

IUGG XXV General Assembly
Earth on the Edge: Science for a Sustainable Planet
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Plenary lecture

Hydrology and Change

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Presentation available online: itia.ntua.gr/1135/

Hydrology is the science of the water on Earth: its occurrence, circulation, distribution, physical and chemical properties, and interaction with the environment and the biosphere



Nile
River



Aswan
Dam



Lake
Nasser



Images from
earthobservatory.nasa.gov/IOTD/view.php?id=2416
earthobservatory.nasa.gov/images/imagerecords/46000/46209/earth_pacific_lrg.jpg
earthobservatory.nasa.gov/images/imagerecords/1000/1234/PIA02647_lrg.jpg
earthobservatory.nasa.gov/images/imagerecords/5000/5988/ISS010-E-14618_lrg.jpg

OUR CHANGING PLANET

The U.S. Climate Change Science Program
for Fiscal Year 2009



OUR CHANGING
OCEANS

A Report by the
Climate Change Science Program and
the Subcommittee on Global Change Research

A Supplement to the President's Budget for Fiscal Year 2009

A Changing World

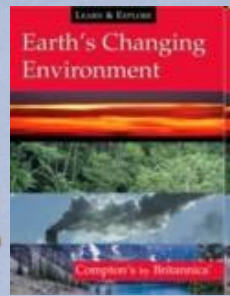


Proceedings of the
NORTH AMERICAN SERIALS
INTEREST GROUP, Inc.

Suzanne McMahon · Miriam Palm · P.
Editors

GLOBAL ENVIRONMENTAL CHANGE

RESEARCH PATHWAYS FOR THE NEXT DECADE



The Economics of Climate Change

The Stern Review



CAMBRIDGE

CHANGING PLANET, CHANGING HEALTH

How the Climate Crisis Threatens Our
Health and What We Can Do about It

PAUL R. EPSTEIN, MD, AND DAN FERBER
FOREWORD BY JEFFREY SACHS

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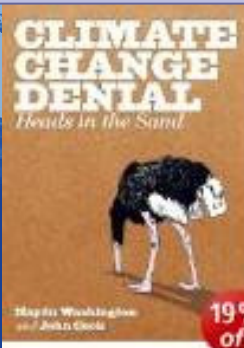


A Changing World

SCIENCE AND SOLUTIONS FOR AUSTRALIA

CLIMATE CHANGE

It looks like, recently, our scientific community has been amazed that things change...



19%
off

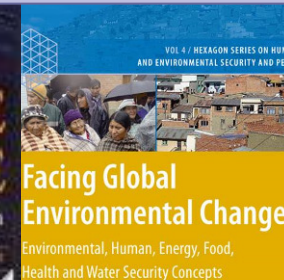


CLIMATE CHANGE

A Multidisciplinary Approach

SECOND EDITION

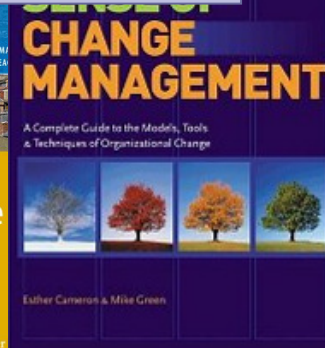
WILLIAM
BURRO



Facing Global Environmental Change

Environmental, Human, Energy, Food,
Health and Water Security Concepts

Springer



CLIMATE CHANGE MANAGEMENT

A Complete Guide to the Models, Tools
& Techniques of Organizational Change

Esther Cameron & Mike Green

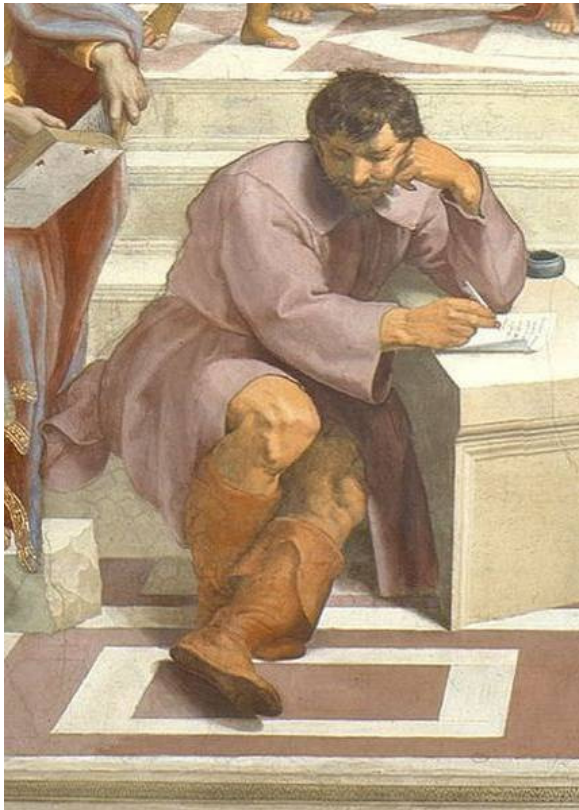
The Government's Vision
for New Priorities in
Denmark's Foreign Policy

June 2003



Raphael's "School of Athens" (1509–1510; Apostolic Palace, Vatican City; en.wikipedia.org/wiki/School_of_Athens)

Heraclitus (ca. 540-480 BC)



Heraclitus (figured by Michelangelo)
in Raphael's School of Athens;
en.wikipedia.org/wiki/Heraclitus

- Πάντα ῥεῖ
Everything flows
 - Alternative versions
 - Τὰ ὄντα ἰέναι τε πάντα καὶ μένειν οὐδέν [from Plato's Cratylus, 401d]
All things move and nothing remains still
 - Πάντα χωρεῖ καὶ οὐδέν μένει [ibid, 402,a]
Everything changes and nothing remains still
- Δις ἐς τὸν αὐτὸν ποταμὸν οὐκ ἂν ἐμβαίης [from Plato's Cratylus, 402a]
You cannot step twice into the same river



Aristotle in Raphael's
"School of Athens";
en.wikipedia.org/wiki/Aristotle

Aristotle (384-322 BC) in Meteorologica

Change

- ὅτι οὔτε ὁ Τάναϊς οὔτε ὁ Νεῖλος ἀεὶ ἔρρει, ἀλλ' ἦν ποτε ξηρὸς ὁ τόπος ὅθεν ῥέουσιν· τὸ γὰρ ἔργον ἔχει αὐτῶν πέρας, ὁ δὲ χρόνος οὐκ ἔχει. ... ἀλλὰ μὲν εἴπερ καὶ οἱ ποταμοὶ γίνονται καὶ φθείρονται καὶ μὴ ἀεὶ οἱ αὐτοὶ τόποι τῆς γῆς ἔνυδροι, καὶ τὴν θάλατταν ἀνάγκη μεταβάλλειν ὁμοίως. τῆς δὲ θαλάττης τὰ μὲν ἀπολειπούσης τὰ δ' ἐπιούσης ἀεὶ φανερόν ὅτι τῆς πάσης γῆς οὐκ ἀεὶ τὰ αὐτὰ τὰ μὲν ἐστὶν θάλαττα τὰ δ' ἥπειρος, ἀλλὰ **μεταβάλλει τῷ χρόνῳ πάντα** [I.14, 353a 16]

Neither the Tanais [River Don in Russia] nor the Nile have always been flowing, but the region in which they flow now was once dry: for their life has a bound, but time has not... But if **rivers are formed and disappear** and the same places were not always covered by water, the sea must change correspondingly. And if the sea is receding in one place and advancing in another it is clear that **the same parts of the whole earth are not always either sea or land, but that all changes in course of time**

Conservation of mass within the hydrological cycle

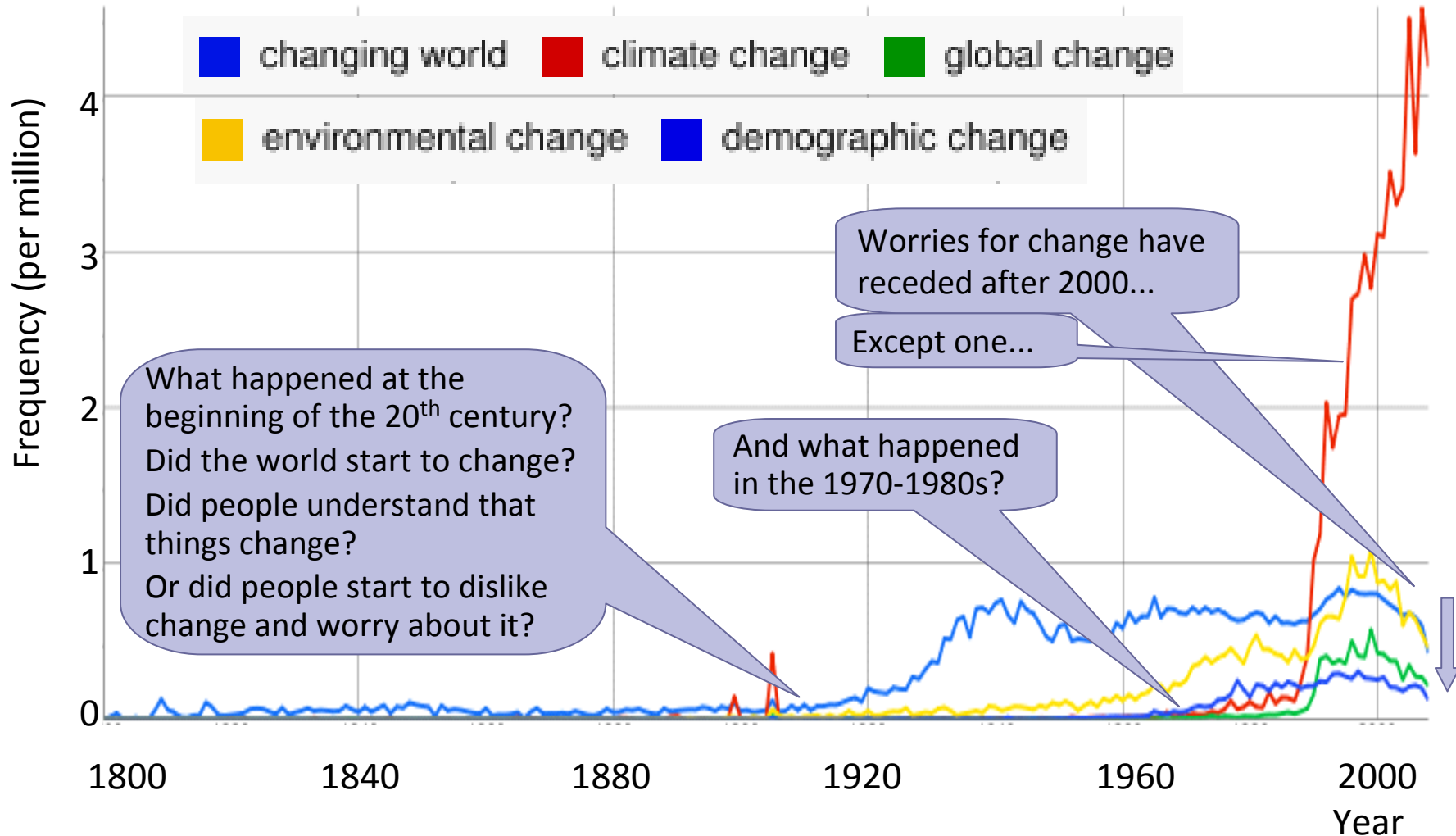
- ὥστε οὐδέποτε ξηρανεῖται· πάλιν γὰρ ἐκεῖνο φθῆσεται καταβὰν εἰς τὴν αὐτὴν τὸ προανεληθόν [II.3, 356b 26]

Thus, [the sea] will never dry up; for [the water] that has gone up beforehand will return to it

- καὶ μὴ κατ' ἐνιαυτὸν ἀποδιδῶ καὶ καθ' ἐκάστην ὁμοίως χώραν, ἀλλ' ἐν γέ τισιν τεταγμένοις χρόνοις ἀποδίδωσι πᾶν τὸ ληφθέν [II.2, 355a 26]

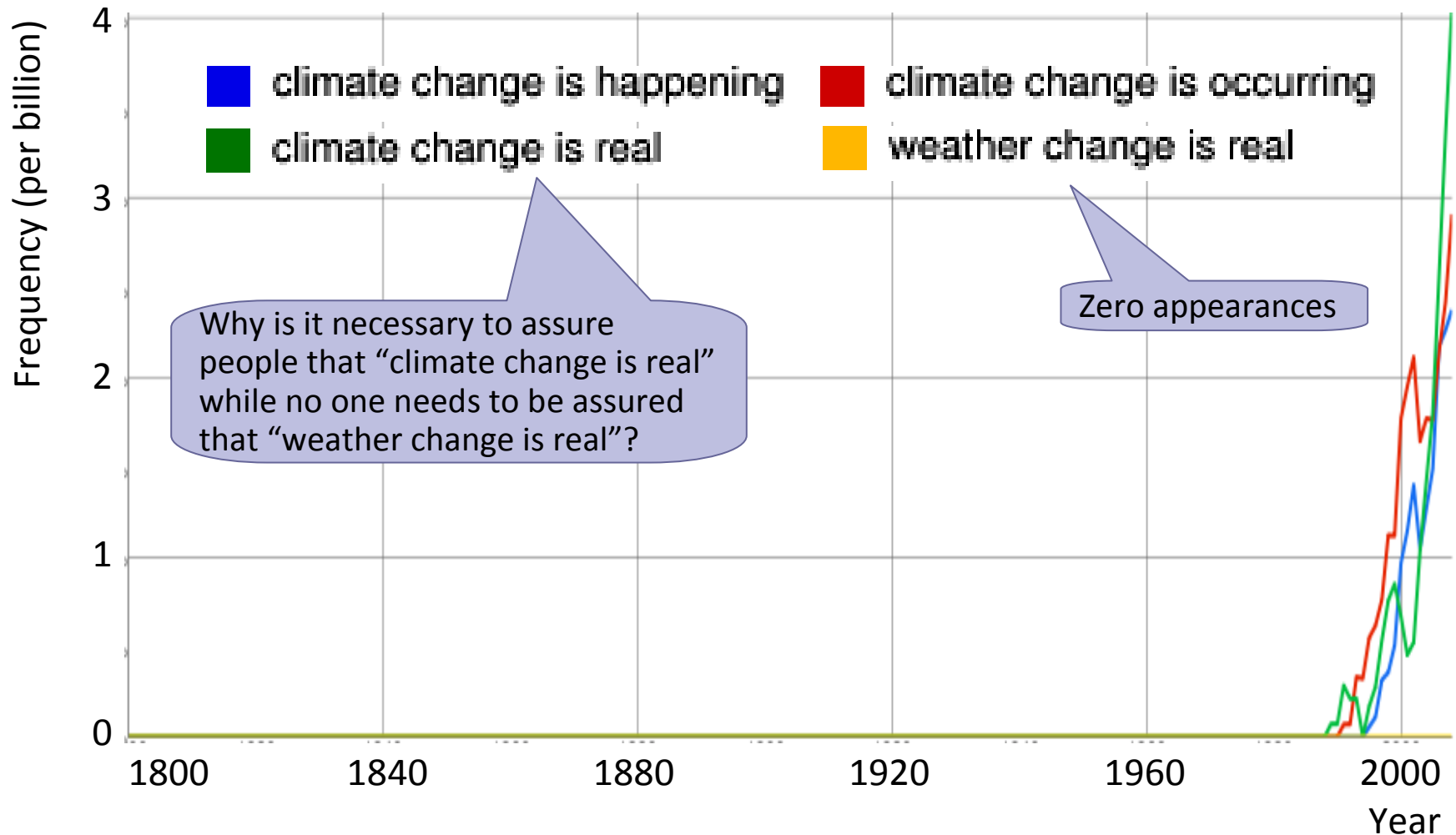
Even if the same amount does not come back every year or in a given place, yet in a certain period all quantity that has been abstracted is returned

Reinventing change... or worrying about it?



