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Statistical properties and Hurst-Kolmogorov dynamics in proxy data and temperature reconstructions

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1. Abstract

The statistical properties of over 300 different proxy records of the last two thousand years derived from the PAGES 2k database years are examined. This includes an estimation of their first four moments and their autocorrelation functions (ACF), as well as the determination of the presence of Hurst-Kolmogorov behavior (known also as long term persistence). The data are investigated in groups according to their proxy type and location, while their statistical properties are also compared to the properties of their corresponding temperature reconstructions.

2. Motivation

- Climatic variability is a global concern due to its impact to ecosystems and human societies. In order to seek for satisfactory answers concerning its magnitude, we track climate variability through the last two millenniums.
- Therefore, we use the using high resolution proxy records and reconstructed temperature time series over the last 2000 years derived from the PAGES 2k database (Ahmed et al., 2013) in an attempt to highlight the similarities and differences between their statistical properties.
- Since statistical analysis is one of the major tools used by climate scientists, we further investigate if there is any evidence of Hurst-Kolmogorov behaviour, known also as long-term persistence (Kolmogorov, 1940; Hurst, 1951). If this holds true, then the use of classical statistics to describe climate is not appropriate, because it would underestimate the climatic variability, as shown by Koutsoyiannis & Montanari (2006).

3. Data set

Our analysis is based on the proxy records and reconstructions provided by the Pages 2k database. Statistical characteristics are found for a fraction of the records and temperature reconstruction data, which are categorized in reference to their type and the region in which the measurements took place. The types of proxies are sediments, tree rings and ice cores. Further discard is made due to the sample size and the compatibility of the methods used to process the proxy measurements. The Hurst coefficient is specifically calculated for time series length up to 400. Figure 1 shows the number of records for each type of proxy and the number that was used for the estimation of the Hurst coefficient while Figure 2 shows the mean length of the time series for each proxy type.

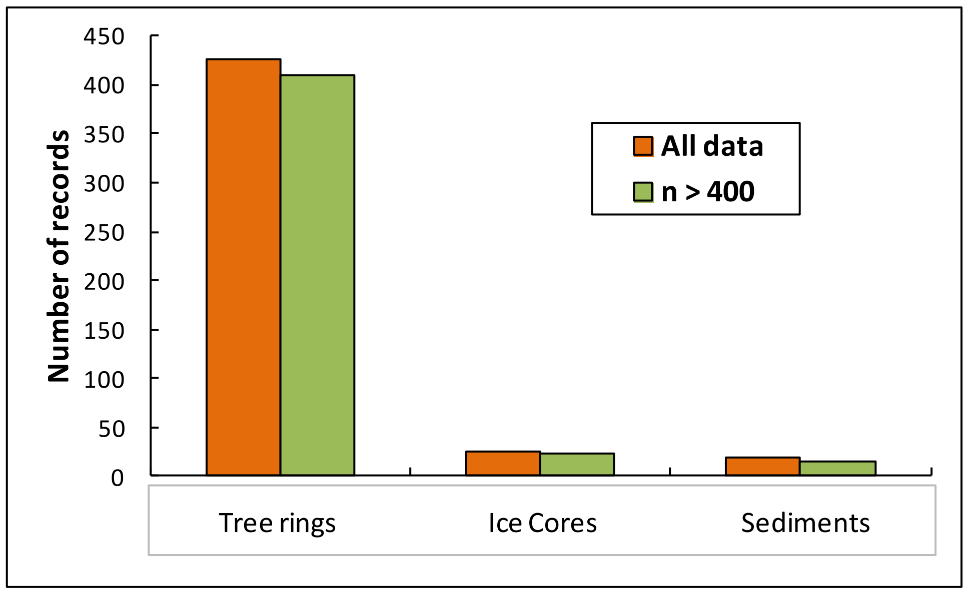


Figure 1

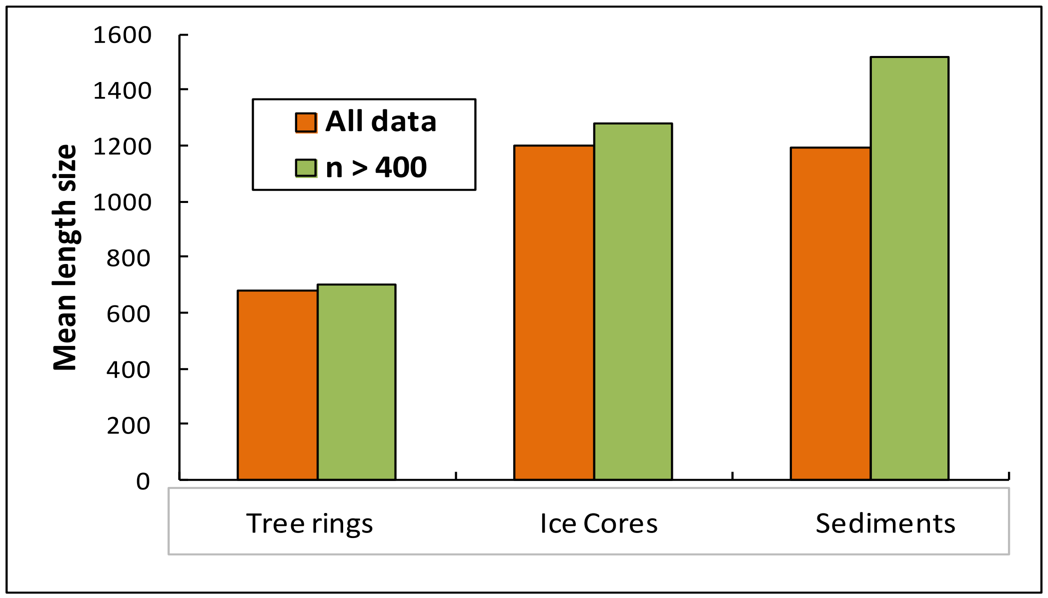
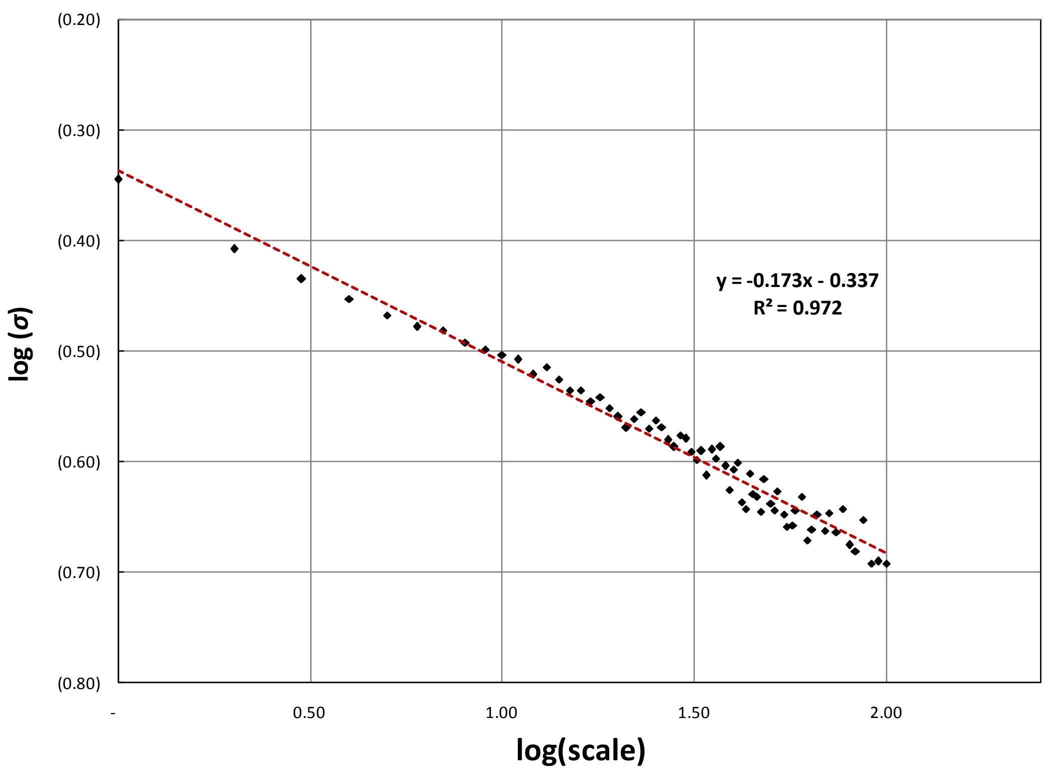


