

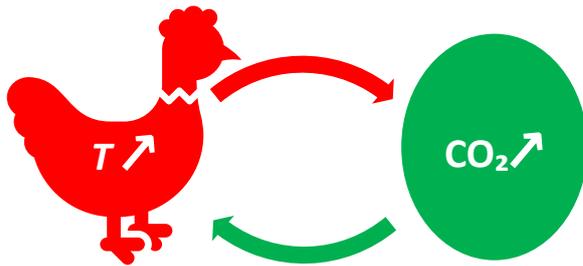
1 Hen-or-egg causality: Atmospheric CO₂ and temperature

2 Demetris Koutsoyiannis¹ and Zbigniew W. Kundzewicz²

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7 Graphical abstract



8 Highlights

- 9 Relation of atmospheric CO₂ and temperature can be seen as a “hen-or-egg” problem.
- 10 Both causality directions exist, but the direction ($T \rightarrow \text{CO}_2$) prevails over ($\text{CO}_2 \rightarrow T$).
- 11 Changes in CO₂ concentration are found to follow changes in global temperature.
- 12 We interpret this postulating a biochemical mechanism in a positive feedback loop.

14 Abstract

15 Relationships between atmospheric concentration of carbon dioxide and the global temperature
16 are widely recognized. It is a common knowledge that increasing CO₂ concentration plays the
17 major role in enhancement of the greenhouse effect and contributes to global warming. The
18 purpose of this study is to complement the conventional and established theory that increased
19 CO₂ concentration due to human emissions cause increase of temperature, by considering the
20 reverse causality. Since increased temperature causes increase in CO₂ concentration, relations of
21 atmospheric CO₂ and temperature may qualify into the category of “hen-or-egg” problems,
22 where it is not always clear which of two interrelated events is the cause and which the effect.
23 We examine the relationship of global temperature and atmospheric carbon dioxide
24 concentration at the monthly time step, covering the time interval 1980-2019, in which reliable
25 instrumental measurements are available. The results of our study support the hypothesis that
26 both causality directions exist, with the direction ($T \rightarrow \text{CO}_2$) being the dominant. Changes in CO₂
27 follow changes in T by about six months on monthly scale, or about one year on annual scale. We
28 attempt to interpret this mechanism by noting the possibility of a positive feedback loop
29 involving biochemical reactions, as soil respiration leads to increasing CO₂ emission at higher
30 temperatures.

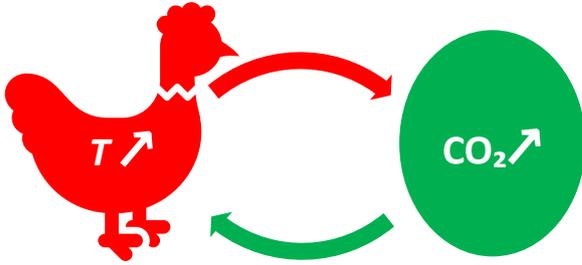
33 **Keywords** temperature; global warming; greenhouse gases; atmospheric CO₂ concentration

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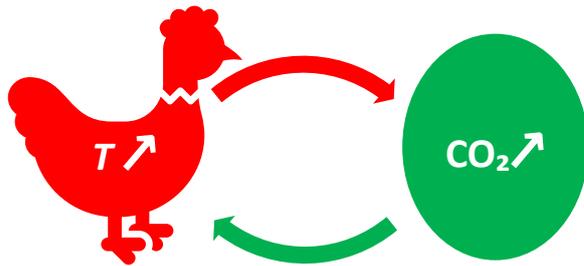
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31 temperatures.

32

33 **Keywords** temperature; global warming; greenhouse gases; atmospheric CO₂ concentration

34 *Πότερον ἡ ὄρνις πρότερον ἢ τὸ ᾠόν ἐγένετο* (Which of the two came first, the hen or the egg?)

35 Πλούταρχος, Ηθικά, Συμποσιακά Β, Πρόβλημα Γ (Plutarch, Moralia, Quaestiones convivales, B, Question III)

36 1 Introduction

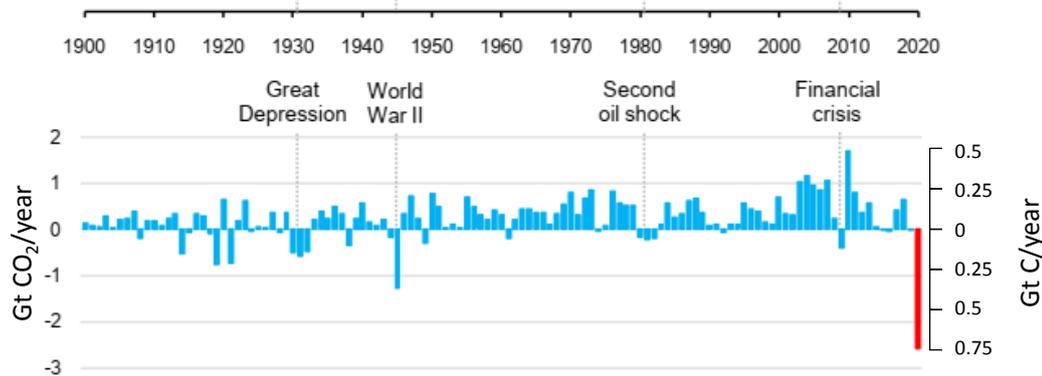
37 The phrase “hen-or-egg” (also known as “chicken-or-egg” or “bird-or-egg”) is a metaphor
38 describing situations where it is not clear which of two interrelated events or processes is the
39 cause and which the effect. Plutarch was the first to pose this type of causality as a philosophical
40 problem using the example of the hen and the egg, as indicated in the motto above.*

41 The objective of the paper is to demonstrate that relations of atmospheric CO₂ and
42 temperature may qualify into the category of “hen-or-egg” problems, where it is not always clear
43 which of two interrelated events is the cause and which the effect. First, we discuss the
44 relationships between temperature and CO₂ concentration and specifically intriguing results
45 from proxy data-based palaeoclimatic study, where change in temperature leads and change in
46 CO₂ concentration follows. Next, we discuss the data bases of modern (instrumental)
47 measurements, related to global temperature and atmospheric CO₂ concentration and introduce
48 a methodology to analyse them. We develop a stochastic framework, introducing useful notions
49 of time irreversibility and system causality. In the results section, we examine the relationship of
50 global temperature and carbon dioxide concentration using the modern data, available at the
51 monthly time step. We juxtapose time series of global temperature and atmospheric CO₂
52 concentration from several sources, covering the common time interval 1980-2019. In our
53 methodology, it is the timing, rather than the magnitude, of changes that is important, being the
54 determinant of causality. When examining time series of global records, we found situations
55 when change in temperature leads and change in CO₂ concentration follows. Interpretation of
56 cross-correlation of time series of global temperature and atmospheric CO₂ supports the “hen-
57 or-egg” hypothesis, indicating that both causality directions exist, while ($T \rightarrow \text{CO}_2$) dominates.
58 We attempt to interpret this mechanism by noting the positive feedback loop—higher
59 temperatures increase soil respiration and, hence, CO₂ emission.

