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Investigation on the stochastic nature of the solar radiation process

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Abstract

A detailed investigation of the variability of solar radiation can be proven useful towards more efficient and sustainable design of renewable resources systems. In this context, we analyze observations from Athens, Greece and we investigate the marginal distribution of the solar radiation process at a daily and hourly step, the long-term behavior based on the annual scale of the process, as well as the double periodicity (diurnal-seasonal) of the process. Finally, we apply a parsimonious double-cyclostationary stochastic model to generate hourly synthetic time series preserving the marginal statistical characteristics, the double periodicity and the dependence structure of the process.

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Keywords: synthetic hourly solar radiation; Kumaraswamy distribution; Hurst parameter; double periodicity

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1. Introduction

Several studies have been conducted to investigate the stochastic simulation of solar radiation for the purpose of renewable energy simulation and management. For example, in the analysis of [1] the Beta distribution is suggested

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for the modelling of hourly solar radiation recorded in Algiers. However, little research has been done in comparing different marginal distributions for the process of the hourly solar radiation. Here, we aim at investigating the marginal distribution for each month at an hourly step (24 hours for 12 months) fitting two of the most suitable distributions for this process. Preliminary analyses in a monthly scale (with a daily step) showed that popular distributions used in geophysics (such as Gamma, Pareto, Lognormal, Pearson etc.), that were fitted through the open-software Hydrognomon (hydrognomon.org), could not adequately fit the right tail of the empirical distribution. This can be explained considering that the solar irradiation process is left and right bounded. Although the left boundary is close to zero, the right boundary varies at a seasonal scale. Therefore, distributions like Gamma and Pareto, although they may exhibit a good fit (based on the Kolmogorov–Smirnov test), they should not be applied for the solar irradiation, since they are not right bounded.

After analysing both scales, hourly and daily, we conclude that the Kumaraswamy distribution [2] describes adequately well the observed distributions of diurnal and monthly solar irradiation and also exhibits certain technical advantages in model building and simulation, as discussed in Section 5.

2. Data

The study area is located in Athens, Greece. We analyze more than 12 year of hourly time series of solar irradiance, that is equivalent to more than 102,920 hours (Fig. 1a) and daily data spanning more than 25 years (Fig. 1b). Hourly data are obtained from the Hydrological Observatory of Athens (http://hoa.ntua.gr/) and daily data from the NASA SSE -Surface meteorology Solar Energy- (http://www.soda-pro.com/web-services/radiation/nasa-sse). From the 288 hourly time series (24 hours × 12 months) we only consider the 170 time series of records of good quality and with a mean solar radiation much larger than zero, i.e. excluding night hours.

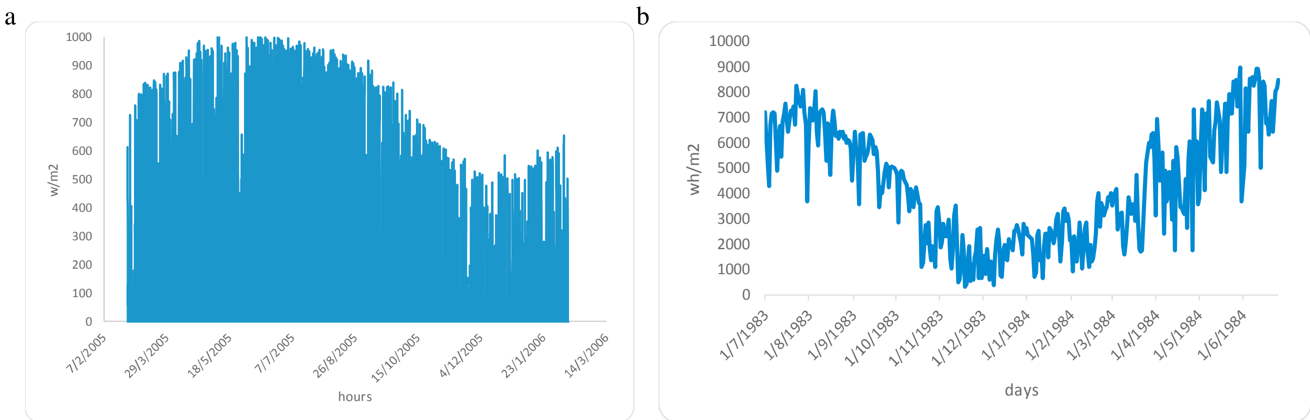

Fig.1. (a) One year of hourly time series of solar irradiance (Athens); (b) One year of daily time series of solar irradiation (Athens).

