KNOWLEDGE FOR THE FUTURE
Joint Assembly IAHS - IAPSO - IASPEI
Gothenburg Sweden 22-26 July 2013

Session HW15:
Testing simulation and forecasting models in non-stationary conditions

In defence of stationarity

Demetris Koutsoyiannis

Department of Water Resources and Environmental Engineering
Faculty of Civil Engineering
National Technical University of Athens, Greece
(dk@itia.ntua.gr, http://www.itia.ntua.gr/dk/)

Presentation available online: http://itia.ntua.gr/1364/

Premature death, premeditated murder or misinformation?

POLICY FORUM

CLIMATE CHANGE

Stationarity Is Dead: Whither Water Management?

Climate change undermines a basic assumption that historically has facilitated management of water supplies, demands, and risks.

P. C. D. Milly,[1*] Julio Betancourt,[2] Malin Falkenmark,[3] Robert M. Hirsch,[4] Zbigniew W. Kundzewicz,[5] Dennis P. Lettenmaier,[6] Ronald J. Stouffer[7]

Systems for management of water throughout the developed world have been designed and operated under the assumption of stationarity. Stationarity—the idea that natural systems fluctuate within an unchanging envelope of variability—is a foundational concept that permeates training and practice in water-resource engineering. It implies that any variable (e.g., annual streamflow or annual flood peak) has a time-invariant (or 1-year–periodic) probability density function (pdf), whose properties can be estimated from the instrument record. Under stationarity, pdf estimation errors are acknowledged, but have been assumed to be reducible by additional observations, more efficient estimators, or regional or paleohydrologic data. The pdfs, in turn, are used to evaluate and manage risks to water supplies, waterworks, and floodplains; annual global invest-

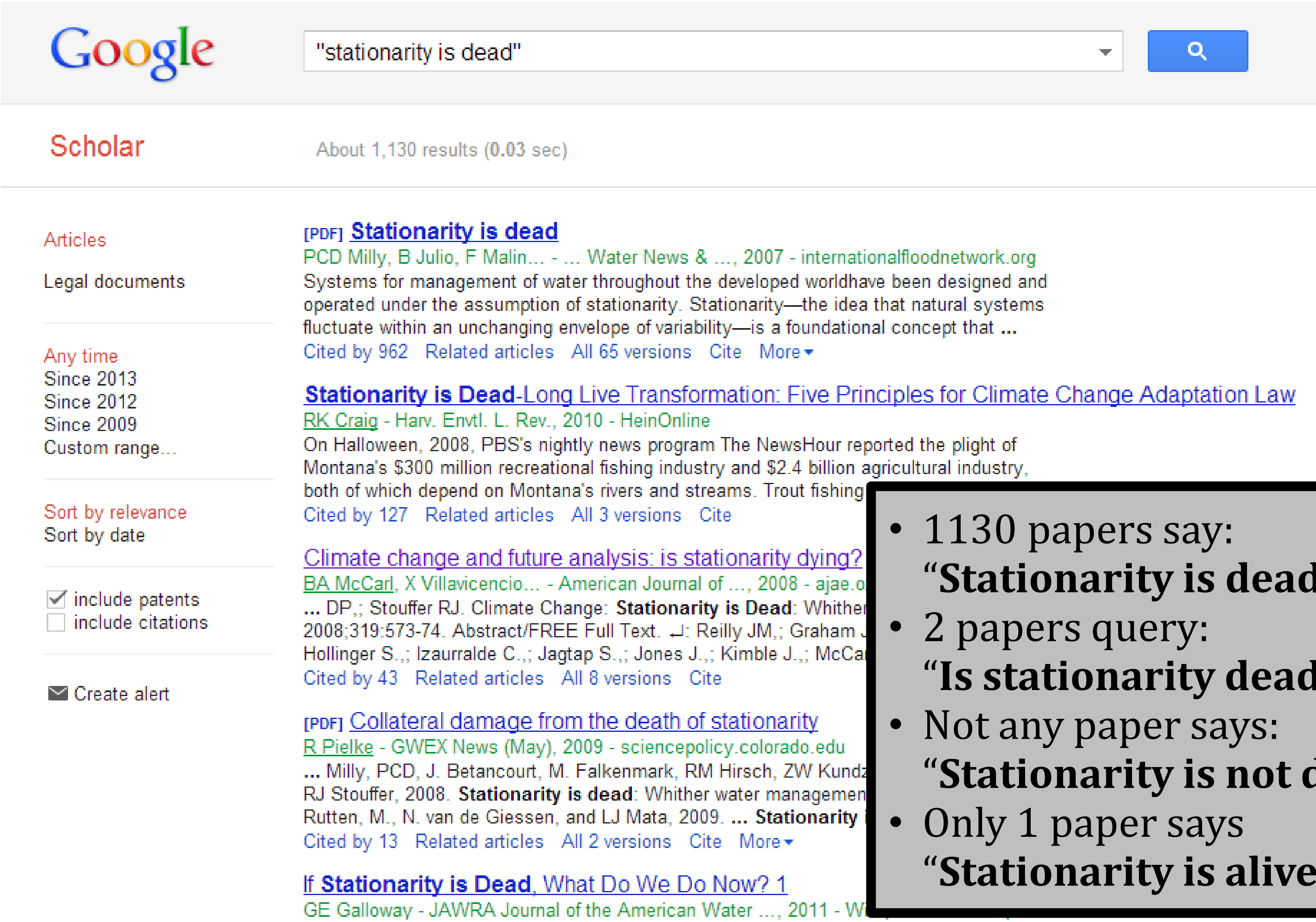
An uncertain future challenges water planners.

that has emerged from climate models (see figure, p. 574).

Why now? That anthropogenic climate change affects the water cycle (*9*) and water supply (*10*) is not a new finding. Nevertheless, sensible objections to discarding stationarity have been raised. For a time, hydroclimate had not demonstrably exited the envelope of natural variability and/or the effective range of optimally operated infrastructure (*11*, *12*). Accounting for the substantial uncertainties of climatic parameters estimated from short records (*13*) effectively hedged against small climate changes. Additionally, climate projections were not considered credible (*12*, *14*).