60 The analysis reported in this paper was prompted by observation of an unexpected (and
61 unfortunate) real-world experiment: during the Covid-19 lockdown in 2020, despite
62 unprecedented decrease in carbon emissions demonstrated in various sources (example given
63 in Figure 1), there was increase in atmospheric CO₂ concentration, which followed a pattern
64 similar to previous years (Figure 2). Indeed, global CO₂ emissions were over 5% lower in the
65 first quarter of 2020 than in that of 2019, mainly due to an 8% decline in emissions from coal,
66 4.5% from oil and 2.3% from natural gas (IEA, 2020). However, the normal pattern of
67 atmospheric CO₂ concentration (increase until May and decrease in June) did not change. Similar
68 was the behaviour after the 2008-09 financial crisis, but the most recent situation is more
69 characteristic because the Covid-19 decline in 2020 is the severest ever, even from those in the

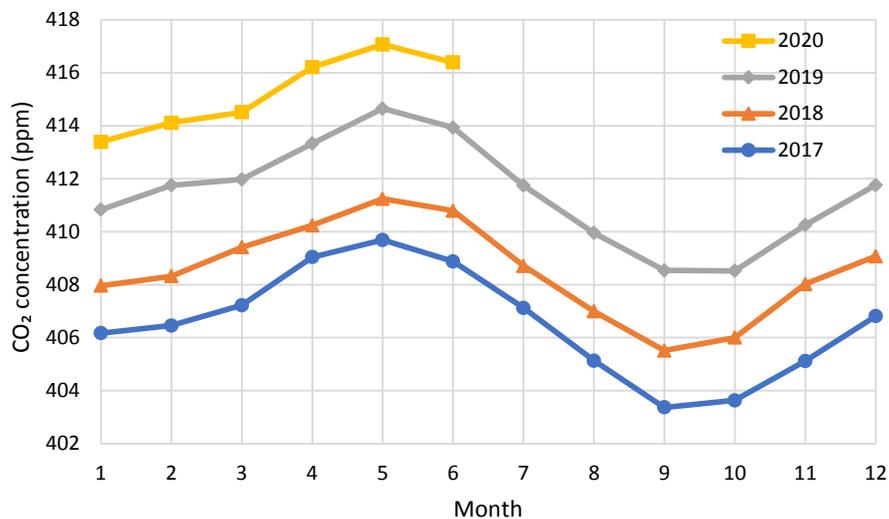
* We note that in the original Greek text “ἡ ὄρνις” is feminine (article and noun) meaning the hen, rather than the chicken. Therefore, here we preferred the form “hen-or-egg” over “chicken-or-egg”, which is more common in English. Very often, in online Greek texts (e.g. [https://el.wikisource.org/wiki/Συμποσιακά Β](https://el.wikisource.org/wiki/Συμποσιακά_Β)), “ἡ ὄρνις” appears as “ἡ ᾠρνις” (again feminine but with an ‘α’ rather than ‘ο’; this form was also reproduced in Koutsoyiannis, 2019). After extended search, we contend that this must be an error, either an old one in manuscript copying (e.g. by monks in monasteries) or a modern one (in OCR, as we met the same error in several other Greek texts). We are confident that the correct word is “ὄρνις”.

70 World Wars. It is also noteworthy in Figure 1 that there were three years in sequel without
 71 major increase in 2010s,* where again there was increase in CO₂ concentration.



72

73 **Figure 1.** Annual change in global energy-related CO₂ emissions (adapted from IEA, 2020)



74

75 **Figure 2.** Atmospheric CO₂ concentration measured in Mauna Loa, Hawaii, USA, in the last four years.

76 **2 Temperature and carbon dioxide – From Arrhenius to palaeo-**
 77 **proxies**

78 Does the relationship of atmospheric carbon dioxide (CO₂) and temperature classify as a “hen-
 79 or-egg” type causality? If we look at the first steps of studying the link between the two, the reply
 80 is clearly negative. Arrhenius (1896), the first scientist who proposed the causal relationship
 81 between atmospheric carbon dioxide concentration and temperature, regarded the changes of
 82 the latter as the cause and the changes of the former as the effect. Specifically, he stated:

83 *Conversations with my friend and colleague Professor Högbom together with the discussions*
 84 *above referred to, led me to make a preliminary estimate of the probable effect of a variation*
 85 *of the atmospheric carbonic acid [meant CO₂] on the temperature of the earth. As this*
 86 *estimation led to the belief that one might in this way probably find an explanation for*

* At first glance, this does not sound reasonable and therefore we have cross-checked the data with another source (Global Carbon Atlas, <http://www.globalcarbonatlas.org/en/CO2-emissions>; see also Our World In Data, <https://ourworldindata.org/grapher/annual-co-emissions-by-region>) and we found only slight differences. Interestingly, **Error! Reference source not found.** also shows a rapid growth in emissions after the 2008–09 global financial crisis, which agrees with Peters et al. (2012).

87 *temperature variations of 5°-10° C, I worked out the calculation more in detail and lay it now*
88 *before the public and the critics.*

89 Furthermore, following the Italian meteorologist De Marchi (1895), whom he cited, he rejected
90 what we call today *Milanković cycles* as possible causes of the glacial periods. In addition, he
91 substantially overestimated the role of CO₂ in the greenhouse effect of the Earth's atmosphere.
92 He calculated the relative weights of absorption of CO₂ and water vapour as 1.5 and 0.88,
93 respectively, a ratio of 1:0.6.

94 Arrhenius (1896) also stated that "*if the quantity of carbonic acid increases in geometric*
95 *progression, the augmentation of the temperature will increase nearly in arithmetic progression*".
96 This Arrhenius's "rule" (which is still in use today) is mathematically expressed as:

$$T - T_0 = \alpha \ln \left(\frac{[\text{CO}_2]}{[\text{CO}_2]_0} \right) \quad (1)$$

97 where T and $[\text{CO}_2]$ denote temperature and CO₂ concentration, respectively, T_0 and $[\text{CO}_2]_0$
98 represent reference states, and α is a constant.

99 However, while the fact that the two variables are tightly connected is beyond doubt, the
100 direction of the simple causal relationship needs to be studied further.

