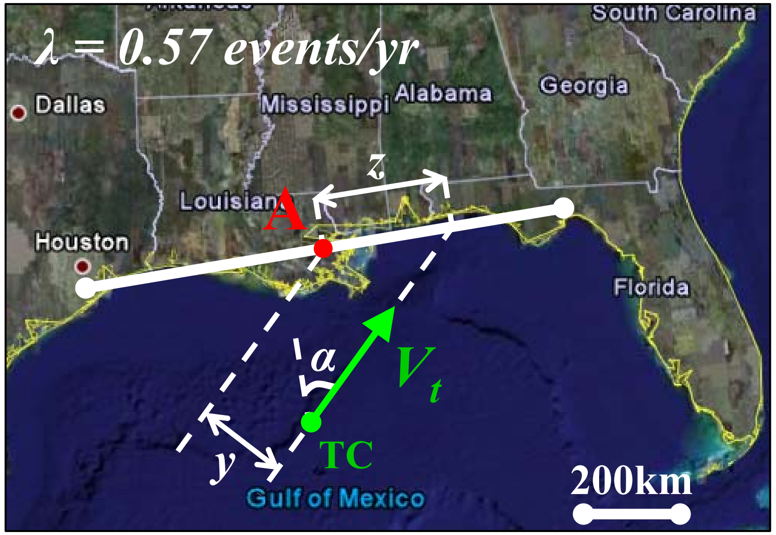
maximum *rainfall intensity at scale l*

parameterize in terms of $\bar{I}$



4. Application to New Orleans

➢ **Recurrence model for** $\omega = [V_{max}, R_{max}\ V_t, y]^T$ ***…and B = 1***



$[V_{max}|\Delta P] \sim$ (m/s) **lognormal** with $m = 4.8\,\Delta P^{0.559}$, $\sigma = 0.15\,m$ (Willoughby and Rahn, 2004)

$[R_{max}|\Delta P] \sim$ (km) **lognormal** with $m = 3.962\text{-}0.00567\Delta P$, $\sigma = 0.313$ (Vickery *et al.*, 2000)

(ind.)

ΔP (mb) ~ **shifted lognormal** with $m_{ln\Delta P} = 3.15$, $\sigma_{ln\Delta P} = 0.68$, *Shift par.* = 18mb (IPET, 2006)

$P[V_{max}, R_{max}]$

$V_t \sim$ **LN with m = 6m/s & σ = 2.5m/s** (Vickery *et al.*, 2000, Chen *et al.* 2006)

$P[V_t]$

$z \sim$ **U[-500km, 500km]**

$\alpha \sim$ **N[-5.4°,(34.9°)²]** (IPET, 2006)

(ind)

$\boxed{y = -z\cos(\alpha)}$

$P[y]$

Joint density $P[\omega]$

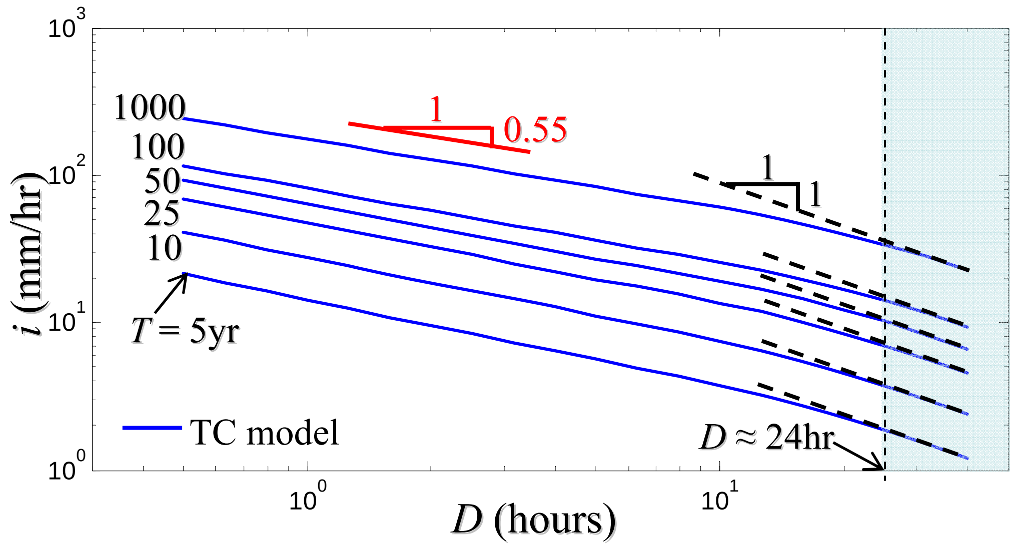
(assuming ind.)