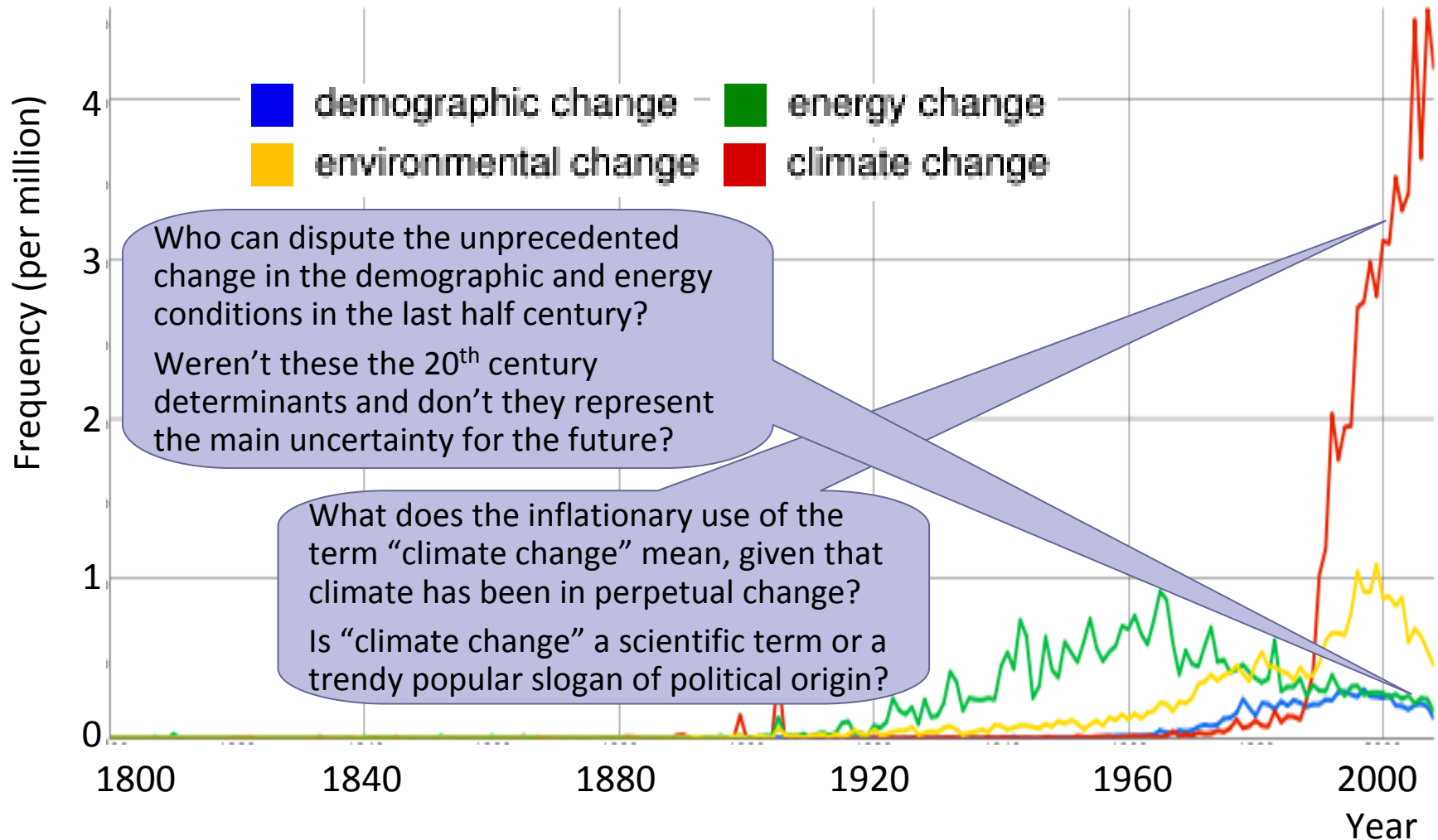
Data and visualization by Google labs; ngrams.googlelabs.com (360 billion words in 3.3 million books published after 1800; see also Mitchel *et al.*, 2011)

An unprecedented disbelief that change is real?



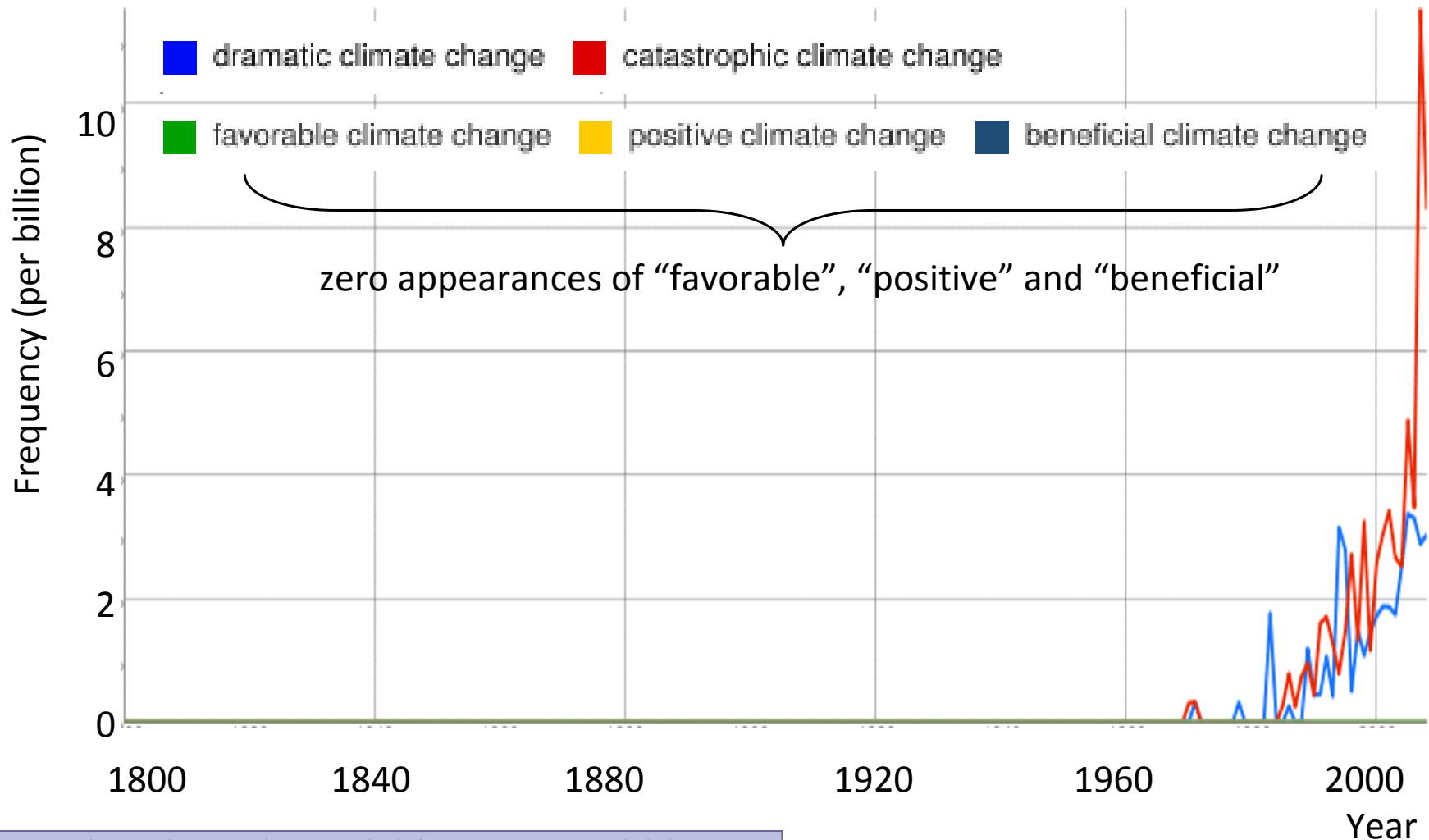
Data and visualization by Google labs; ngrams.googlelabs.com

Is our focus opposite to importance?



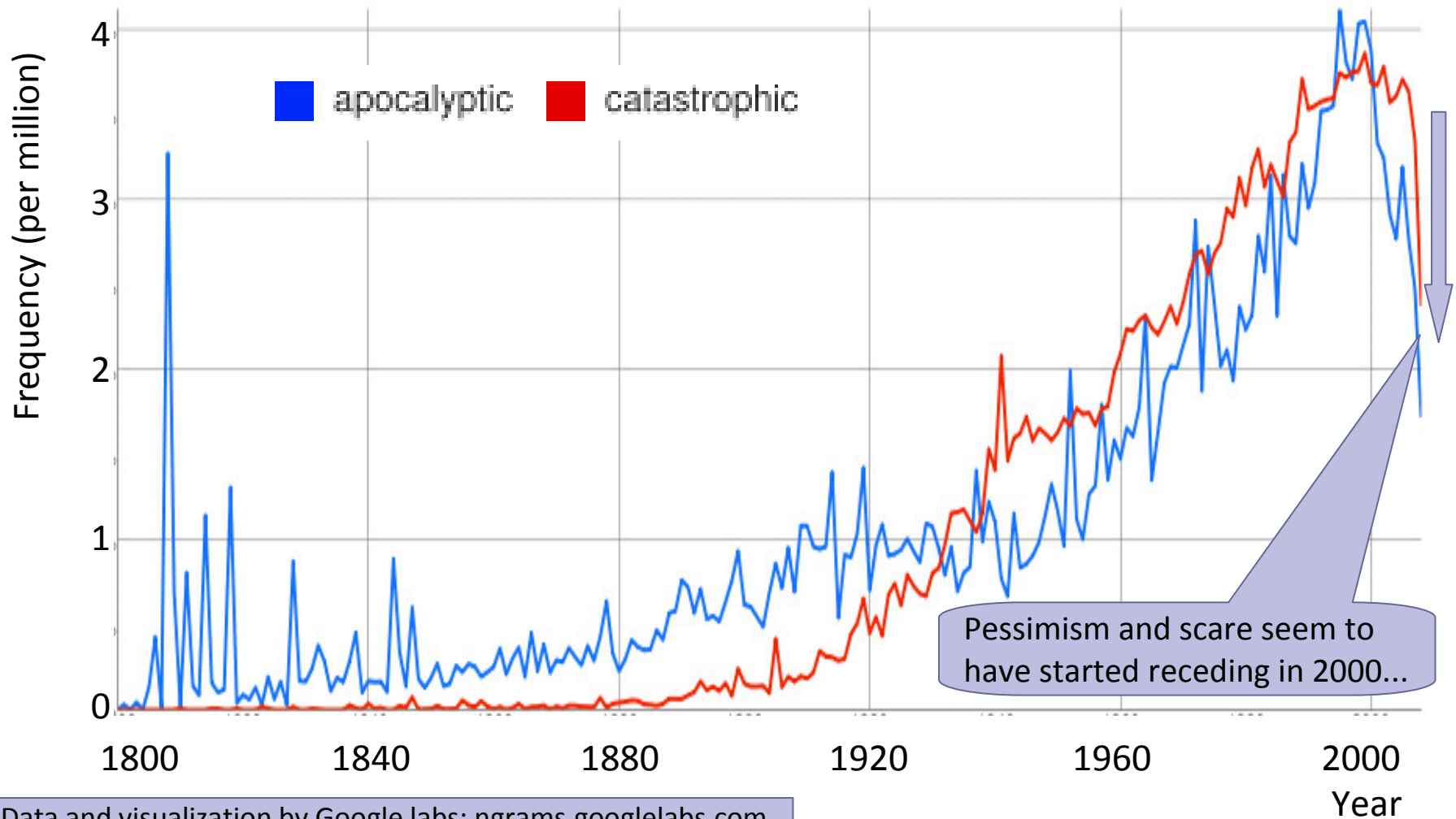
Data and visualization by Google labs; ngrams.googlelabs.com

Is change only negative?