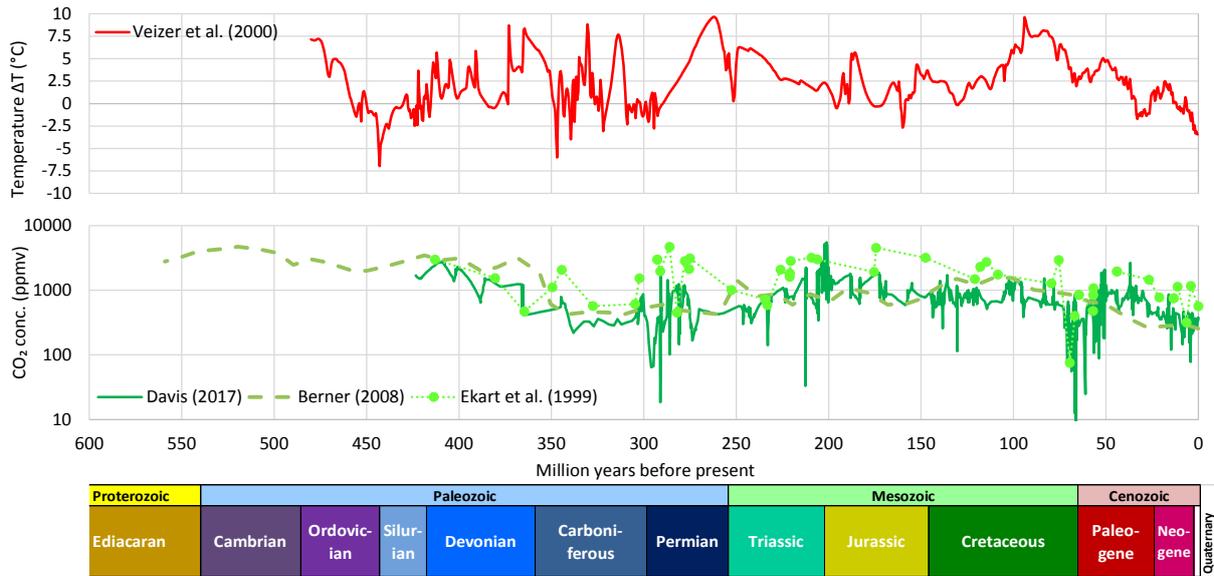
101 Today additional knowledge has been accumulated, particularly from palaeoclimatic
102 studies, which allow us to examine Arrhenius's hypotheses on a sounder basis. In brief, we can
103 state the following:

- 104 • Indeed, CO₂ plays a substantial role as a greenhouse gas. However, modern estimates of
105 the CO₂ contribution to the greenhouse effect largely differ from Arrhenius's results,
106 attributing 19% of the long-wave radiation absorption to CO₂ against 75% of water
107 vapour and clouds (Schmidt et al., 2010), a ratio of 1:4.
- 108 • During the Phanerozoic Eon, Earth's temperature has varied by even more than 5-10 °C,
109 which was postulated by Arrhenius (see Figure 3). The link of temperature and CO₂ is
110 beyond doubt, even though it is not clear in Figure 3. It becomes more legible in proxy
111 data of the Quaternary (see Figure 4). As seen in Figure 3, the CO₂ concentration has
112 varied by about two orders of magnitude.
- 113 • It has been demonstrated in a persuasive manner (Roe, 2006) that in the Quaternary it is
114 the effect of Milanković cycles (variations in eccentricity, axial tilt, and precession of
115 Earth's orbit), rather than of atmospheric CO₂ concentration, that explains the glaciation
116 process. Specifically (quoting Roe, 2006):

117 *variations in atmospheric CO₂ appear to lag the rate of change of global ice volume.*
118 *This implies only a secondary role for CO₂ —variations in which produce a weaker*
119 *radiative forcing than the orbitally-induced changes in summertime insolation— in*
120 *driving changes in global ice volume.*

121 Despite falsification of some of Arrhenius's hypotheses, his line of thought remained
122 dominant.

123



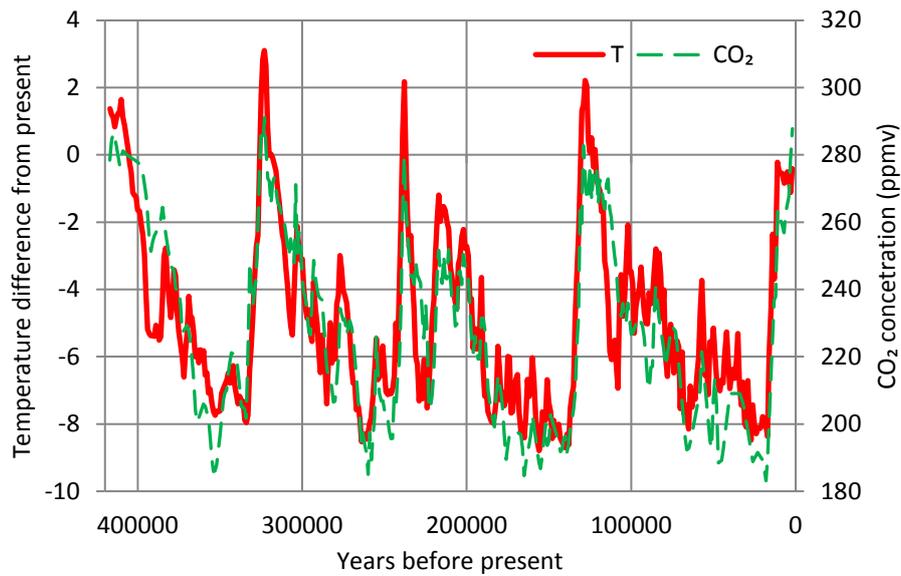
124

125 **Figure 3.** Proxy-based reconstructions of temperature, and CO₂ concentration during the Phanerozoic
 126 Eon. The original figures by Veizer et al. (2000), Davis (2017), Berner (2008) and Ekart et al. (1999) were
 127 digitized in this study. The chronologies of geologic eras shown in the bottom of the figure have been
 128 taken from the International Commission on Stratigraphy (<https://stratigraphy.org/chart>).

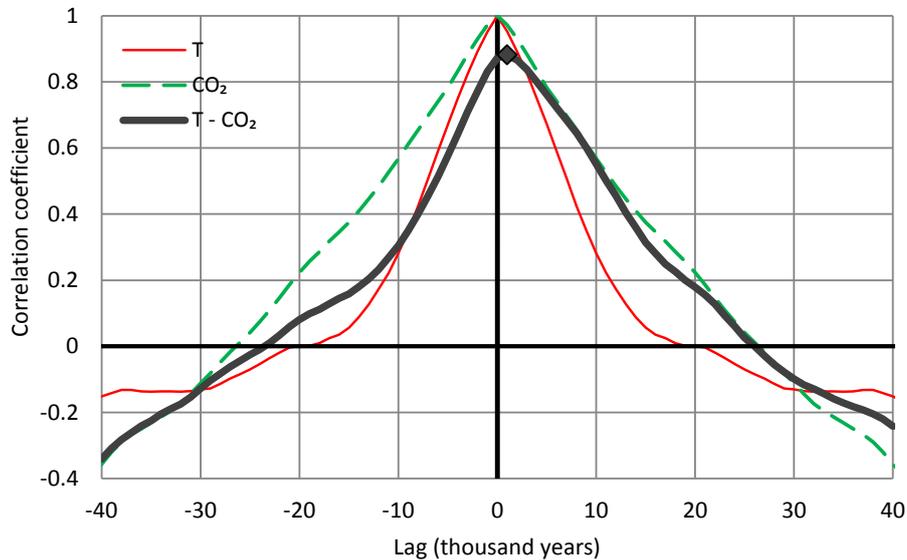
129 Yet there have been some important studies, based on palaeoclimatological
 130 reconstructions (mostly the Vostok ice cores, Jouzel et al., 1987; Petit et al. 1999), which have
 131 pointed to the opposite direction of causality, i.e. the change of temperature as the cause and
 132 that in the CO₂ concentration as the effect. Such claims have explained the fact that temperature
 133 change leads and CO₂ concentration change follows. In agreement with Roe (2006), several
 134 papers have found the time lag positive, with estimates varying from 50 to 1000 years,
 135 depending on the time period and the particular study (Caillon et al., 2003; Soon, 2007; Pedro et
 136 al., 2012; Koutsoyiannis, 2019; Chowdhry Beeman et al., 2019). Claims that CO₂ concentration
 137 leads (i.e., a negative lag) have not been generally made by these authors. At most a synchrony
 138 claim has been sought, on the basis that the estimated positive lags are often within the 95%
 139 uncertainty range (Chowdhry Beeman et al., 2019), while Pedro et al. (2012) has asserted that a
 140 “short lead of CO₂ over temperature cannot be excluded”.