Figure 2

4. Hurst-Kolmogorov dynamics



Hurst-Kolmogorov (HK) dynamics describes the effect of scale on evolution of natural processes. It is understood as the tendency of high or low values of natural events to group. Scaling behavior can indicate frequent and sometimes strong "trends" in a process. This behaviour can be mathematically described by the invariance properties of a time series aggregated on different time scales, and then computed through the Hurst exponent, symbolized with the letter *H*, which is described by the relationship:

$$\sigma\,(k) = k^{H-1}\,\sigma$$

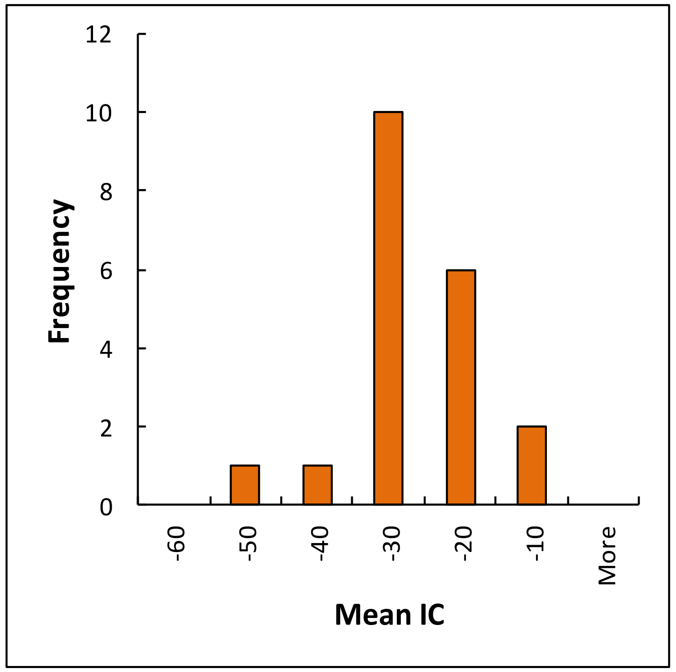
where $\sigma(k)$ and σ are the standard deviations at time scales k and 1, respectively. In a white noise series H is 0.5, whereas in real-world time series H is usually greater.

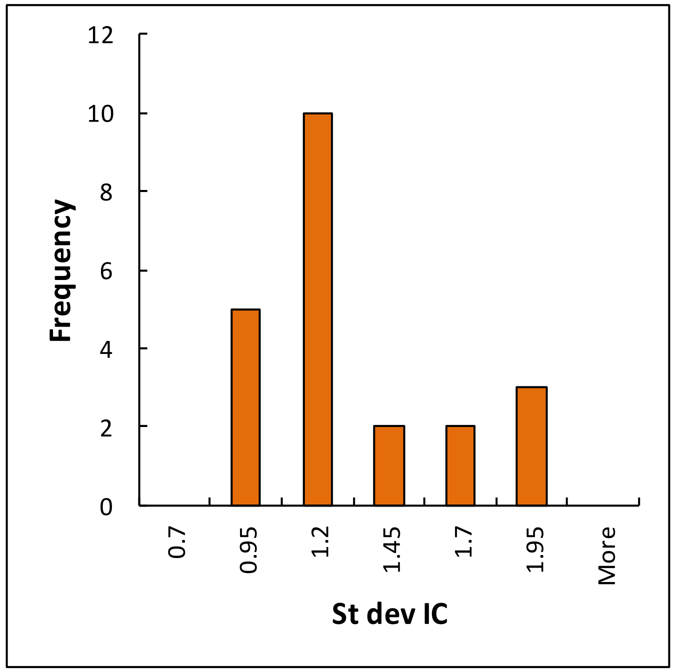
5. Methodology

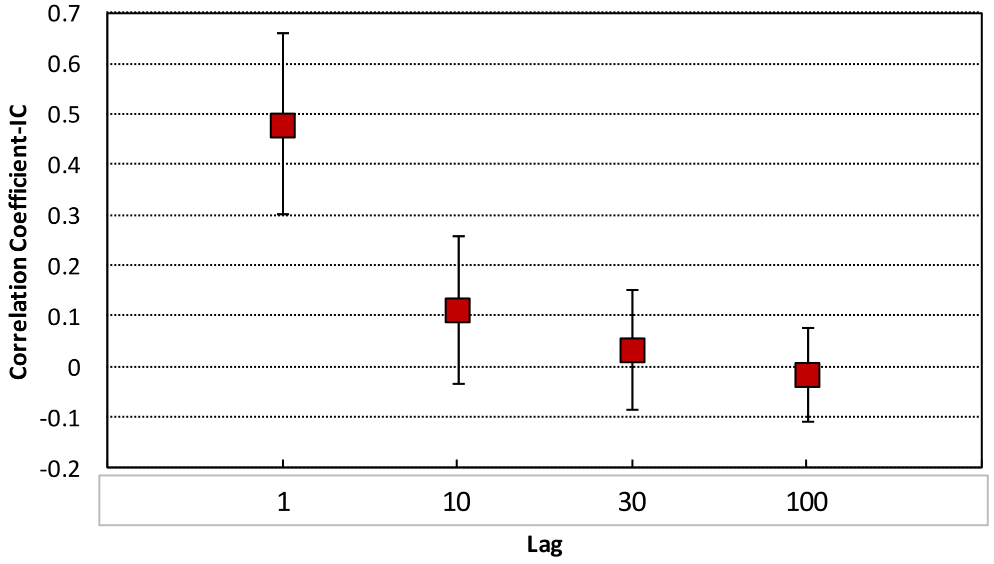
In order to investigate the existence of Hurst-Kolmogorov behavior in the proxy data and in time reconstructions and the consistency of the proxy measurements we followed these steps:

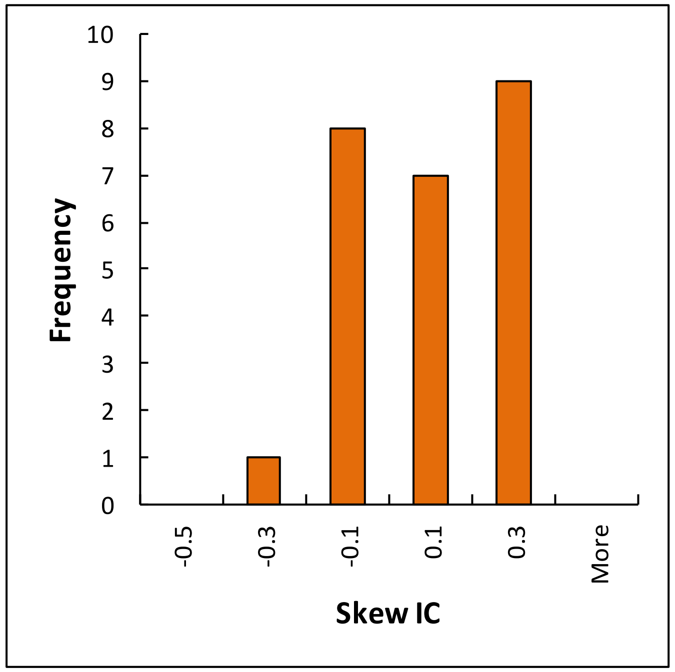
- We estimate the first four moments for the proxy data and temperature reconstruction data.
- We calculate the autocorrelation coefficient for lags 1, 10, 30, 100 years.
- We estimate the Hurst coefficient using the climacogram.
- We also examine the continental variation of standard deviation, skewness and kurtosis for the tree rings.
- Finally, we applied the above steps for the temperature reconstructions.

