Efficient discretization in finite difference method

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MOTIVATION

The method of FD (FDM) is a plausible and simple method for solving partial differential equations. The standard practice is to use an orthogonal discretization to form algebraic approximate formulations of the derivatives of the unknown function and a grid, much like raster maps, to represent the properties of the function domain. Unfortunately, this simple approach to describe the topology comes along with the known disadvantages of the FDM (rough representation of the geometry). To overcome these disadvantages, Hunt (1983) has suggested an alternative approach to describe the topology including a) an index with the neighbour nodes of each node, and b) vector representation of the geometry of the boundaries. This enables graded meshes, which are capable of restricting grid refinement only in the areas of interest. This can result in increased computational speed, which is crucial for integrated hydrological modeling (Nalbantis et al., 2002).

DISCRETIZATION AND GROUNDWATER MODELS

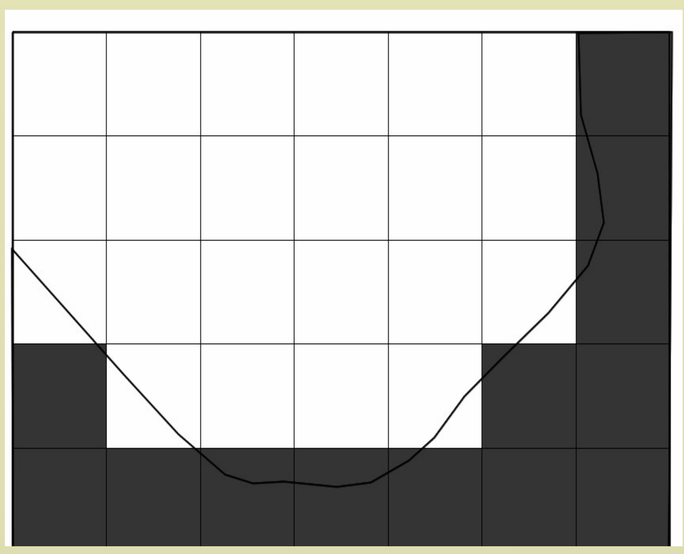
MODFLOW, grid refinements extend to the grid edges (all cells of the same row/column). Adequately dense discretization is required to properly represent the boundaries' geometry.

Hunt model supports localized refinement (see Figure 4.6 of Hunt (1983)) and provides accurate description of the boundaries' geometry.

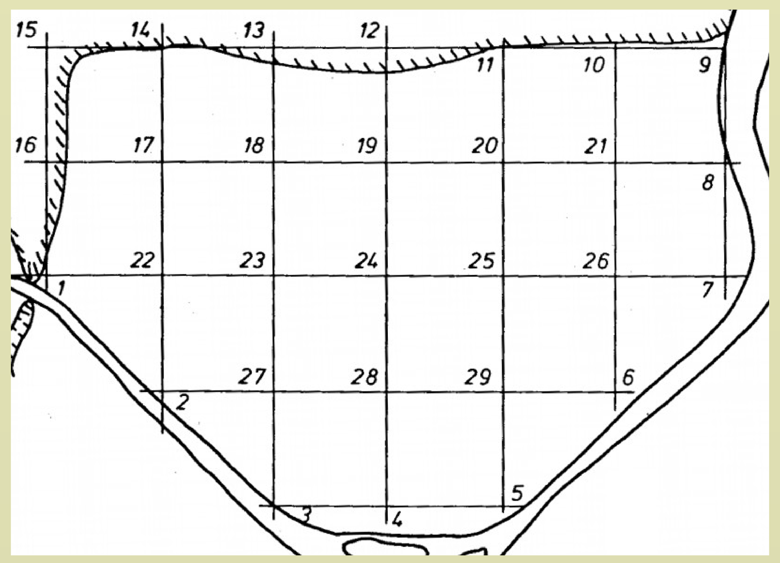
FVMSI is a simplification of the Narasimhan's and Witherspoon's (1976) concept of unstructured grids. It requires prior information concerning the flow domain and careful design of the discretization mesh, which should comply with two discretization conditions (Rozos and Koutsoyiannis, 2010, 2005).

CASE STUDY

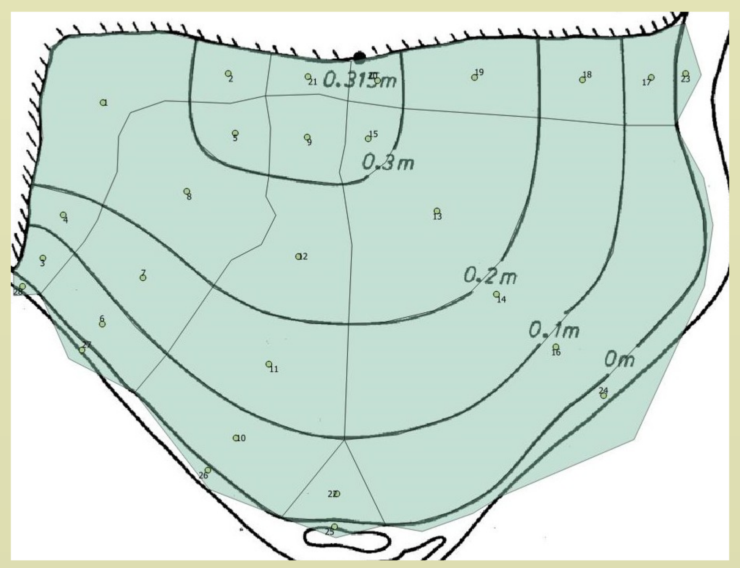
An alluvial plain is bounded on the north and west by impermeable embankments. The other boundaries are along rivers with constant elevation equal to 0. The aquifer is confined with homogeneous transmissivity equal to 0.01 m²/s and uniform recharge equal to 263 mm/month (from Hunt (1983) page 248).