141 Surprisingly however, to our knowledge, there have been no studies of this type (i.e. by
 142 exploring the wealth of existing data rather than fitting models), about the causal relation
 143 between temperature and CO₂, based on the rich body of modern datasets.

144 Since palaeoclimatic data suggest a direction opposite to that assumed by Arrhenius,
 145 Koutsoyiannis (2019), using palaeoclimatic data from the Vostok ice cores at a time resolution of
 146 1000 years and a stochastic framework similar to that of the present study (see section 3.2)
 147 concluded that change in temperature precedes that of CO₂ by one time step (1000 years), as
 148 illustrated in Figure 7. He also noted that this “causality condition holds for a wide range of time
 149 lags, up to 26 000 years, and hence the time lag is positive and most likely real.” He asserted that
 150 the problem is obviously more complex than that of exclusive roles of cause and effect,
 151 classifying it in the hen-or-egg causality problems. Obviously, however, the proxy character of
 152 these data and the too large time step of the time series reduce the reliability and accuracy of the
 153 results.



154



155

156 **Figure 4.** (Upper) Time series of temperature and CO₂ concentration from the Vostok ice core, covering
 157 part of the Quaternary (420 000 years) with time step of 1000 years. (Lower) Auto- and cross-
 158 correlograms of the two time series. The maximum value of the cross-correlation coefficient is 0.88 and
 159 appears at lag 1 thousand years (Adapted from Koutsoyiannis, 2019).

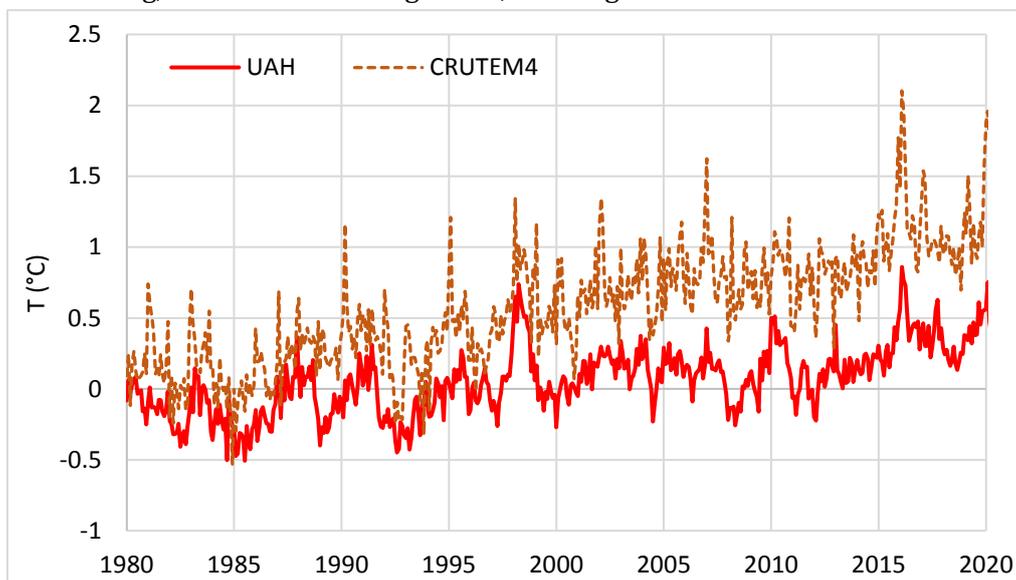
160 3 Data and methods

161 3.1 Data sets

162 Our investigation of the relationship of temperature and concentration of carbon dioxide in the
 163 atmosphere is based on two time series of the former process and four of the latter. Specifically,
 164 the temperature data are of two origins, satellite and ground based. The satellite dataset,
 165 developed at the University of Alabama in Huntsville (UAH), infers the temperature, T , of three
 166 broad levels of the atmosphere from satellite measurements of the oxygen radiance in the
 167 microwave band, using advanced (passive) microwave sounding units on NOAA and NASA
 168 satellites (Spencer and Christy, 1990; Christy et al., 2007). The data are publicly available on
 169 monthly scale in the forms of time series of “anomalies” (defined as differences from long-term
 170 means) for several parts of earth, as well as in maps. Here we use only the global average on
 171 monthly scale for the lowest level, referred to as the lower troposphere. The ground-based data

172 series we used is the CRUTEM.4.6.0.0 global T2m land temperature (Jones et al., 2012). This
173 originates from a gridded dataset of historical near-surface air temperature anomalies over land.
174 Data are available for each month from January 1850 to present. The dataset is a collaborative
175 product of the Met Office Hadley Centre and the Climatic Research Unit at the University of East
176 Anglia.

177 The two temperature series used in the study are depicted in Figure 5. They are consistent
178 to each other (and correlated, $r = 0.8$), yet the CRUTEM4 series shows a larger increasing trend
179 than the UAH series. The differences are explainable by three reasons: (a) the satellite series
180 includes both land and sea, while the ground based is for land only, in which the increasing trend
181 is substantially higher than in sea; the satellite series refers to some high altitude in the
182 troposphere (see Koutsoyiannis, 2020), while the ground-based series refers to the ground level;
183 and (c) the ground-based series is affected by urbanization (a lot of ground stations are located
184 in urban areas). In any case, the difference in the increasing trends is irrelevant for the current
185 study, as the timing, rather than the magnitude, of changes is the determinant of causality.