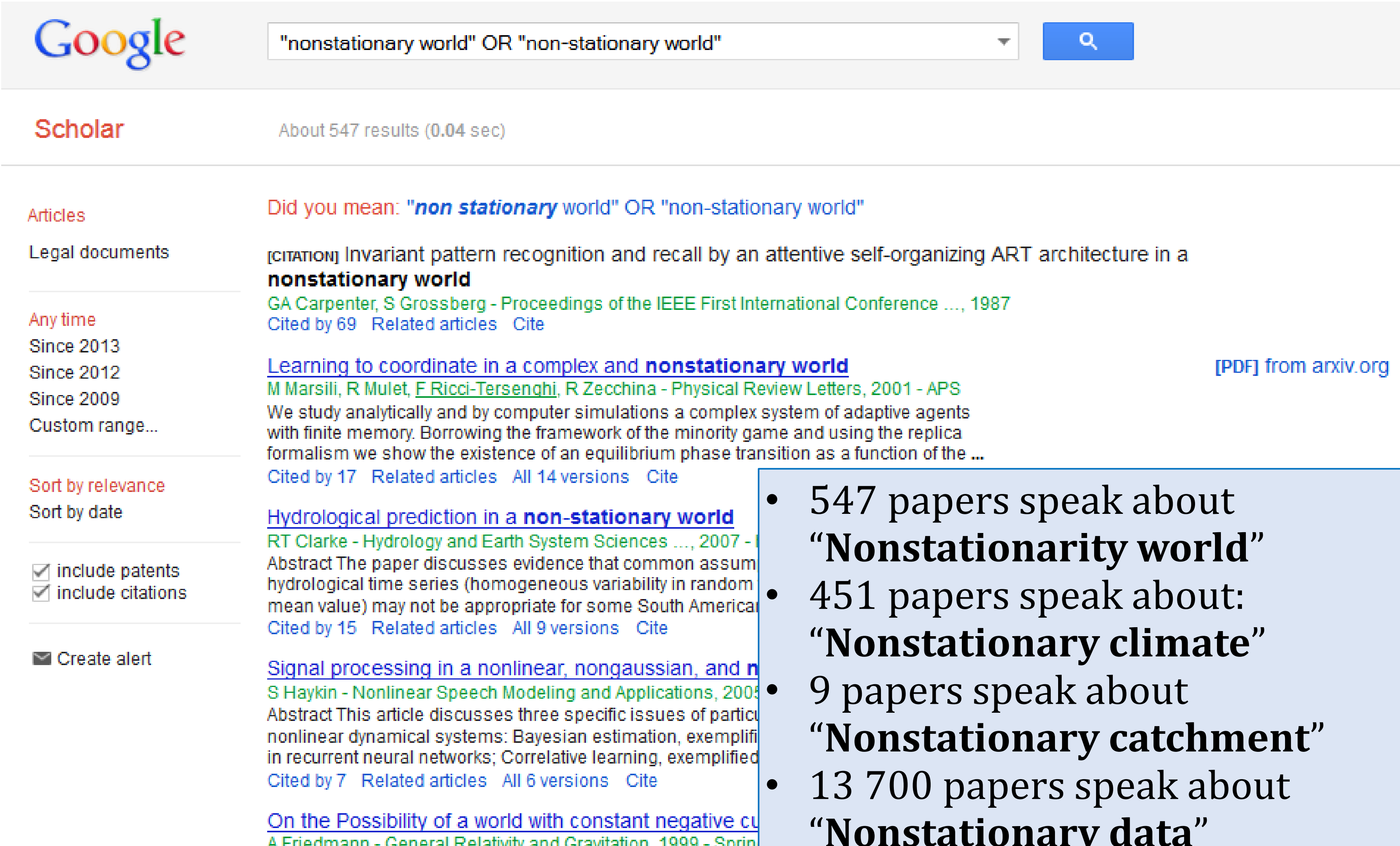
Recent developments have led us to the opinion that the time has come to move beyond the wait-and-see approach. Projections of runoff changes are bolstered by the recently demonstrated retrodictive skill of cli-

The consensus on the death of stationarity



- 1130 papers say: **"Stationarity is dead"**
- 2 papers query: **"Is stationarity dead?"**
- Not any paper says: **"Stationarity is not dead"**
- Only 1 paper says **"Stationarity is alive"**

Is the world nonstationary?

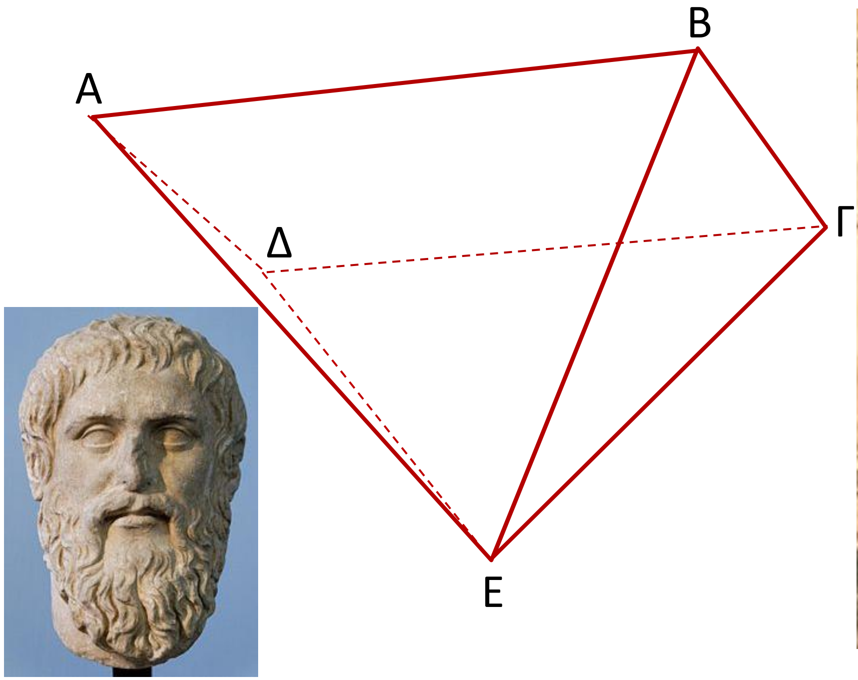


- 547 papers speak about “**Nonstationarity world**”
- 451 papers speak about: “**Nonstationary climate**”
- 9 papers speak about “**Nonstationary catchment**”
- 13 700 papers speak about “**Nonstationary data**”

