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To
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with compliments,
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GEOPHYSICAL TIME SERIES AND CATASTROPHISM

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Introduction

The old goal of a great many Egyptian rulers and governments to harness the Nile River and 'make use of the Nile water to the fullest possible extent' (*Hurst et al.,* 1965) has led to extensive studies of hydrological records of the Nile River and, in particular, of the long series of maximum flood stages recorded on the Roda gauge in Cairo which dates back to 641 A.D.

Since about 1936 these studies were conducted by H.E. Hurst and were aimed at finding a storage reservoir capacity that would be necessary to make the mean annual discharge of the Nile River constant and equal to its long-term mean. 'The method for doing this is well known and consists in taking the departures of the yearly discharges from the mean in order and forming their continued sums. The difference (range), R, between the maximum and minimum of these sums is the storage which would have been required to maintain the mean discharge throughout the period' (*Hurst et al.,* 1965). The method is illustrated in Figure 1. In order that the values of R_n corresponding to a given series length n could be compared for different time series, the range is normalized by the standard deviation S_n of the series to form a variable R_n/S_n called the 'rescaled range'.

Hurst (1951) found that for a random series

$$E\left[R_n/S_n\right] \sim n^{0.5} \tag{1}$$

where E signifies the arithmetic mean and the sign - indicates proportionality. This result was confirmed by *Feller* (1951) and was later shown to hold asymptotically not only for purely random processes but also for all the common types of stationary stochastic processes. It was therefore expected that for large n the plot $\log E\,(R_n/S_n)$ versus $\log n$ (let us call it the 'range plot') of empirical time series should have an asymptote with slope equal to 0.5. It thus came as a great surprise to theoreticians when Hurst showed that the range plot of the Roda gauge data as well as range plots of long series of precipitation, river discharges, annual

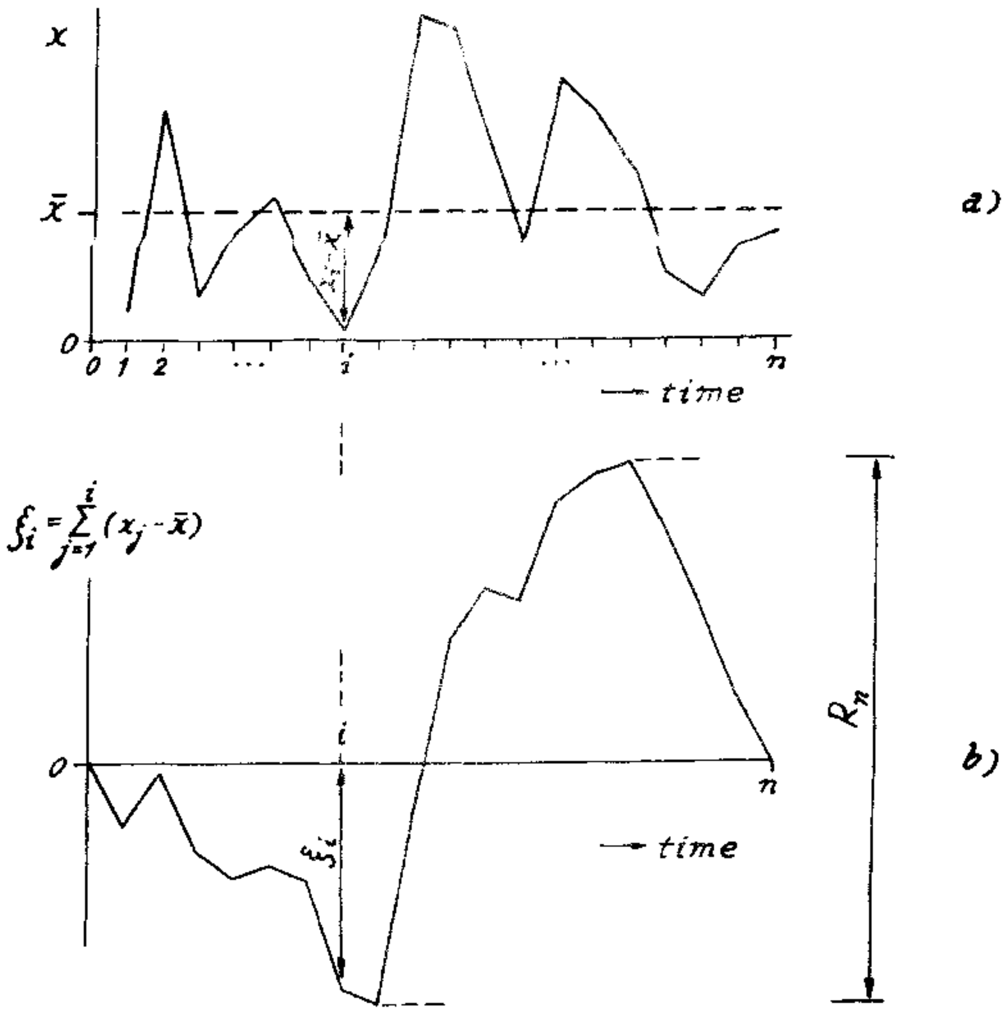

Fig. 1 a) Time series of variable x
b) Time series of cumulative departures from the mean x