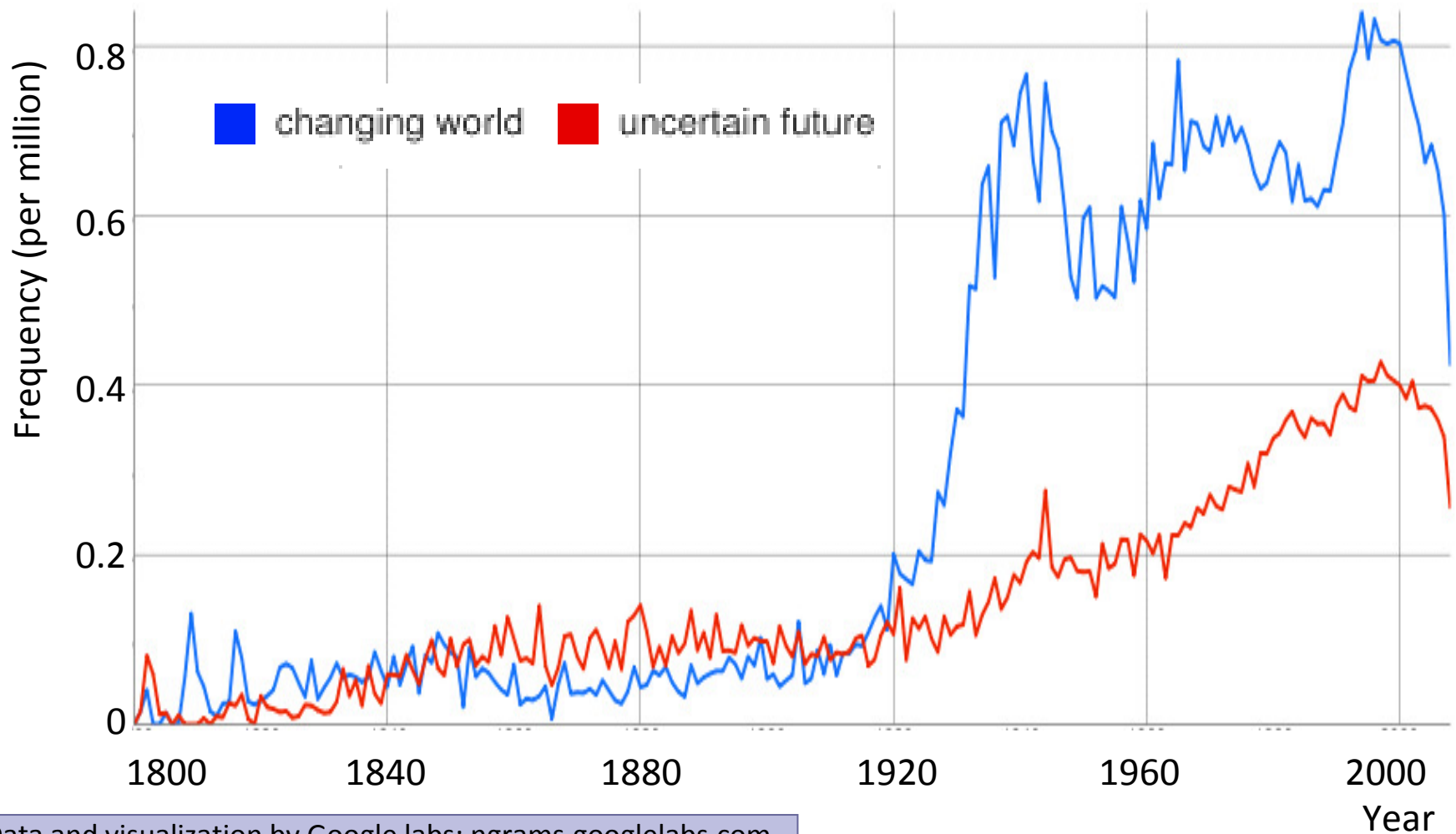
Data and visualization by Google labs; ngrams.googlelabs.com

Are we becoming more and more susceptible to scare and pessimism?



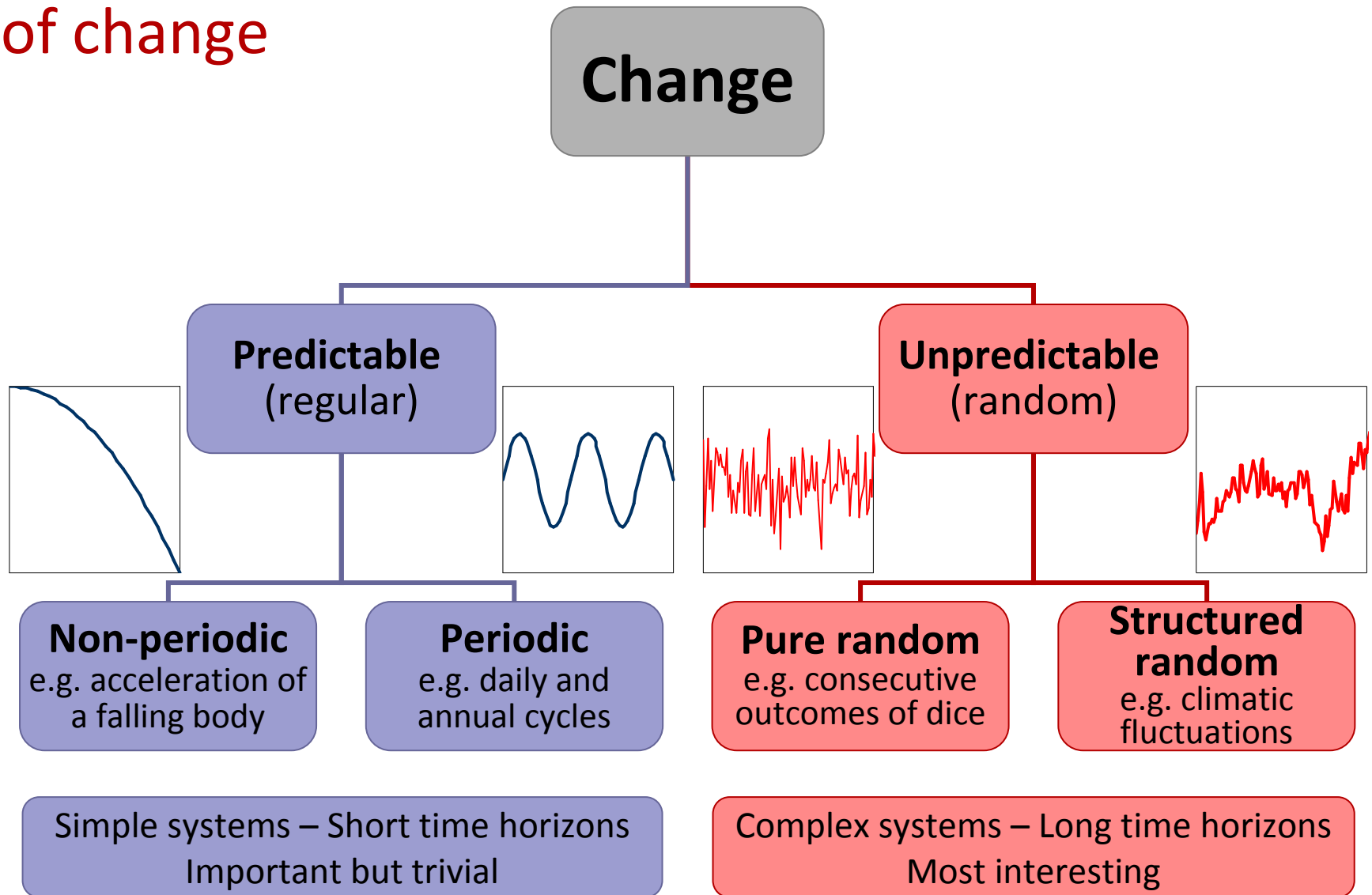
Data and visualization by Google labs; ngrams.googlelabs.com

Change is tightly linked to uncertainty



Data and visualization by Google labs; ngrams.googlelabs.com

Predictability of change



What is randomness?

- Common dichotomous view:
 - Natural process are composed of two different, usually additive, parts or components—deterministic (signal) and random (noise)
 - Randomness is cancelled out at large scales and does not produce change; only an exceptional forcing can produce a long-term change
- My view (explained in Koutsoyiannis, 2010):
 - Randomness is none other than unpredictability
 - Randomness and determinism coexist and are not separable
 - Deciding which of the two dominates is simply a matter of specifying the time horizon and scale of the prediction
 - At long time horizons (where length depends on the system) **all is random**



Heraclitus (Fragment 52)

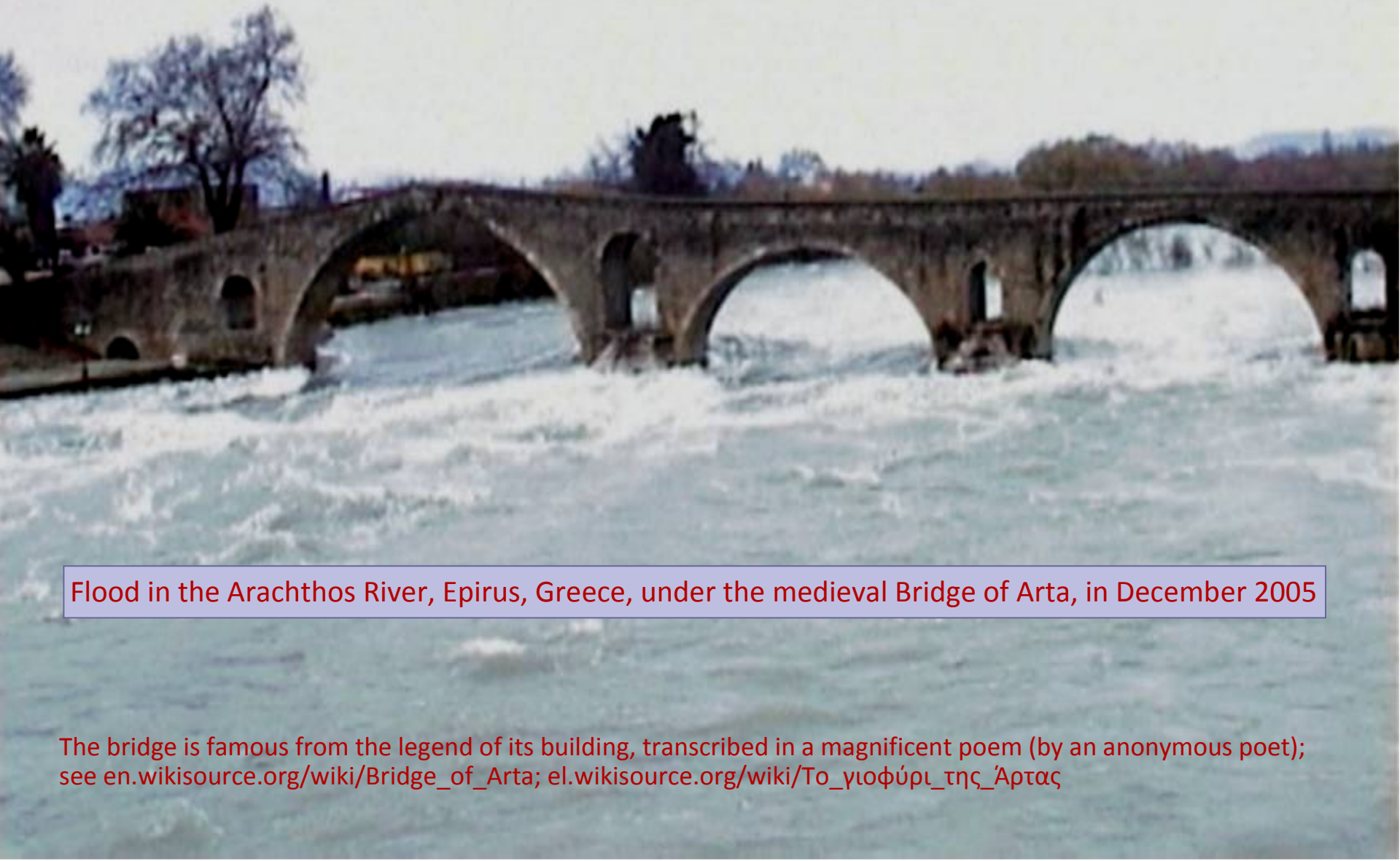
Αἰὼν παῖς ἐστὶ παίζων πεσσεύων· παιδὸς ἡ βασιληίη
Time is a child playing, throwing dice; the ruling power is a child's

Vs. **Einstein** (in a letter to Max Born in 1926)

I am convinced that He does not throw dice

This die is from 580 BC (photo from the Kerameikos Ancient Cemetery Museum, Athens)

Contemplating the change in rivers: From mixing and turbulence to floods and droughts



Flood in the Arachthos River, Epirus, Greece, under the medieval Bridge of Arta, in December 2005

The bridge is famous from the legend of its building, transcribed in a magnificent poem (by an anonymous poet); see en.wikisource.org/wiki/Bridge_of_Arta; el.wikisource.org/wiki/Το_γιοφύρι_της_Άρτας

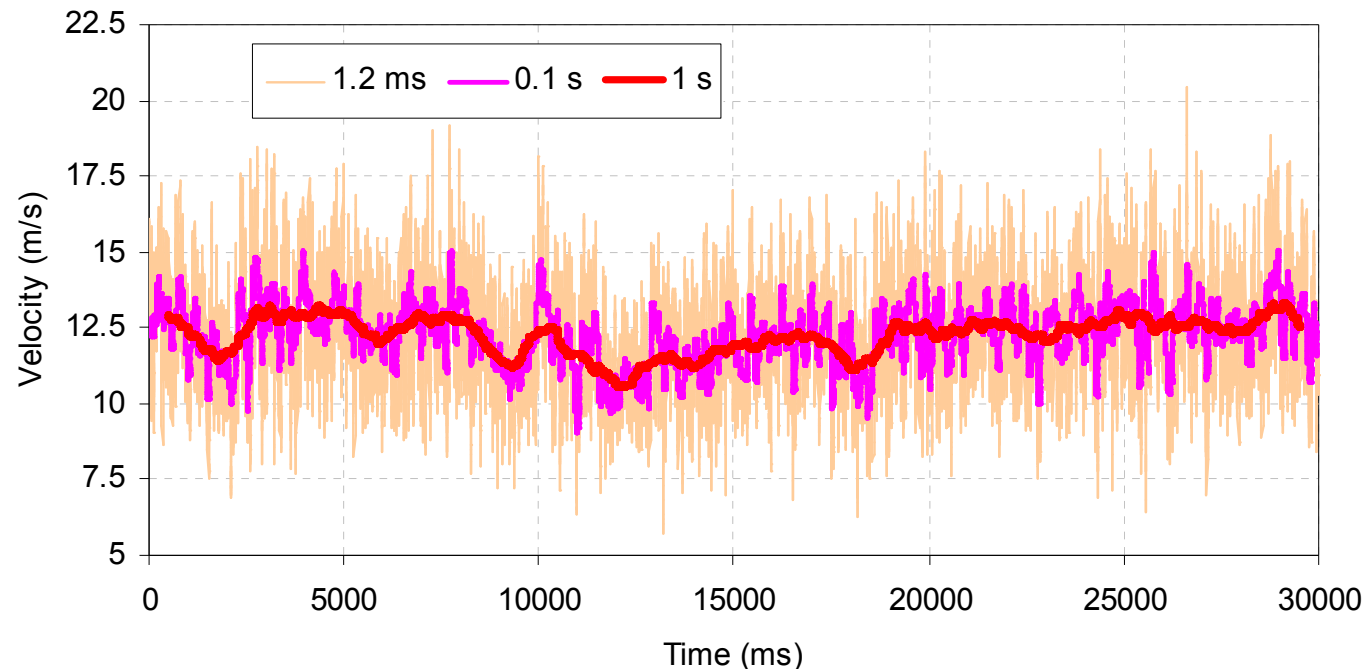
Turbulence: macroscopic motion at millisecond scale