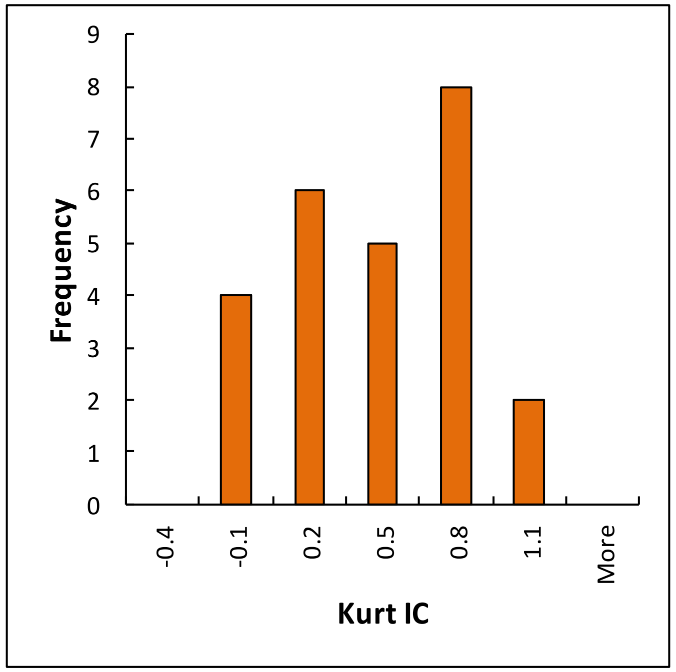
6. Statistical properties of proxy data: Ice cores