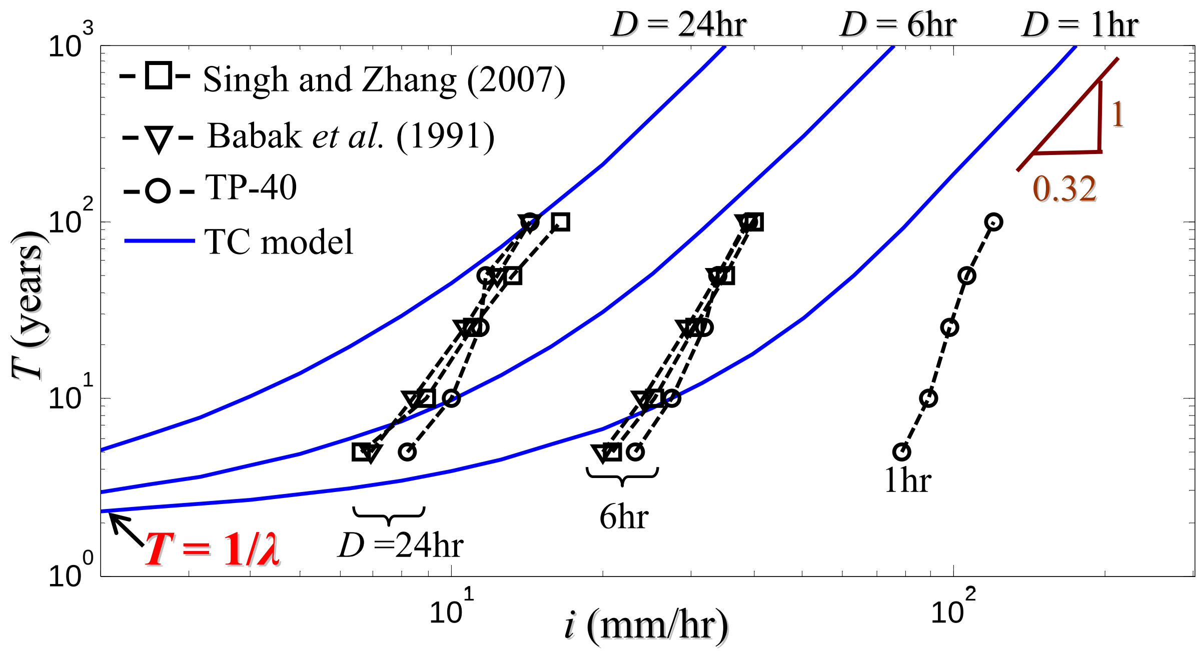
Application to New Orleans: IDF curves

Rainfall Risk and IDF curves:

$$\lambda_D(i) = \lambda \int_{\text{all } \omega} P[I_{max}(D) > i|\omega]\, P[\omega]\, d\omega$$

IDFs: *plots of i against* ***D*** *and*
$T = 1/\lambda_D(i)$ (years)