- Laboratory measurements of nearly isotropic turbulence in Corrsin Wind Tunnel (section length 10 m; cross-section 1.22 m by 0.91 m) at a high-Reynolds-number (Kang *et al.*, 2003)
- Measurements by X-wire probes; Sampling rate of 40 kHz, here aggregated at 0.833 kHz—each point is the average of 48 original values



When I meet God, I am going to ask him two questions: Why relativity? And why turbulence? I really believe he will have an answer for the first

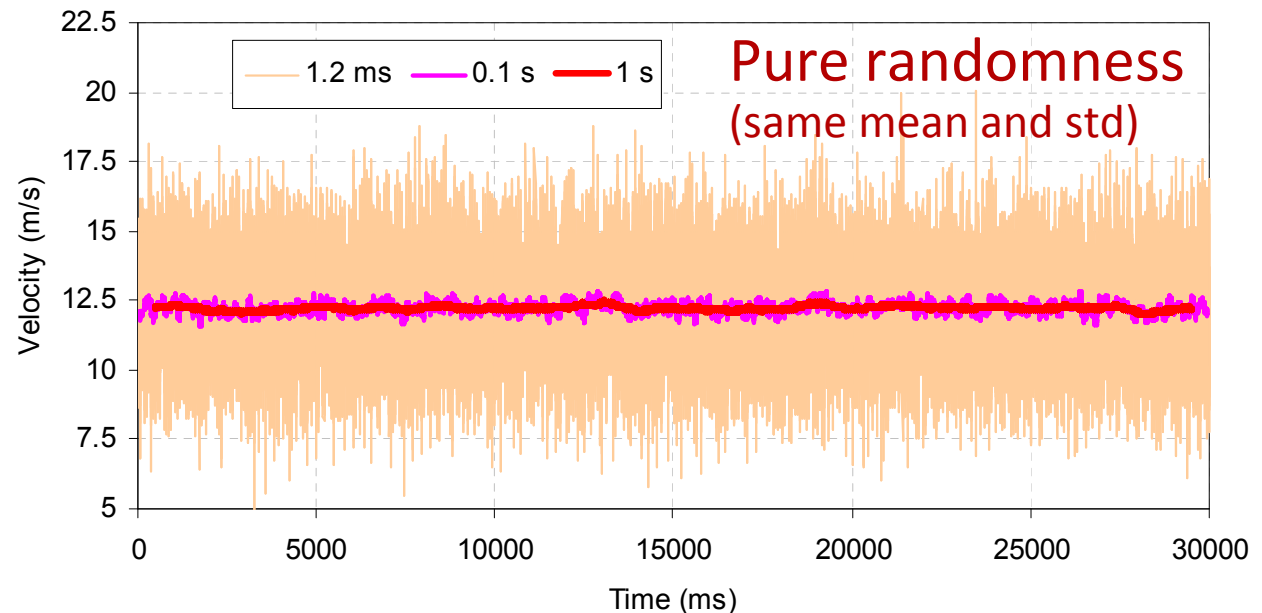
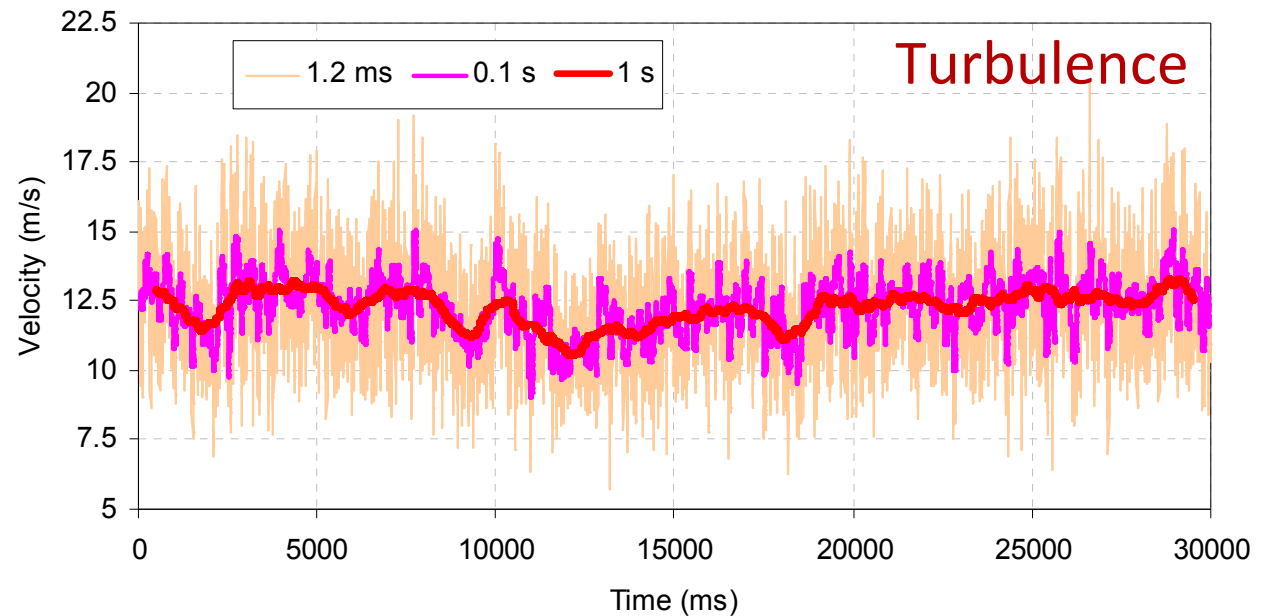
(attributed to **Werner Heisenberg** or, in different versions, to **Albert Einstein** or to **Horace Lamb**)



Data downloaded from www.me.jhu.edu/meneveau/datasets/Activegrid/M20/H1/m20h1-01.zip

Turbulence vs. pure randomness

- Pure random processes, assuming independence in time (white noise), have been effective in modelling microscopic motion (e.g. in statistical thermodynamics)
- Macroscopic random motion is more complex
- In pure randomness, change vanishes at large scales
- In turbulence, change occurs at all scales



A river viewed at different time scales—from seconds to millions of years

- Next second: the hydraulic characteristics (water level, velocity) will change due to turbulence
- Next day: the river discharge will change (even dramatically, in case of a flood)
- Next year: The river bed will change (erosion-deposition of sediments)
- Next century: The climate and the river basin characteristics (e.g. vegetation, land use) will change
- Next millennia: All could be very different (e.g. the area could be glacialized)
- Next millions of years: The river may have disappeared

- None of these changes will be a surprise
- Rather, it would be a surprise if things remained static
- These changes are not predictable
- Most of these changes can be mathematically modelled in a stochastic framework admitting stationarity!

The Roda Nilometer and over-centennial change

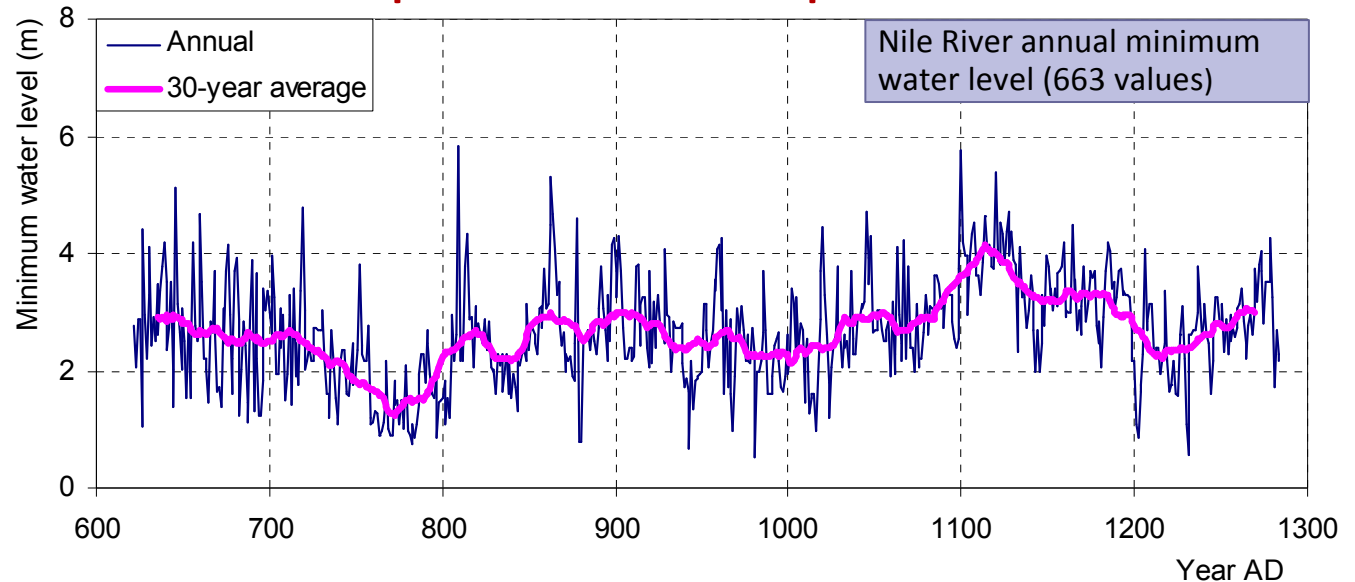


- ❑ The Roda Nilometer as it stands today; water entered and filled the Nilometer chamber up to river level through three tunnels
- ❑ In the centre of the chamber stands a marble octagonal column with a Corinthian crown; the column is graded and divided into 19 cubits (a cubit is slightly more than half a meter) and could measure floods up to about 9.2 m
- ❑ A maximum level below the 16th mark could portend drought and famine; a level above the 19th mark meant catastrophic flood
(Credit: Aris Georgakakos)