3. Marginal distribution

3.1 Double periodicity

One of the most common characteristic of atmospheric processes, such as the solar radiation process (Fig. 2), is the double periodicity, i.e., the diurnal and seasonal variation of the process. Therefore, for a robust generation of a synthetic time series we have to analyze the hourly and monthly statistical characteristic of solar radiation (such as the double periodic statistical mean and standard deviation).

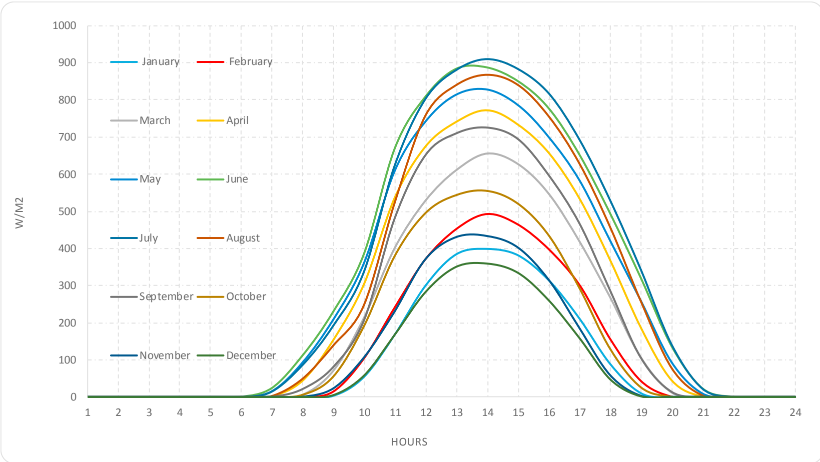

Fig.2. Solar radiation (in W/m²) for each month and daily hour (Athens).

3.2 Kumaraswamy and Beta distributions

The Kumaraswamy distribution is a probability distribution suitable for double bounded random processes. It is very familiar to the Beta distribution, allowing us to generate a large variety of probability distribution shapes of processes. Moreover, the Kumaraswamy distribution, has the advantage of an invertible closed form of the cumulative distribution function [3], as shown in Eqn. 1. Therefore, using Kumaraswamy rather than Beta for simulation purposes may be proven less computationally intensive.

In particular, the Kumaraswamy cumulative density function can be expressed as:

$$f(x;a,b) = \int_0^x f(\xi;a,b)d\xi = 1-(1-z^{\alpha})^{\beta} \quad (1)$$

where $z \in [0,1]$ is standardized according to $z = \frac{z-z_{min}}{z_{max}-z_{min}}$, with z_{min} and z_{max} are the minimum and maximum values of the empirical time series.

Also, the Beta Cumulative density function is given by:

$$f(z;a,b) = \frac{B(z;a,b)}{B(a,b)} \quad (2)$$

where $B(z;a,b)$ is the incomplete Beta function $B(z;a,b) = \int_0^x t^{a-1}(1-t)^{b-1}\,\mathrm{d}t$ and $B(a,b)$ is the Beta function $B(a,b) = \int_0^1 t^{a-1}\,(1-t)^{b-1}\mathrm{d}t$.

3.3 Comparison between the Kumaraswamy and Beta distributions for the monthly scale at the daily and hourly step

In order to examine whether the marginal distribution of solar irradiance can be adequately fitted from the Kumaraswamy or the Beta distribution, we apply three tests of goodness of fit. Also, we employ one model selection criterion, i.e., the Akaike information criterion [4] which is a function of the number of parameters in the model and the resulting log-likelihood value. However, since the Kumaraswamy and the Beta distributions have only two parameters, the above test compares only the likelihood value of each distribution. For the goodness of fit we use the Kolmogorov-Smirnov [5,6], Cramer von Misses [7] and the Anderson Darling tests [8]. Computations are carried out in the R statistical environment [14].