temperatures, and pressures, annual sediment deposits (varves) in lakes, and tree ring thicknesses exhibited a relationship

$$E\left[R_n/S_n\right] \sim n^h \tag{2}$$

where h was close to 0.7. Similar empirical findings were later made with regard to economic and other historic time series.

The failure of the exponent *h* to accord with theory underlying eq. (1) has been labelled the Hurst phenomenon. It has long resisted an adequate theoretical explanation and geophysicists and mathematicians still have not reached agreement as to its cause.

One theoretical model that accounts for the Hurst phenomenon is a stochastic process called fractional Brownian noise (fBn) put forward by *Mandelbrot* (1965) and later developed by himself and his collaborators (a detailed bibliography is given, for instance, in *Klemeš*, 1974). The fBn is a time series, each term of which, $x(t)$, has been constructed as a weighted sum of an infinite number of Gaussian random quantities, $\varepsilon(t-1), \varepsilon(t-2), \ldots, \varepsilon(t-\tau)$; $\tau \to \infty$ where the weights are τ^{-k}, with $0.5<k<1.5$. The fBn can possess any value of *h* observed in historic series and it (or rather its approximations) has been successfully used as an operational model for hydrologic series. The problem is, however, that this mathematical model implies that the modelled physical process has an infinite memory of a peculiar kind: the formation of each new term of the series requires that all the past values be available (i.e. stored) since their original values are repeatedly used and modified by different coefficients (weights) in every step. Such a mechanism can be easily visualized in economic time series where the original values of variables (prices, volumes of stock, etc.) are preserved in the book-keeping process, and in some biological processes where the long-term memory could be related to the mechanism of genetic coding. It, however, does not seem likely that the model has universal validity, especially in geophysical processes which tend to have the so called Markovian property, it means the past history of the process affects its present state through its cumulative effect which is fully reflected in a small number of the most recent states of the process. For instance, the rate of outflow from a lake depends on the instant elevation of its water level and the slope of the surface, and it is immaterial through what sequence of past events their current values have been reached. It is hard to visualize a physical mechanism through which, for instance, this month's mean temperature could have been directly influenced by that of, say, May 1950 or 1850.

The present author (Klemeš, 1974) has suggested two different processes that account for the Hurst phenomenon and whose mechanisms are compatible with geophysical processes. One of them is a compounded cumulative process arising in a series of semi-infinite storage reservoirs where output from one becomes the input into the next. This process, although quite common in hydrologic and related processes, will not be discussed here as it has no bearing on the concept of catastrophism. It is the other process that can be of interest in this connection.

Process with time-variant central tendency

Consider a historic record of a time series describing fluctuations of some geophysical variable. For the sake of simplicity the variable concerned will be one defined on the yearly basis, for instance the annual precipitation total or maximum air temperature at a particular geographic location, date of freeze-up of a particular lake, etc.

In trying to develop a mathematical model for such series we must first introduce certain assumptions which may be based on our knowledge (or the lack of it) of the physical laws governing the process, on the geometry of the historic series, on the tools available for the analysis. In any case, these assumptions serve to simplify the complex reality and to make it fit into our preconceived schemes called 'organized system of knowledge'. It should be pointed out that these assumptions are to a great extent arbitrary.

The first assumption will probably be one concerning the stochastic nature of the series. It implies an admission on our part of not being able to predict the future values of the series accurately. The degree of uncertainty involved can be ascertained only by making predictions and testing their accuracy. To make predictions, or estimates (they need not relate to specific values of the variable but to their dispersion corresponding to a given probability), specific assumptions, or sets of assumptions, about the pattern of the series behaviour are necessary.

Here we must first decide the following fundamental question: does the series represent fluctuations within some firm, stable general pattern, or does the pattern, the master plan itself, change? In other words, is the series a stationary one or a nonstationary one?