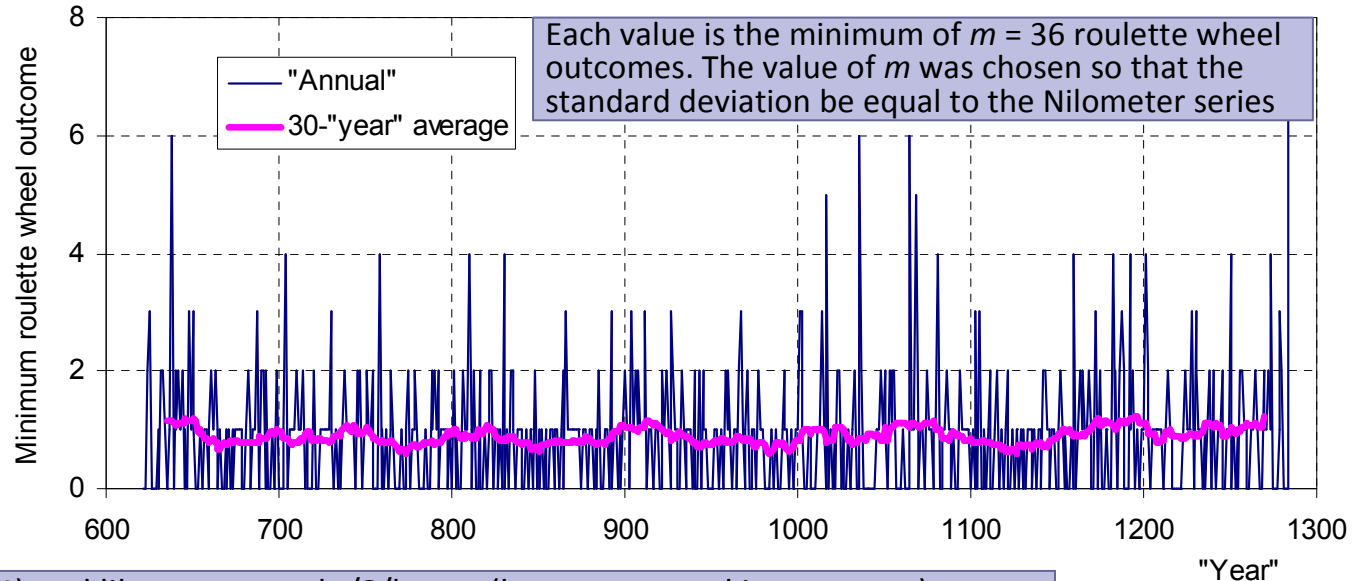


The Nilometer record vs. a pure random processes

A real-world process



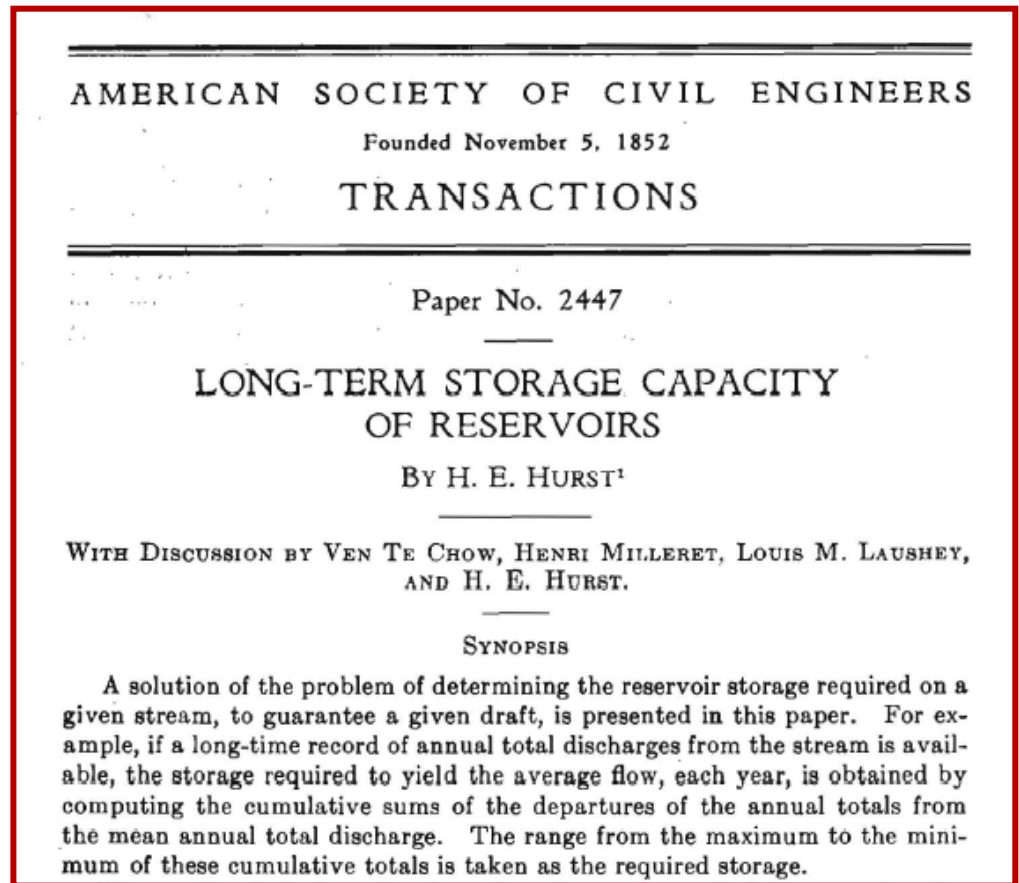
A "roulette" process



Nilometer data: Beran (1994) and lib.stat.cmu.edu/S/beran (here converted into meters)

Hurst's (1951) seminal paper

- The motivation of Hurst was the design of the High Aswan Dam on the **Nile** River
- However the paper was theoretical and explored numerous data sets of diverse fields
- Hurst observed that:
*Although in random events **groups of high or low values** do occur, their tendency to occur in natural events is **greater**. This is the main difference between natural and random events*



Obstacles in the dissemination and adoption of Hurst's finding:

- Its direct connection with reservoir storage
- Its tight association with the Nile
- The use of a complicated statistic (the rescaled range)

Kolmogorov (1940)

- Kolmogorov studied the stochastic process that describes the behaviour to be discovered a decade later in geophysics by Hurst
- The proof of the existence of this process is important, because several researches, ignorant of Kolmogorov's work, regarded Hurst's finding as inconsistent with stochastics and as numerical artefact

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1940. Volume XXVI, № 2

MATHEMATIK

WIENERSCHE SPIRALEN UND EINIGE ANDERE INTERESSANTE KURVEN IM HILBERTSCHEN RAUM

Von A. N. KOLMOGOROFF, Mitglied der Akademie

Wir werden hier einige Sonderfälle von Kurven betrachten, denen meine vorhergehende Note «Kurven im Hilbertschen Raum, die gegenüber einer einparametrischen Gruppe von Bewegungen invariant sind» ⁽¹⁾ gewidmet ist.

Unter einer Ähnlichkeitstransformation im Hilbertschen Raum H werden wir eine beliebige

und $y \neq x$ der Punkte, die auf derselben Kurve liegen, übergeht.

Satz 6. Die Funktion $B_{\xi}(\tau_1, \tau_2)$, die der Funktion $\xi(t)$ der Klasse \mathfrak{A} entspricht, kann in der Form

$$B_{\xi}(\tau_1, \tau_2) = c [|\tau_1|^Y + |\tau_2|^Y - |\tau_1 - \tau_2|^Y]$$

115

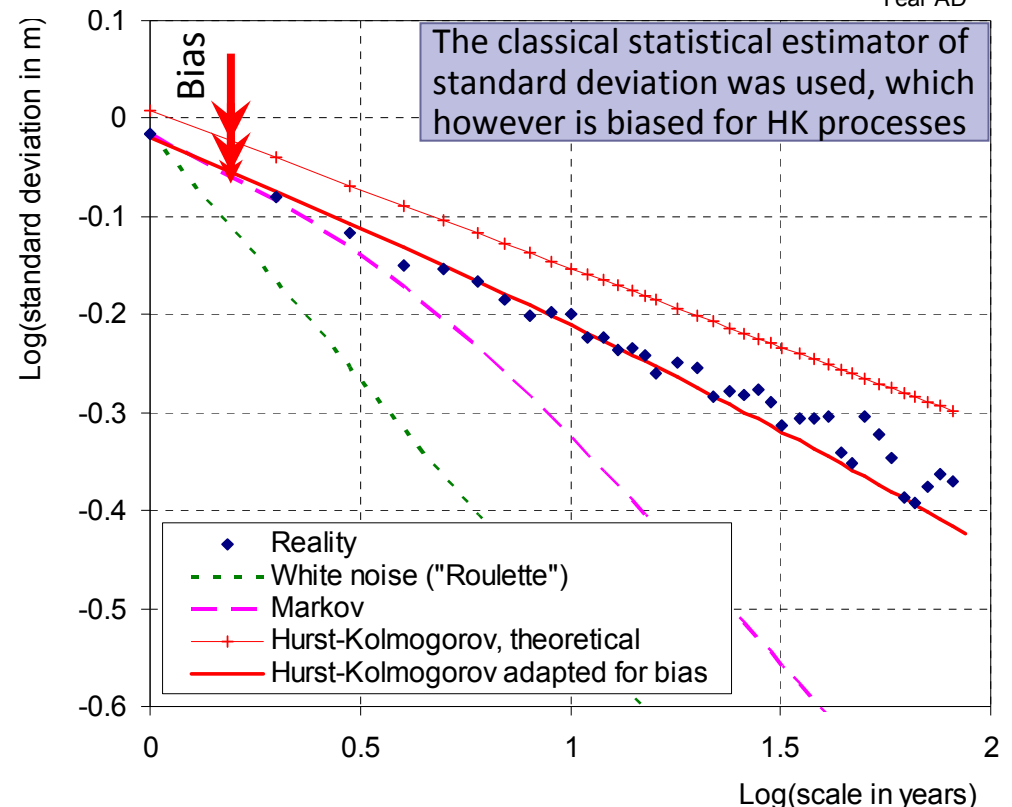
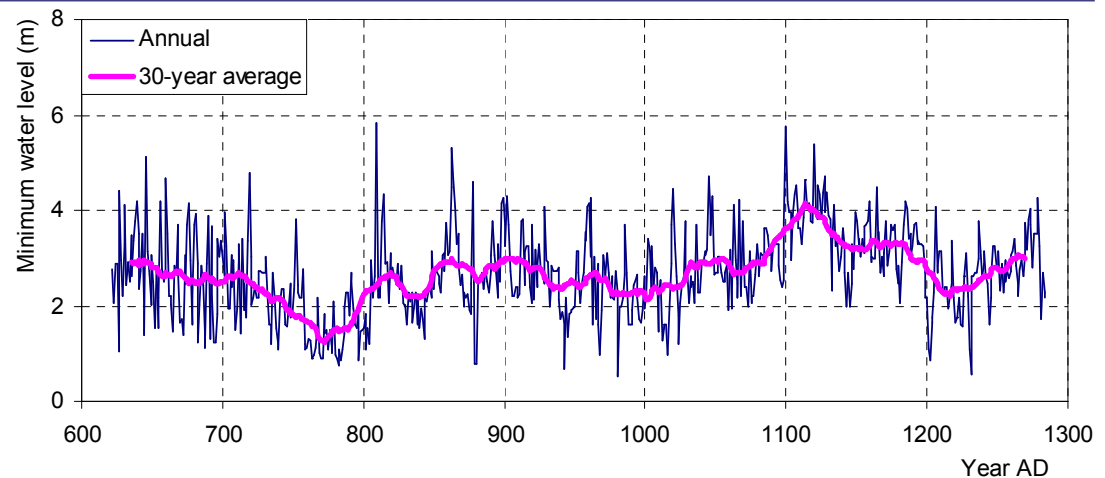
- ❑ Kolmogorov's work did not become widely known
- ❑ The process was named by Kolmogorov "Wiener's Spiral" (Wienersche Spiralen) and later "Self-similar process", or "fractional Brownian motion" (Mandelbrot and van Ness, 1968); here it is called the **Hurst-Kolmogorov (HK) process**

The climacogram: A simple statistical tool to quantify the change across time scales

- Take the Nilometer time series, x_1, x_2, \dots, x_{663} , and calculate the sample estimate of standard deviation $\sigma^{(1)}$, where the superscript (1) indicates time scale (1 year)
- Form a time series at time scale 2 (years):
 $x^{(2)}_1 := (x_1 + x_2)/2, x^{(2)}_2 := (x_3 + x_4)/2, \dots, x^{(2)}_{331} := (x_{661} + x_{662})/2$
and calculate the sample estimate of standard deviation $\sigma^{(2)}$
- Form a time series at time scale 3 (years):
 $x^{(3)}_1 := (x_1 + x_2 + x_3)/3, \dots, x^{(3)}_{221} := (x_{661} + x_{662} + x_{663})/3$
and calculate the sample estimate of standard deviation $\sigma^{(3)}$
- Repeat the same procedure up to scale 66 (1/10 of the record length) and calculate $\sigma^{(66)}$
- The **climacogram** is a logarithmic plot of standard deviation $\sigma^{(k)}$ vs. scale k
- If the time series x_i represented a pure random process, the climacogram would be a straight line with slope -0.5 (the proof is very easy)
- In real world processes, the slope is different from -0.5 , designated as $H - 1$, where H is the so-called Hurst coefficient ($0 < H < 1$)
- The scaling law $\sigma^{(k)} = \sigma^{(1)} / k^{1-H}$ defines the **Hurst-Kolmogorov (HK) process**
- High values of H (> 0.5) indicate **enhanced change** at large scales, else known as **long-term persistence**, or strong **clustering** (grouping) of similar values

The climacogram of the Nilometer time series

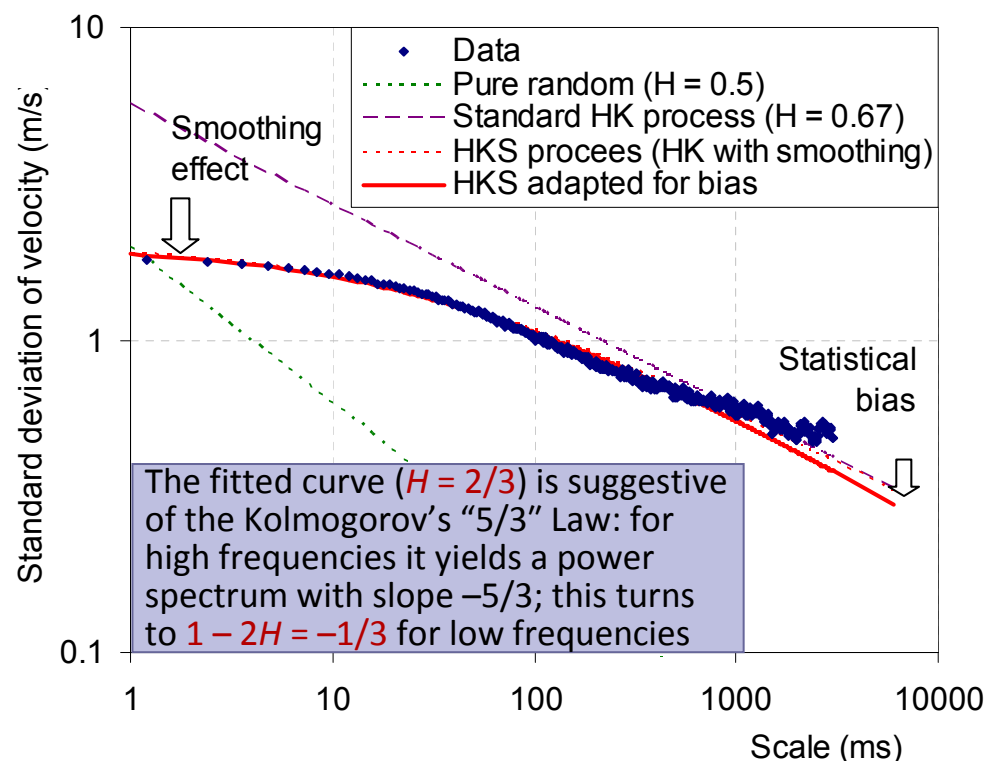
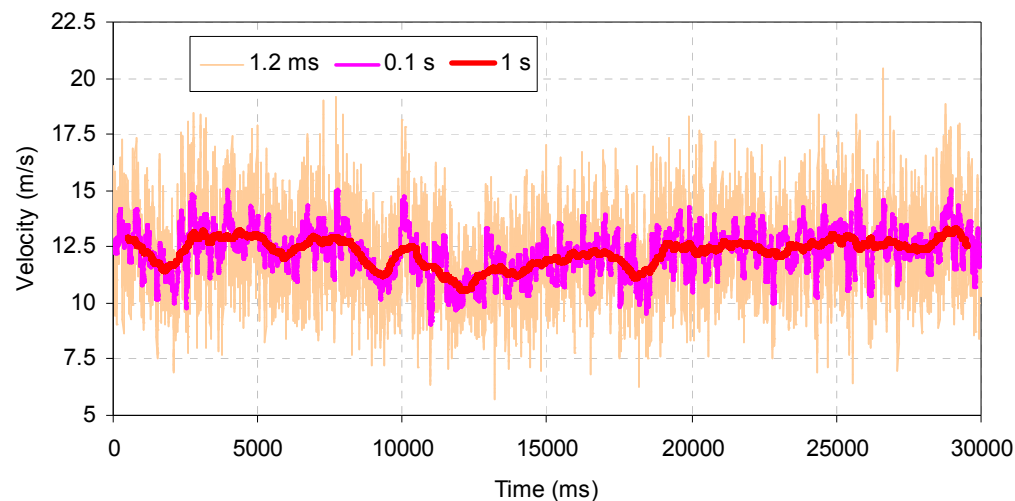
- The Hurst-Kolmogorov process seems consistent with reality
- The Hurst coefficient is $H = 0.89$ (Similar H values are estimated from the simultaneous record of maximum water levels and from the modern, 131-year, flow record of the Nile flows at Aswan)
- Essentially, the Hurst-Kolmogorov behaviour, depicted in the climacogram, manifests that long-term changes are much more frequent and intense than commonly perceived and, simultaneously, that the future states are much more uncertain and unpredictable on long time horizons than implied by pure randomness