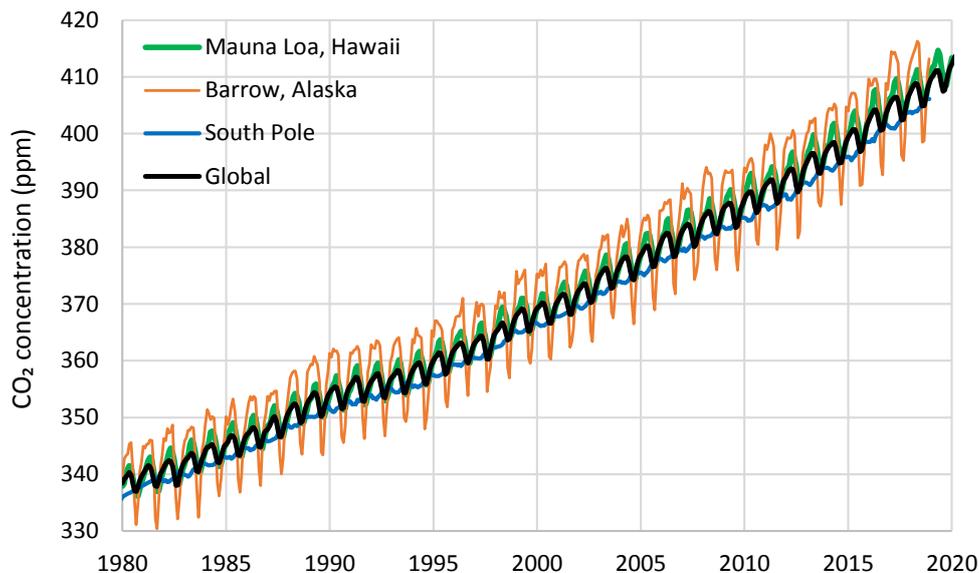
186
187 **Figure 5.** Plots of the data series of global temperature anomalies since 1980, as used in the study, from
188 satellite measurements over the globe (UAH) and from ground measurements over land (CRUTEM4).

189 The most famous CO₂ data set is that of Mauna Loa Observatory (Keeling et al., 1976). The
190 Observatory, located on the north flank of Mauna Loa Volcano, on the Big Island of Hawaii, USA,
191 at an elevation of 3397 m above sea level, is a premier atmospheric research facility that has
192 been continuously monitoring and collecting data related to the atmosphere since the 1950s.
193 The NOAA has also other stations that systematically measure atmospheric CO₂ concentration,
194 namely at Barrow, Alaska, USA and at South Pole. The NOAA's Global Monitoring Laboratory
195 Carbon Cycle Group also computes global mean surface values of CO₂ concentration using
196 measurements of weekly air samples from the Cooperative Global Air Sampling Network. The
197 global estimate is based on measurements from a subset of network sites. Only sites where
198 samples are predominantly of well-mixed marine boundary layer air, representative of a large
199 volume of the atmosphere, are considered (typically at remote marine sea level locations with
200 prevailing onshore winds). Measurements from sites at high altitude (such as Mauna Loa) and

201 from sites close to anthropogenic and natural sources and sinks are excluded from the global
202 estimate.*

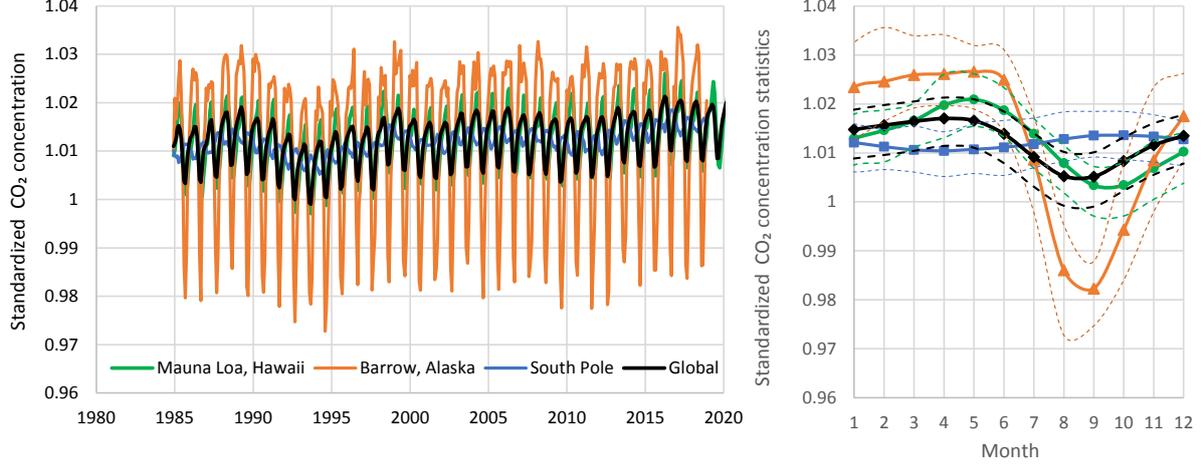
203 The period of data coverage varies, but all series cover the common 40-year period 1980-
204 2019, which hence constituted the time reference of all our analyses. As a slight exception, the
205 Barrow (Alaska) and South Pole measurements have not yet been available in final form for
206 2019 and, thus, this year was not included in our analyses of these two time series. The data of
207 the latter two stations are given in irregular-step time series, which was regularized to monthly
208 in this study. All other data series have already been available on monthly scale.

209 All four CO₂ time series used in the study are depicted in Figure 6. They show a
210 superposition of increasing trends and annual cycles whose amplitudes increase as we head
211 from the South to the North Pole. The South Pole series has opposite phase of oscillation
212 compared to the other three. The annual cycle is better seen in Figure 7, where we have
213 removed the trend with standardization, namely by dividing each monthly value by the
214 geometric average of the 5-year period before it. The reason why we used division rather than
215 subtraction and geometric rather than arithmetic average (being thus equivalent to subtracting
216 or averaging the logarithms of CO₂ concentration), will become evident in section 4. In the right
217 panel of Figure 7, which depicts monthly statistics of the time series of the left panel, it is seen
218 that in all sites but the South Pole the annual maximum occurs in May; that of the South Pole
219 occurs in September.



220
221 **Figure 6.** Plots of the data series of atmospheric CO₂ concentration measured in Mauna Loa (Hawaii, USA),
222 Barrow (Alaska, USA) and South Pole, and global average.

* Details about this data set are provided in https://www.esrl.noaa.gov/gmd/ccgg/about/global_means.html.



223

224 **Figure 7.** Plots of atmospheric CO₂ concentration after standardization: (Left) Each monthly value is
 225 standardized by dividing with the geometric average of the 5-year period before it. (Right) Monthly
 226 statistics of the values of the left panel; for each month the average is shown in continuous line and the
 227 minimum and maximum in thin dashed lines of the same colour as the average.

228 3.2 Stochastic framework

229 A recent study (Koutsoyiannis, 2019) has investigated time irreversibility in
 230 hydrometeorological processes and developed a theoretical framework in stochastic terms. It
 231 also studied necessary conditions for causality, which is tightly tied to time irreversibility. A
 232 simple definition of time reversibility within stochastics is the following, where underlined
 233 symbols denote stochastic (random) variables and non-underlined ones denote values thereof
 234 or regular variables.