The geometry of the historical record can seldom provide a clear-cut answer. For instance, almost any series can be equally well visualized as either a sequence of large fluctuations about a constant mean or a sequence of small fluctuations about a sharply fluctuating mean (Figure 2). Mathematical convenience dictates the acceptance of the stationary hypothesis whereas the physical evidence testifies to the contrary. It seems to make little sense to assume, for instance, a constant mean discharge at a certain point of a river if we know that the slope of the river and the topography of the basin undergo permanent changes due to erosion and sedimentation; it would seem to be even less reasonable to assume a constant mean discharge over a period containing a glacial age during which the river ceased to exist.

In this connection the following two things should be noted, (1) that the failure to account for the Hurst phenomenon is observed in the common types of stationary stochastic process which, by definition, all possess a constant long-term mean, (2) that the variables R_n and S_n, on the basis of which the Hurst phenomenon is defined, are both functions of deviations from a constant long-term mean $\bar{x}$.

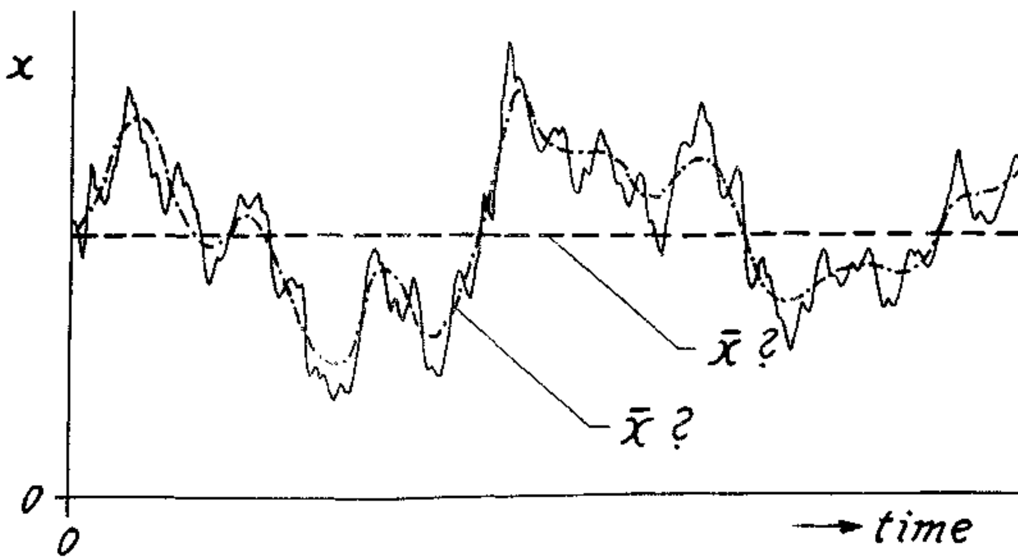

Fig. 2 Illustration of constant and time-variant mean of a time series

These two observations have lead the author to experiments with processes which have a time-variant mean. It was found that the simplest type of process exhibiting the Hurst phenomenon is one where the mean has constant value $\bar{x}_i$ within an epoch of some finite duration t_i, then suddenly takes on a different value $\bar{x}_j$ which is maintained during an epoch of duration t_j, after which another sudden change takes place, etc. Another type of process which leads to almost identical results is one where the values of the means x_i, x_j, ..., within epochs t_i, t_j, ..., are not constant but vary as linear trends. Sample series of these two processes are shown in Figure 3.