The climacogram of the turbulent velocity time series

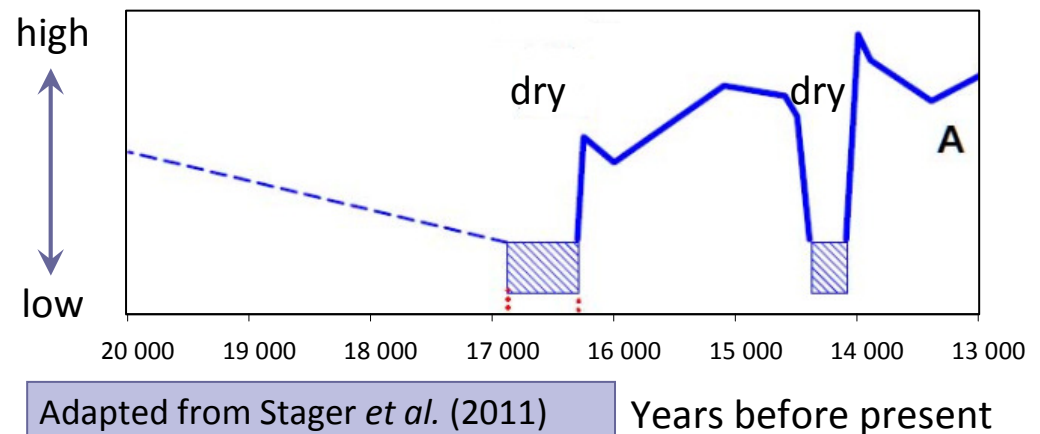
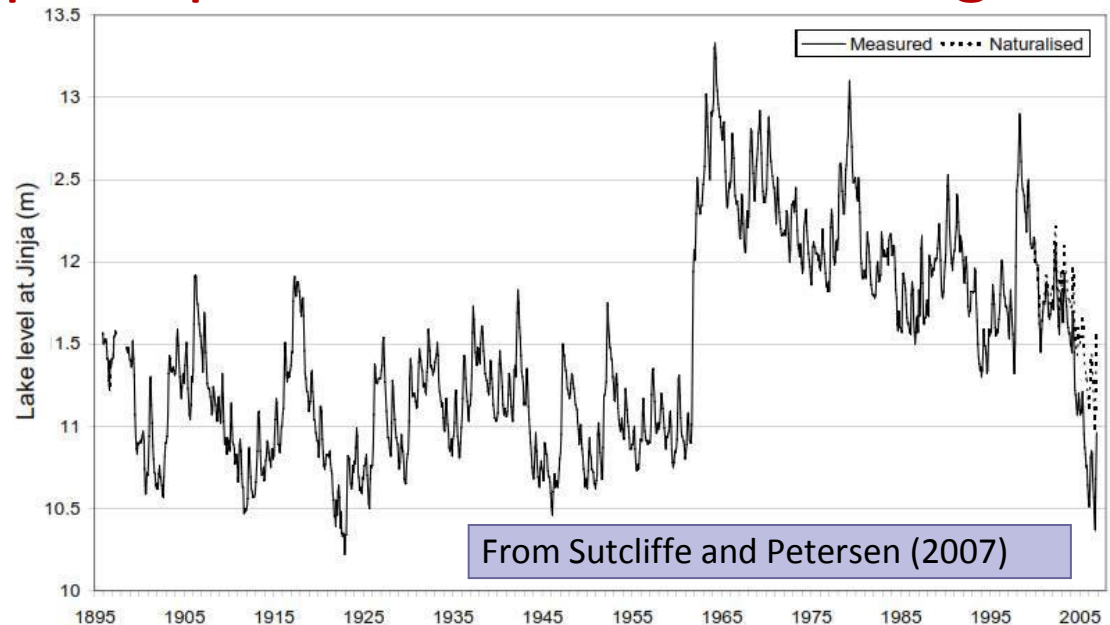
- The simple-scaling HK process is appropriate for scales > 50 ms, but not for scales smaller than that
- For small scales, a smoothing effect reduces variability (in comparison to that of the HK process)
- A Hurst-Kolmogorov process with Smoothing (HKS) is consistent with turbulence measurements at the entire range of scales
- The HKS process, in addition to the Hurst coefficient, involves a smoothing parameter (α), and can be defined by

$$[\sigma^{(k)}]^2 = [\sigma^{(1)}]^2 / (\alpha^{2-2H} + k^{2-2H})$$

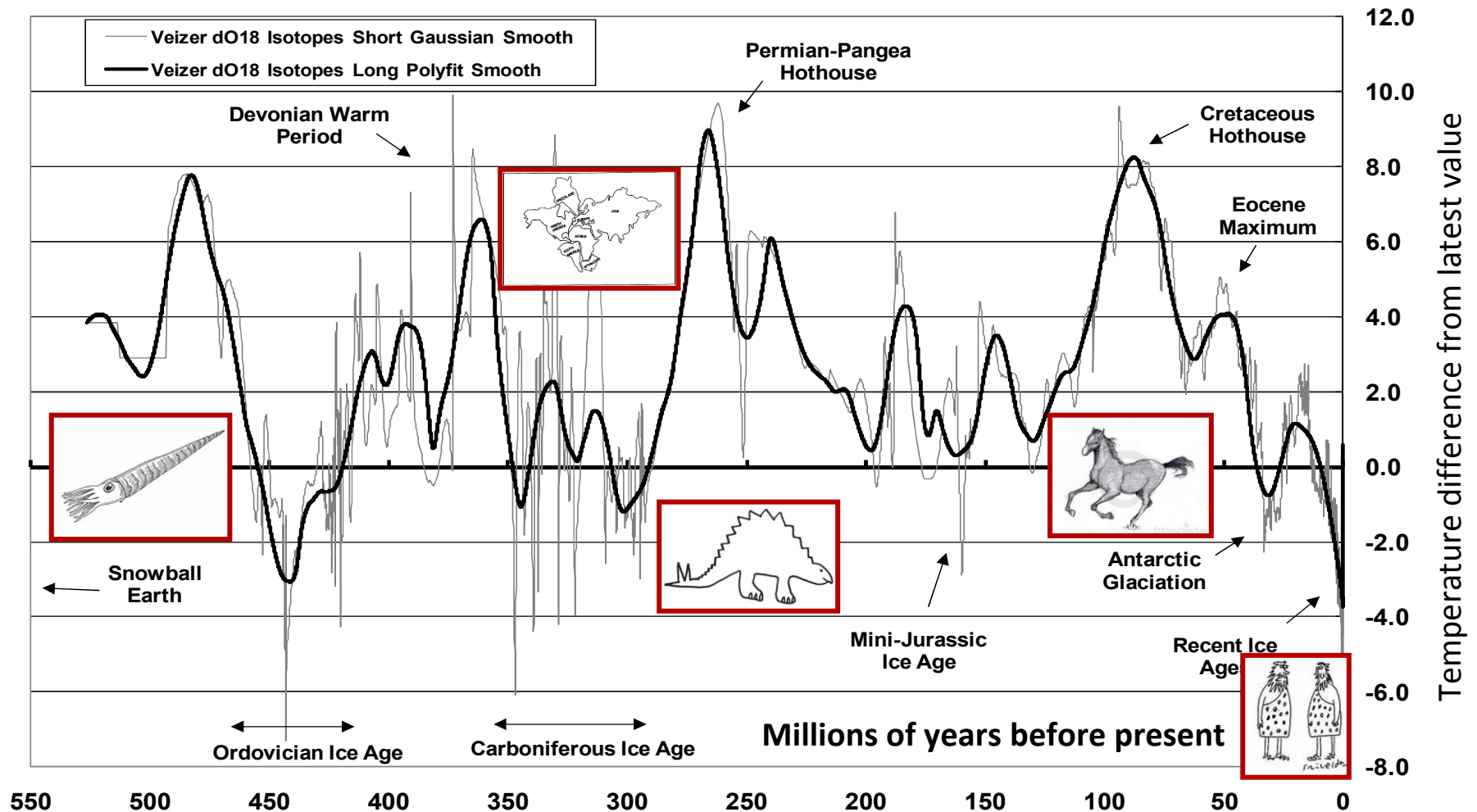


Are reconstructions of past hydroclimatic behaviours consistent with the perception of enhanced change?

- Lake Victoria is the largest tropical lake in the world (68 800 km²) and is the headwater of the White Nile
- The contemporary record of water level (covering a period of more than a century) indicates huge changes
- Reconstructions of water level for past millennia from sediment cores (Stager *et al.*, 2011) suggest that the lake was even dried for several centuries



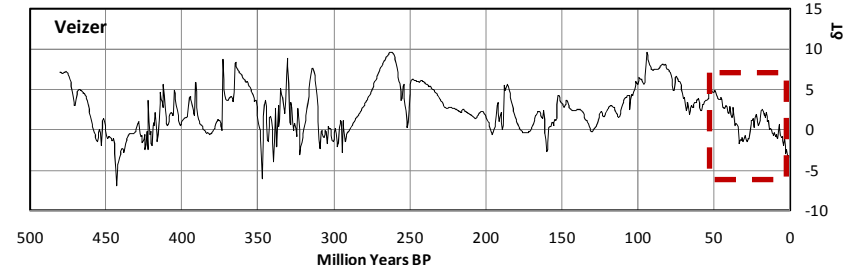
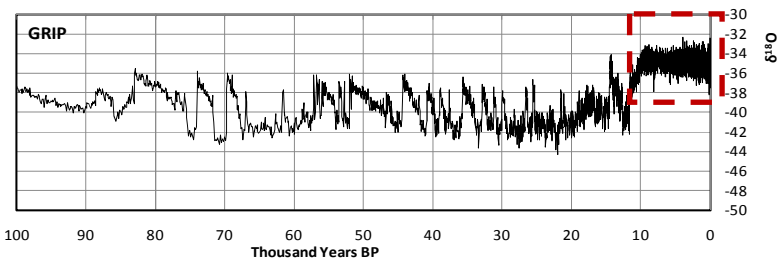
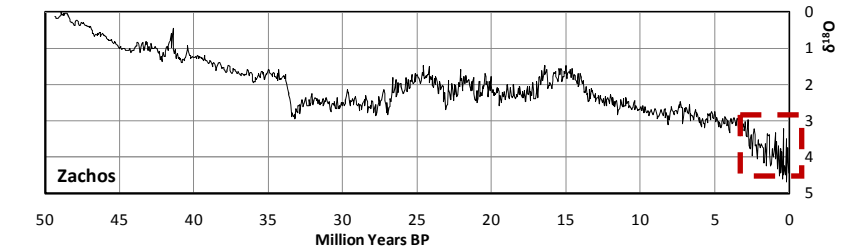
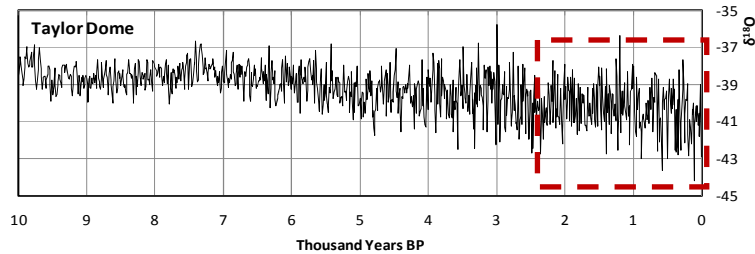
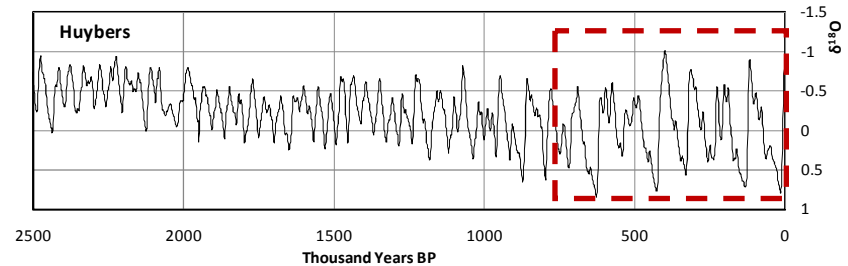
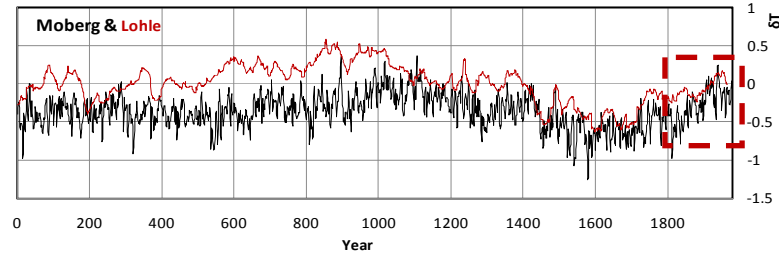
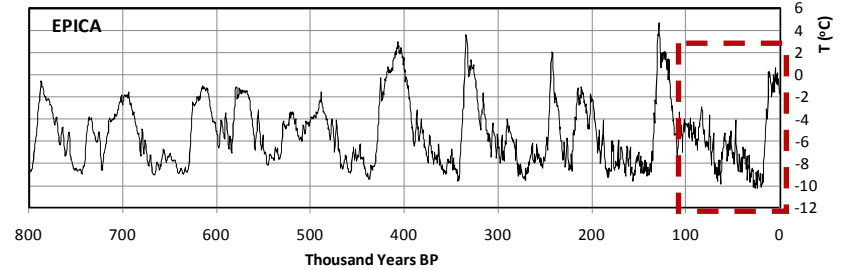
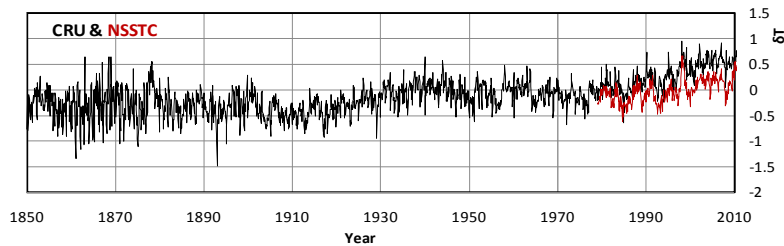
Co-evolution of climate with tectonics and life on Earth over the last half billion years