After applying all the tests, we conclude that at the monthly scale (using time series of daily irradiation), for the AIC test (which does not provide information on the goodness of fit of the model) the Kumaraswamy distribution performs better than the Beta distribution. Note that, we adopt the suggestion of [9] that for a difference below 2 points between the AIC values of the two models, both models have good support. Considering the latter, our results show that the Kumaraswamy distribution is always selected by the test contrariwise to the Beta distribution. However, for three time series (for April, May and December) both distributions are selected by the test (Fig. 3a).

For the goodness of fit, we set a 5% confidence level. According to the Kolmogorov-Smirnov test, Crammer von Misses and the Anderson & Darling test, the Kumaraswamy distribution is rejected in fewer months than the Beta distribution (Fig. 3b). Nevertheless, all hypothesis tests reject both the Kumaraswamy and the Beta distributions for the summer months (see the difference between Fig. 3c and 3d). For the hourly step, according to AIC criterion, the Kumaraswamy distribution is again preferred to the Beta distribution. However, based on the goodness of fit tests, the Kumaraswamy and the Beta distributions are not rejected only for the 44 out of the 170 time series (mostly at the midday hours). This inability of both distributions to adequately fit mainly the hours with the potential highest solar radiation within the day (e.g., 15:00 during the summer months), may be due to the variability induced by the clearness index process K_T (a measure of the ratio of measured irradiation in a locale relative to the extraterrestrial irradiation calculated at the given locale i.e. for $K_T \rightarrow 1$: atmosphere is clear and for $K_T \rightarrow 0$: atmosphere is cloudy), which highly affects the behaviour of the marginal distribution.