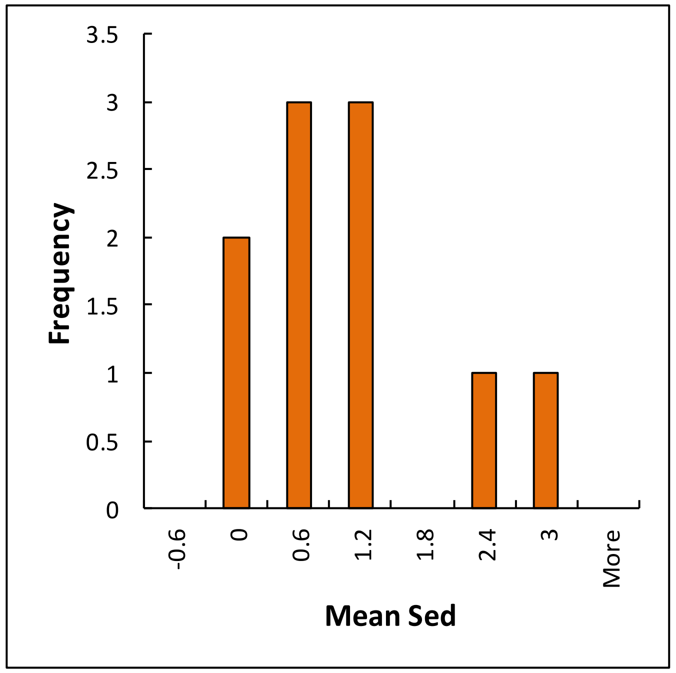


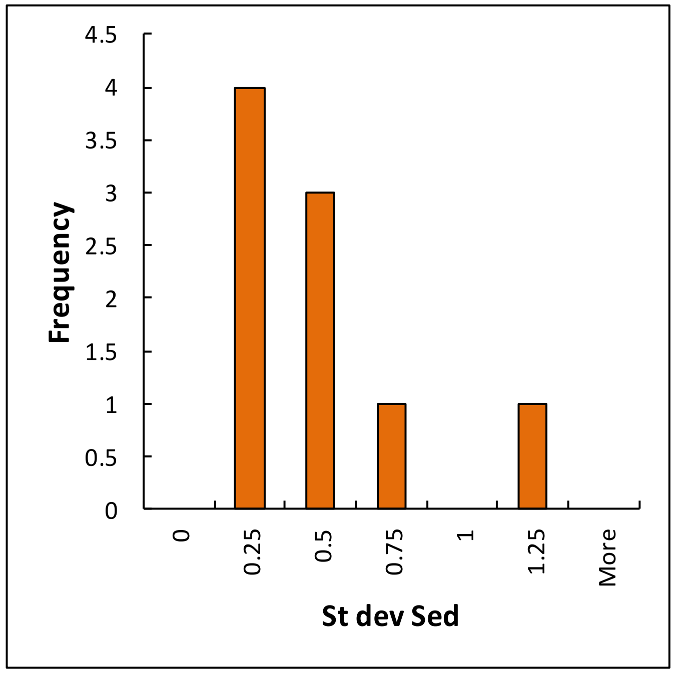


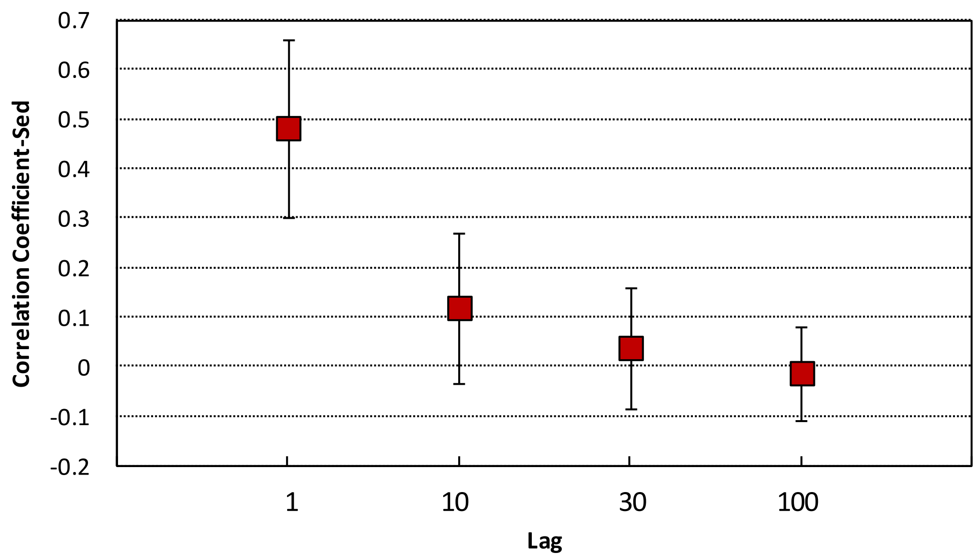


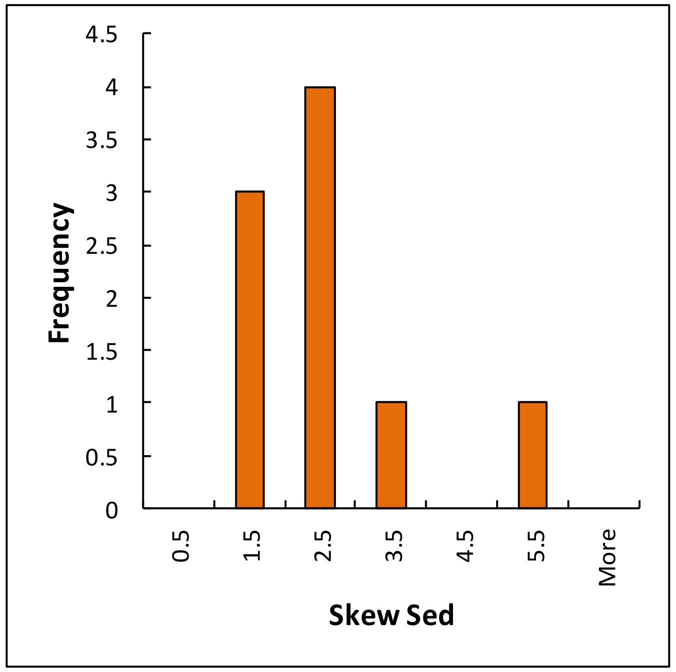
St dev: Standard Deviation
Skew: Skewness
Kurt: Kurtosis
IC: Ice Cores

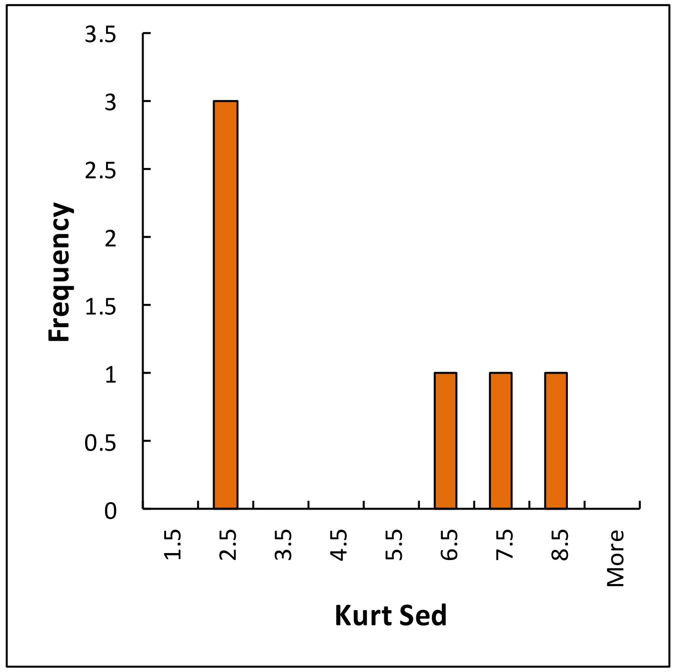
7. Statistical properties of proxy data: Sediments





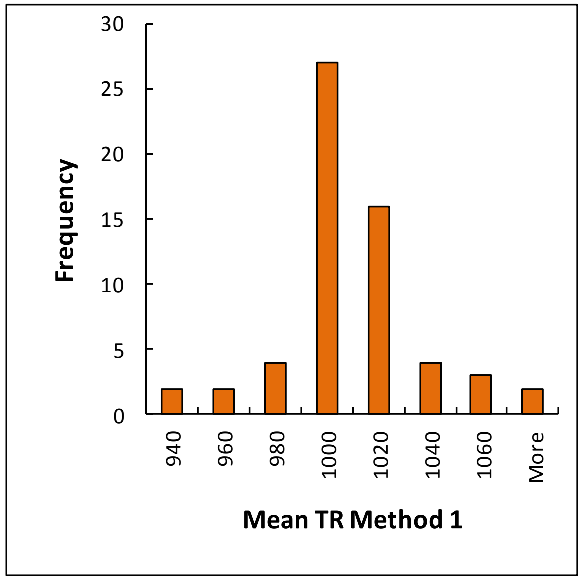


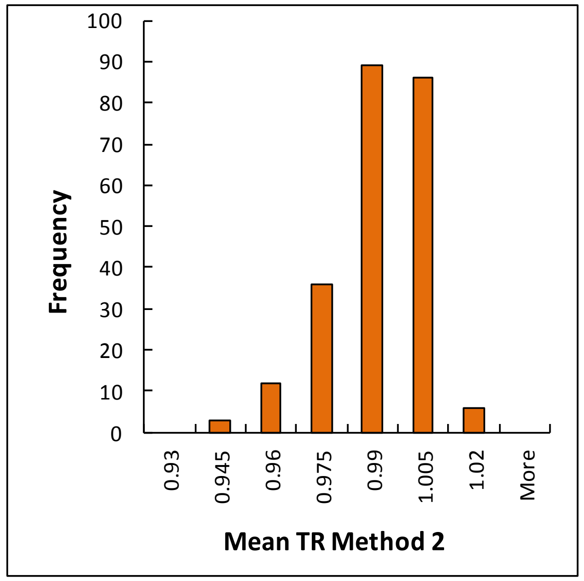


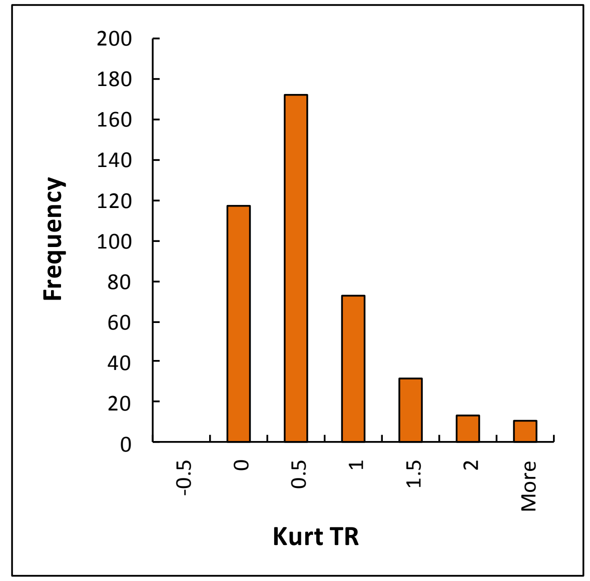


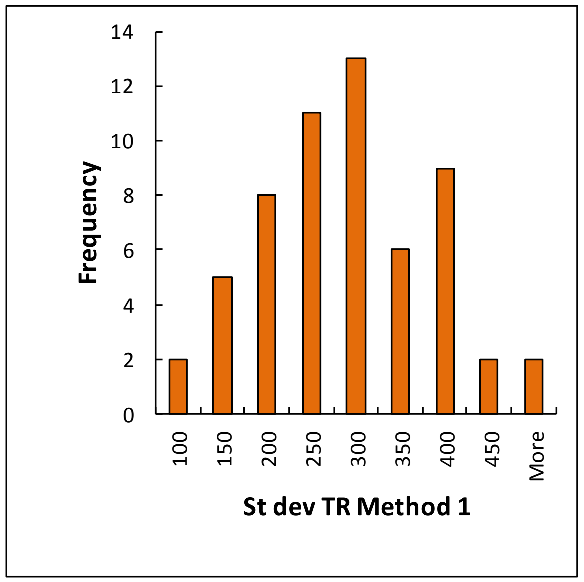
St dev: Standard Deviation
Skew: Skewness
Kurt: Kurtosis
Sed: Sediments