Plato's metaphysical theory

- The real world: A world of ideal or perfect forms (archetypes, αρχέτυπα)
- It is unchanging and unseen
- It can only be perceived by reason (νοούμενα, nooumena)

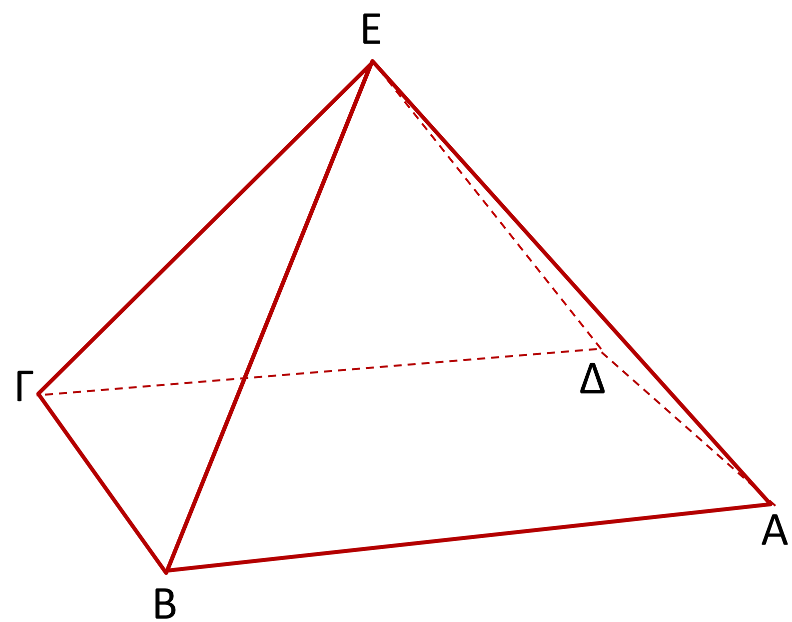
- The physical world: an imperfect image of the world of archetypes
- Physical objects and events are "shadows" of their ideal forms and are subject to change
- They can be perceived by senses (φαινόμενα, phenomena)



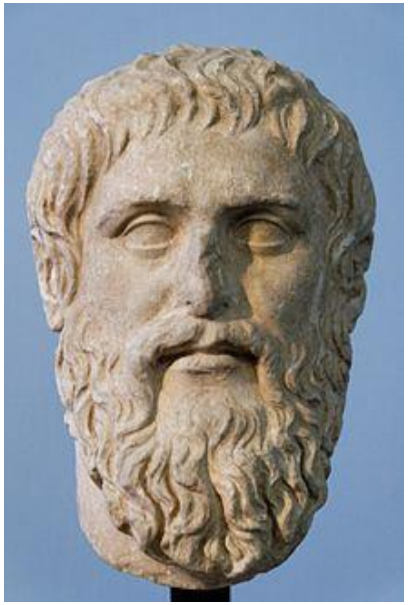
An upside-down turn of Plato's theory

- The physical world is the real world
- It is perfect
- It is perpetually changing

- Abstract representations or models of the real world are imperfect
- But can be useful to describe the real world



Merging lessons from Plato and Heraclitus



It is important to make the distinctions:

Physical world ≠ models,

Phenomena ≠ nooumena

It is important to recognize that in the physical world:

Πάντα ῥεῖ (Panta rhei, Everything flows)

Where do stationarity and nonstationarity belong?

Real world

Physical system

Unique evolution

Time series (observed)

Perpetual change

Many different models can be constructed

Mental copies depend on model constructed

The observed time series is unique; the simulated can be arbitrarily many

An important consequence: **Stationarity is immortal**

Abstract representation

Model (Stochastic process)

Ensemble (Gibbs's idea): **mental copies** of natural system

Time series (simulated)

Both stationarity and nonstationarity apply here (not in the real world)

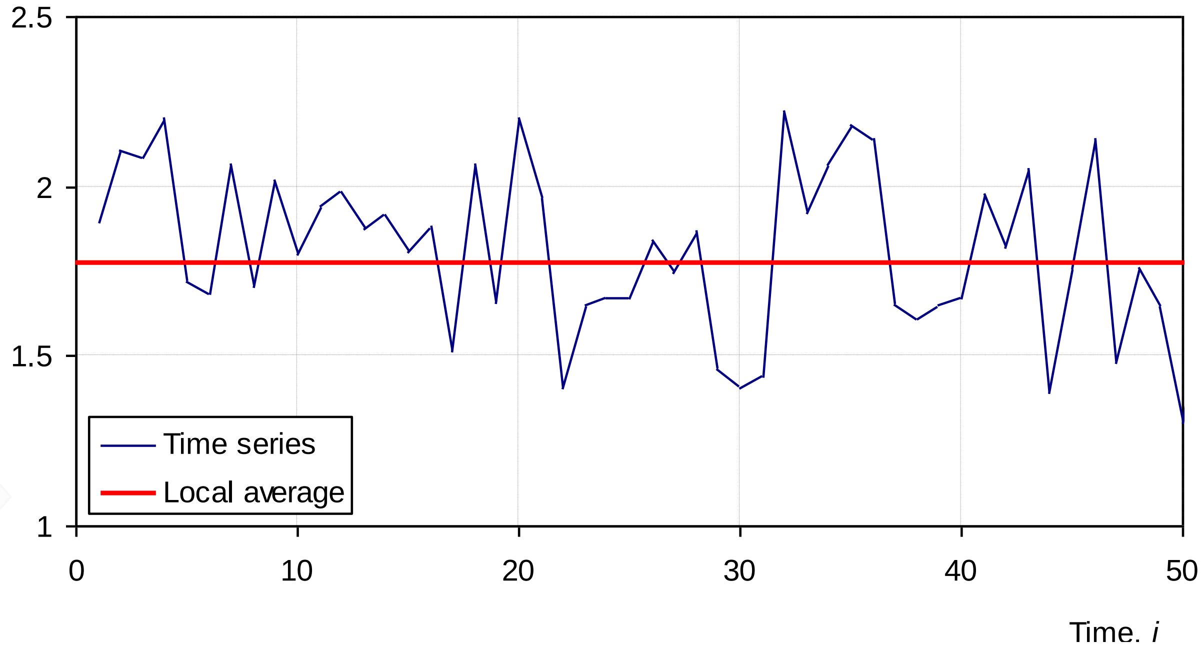
Back to Plato: Seeking invariant properties within change—simple systems