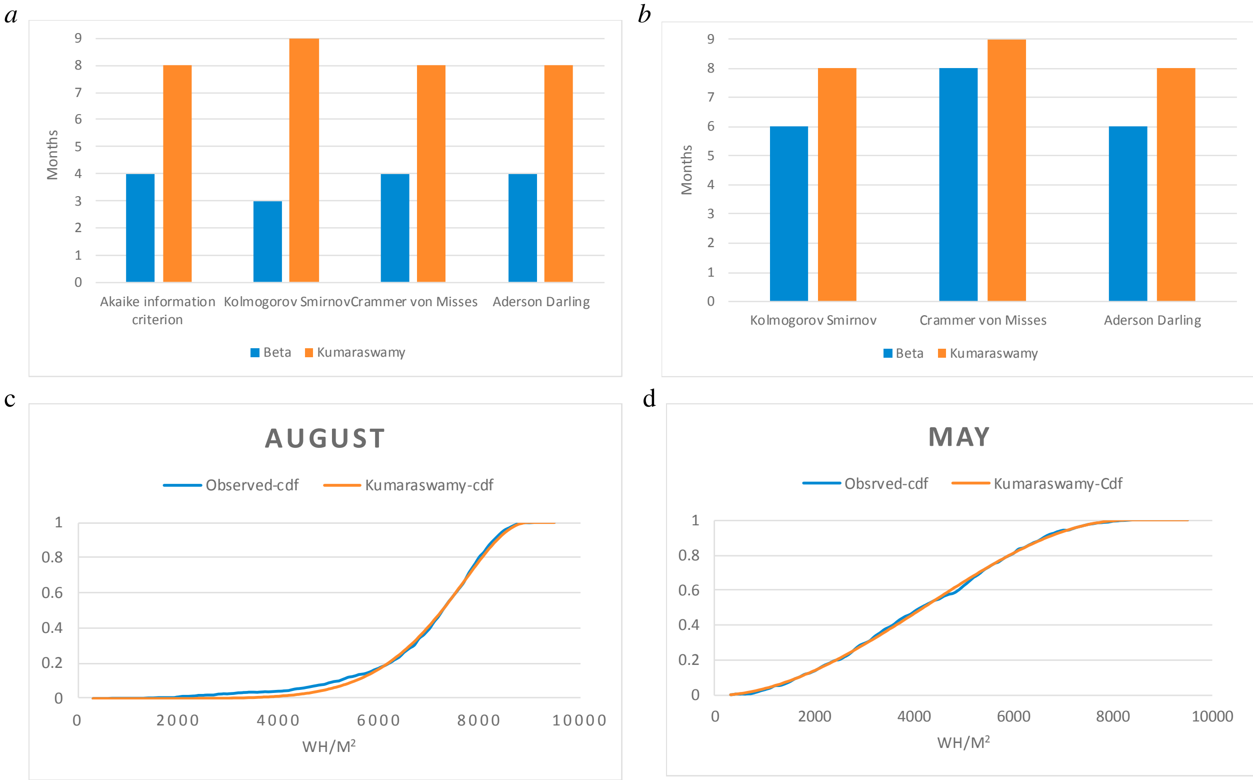

Fig.3. (a) Selected model for marginal distribution of the monthly scale based on the AIC, KS, CvM and AD tests; (b) results from the goodness of fit of the monthly scale of the marginal distribution base on the KS, CvM and AD tests; (c) plot of the Kumaraswamy distribution drawn from the August time series which is rejected from the above tests; (d) plot of the Kumaraswamy distribution from a non-rejected time series in May.

3.4 Kumaraswamy distribution parameters for the monthly scale in a daily step.

We estimate the parameters of the Kumaraswamy distribution applying four methods, i.e., the maximum likelihood, the L-moments and two least square methods (one based on quartiles and the other on the cumulative distribution) (Fig 4.a, b).

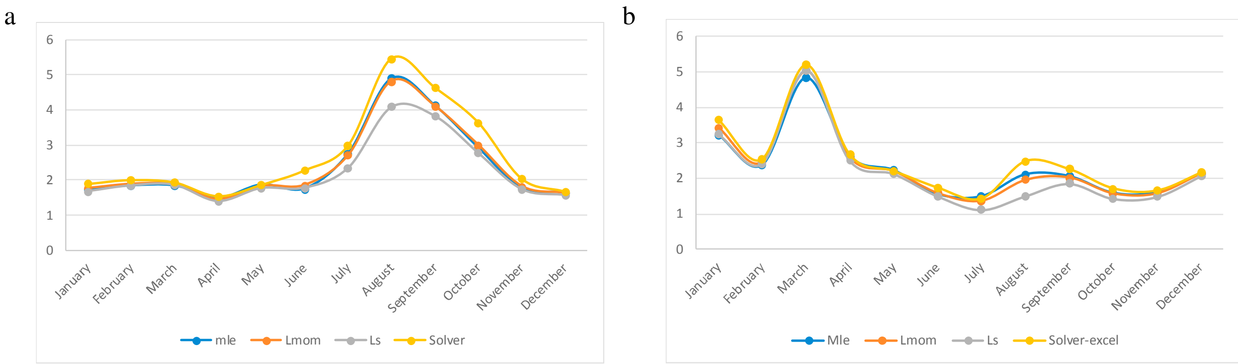

Fig. 4. (a) Plot of Kumaraswamy's *a* parameter calculated from four different methods; (b) plot of Kumaraswamy's {b} parameter calculated from four different.

4. Dependence structure at the hourly scale

For the estimation of the dependence structure of the process, we analyze a time series of more than 17 years of hourly solar radiation values. To quantify the persistence behavior of the process, we estimate the Hurst parameter (H = 0.83) via the climacogram introduced in [10] (i.e. plot of standard deviation σ (k) vs. averaging scale k), as shown in Fig. 5. We justify the use of climacogram to estimate the stochastic structure of the process instead of the commonly used autocorrelation functions and power spectrums as explained in [11], where it is shown that the climacogram has always a smaller statistical uncertainty from the other tools for common processes such as Markov and Hurst-Kolmogorov (HK). Since $H > 0.5$, we conclude that the examined process follows a HK behavior and the annual solar radiation is strongly correlated and cannot be considered as a white noise process ($H = 0.5$).