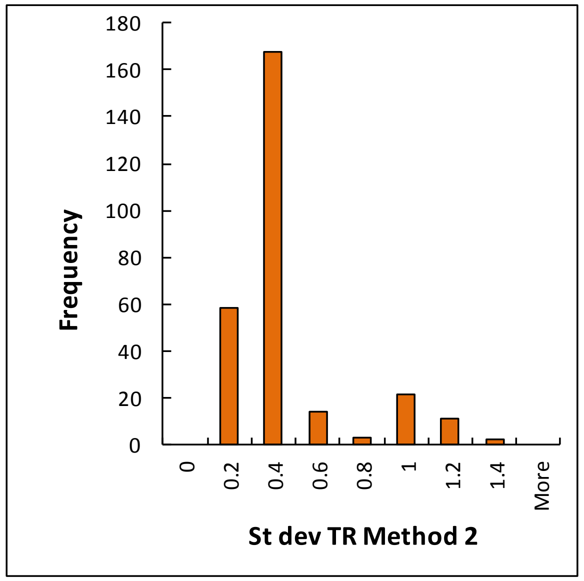
8. Statistical properties of proxy data: Tree rings

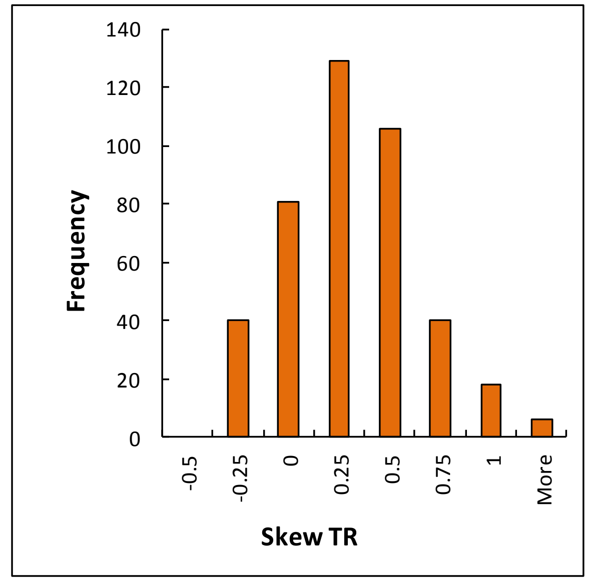


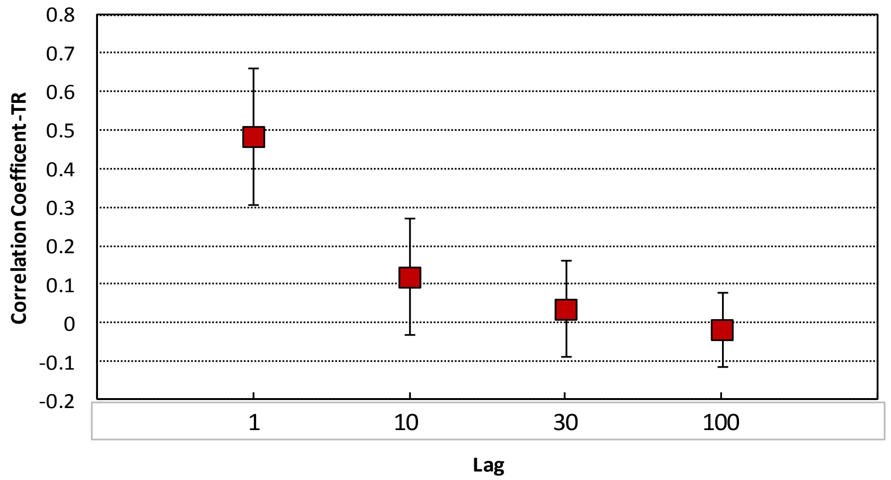








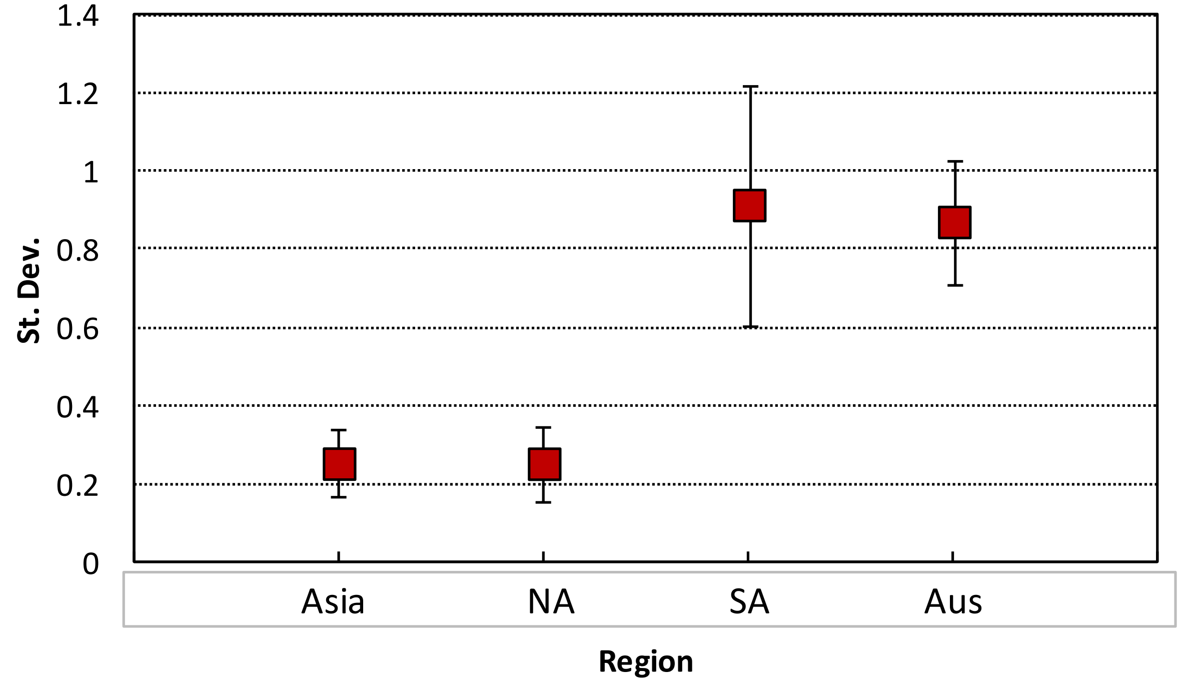


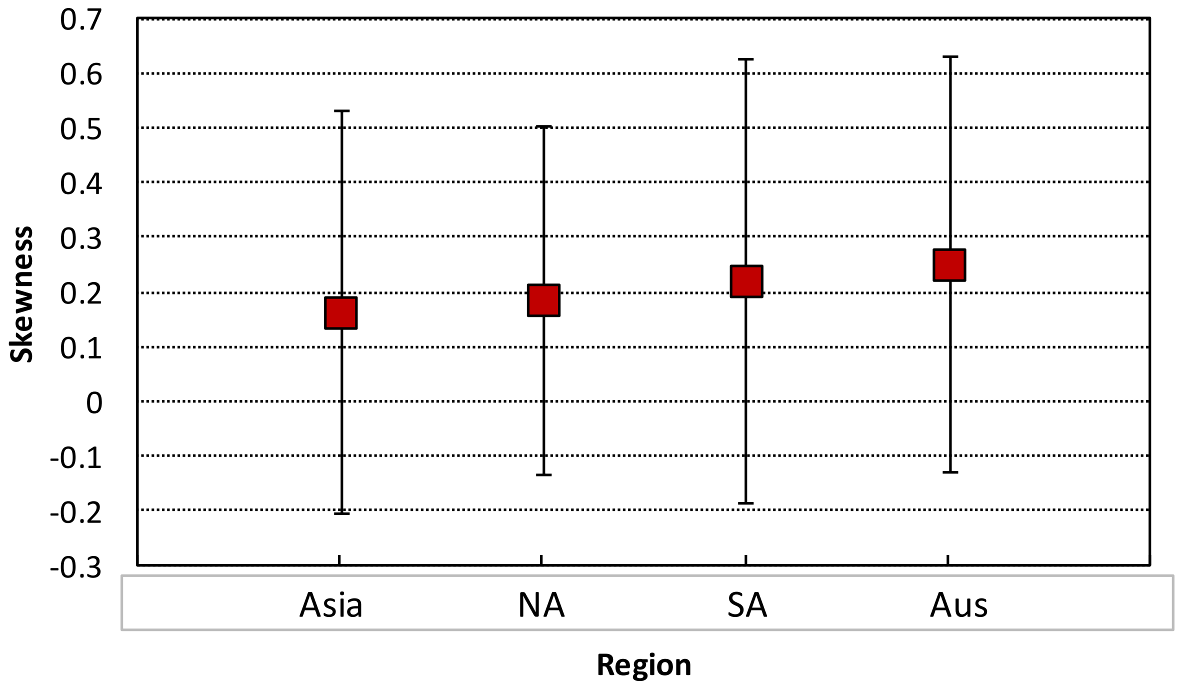


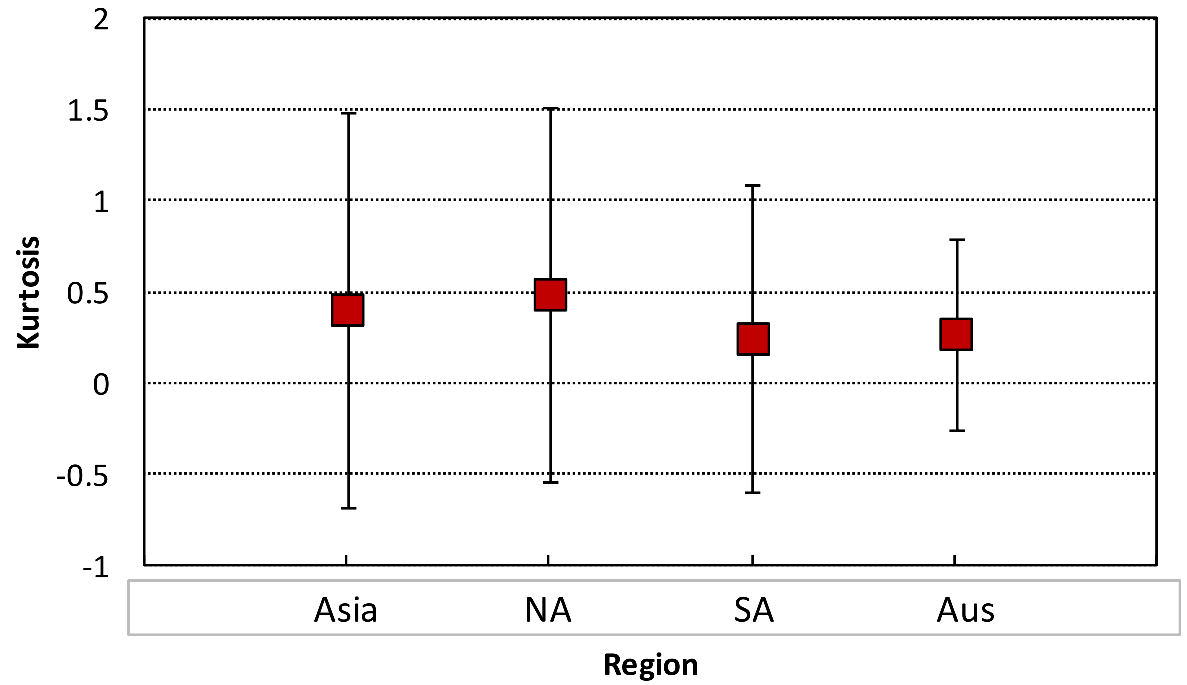