- Newton's first law: Position changes but **velocity is constant** (in absence of an external force)
  - $u = \mathrm{d}x/\mathrm{d}t$ = constant
    Huge departure from the Aristotelian view that bodies tend to rest
- Newton's second law: In presence of a constant force, the velocity changes but the **acceleration is constant**
  - $a = \mathrm{d}u/\mathrm{d}t = F/m$ = constant
  - For the weight W of a body $a = g = W/m$ = constant
- Newton's law of gravitation: The weight W (the attractive force exerted by a mass M) is not constant but inversely proportional to the square of distance; thus other **constants** emerge, i.e.,
  - $a\, r^2 = -\, G\, M$ = constant
  - $\frac{d\theta}{dt} r^2$ = constant (angular momentum per unit mass; θ = angle)

The stationarity concept: Seeking invariant properties in complex systems

- Complex natural systems are impossible to describe in full detail and to predict their future evolution with precision
- The great scientific achievement is the invention of macroscopic descriptions that need not model the details
- Essentially this is done using probability theory (laws of large numbers, central limit theorem, principle of maximum entropy)
- Related concepts are: stochastic processes, statistical parameters, stationarity, ergodicity

Example 1:
50 terms of a synthetic time series (to be discussed later)



What is stationarity and nonstationarity?

Stationary Processes

A stochastic process $\mathbf{x}(t)$ is called *strict-sense stationary* (abbreviated SSS) if its statistical properties are invariant to a shift of the origin. This means that the processes $\mathbf{x}(t)$ and $\mathbf{x}(t+c)$ have the same statistics for any c.

WIDE SENSE. A stochastic process $\mathbf{x}(t)$ is called *wide-sense stationary* (abbreviated WSS) if its mean is constant

$$E\{\mathbf{x}(t)\} = \eta \tag{10-41}$$

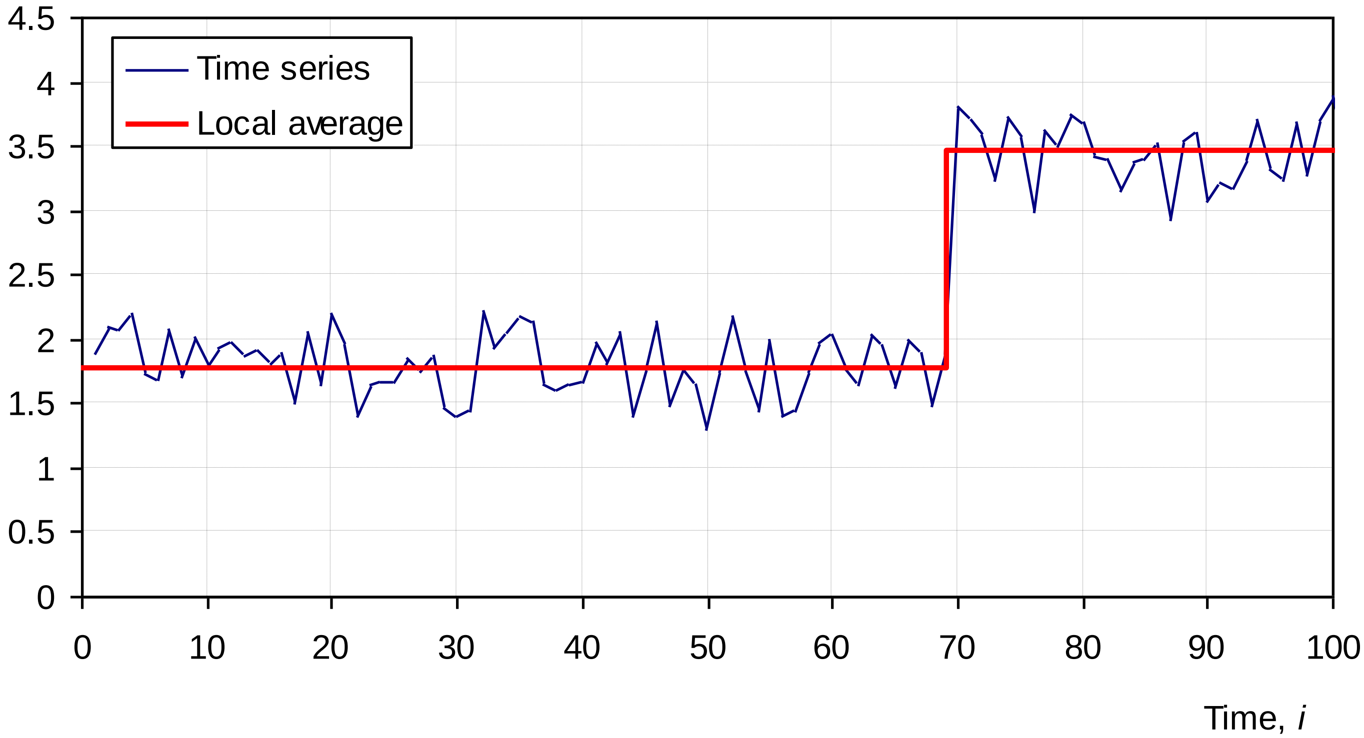
and its autocorrelation depends only on $\tau = t_1 - t_2$:

$$E\{\mathbf{x}(t+\tau)\mathbf{x}^*(t)\} = R(\tau) \tag{10-42}$$

- Definitions copied from Papoulis (1991).
- Note 1: Definition of **stationarity** applies to a **stochastic process**
- Note 2: Processes that are not stationary are called **nonstationary**; some of their statistical properties are **deterministic** functions of time

Does this example suggest stationarity or nonstationarity?