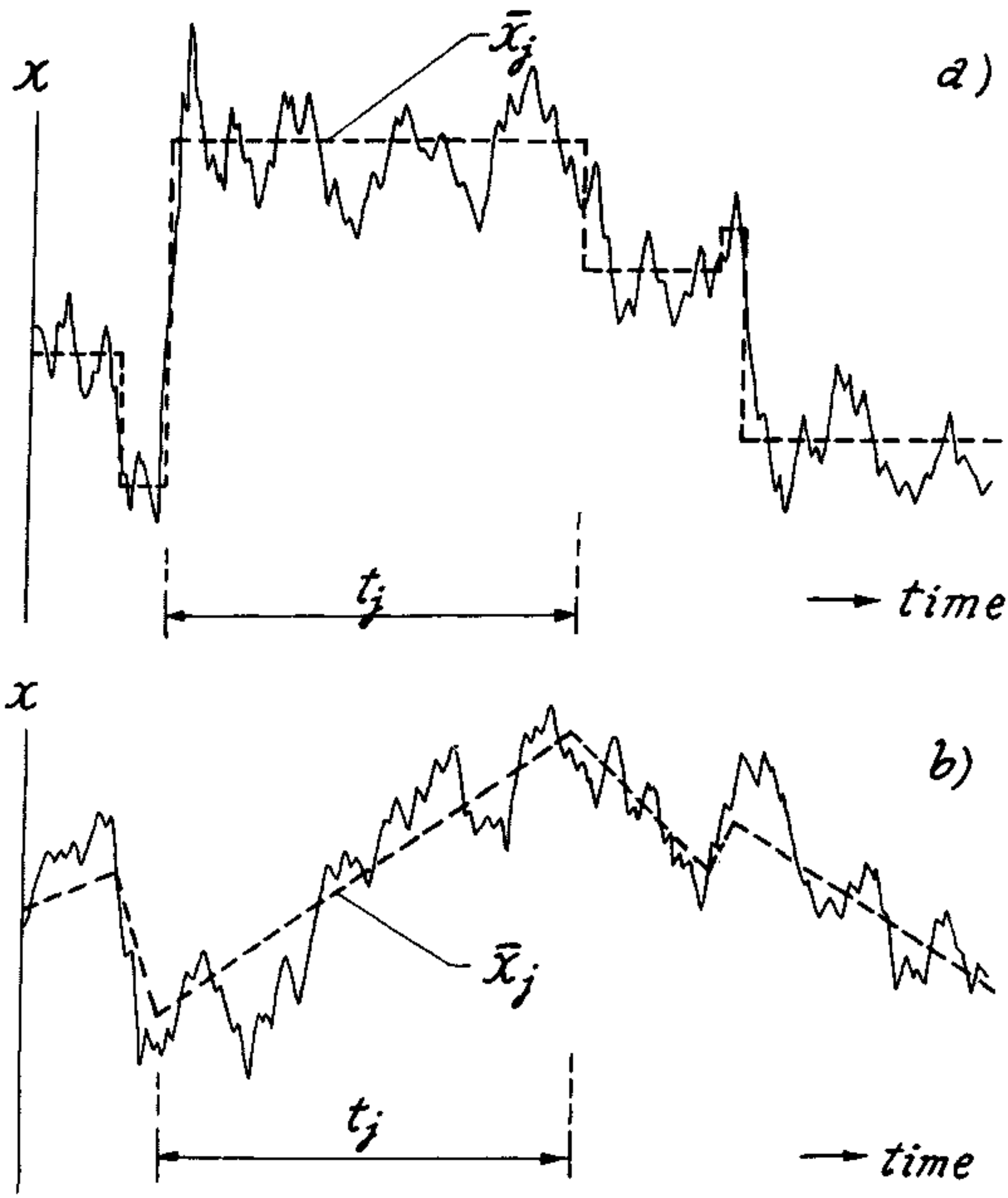

Fig. 3 Two types of constationary processes exhibiting the Hurst phenomenon

a) constant means within epochs
b) linearly changing means within epochs.

When such processes are treated as if they were stationary, i.e., if a fictitious constant long-term mean is assumed and all the deviations entering R and S are taken from this value, both processes exhibit the Hurst phenomenon. If, however, the deviations are properly taken from the actual time-variant mean, then the Hurst phenomenon disappears and the exponent h in eq. 2 approaches assymptotically the value of 0.5. This indicates that the Hurst phenomenon may arise from an incorrect interpretation of a process, specifically from assuming stationarity when the process is in fact nonstationary in the mean.

The type of probabilistic distribution of the mean $\bar{x}$ itself and of the deviation x-$\bar{x}$ does not seem to be important for the result nor does the stochastic structure of the process within a single epoch. The crucial thing is the distribution of epoch length t. It was found that the probabilities of epochs must be roughly inversely proportional to epoch lengths which implies that long stationary-looking periods exist but are very rare. A reasonably good approximation of the probability density of t appears to be

$$g(t) = \frac{\bar{t}}{\bar{t} - 1}\, t^{-\frac{2\bar{t} - 1}{\bar{t}}} \qquad (3)$$

where the mean epoch length $\bar{t}$ can vary from 1 to ∞, its different values giving rise to different values of the power h in eq. 2.

In physical terms the central tendency of a series reflects the overall energy level of the process. Physical causes of sudden changes in this energy level can be very diverse and can range from man-made effects on the environment, through processes in the earth crust, to extraterrestrial phenomena.

Conclusions

The purpose of this paper has been to point out that the behaviour of time series of long historic records of geophysical processes is compatible with the concept of catastrophic evolution, more specifically with the occurrence of sudden changes in the environment which take place at irregular time intervals. While it does not exclude other interpretations, for instance the presence of long persistence or storage effects which can be associated with certain types of processes, the concept of time-variant central tendency offers, in the author's opinion, the physically most plausible explanation, especially as far as long geological time scales are concerned.

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