St dev: Standard Deviation
Skew: Skewness
Kurt: Kurtosis
TR: Tree Rings

9. Statistical properties of proxy data: Tree rings in space

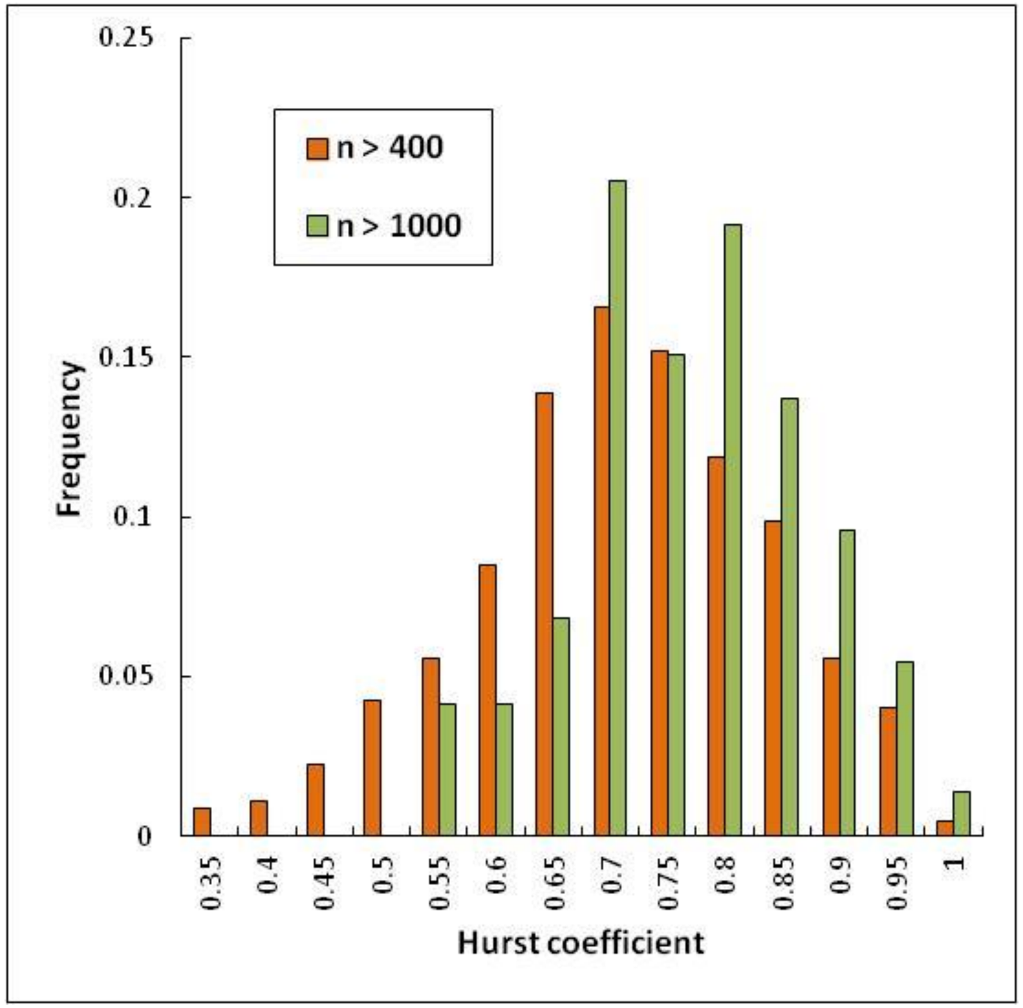
St dev: Standard Deviation
Skew: Skewness
Kurt: Kurtosis
NA: North America
SA: South America
Aus: Australia