Example 1 extended up to time 100



Mean m (red line) of time series (blue line) is:

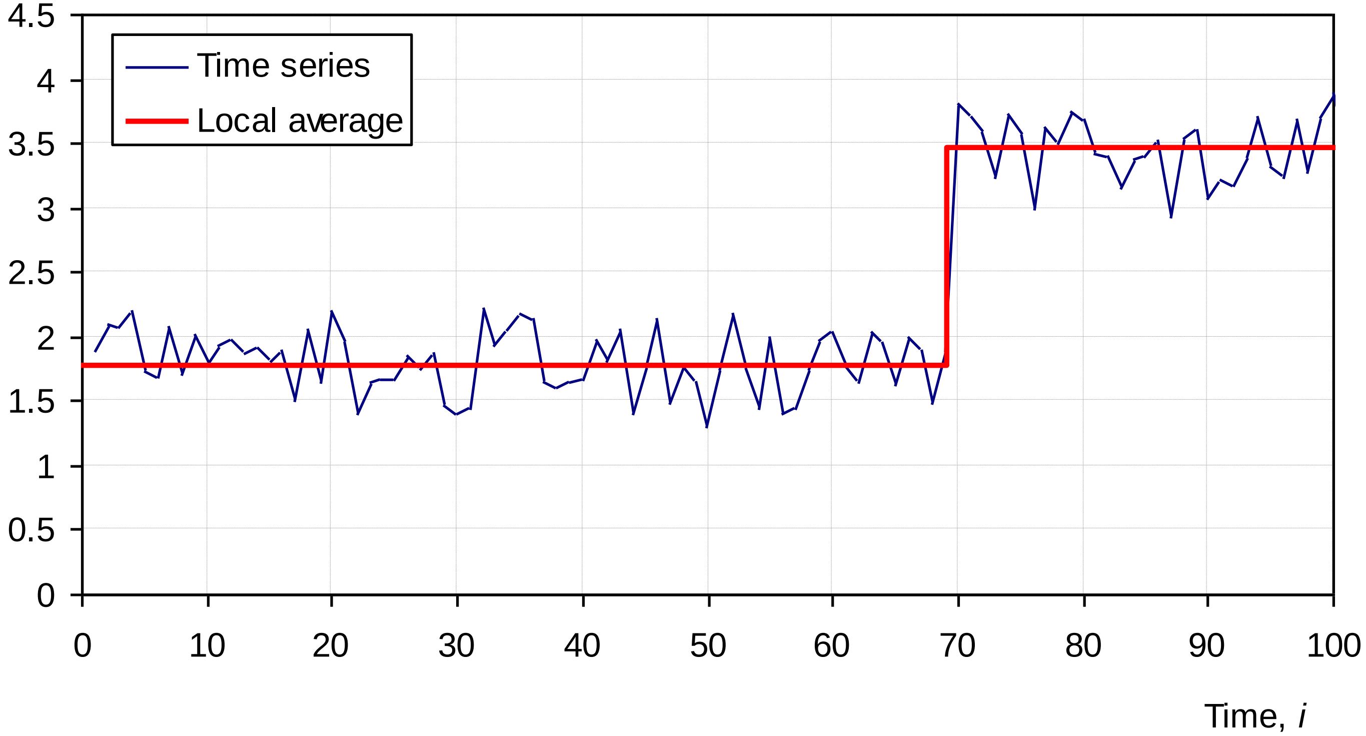
$m = 1.8$ for $i < 70$

$m = 3.5$ for $i \geq 70$

See details of this example in Koutsoyiannis (2011)

Reformulation of question:
Does the red line reflect a **deterministic** function?

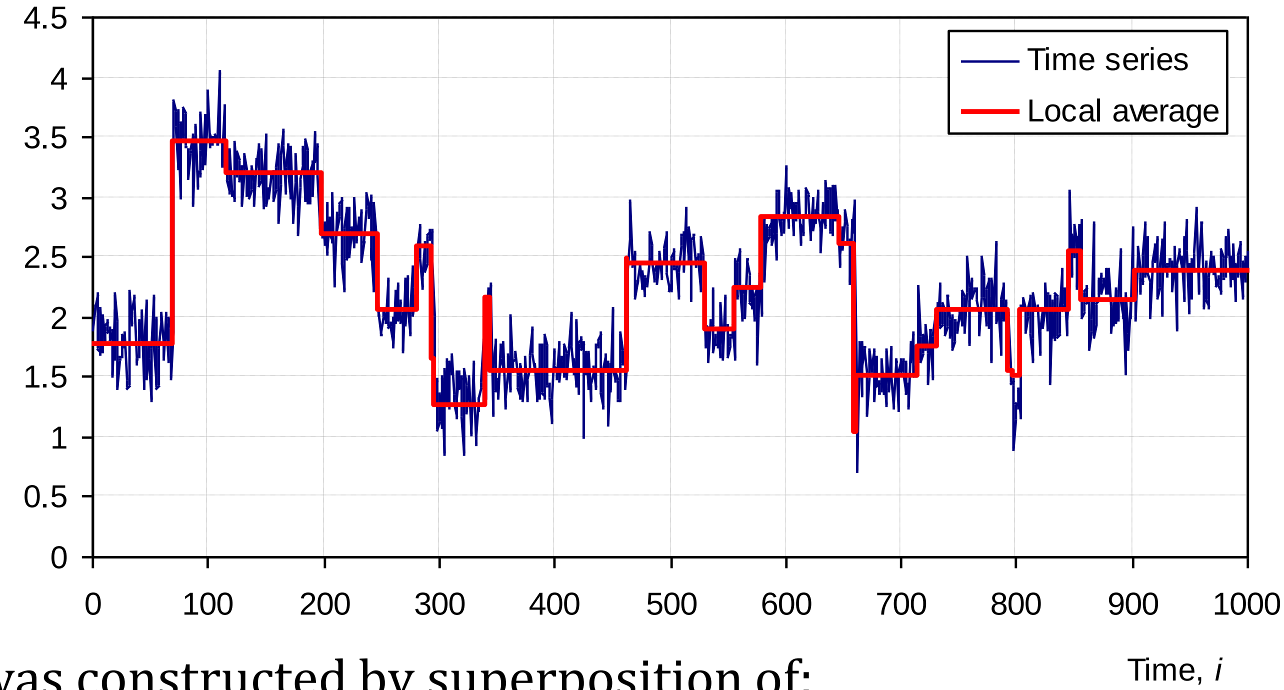
Example 1 extended up to time 100



- If the red line is a deterministic function of time:
  → **nonstationarity**
- If the red line is a random function (realization of a stationary stochastic process) → **stationarity**