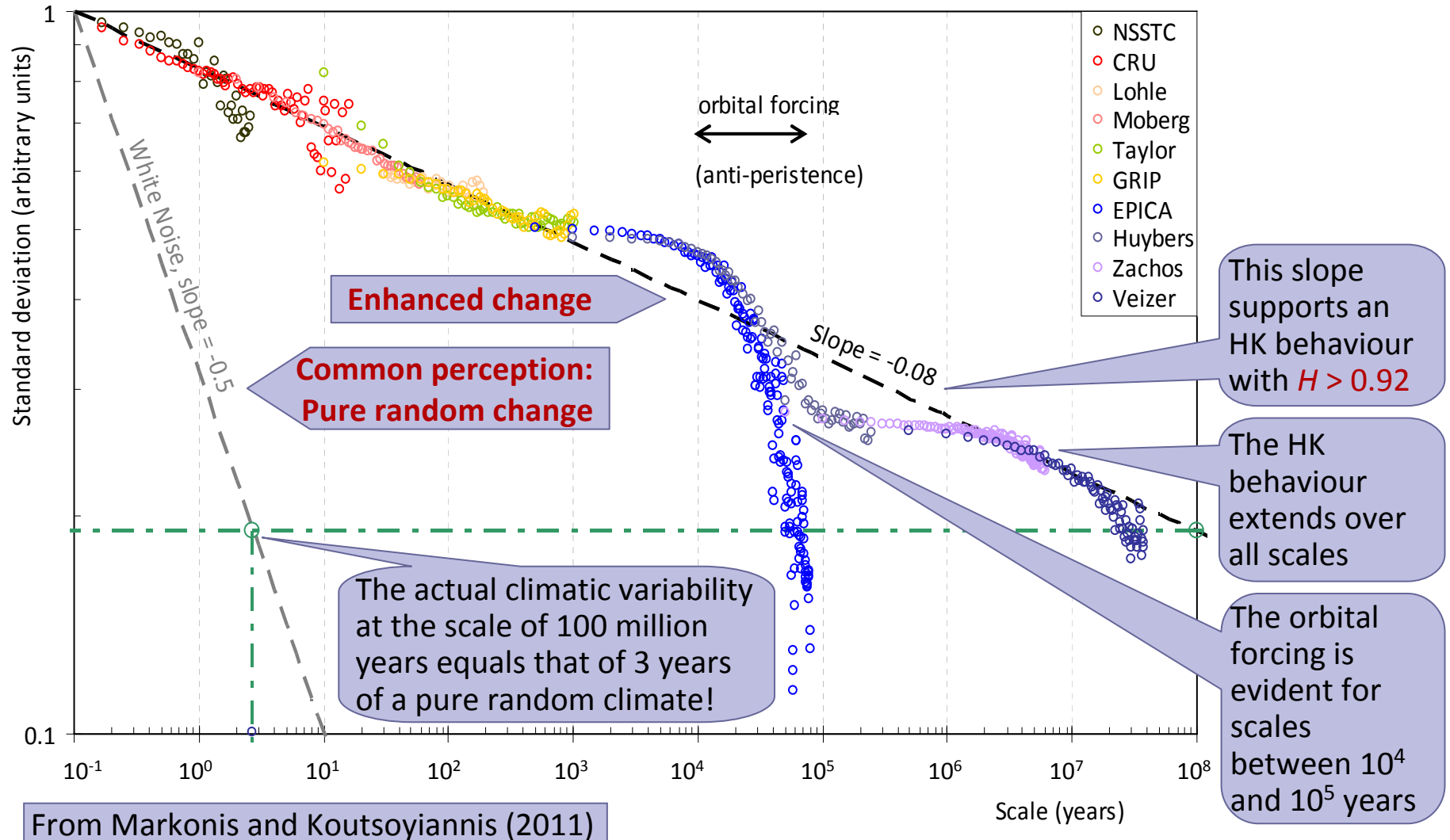
Palaeoclimate temperature estimates based on $\delta^{18}\text{O}$; adapted from Veizer *et al.* (2000)

Temperature change on Earth based on observations and proxies

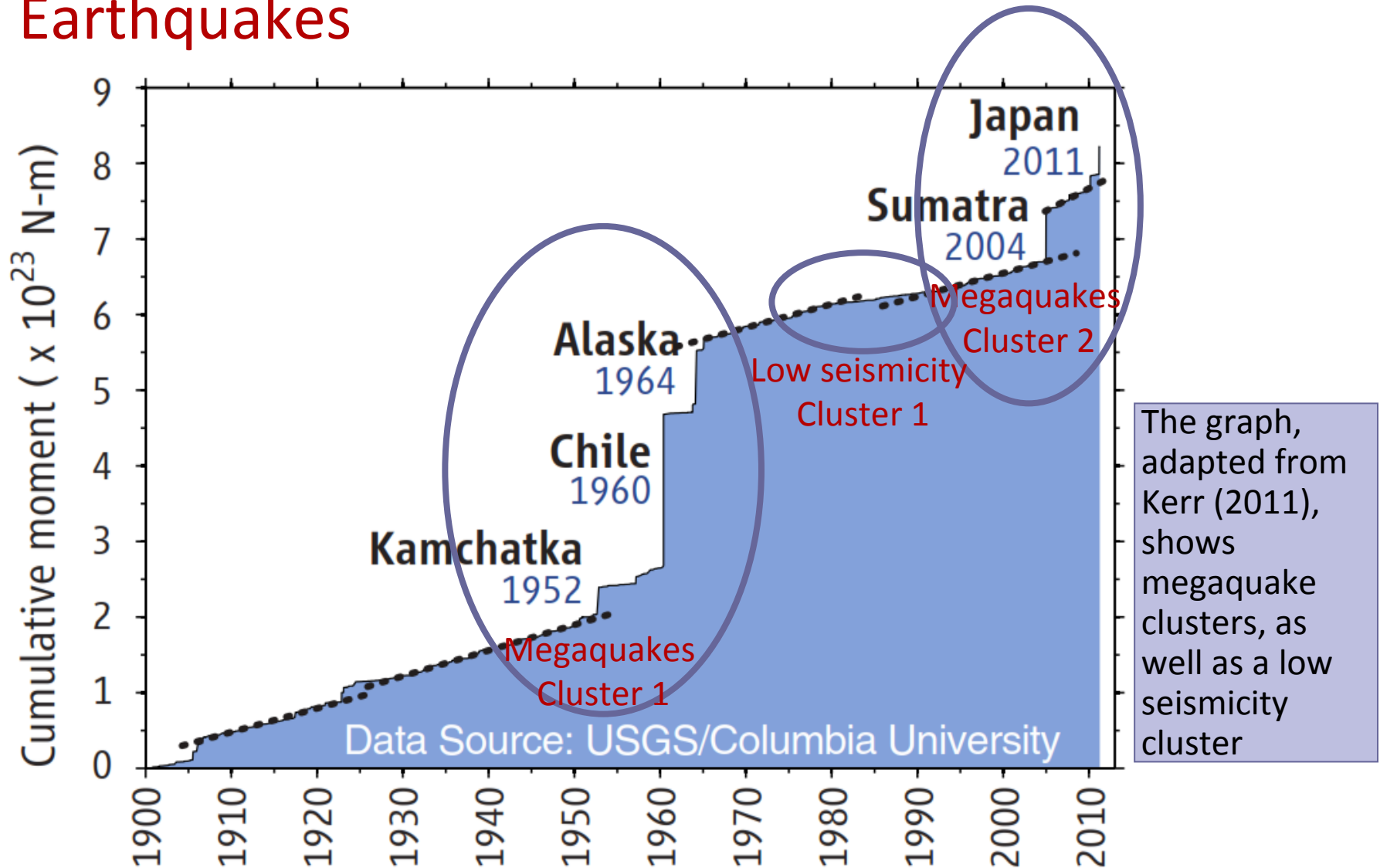
From Markonis and Koutsoyiannis (2011)



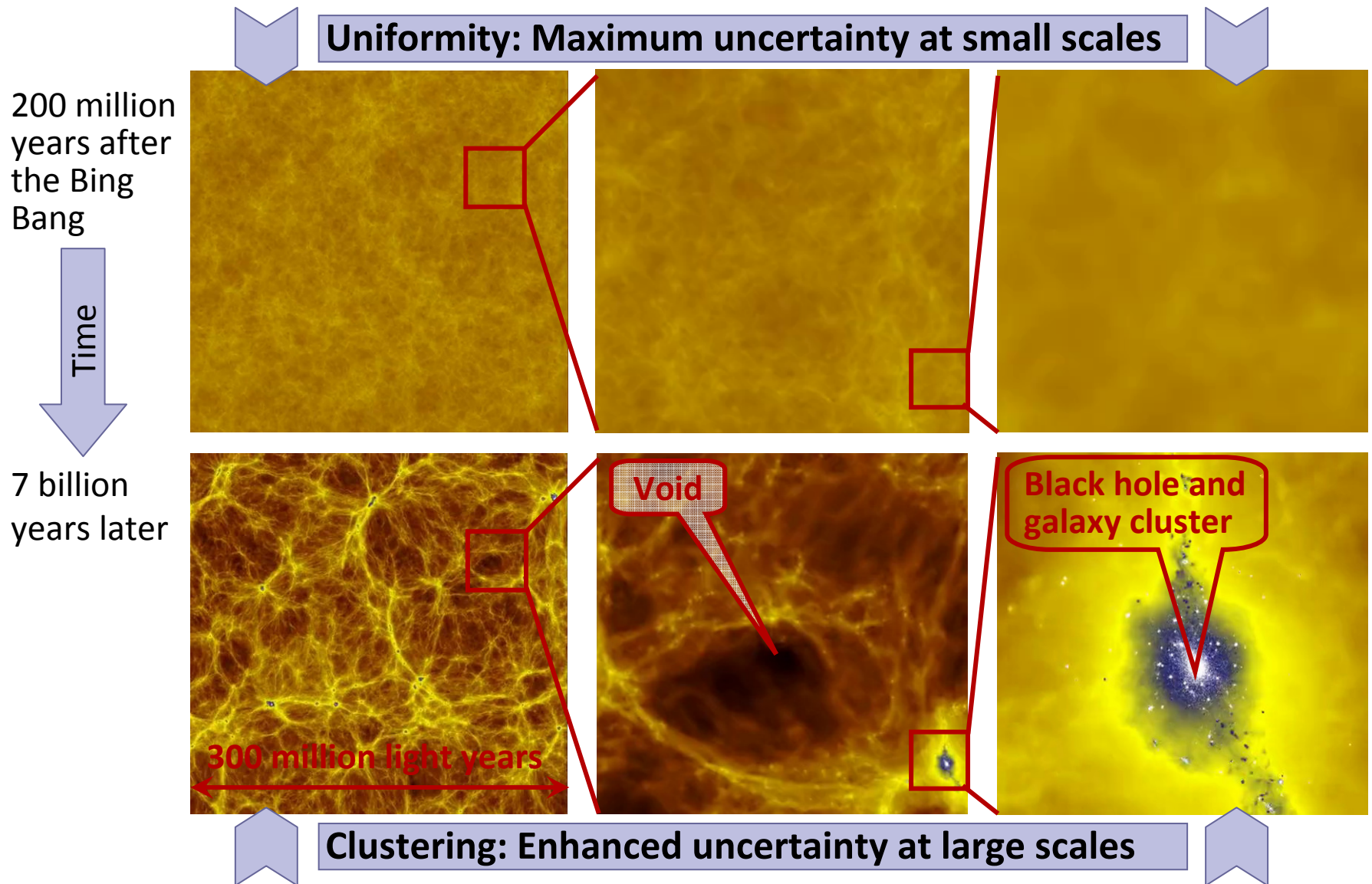
A combined climacogram of all 10 temperature observation sets and proxies