MODFLOW 5×7



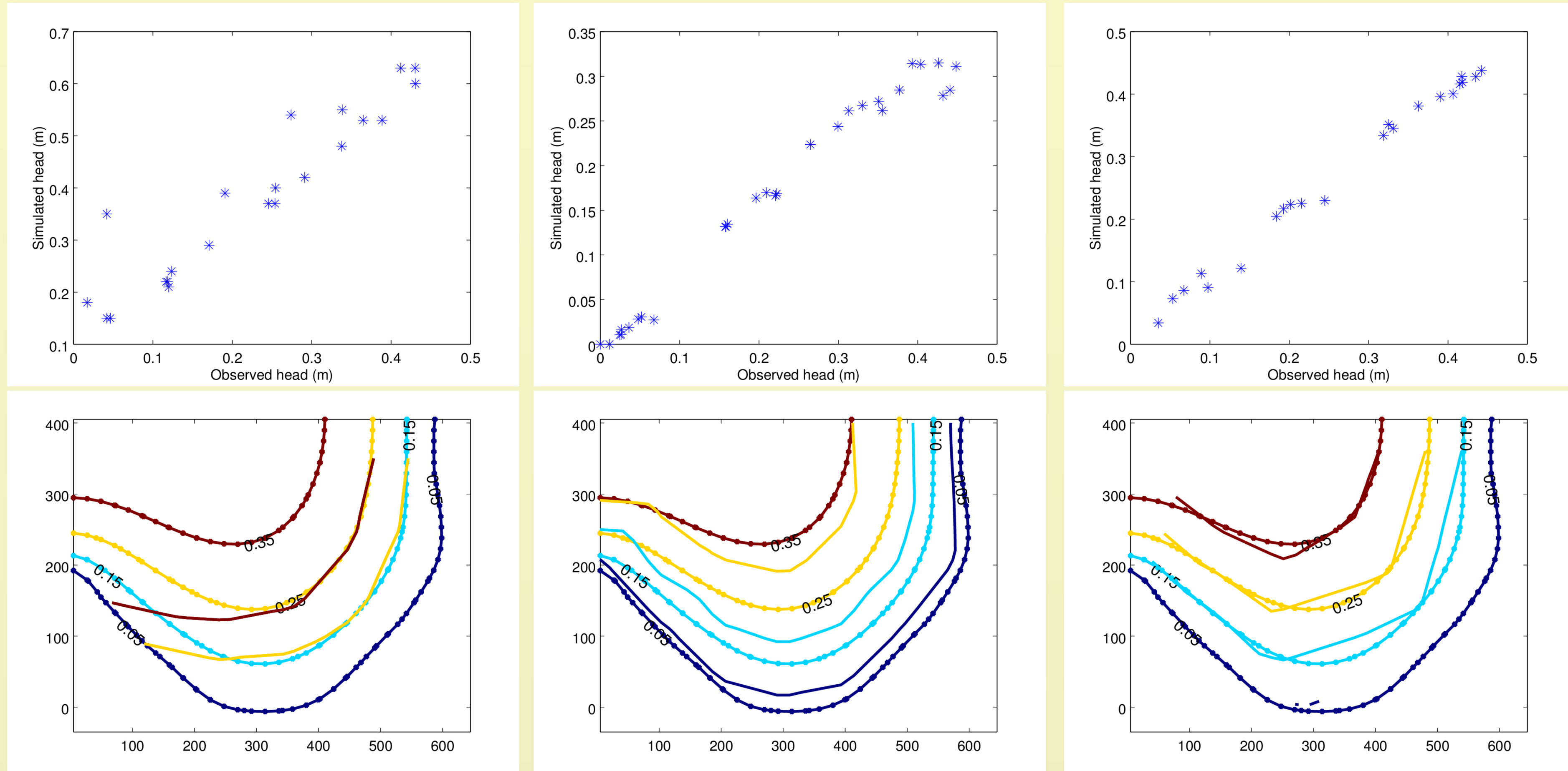
Hunt 4×6



FVMSI

RESULTS, DISCUSSION AND CONCLUSION

MODFLOW 5×7 | Hunt 4×6 | FVMSI



Virtual observations were prepared using MODFLOW with dense discretization (45×65). The performance of the three models was assessed using plots of simulated vs reference values and contour plots (reference contours with marked lines on the figures left).

MODFLOW 5×7 achieved the worst performance amongst the tested models (Nash–Sutcliffe -0.55). This happened because of the rough representation of the boundaries' geometry. Recently, MODFLOW-USG reintroduced the Narasimhan's and Witherspoon's (1976) concept of unstructured grids to overcome these issues (Panday et al., 2013). **Hunt** method achieved a quite decent performance (NS = 0.79) despite using a grid of only 4×6 cells. Finally, **FVMSI** achieved the best performance (NS = 0.99). However, such performances are not expected in cases of transient flows characterized by significant flow-geometry changes.

Conclusion: Hunt's method inherits the plausibility of FDM, but also allows flexible discretizations, similar to FEM, which helps to improve the modelling accuracy.

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