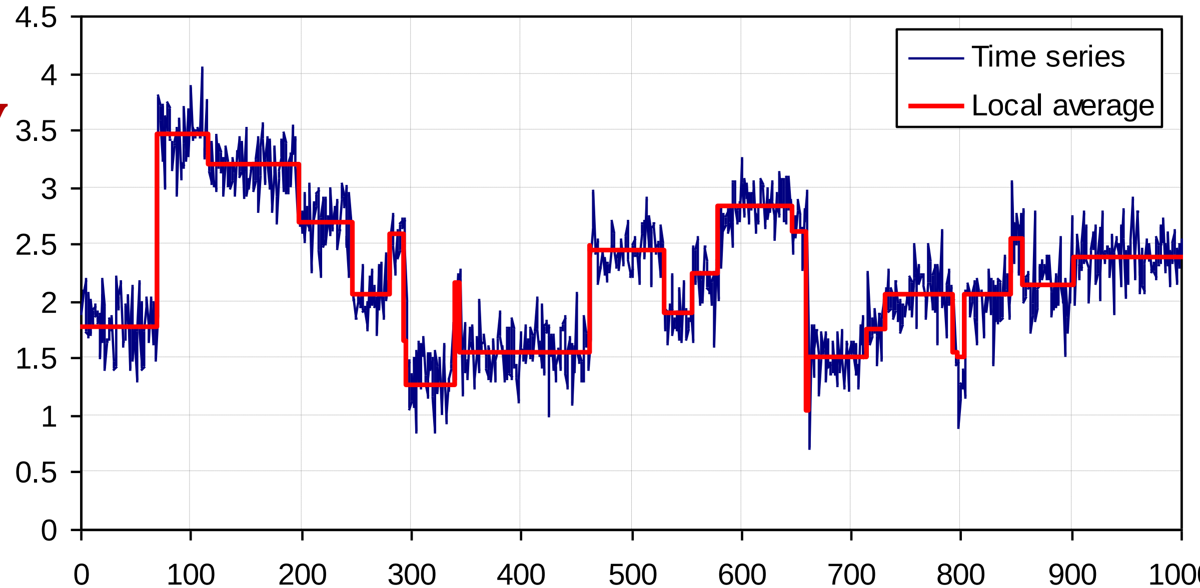
Answer of the last question: the red line is a realization of a stochastic process

Example 1 extended up to time 1000

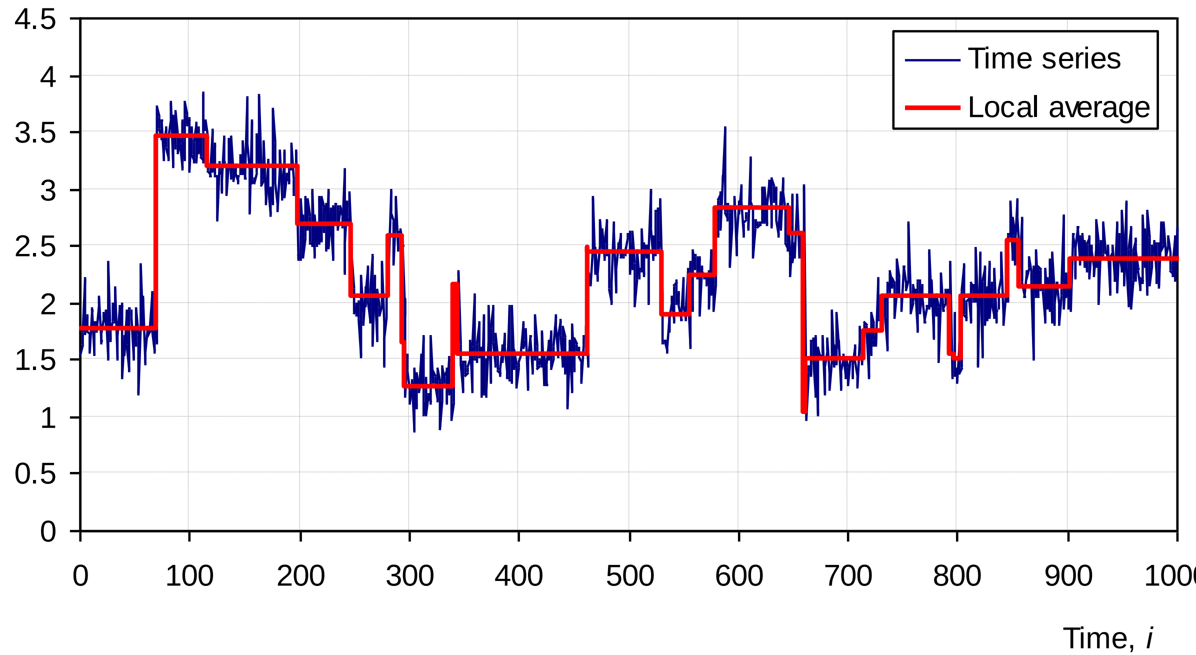


- The time series was constructed by superposition of:
  - A stochastic process with values $\underline{m}_j \sim N(2, 0.5)$ each lasting a period $\underline{\tau}_j$ exponentially distributed with $E[\underline{\tau}_j] = 50$ (red line)
  - White noise $N(0, 0.2)$
- Nothing in the model is nonstationary
- The process of our example is **stationary**

The big difference of **nonstationarity** and stationarity (1)



The initial time series



A mental copy generated by a **nonstationary** model (assuming the red line is a deterministic function)

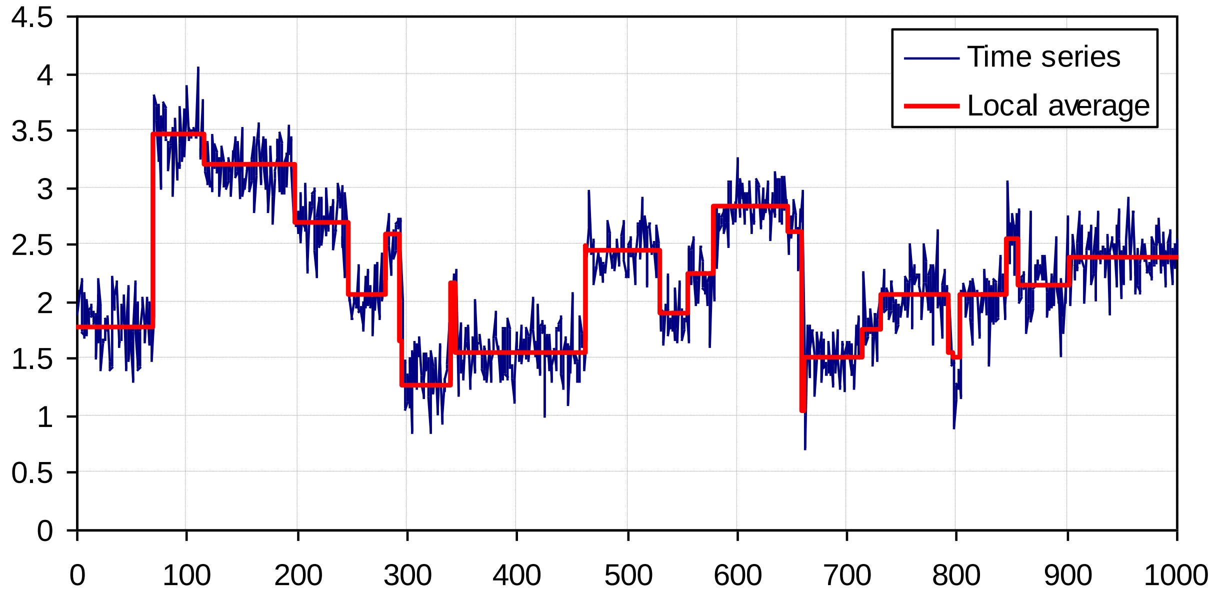
Unexplained variance (differences between blue and red line): $0.2^2 = 0.04$