235 A stochastic process $\underline{x}(t)$ at continuous time t , with n th order distribution function:

$$F(x_1, x_2, \dots, x_n; t_1, t_2, \dots, t_n) := P\{\underline{x}(t_1) \leq x_1, \underline{x}(t_2) \leq x_2, \dots, \underline{x}(t_n) \leq x_n\} \quad (2)$$

236 is time-symmetric or time-reversible if its joint distribution does not change after reflection of
 237 time about the origin, i.e., if for any n, t_1, t_2, \dots, t_n ,

$$F(x_1, x_2, \dots, x_n; t_1, t_2, \dots, t_n) = F(x_1, x_2, \dots, x_n; -t_1, -t_2, \dots, -t_n) \quad (3)$$

238 If times t_i are equidistant, i.e. $t_i - t_{i-1} = D$, the definition can be also written by reflecting the
 239 order of points in time, i.e.:

$$F(x_1, x_2, \dots, x_{n-1}, x_n; t_1, t_2, \dots, t_{n-1}, t_n) = F(x_1, x_2, \dots, x_{n-1}, x_n; t_n, t_{n-1}, \dots, t_2, t_1) \quad (4)$$

240 A process that is not time-reversible is called time-asymmetric, time-irreversible or time-
 241 directional. Important results related to time (ir)reversibility are the following:

- 242 • A time reversible process is also stationary (Lawrance, 1991).
- 243 • If a scalar process $\underline{x}(t)$ is Gaussian (i.e., all its finite dimensional distributions are
 244 multivariate normal) then it is reversible (Weiss, 1975). The consequences are: (a) a
 245 directional process cannot be Gaussian; (b) a discrete-time ARMA process (and a
 246 continuous-time Markov process) is reversible if and only if it is Gaussian.
- 247 • However, a vector (multivariate) process can be Gaussian and irreversible at the same
 248 time. A multivariate Gaussian linear process is reversible if and only if its autocovariance
 249 matrices are all symmetric (Tong and Zhang, 2005).

250 Time asymmetry of a process can be studied more conveniently (or even exclusively in a
251 scalar process) through the differenced process, i.e.:

$$\tilde{x}_{\tau,\nu} := x_{\tau+\nu} - x_{\tau} \quad (5)$$

252 for an appropriate time-step ν of differencing. The differenced process represents change of the
253 original process within a time period of length ν . We further define the cumulative process of \tilde{x}_{τ}
254 for discrete time κ as:

$$\underline{X}_{\kappa} := \underline{x}_1 + \underline{x}_2 + \dots + \underline{x}_{\kappa} \quad (6)$$

255 The time average of the original process \underline{x}_{τ} for discrete time scale κ is

$$\underline{x}_{\tau}^{(\kappa)} := \frac{\underline{x}_{(\tau-1)\kappa+1} + \underline{x}_{(\tau-1)\kappa+2} + \dots + \underline{x}_{\tau\kappa}}{\kappa} = \frac{\underline{X}_{\tau\kappa} - \underline{X}_{(\tau-1)\kappa}}{\kappa} \quad (7)$$

256 Similar equations for the cumulative and averaged processes for the differenced process $\tilde{x}_{\tau,\nu}$ are
257 given in Appendix A.

258 The variance of the process $\underline{x}_{\tau}^{(\kappa)}$ is a function of the time scale κ which is termed the
259 climacogram of the process:

$$\gamma_{\kappa} := \text{var} \left[\underline{x}_{\tau}^{(\kappa)} \right] \quad (8)$$

260 The autocovariance function for time lag η is derived from the climacogram through the
261 relationship (Koutsoyiannis, 2016):

$$c_{\eta} = \frac{(\eta + 1)^2 \gamma_{|\eta+1|} + (\eta - 1)^2 \gamma_{|\eta-1|}}{2} - \eta^2 \gamma_{|\eta|} \quad (9)$$

262 For sufficiently large κ (theoretically as $\kappa \rightarrow \infty$), we may approximate the climacogram as:

$$\gamma_{\kappa} \propto \kappa^{2H-2} \quad (10)$$

263 where H is termed the *Hurst parameter*. The theoretical validity of such (power-type) behaviour
264 of a process was implied by Kolmogorov (1940). The quantity $2H - 2$ is visualized as the slope of
265 the double logarithmic plot of the climacogram for large time scales. In a random process, $H =$
266 $1/2$, while in most natural processes $1/2 \leq H \leq 1$, as first observed by Hurst (1951). This natural
267 behaviour is known as (long-term) *persistence* or *Hurst-Komogorov (HK) dynamics*. A high value
268 of H (approaching 1) indicates enhanced presence of patterns, enhanced change and enhanced
269 uncertainty (e.g. in future predictions). A low value of H (approaching 0) indicates enhanced
270 fluctuation or *antipersistence* (sometimes misnamed as quasi-periodicity, as the period is not
271 constant).

272 For a stationary stochastic process \underline{x}_{τ} , the differenced process \tilde{x}_{τ} has mean zero and
273 variance:

$$\tilde{\gamma}_{\nu,1} := \text{var}[\tilde{x}_{\tau,\nu}] = \text{var}[x_{\tau+\nu}] + \text{var}[x_{\tau}] - 2 \text{cov}[x_{\tau+\nu}, x_{\tau}] = 2(\gamma_1 - c_{\nu}) \quad (11)$$

274 where γ_1 and c_{ν} are the variance and lag ν autocovariance, respectively, of \underline{x}_{τ} . Furthermore, as
275 demonstrated by Koutsoyiannis (2019), the Hurst coefficient of the differenced process \tilde{x}_{τ}
276 precisely equals zero, which means that \tilde{x}_{τ} is completely antipersistent, irrespective of γ_{κ} .

277 As the first moment (mean) of the differenced process is always zero (provided that the
278 original process is stationary), while the second one (variance) is always positive and thus it
279 does not provide indications on time asymmetry, in a scalar process the least-order moment that
280 can be used to detect reversibility is the third, $\mu_3[\tilde{x}_{\tau,\nu}]$, or equivalently, the skewness coefficient:

$$\tilde{C}_{S_v} := \frac{\mu_3[\tilde{x}_{t,v}]}{(\text{var}[\tilde{x}_{t,v}])^{3/2}} \quad (12)$$

281 Processes with large \tilde{C}_{S_v} signify high (positive) time irreversibility. Gaussian processes, in which
 282 the skewness is zero, are necessarily time symmetric, as already mentioned.

283 However, in vector processes, to study irreversibility we can use second order moments,
 284 and in particular cross-covariances among the different components of the vector. In particular
 285 (simplifying the analyses and results in Koutsoyiannis, 2019), given two processes \underline{x}_t and \underline{y}_t we
 286 could study the cross-correlations:

$$r_{\tilde{x}\tilde{y}}[v, \eta] = \text{corr}[\tilde{x}_{t,v}, \tilde{y}_{t+\eta,v}] \quad (13)$$

287 Time (ir)reversibility could then be characterized by studying the properties of symmetry or
 288 asymmetry of $r_{\tilde{x}\tilde{y}}(v, \eta)$ as a function of the time lag η . In a symmetric bivariate process, $r_{\tilde{x}\tilde{y}}[v, \eta]$
 289 = $r_{\tilde{x}\tilde{y}}[v, -\eta]$ and if the two components are positively correlated, the maximum of $r_{\tilde{x}\tilde{y}}[v, \eta]$ will
 290 appear at lag $\eta = 0$. If the bivariate process is irreversible, this maximum will appear at a
 291 lag $\eta_1 \neq 0$ and its value will be $r_{\tilde{x}\tilde{y}}[v, \eta_1]$.