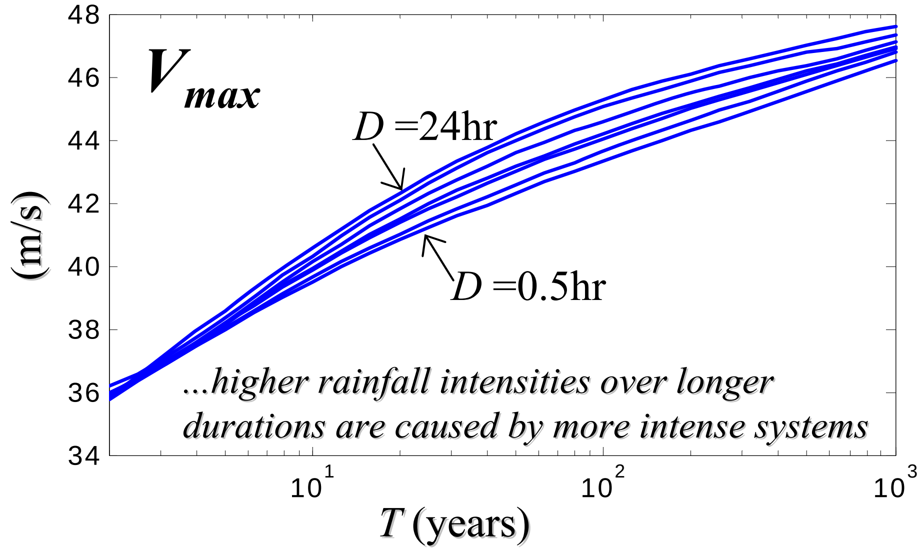


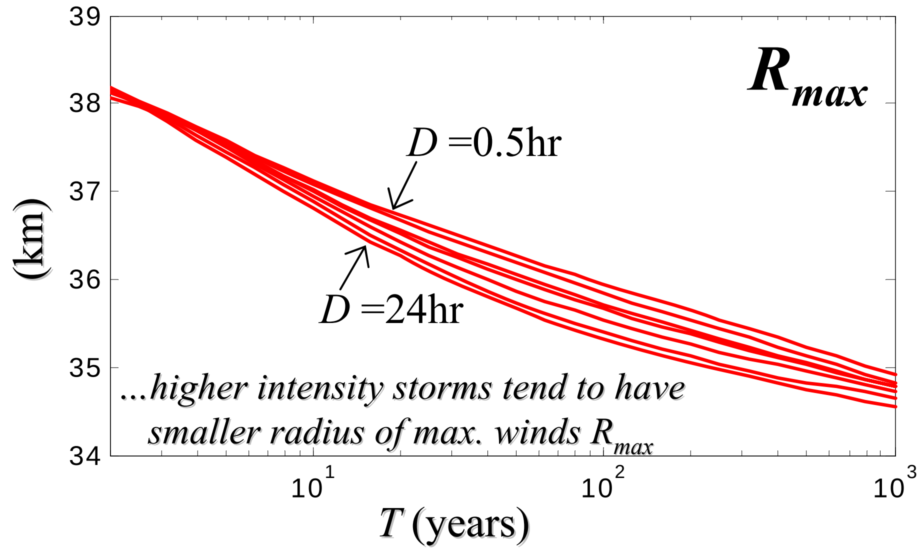


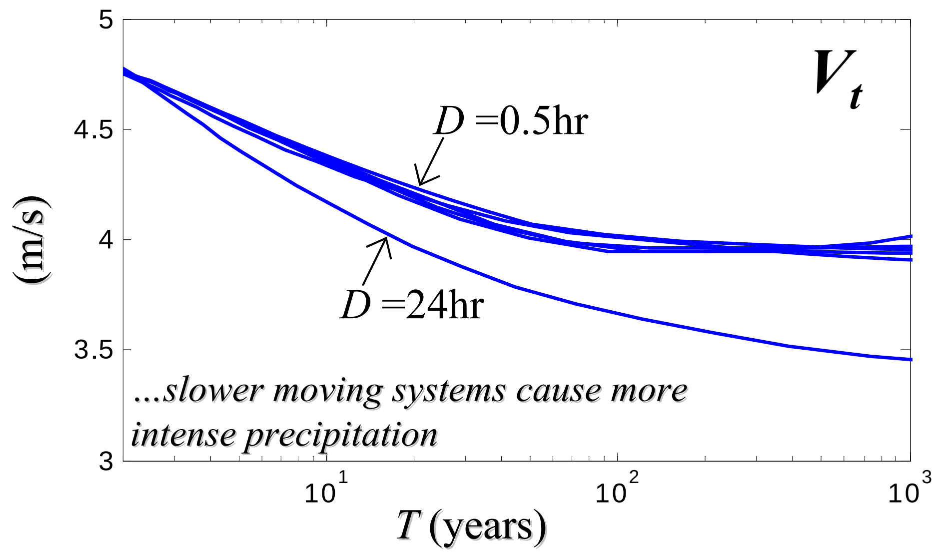
- For ***large D*** and ***T*** TCs dominate risk.
- For ***small D*** applies the rule: *"convection is convection"*

Design storms for New Orleans

Modal values of [$\omega|D,T$]:







$$\boxed{y \approx R_{max}}$$

MSR *model maximum*

Conclusions (1)

- *We developed a model of* ***peak rainfall intensities*** *from TCs with the following characteristics:*

  - *Explicit parameterization of the hurricane:* $\boldsymbol{\omega}=[V_{max}, R_{max}, V_t, y]^T$
  - *Physical model* (MSR) *to obtain* ***large-scale rainfall*** *given* $\boldsymbol{\omega}$
  - *Statistical model for* ***large-scale*** *(storm-to-storm) rainfall fluctuations:* $\boldsymbol{\beta}$
  - *Statistical model for* **small-scale** *variability on rainfall maxima:* $\gamma_{max}(l)$
  - ***Calibration*** *and* ***validation*** *using PR/TRMM data*