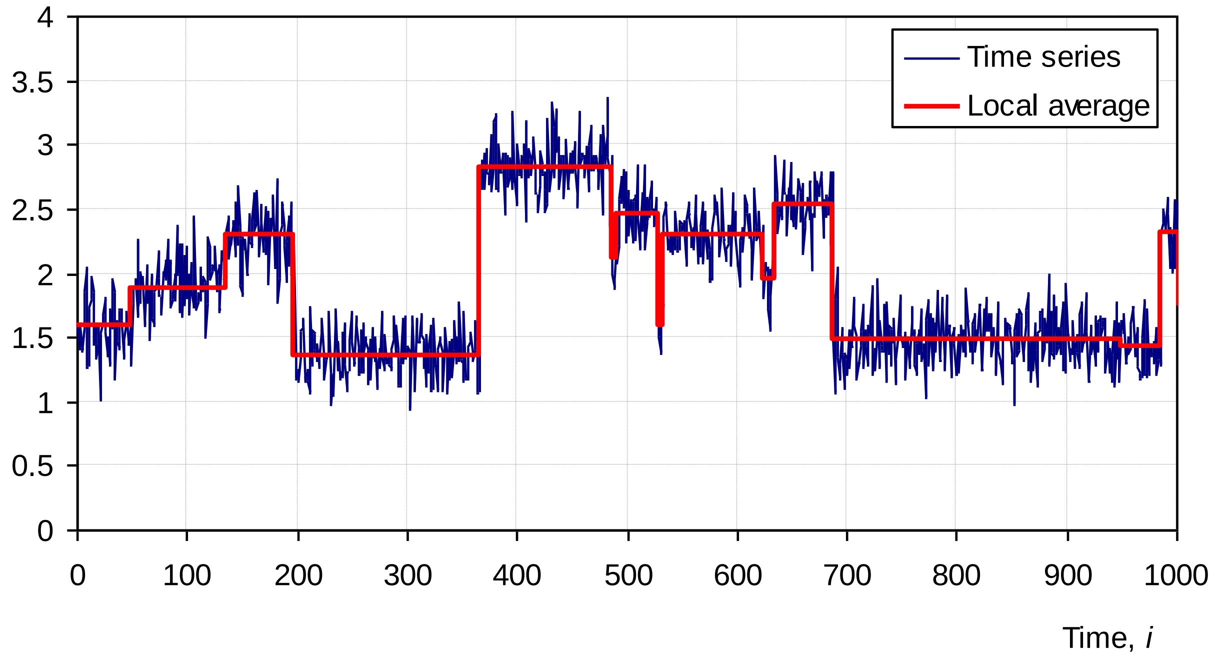
The big difference of nonstationarity and **stationarity** (2)

The initial time series

A mental copy generated by a **stationary** model (assuming the red line is a stationary stochastic process)

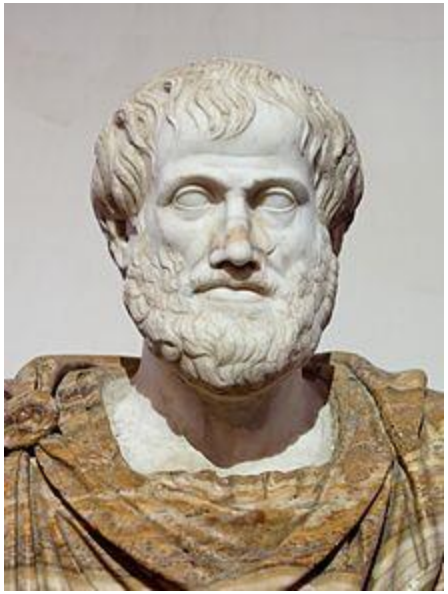
Unexplained variance (the "undecomposed" time series): 0.38 (~10 times greater)





Caution in using the term “nonstationarity”

- **Stationary** is not synonymous to **static**
- **Nonstationary** is not synonymous to **changing**
- In a nonstationary process, the change is described by a deterministic function
- A deterministic description should be constructed:
  - by deduction (the Aristotelian **apodeixis**),
  - not by induction (the Aristotelian **epagoge**), which makes direct use use of the data
- To claim nonstationarity, we must :
  1. Establish a causative relationship
  2. Construct a quantitative model describing the change as a deterministic function of time
  3. Ensure applicability of the deterministic model in future time
- Nonstationarity reduces uncertainty!!! (it explains part of variability)
- Unjustified/inappropriate claim of nonstationarity results in underestimation of variability, uncertainty and risk!!!