292 Time asymmetry is closely related to causality, which presupposes irreversibility. Thus,
 293 “no causal process (i.e., such that of two consecutive phases, one is always the cause of the other)
 294 can be reversible” (Heller 1983; see also Kline 1980). In probabilistic definitions of causality,
 295 time asymmetry is determinant. Thus, Suppes (1970) defines causation as “An event B_t
 296 [occurring at time t] is a *prima facie* cause of the event A_t if and only if (i) $t' < t$, (ii) $P\{B_{t'}\} > 0$,
 297 (iii) $P(A_t|B_{t'}) > P(A_t)$.” Also, Granger’s (1980) first axiom in defining causality reads “The past
 298 and present may cause the future, but the future cannot.”

299 Consequently, in simple causal systems, in which the process component \underline{x}_t is the cause of
 300 \underline{y}_t (like in the case of rainfall (\underline{x}_t) and runoff (\underline{y}_t)), it is reasonable to expect $r_{\tilde{x}\tilde{y}}[v, \eta] \geq 0$ for any
 301 $\eta \geq 0$, while $r_{\tilde{x}\tilde{y}}[v, \eta] = 0$ for any $\eta = 0$. However, in hen-and-egg causal systems, this will not be
 302 the case and we reasonably expect $r_{\tilde{x}\tilde{y}}[v, \eta] \neq 0$ for any η . Yet, we can define a dominant
 303 direction of causality based on the cross-correlation maximizing time lag η_1 , formally defined for
 304 a specified v as:

$$\eta_1 := \arg \max |r_{\tilde{x}\tilde{y}}(v, \eta)| \quad (14)$$

305 We can thus distinguish the following three cases:

- 306 • If $\eta_1 = 0$ then there is no dominant direction.
- 307 • If $\eta_1 > 0$ then the dominant cause is \underline{x}_t .
- 308 • If $\eta_1 < 0$ then the dominant cause is \underline{y}_t .

309 Further explanations are provided in Appendix B. It must be stressed that the above conditions
 310 are put as necessary and not sufficient conditions for a causative relationship between the
 311 processes \underline{x}_t and \underline{y}_t . Following Koutsoyiannis (2019), we avoid seeking sufficient conditions, a
 312 task that would be too difficult or impossible due to its deep philosophical complications.
 313 Additional necessary conditions can be found in Koutsoyiannis (2019).

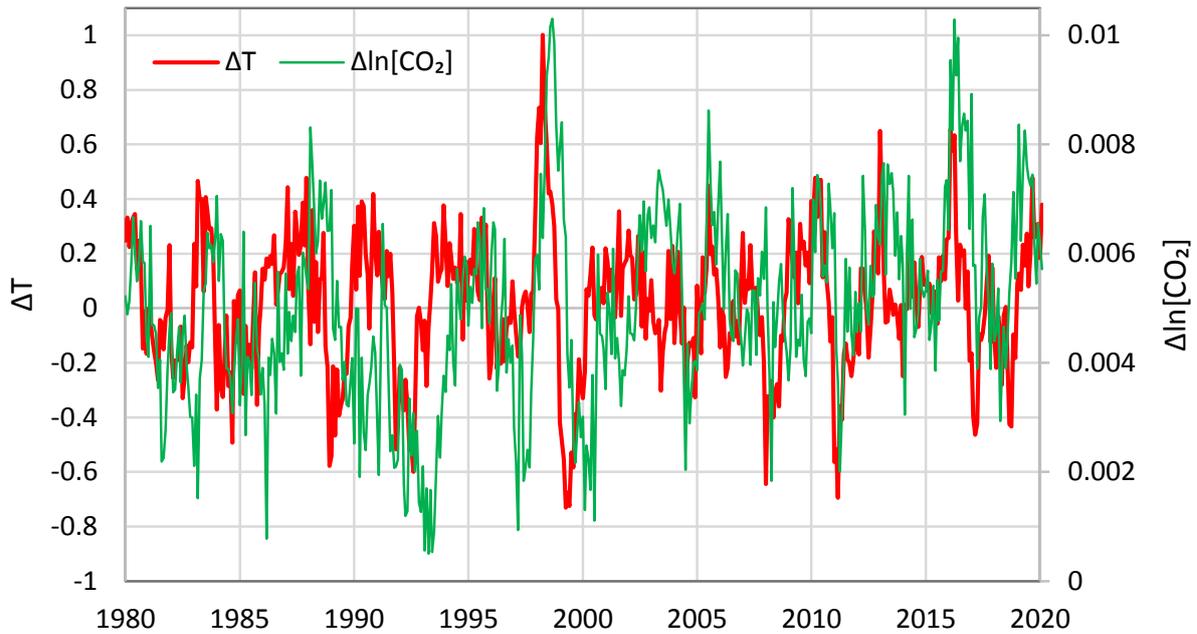
314 4 Results

315 Here we examine the relationship of atmospheric temperature and carbon dioxide concentration
 316 using the modern data (observations rather than proxy), available at the monthly time step, as

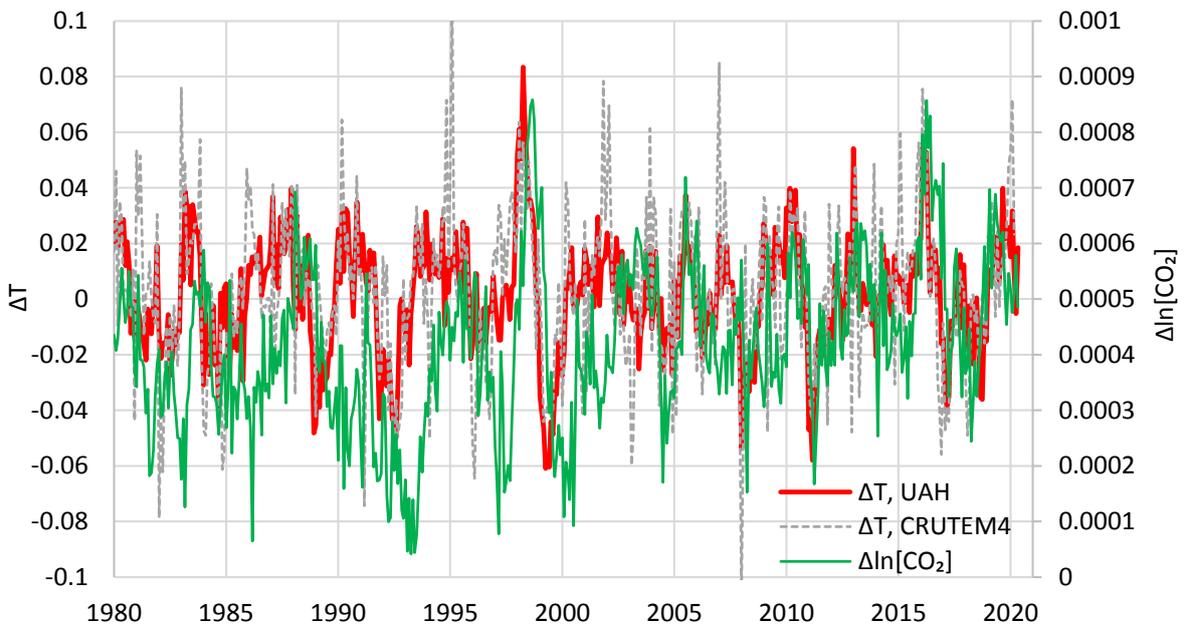
317 described in section 3. To apply our stochastic framework, we must first make the two time
318 series linearly compatible. Specifically, based on Arrhenius's rule (equation (1)), we take the
319 logarithms of CO₂ concentration, while we keep T untransformed. We then construct the
320 differenced processes, which quantify changes. Taking differences is physically meaningful as
321 both CO₂ concentration and temperature (equivalent to thermal energy) represent "stocks", i.e.
322 stored quantities, and, thus, indeed the mass and energy fluxes are represented by differences.