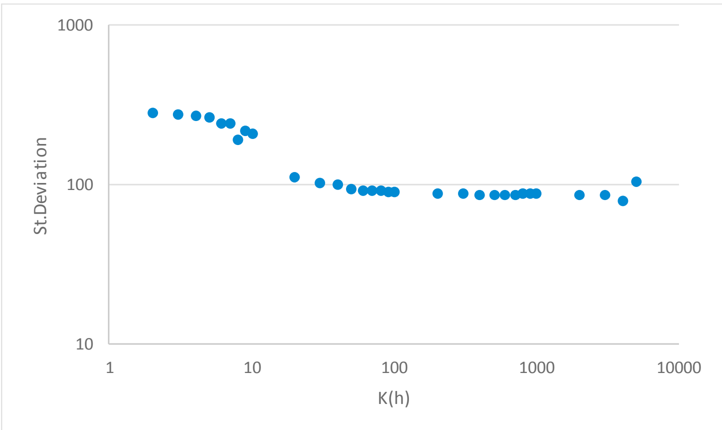

Fig. 5. Standardized climacogram (i.e., standard deviation of the scaled process).

5. Generation of synthetic time series

In this section, we apply a double periodic model, to generate synthetic hourly solar irradiance time series. The mean hourly synthetic time series is produced using the methodology of [12] suitable for double cyclostationary processes such as the ones examined in this study. Particularly, this methodology preserves the double periodicity (i.e., diurnal and seasonal) of a process through the hourly-monthly marginal distributions, including intermittent characteristics such as probability of zero values (i.e. during night time), as well as the dependence structure of the process through the climacogram. For the dependence structure, we apply an HK model based on the empirical climacogram of the solar irradiance as estimated in the previous section. Finally, for the generation scheme we use the CSAR algorithm (Cyclostationary Sum of finite independent AR(1) processes, [13]) capable of generating any length of time series following an HK, or various other processes, and with arbitrary distributions of each internal stationary process of the double cyclostationary process. In Figures 6 we compare the synthetic time series with the observed one.

a b

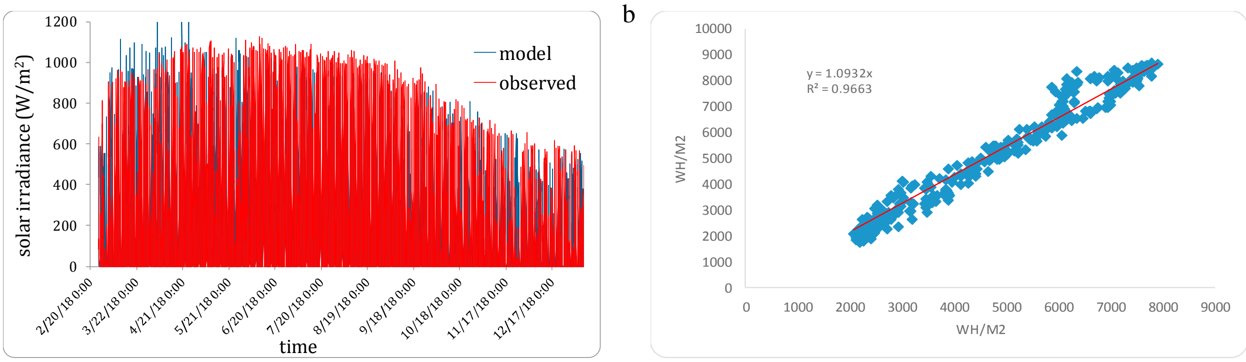

Fig.6. (a) Synthetic and observed hourly synthetic time series of daily irradiation; (b) Yearly average of daily observed and synthetic values.

6. Conclusions

In this study, we investigate the statistical properties of the solar radiation process at a monthly scale for both a daily and an hourly step. Regarding the marginal distribution, we conclude that the Kumaraswamy distribution can adequately describe the (daily step) monthly solar radiation and is generally preferred to the Beta distribution based on the three proposed tests of goodness of fit and on one model selection criterion. However, further research needs to be conducted in order to investigate the impact of the clearness index to the hourly process. Also, we calculate the parameters of the marginal distribution of the monthly solar radiation in Athens according to four different statistical methods (maximum likelihood, L-moments and two least square methods). An important result is that, solar radiation is found to exhibit a strong Hurst-Kolmogorov behaviour since the Hurst parameter is estimated as high as 0.83, that implies high correlation between successive years. Finally, we present a double periodicity model for generating hourly solar radiation time series which reproduces exceptionally well all the above statistical characteristics of the examined process.

Acknowledgment

The statistical analyses were performed in the R statistical environment [14] by also using the contributed packages VGAM [15], fitdistrplus [16], goftest [17] and lmomco [18]

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