Clustering in time in other geophysical processes: Earthquakes



Cosmological evolution: from uniformity to clustering

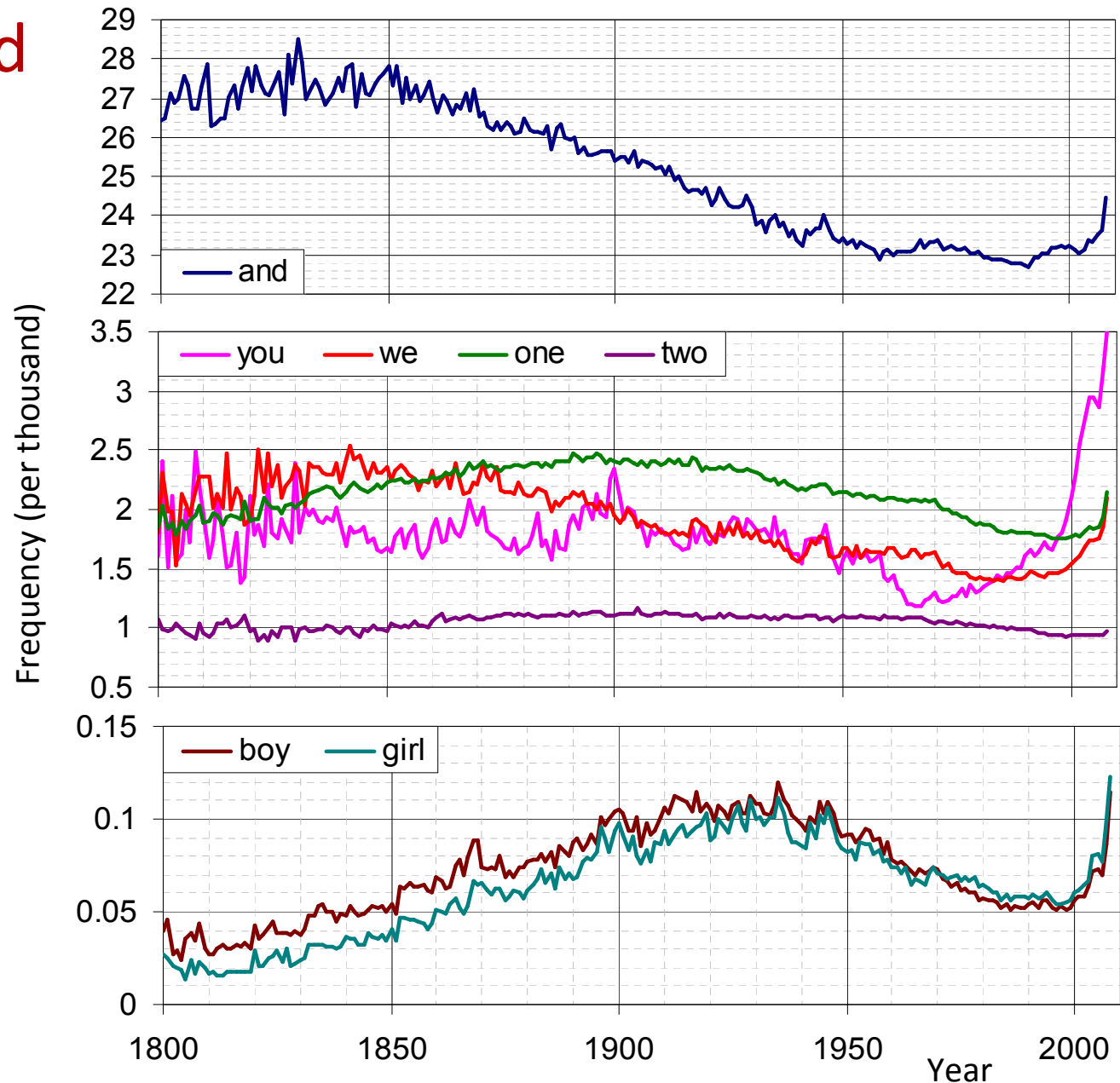


Images are snapshots from videos depicting cosmological simulations, timemachine.gigapan.org/wiki/Early_Universe; Di Matteo et al. (2008)

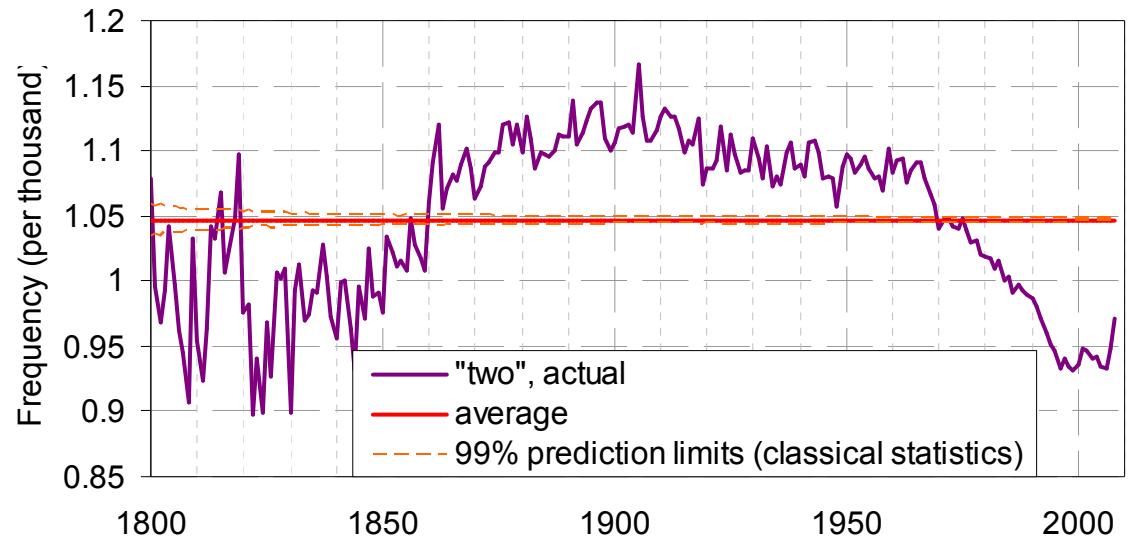
Clustering and change in human related processes

Even the use of neutral words, like “and”, “you”, “we”, etc., is subject to spectacular change through the years

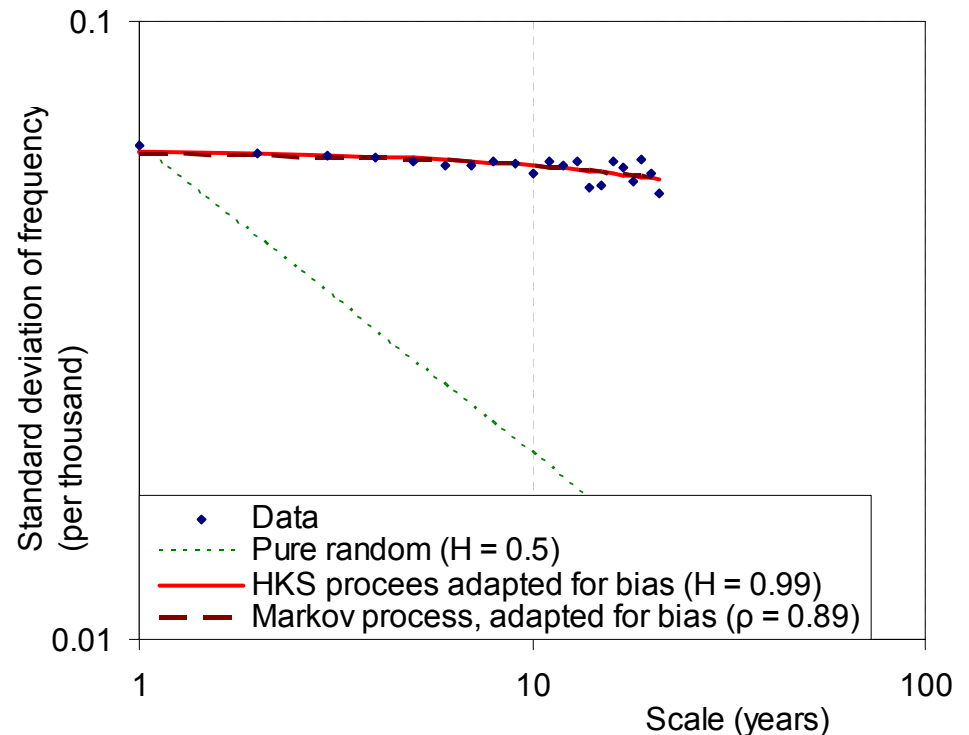
Data from Google labs;
ngrams.googlelabs.com



The climacogram of the frequency of a very neutral word: “two”



- The pure random approach is a spectacular failure
- The climacogram is almost flat
- This indicates an HKS process with H close to 1
- Even a Markov (AR1) model with high autocorrelation can match this climacogram
- The two models (HKS and Markov) are indistinguishable in this case (the available record is too short)



Seeking an explanation of long-term change: Entropy

- Definition of **entropy** of a random variable \underline{z} (adapted from Papoulis, 1991)

$$\Phi[\underline{z}] := E[-\ln[f(\underline{z})/I(\underline{z})]] = -\int_{-\infty}^{\infty} f(z) \ln [f(z)/I(z)] dz \quad [\text{dimensionless}]$$

where $f(z)$ the probability density function, with $\int_{-\infty}^{\infty} f(z) dz = 1$, and $I(z)$ a Lebesgue density (numerically equal to 1 with dimensions same as in $f(z)$)

- Definition of **entropy production** for the stochastic process $\underline{z}(t)$ in continuous time t (from Koutsoyiannis, 2011)

$$\Phi'[\underline{z}(t)] := d\Phi[\underline{z}(t)] / dt \quad [\text{units } T^{-1}]$$

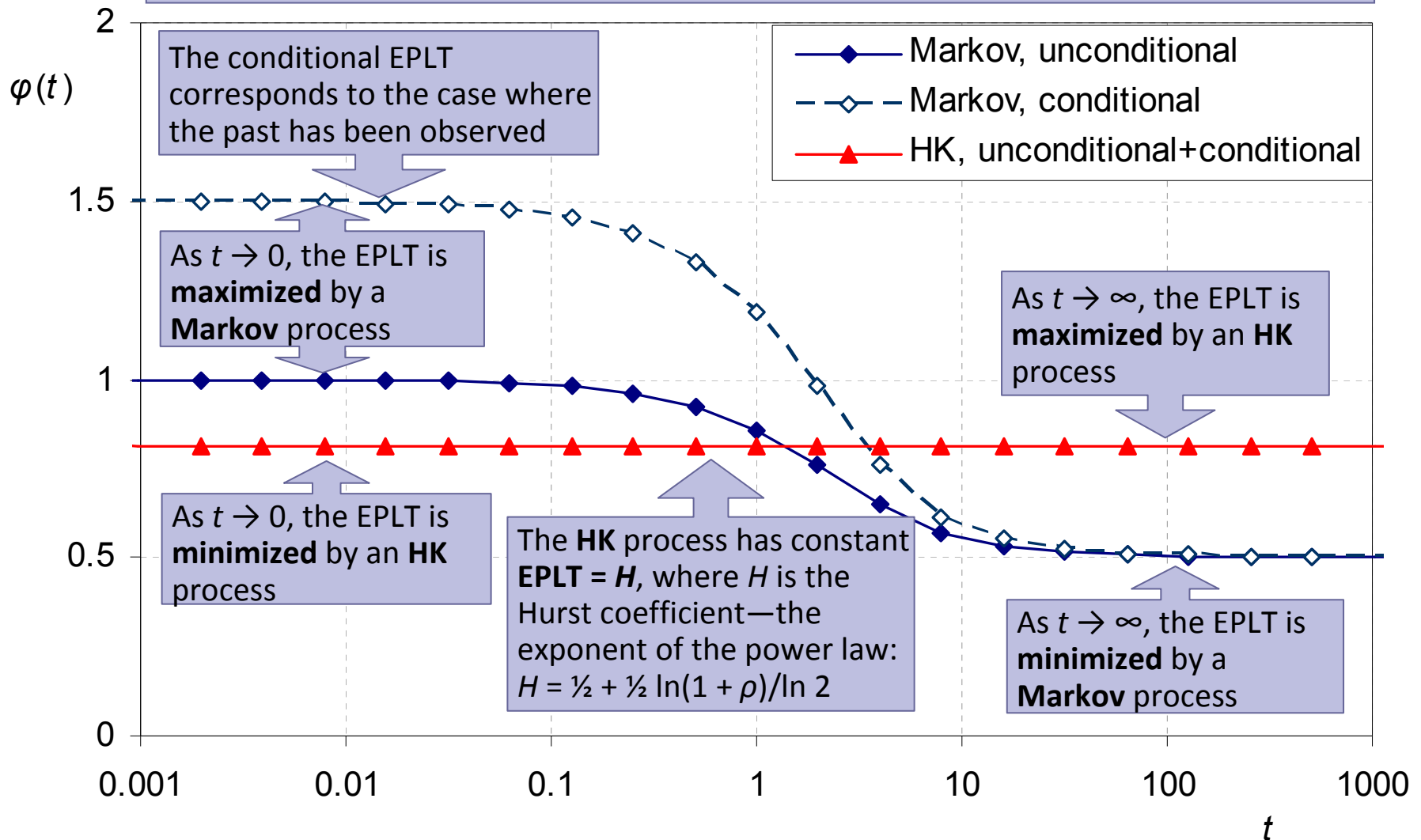
- Definition of **entropy production in logarithmic time (EPLT)**

$$\varphi[\underline{z}(t)] := d\Phi[\underline{z}(t)] / d(\ln t) \equiv \Phi'[\underline{z}(t)] t \quad [\text{dimensionless}]$$

- Note 1: Starting from a stationary stochastic process $\underline{x}(t)$, the cumulative (nonstationary) process $\underline{z}(t)$ is defined as $\underline{z}(t) := \int_0^t \underline{x}(\tau) d\tau$; consequently, the discrete time process $\underline{x}_i^\Delta := \underline{z}(i\Delta) - \underline{z}((i-1)\Delta)$ represents stationary intervals (for time step Δ in discrete time i) of the cumulative process $\underline{z}(t)$
- Note 2: For any specified t and any two processes $\underline{z}_1(t)$ and $\underline{z}_2(t)$, an inequality relationship between entropy productions, such as $\Phi'[\underline{z}_1(t)] < \Phi'[\underline{z}_2(t)]$ holds also true for EPLTs, e.g. $\varphi[\underline{z}_1(t)] < \varphi[\underline{z}_2(t)]$

Extremizing entropy production (EPLT)

The solutions depicted are generic, valid for any Gaussian process, independent of μ and σ , and depended on ρ only (the example is for $\rho = 0.543$)—see Koutsoyiannis (2011)



Concluding remarks

- The world exists only in change
- Change occurs at all time scales
- Change is hardly predictable in deterministic terms
- Humans are part of the changing Nature—but change is hardly controllable by humans (fortunately)
- Hurst-Kolmogorov dynamics is the key to perceive multi-scale change and model the implied uncertainty and risk
- Hydrology has greatly contributed in discovering and modelling change—however, lately, following other geophysical disciplines, it has been affected by 19th-century myths of static or clockwork systems, deterministic predictability (*cf.* climate models) and elimination of uncertainty
- A new change of perspective is thus needed in which change and uncertainty are essential parts

Both classical physics and quantum physics are indeterministic

Karl Popper (in his book “Quantum Theory and the Schism in Physics”)

The future is not contained in the present or the past

W. W. Bartley III (in Editor’s Foreward to the same book)

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