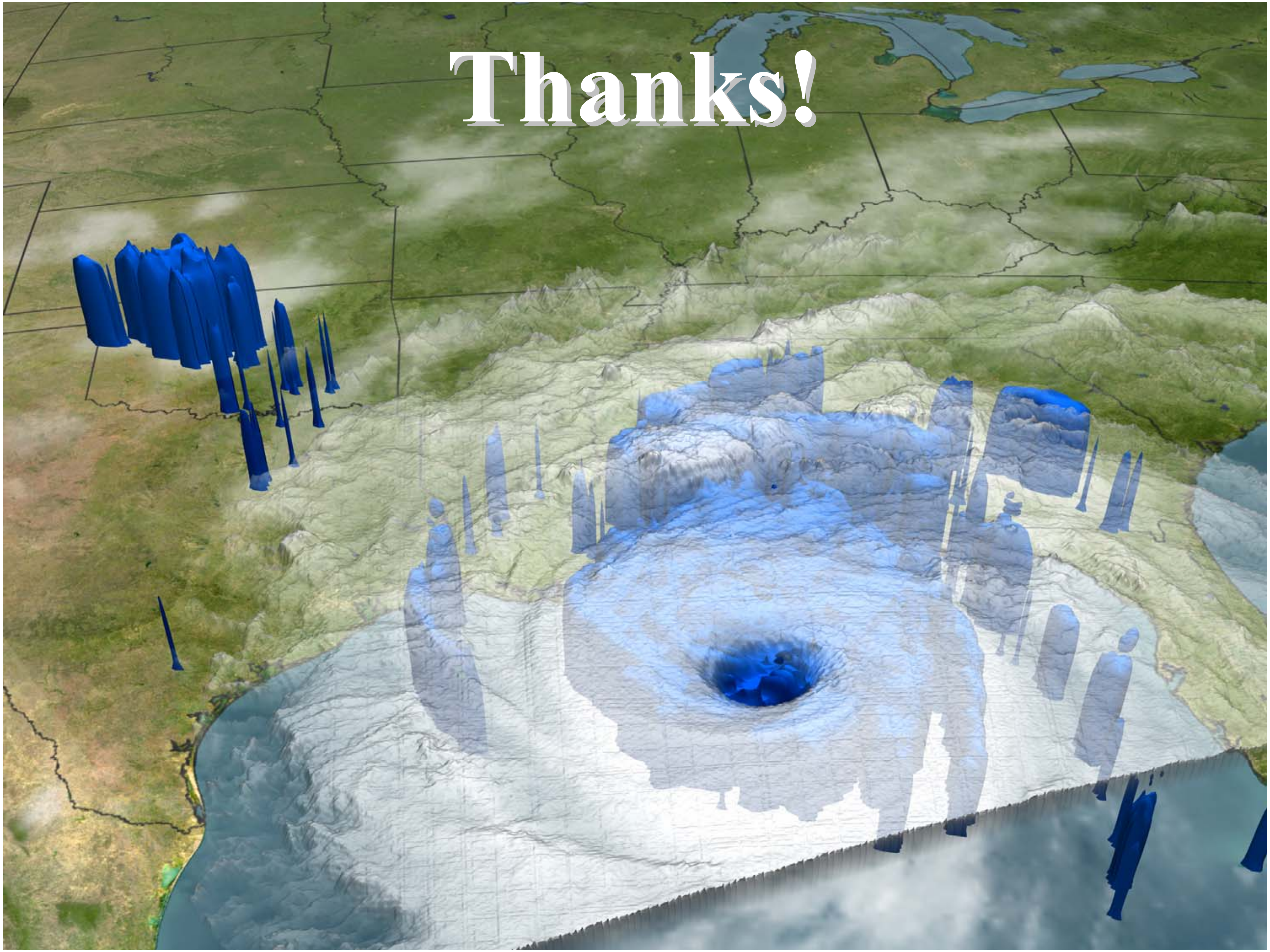
Conclusions (2)

Uses of Model:

- *Mean **wind** field characterization:* **MS model**
- *Obtain distribution of **maximum rainfall** intensity given storm parameters* **ω**: **MSR** + **Stat. model**
- *Obtain **design** rainfall **intensities** i for given* (D,T)
- *Obtain **design** storm parameters* **ω** *for given* (D,T)
- ***Assess** relative importance of TCs and other rainstorms*
- ***Complement** wind, surge, and wave risk models with a rain model*

Future Directions

- Develop a simple parameterization for $\bar{I}_{MSR}$
- Extend to locations farther inland
- Short-term rainfall forecasting
- Apply a similar approach to assess risk from TC winds


Thanks!