A note on aleatory and epistemic uncertainty

- We often read that epistemic uncertainty is inconsistent with stationarity or even not describable in probabilistic terms
- The separation of uncertainty into aleatory and epistemic is subjective (arbitrary) and unnecessary (misleading)
- In macroscopic descriptions/models there are no demons of randomness producing aleatory uncertainty: all uncertainty is epistemic, yet not subject to elimination (see clarifications in Koutsoyiannis 2010)
- From a probabilistic point of view classification of uncertainty into aleatory or epistemic is indifferent; the obey the same probabilistic laws

Important note: A **random variable** is not a variable infected by the virus of randomness; it is a variable that is not precisely known or cannot be precisely predicted

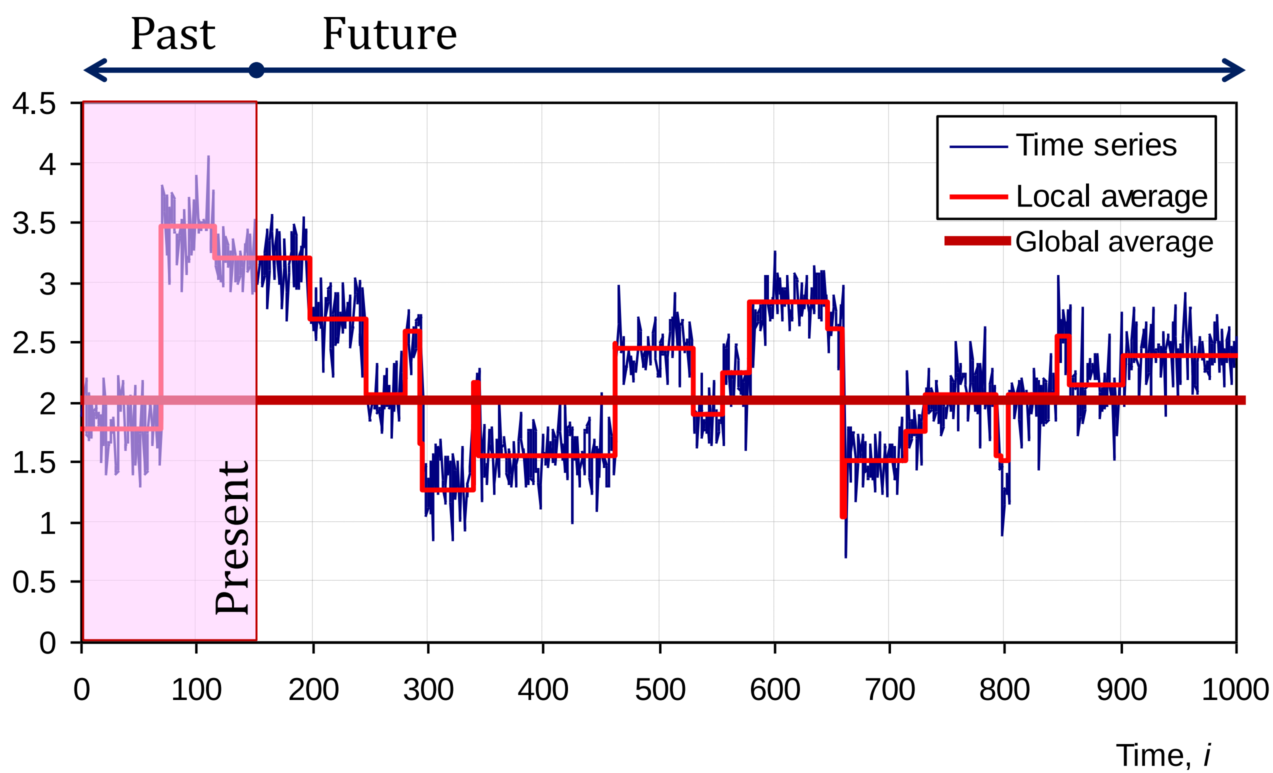
Justified use of nonstationary descriptions: Models for the past

- Changes in catchments happen all the time, including in quantifiable characteristics of catchments and conceptual parameters of models
- If we know the evolution of these characteristics and parameters (e.g. we have information about how the percent of urban area changed in time; see poster paper by Efstratiadis et al. tomorrow), then we build a nonstationary model
    - Information → Reduced uncertainty → Nonstationarity
- If we do not have this quantitative information, then:
    - We treat catchment characteristics and parameters as random variables
    - We build stationary models entailing larger uncertainty

Justified uses of nonstationary descriptions: Models for the future

- It is important to distinguish explanation of observed phenomena in the past from modelling that is made for the future
- Except for trivial cases the future is not easy to predict in deterministic terms
- If changes in the recent past are foreseen to endure in the future (e.g. urbanization, hydraulic infrastructures), then the model of the future should be adapted to the most recent past
  - This may imply a stationary model of the future that is different from that of the distant past (prior to the change)
  - It may also require "stationarizing" of the past observations, i.e. adapting them to represent the future conditions
- In the case of planned and controllable future changes (e.g. catchment modification by hydraulic infrastructures, water abstractions), which indeed allow prediction in deterministic terms, nonstationary models are justified

Conditional nonstationarity arising from stationarity models



- If the prediction horizon is long, then in modelling we will use the global average and the global variance
- If the prediction horizon is short, then we will use the local average at the present time and a reduced variance
- This is not called nonstationarity; it is dependence in time
- When there is dependence (i.e. always) observing the present state and conditioning on it looks like local nonstationarity

Concluding remarks

- Πάντα ῥεῖ (or: Change is Nature's style)
- Change occurs at all time scales
- Change is not nonstationarity
- Stationarity and nonstationarity apply to models, not to the real world, and are defined within stochastics
- Nonstationarity should not be confused with dependence
- Nonstationary descriptions are justified only if the future can be predicted in deterministic terms and are associated with reduction of uncertainty
- In absence of credible predictions of the future, admitting stationarity (and larger uncertainty) is the way to go

Long live stationarity!!!

References

- Efstratiadis, A., I. Nalbantis and D. Koutsoyiannis, Hydrological modelling in presence of non-stationarity induced by urbanization: an assessment of the value of information, IAHS - IAPSO – IASPEI Joint Assembly 2013, Gothenburg, Sweden, 2013
- Koutsoyiannis, D., A random walk on water, *Hydrology and Earth System Sciences*, 14, 585–601, 2010
- Koutsoyiannis, D., Hurst-Kolmogorov dynamics and uncertainty, *Journal of the American Water Resources Association*, 47 (3), 481–495, 2011
- Milly, P. C. D., J. Betancourt, M. Falkenmark, R. M. Hirsch, Z. W. Kundzewicz, D. P. Lettenmaier and R. J. Stouffer, Stationarity is dead: whither water management?, *Science*, 319, 573-574, 2008
- Papoulis, A., *Probability, Random Variables, and Stochastic Processes*, 3rd ed., McGraw-Hill, New York, 1991