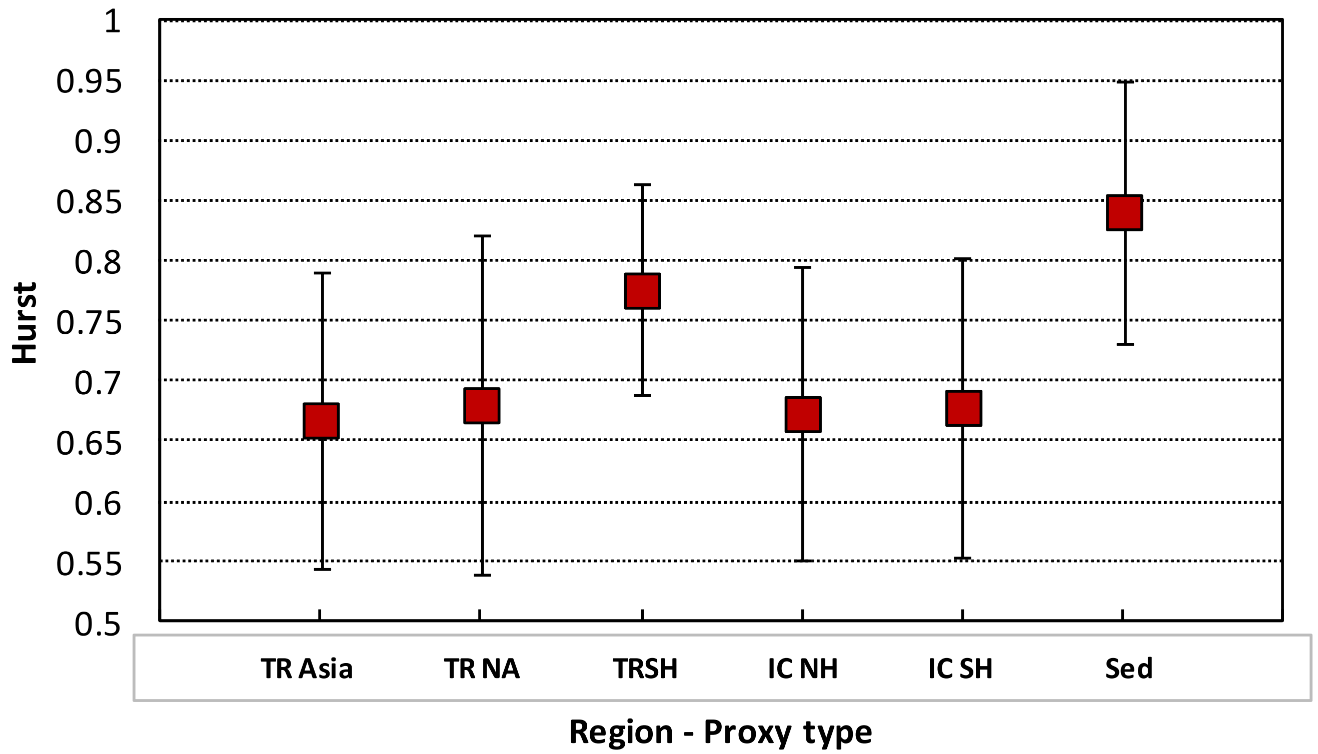


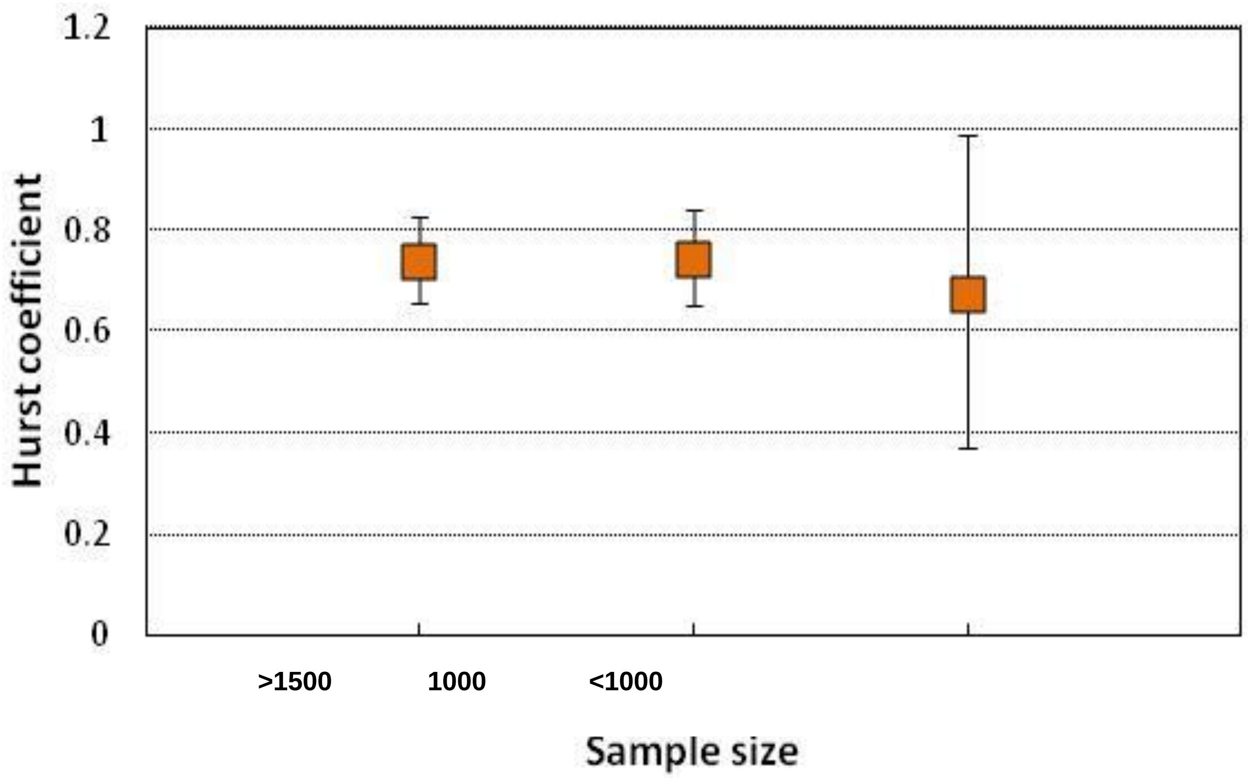




10. Hurst coefficient for proxy records

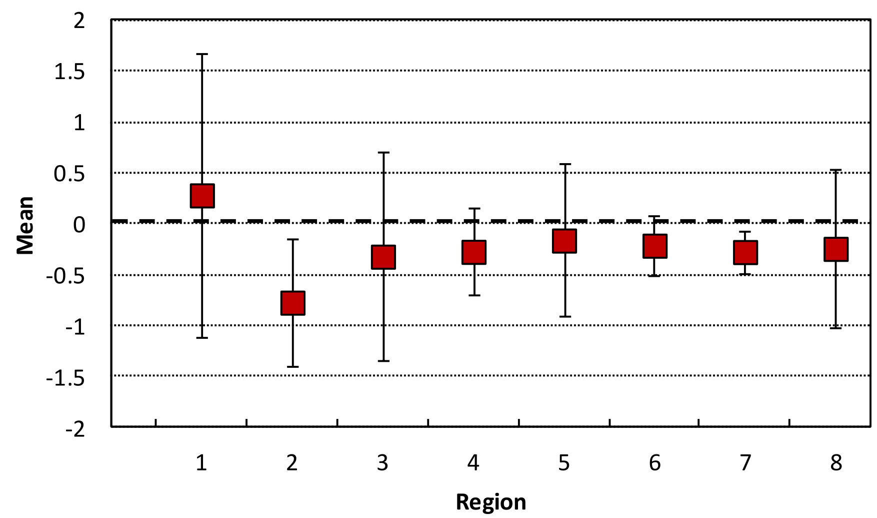


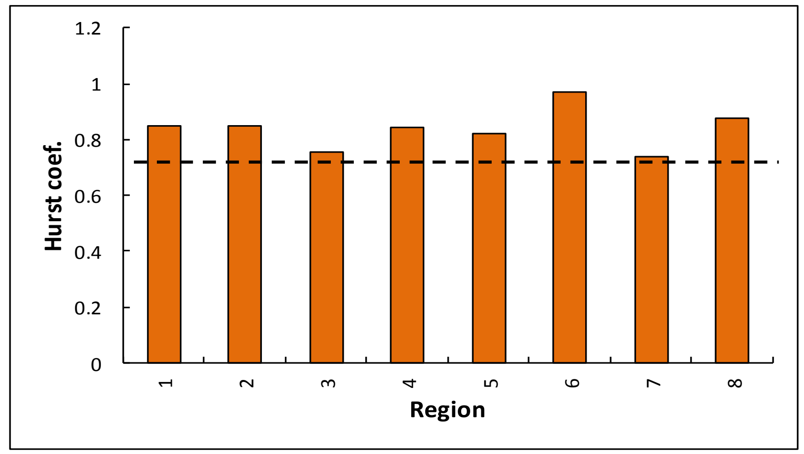


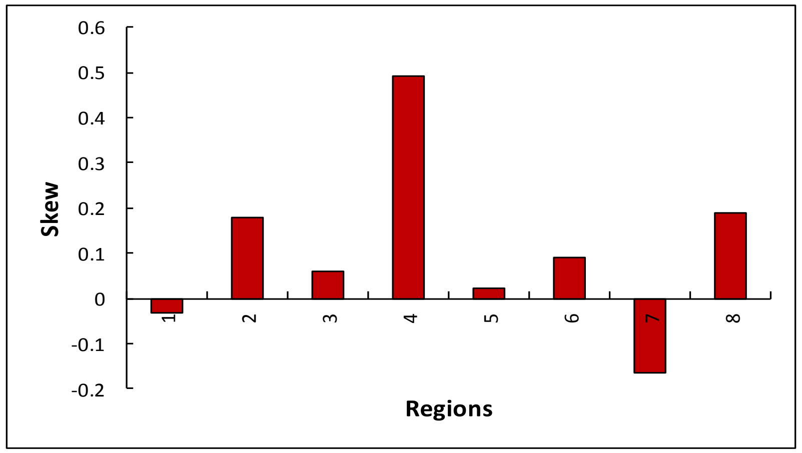


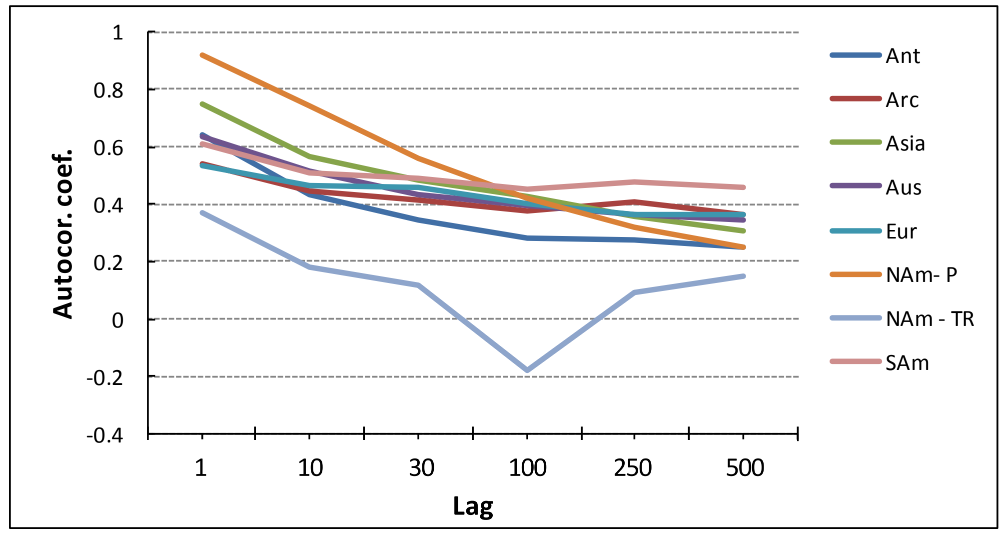
NA: North America
SA: South America
Aus: Australia
SH: South Hemisphere
NH: North Hemisphere
IC : Ice Cores
Sed: Sediments
TR: Tree rings

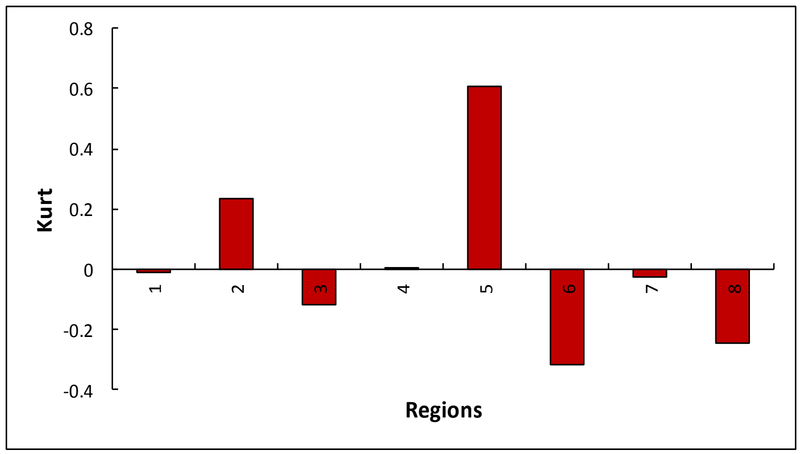
11. Statistical characteristics and Hurst coefficient for temperature reconstructions











St dev: Standard Deviation
Skew: Skewness
Kurt: Kurtosis
1: Antartica
2: Arctic
3: Asia
4: Australasia
5: Europe
6: North America
7: North America
8: South America

12. Conclusions

1. There are substantial differences in standard deviations of tree ring records between northern and southern hemisphere. This could be attributed to the smaller record length of the latter.
2. There is a steady decrease in the ACF of raw proxy data, which converses to zero near lag 30 years. Notably, the decrease in all reconstructed time series of temperature was milder, with convergence to $\rho = 0.2$ at lag 500 years.
3. HK dynamics is evident. The biased value for Hurst coefficient is estimated at 0.7 for 451 proxy records with sample size above 400.
4. For longer proxy records (sample size above 1500) the biased Hurst coefficient rises to the value of 0.77.
5. The mean value of the Hurst coefficient estimated for the temperature reconstructions manifests a higher value than for the corresponding proxy record.
6. Most regions manifest negative mean temperature differences with the corresponding mean of the 1960-1990 period, indicating a recent rise in the temperature near -0.26.

References

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