323 The time step of differencing was chosen equal to one year ($\nu = 12$ for the monthly time
324 step of the time series). For instance, from the value of January of a certain year we subtract the
325 value of January of the previous year and so forth. A first reason for this choice is that it almost
326 eliminates the effect of the annual cycle (periodicity). A second reason is that the temperature
327 data are given in terms of "anomalies", i.e., differences from an average which varies from month
328 to month. By taking $\nu = 12$, the varying means are eliminated and "anomalies" are effectively
329 replaced by the actual processes (as the differences in the actual values equal the differences of
330 "anomalies").

331 We perform all analyses on monthly and annual time scales. Figure 8 shows the
332 differenced time series for the UAH temperature and Mauna Loa CO₂ concentration at monthly
333 scale; the symbols ΔT and $\Delta \ln[\text{CO}_2]$ are used interchangeably with $\tilde{x}_{\tau,12}$ and $\tilde{y}_{\tau,12}$, respectively. It
334 may be observed that most often the temperature curve leads and that of CO₂ follows. However,
335 there are cases where the changes in the two processes synchronize in time or even become
336 decoupled.



337

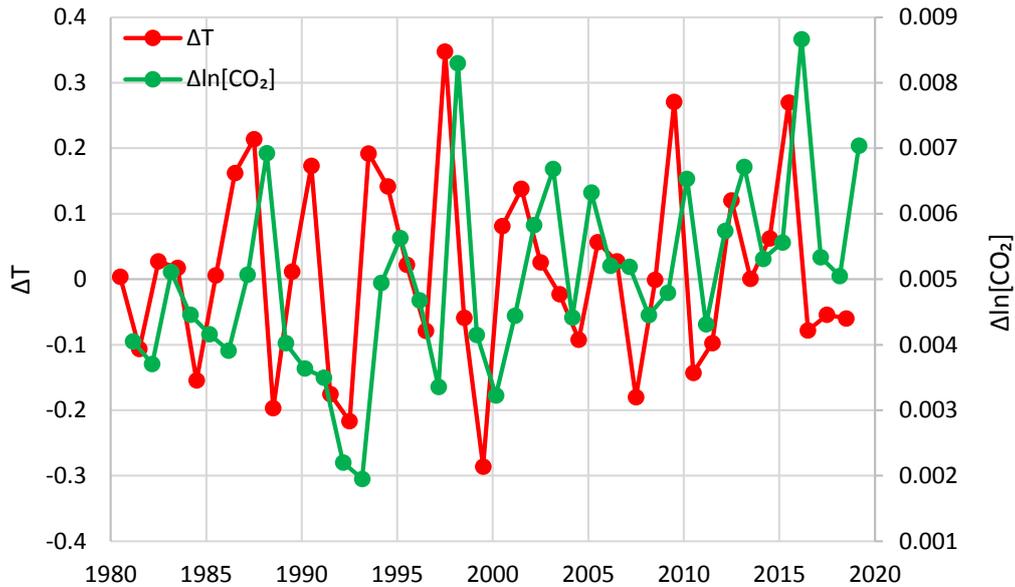


338

339 Figure 8. Differenced time series of UAH temperature and logarithms of CO₂ concentrations at Mauna Loa
 340 at monthly scale. The graph in the upper panel was constructed in the manner described in the text. The
 341 graph in the lower panel is given for comparison and was constructed differently, by taking differences of
 342 the values of each month with the previous month and then averaging over the previous 12 months (to
 343 remove periodicity); in addition, the lower graph includes the CRUTEM4 land temperature series.

344 Figure 9 shows the same time series at the annual time scale, with the year being defined
 345 as July-June for ΔT and February-January for $\Delta \ln[\text{CO}_2]$. The reason for this differentiation will be
 346 explained in the next section. Here it is more evident that most of the time the temperature
 347 change leads and that of CO₂ follows.

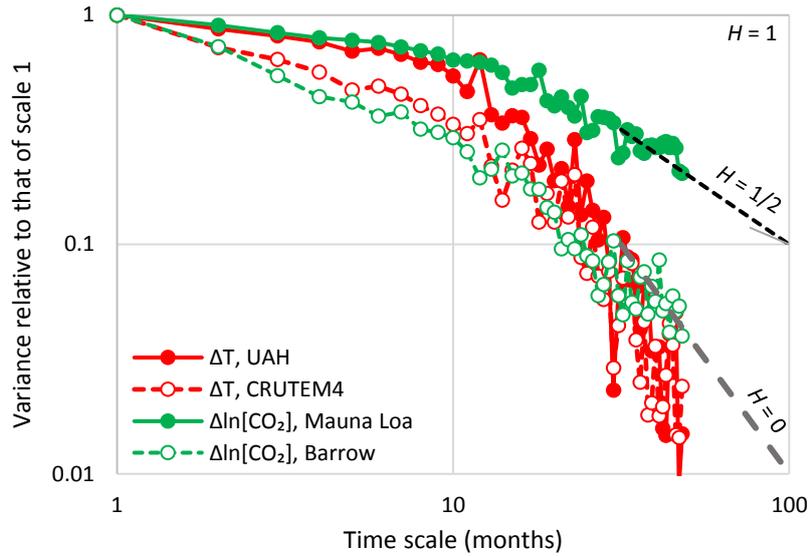
348 It is of interest here that the variability of global mean annual temperature is significantly
 349 influenced by the rhythm of ocean-atmosphere oscillations, such as ENSO, AMO and IPO
 350 (Kundzewicz et al., 2020). This mechanism may be a complicating factor, in turn influencing the
 351 link between temperature and CO₂ concentration.



352

353 Figure 9. Annually averaged time series of differenced temperatures (UAH) and logarithms of CO₂
 354 concentrations (Mauna Loa). Each dot represents the average of a one-year duration ending at the time of
 355 its abscissa.

356 The climacograms of the differenced time series used (actually four of the six to avoid an
 357 overcrowded graph) are shown in Figure 10. It appears that the differenced temperature time
 358 series are consistent with the condition implied by stationarity, i.e., $H = 0$ for the differenced
 359 process. The same does not look to be the case for the CO₂ time series, particularly for the Mauna
 360 Loa time series, in which the Hurst parameter appears to be close to 1/2. Based on this, one
 361 would exclude stationarity for the Mauna Loa CO₂ time series. However, a simpler interpretation
 362 of the graph is that the data record is not long enough to reveal that $H = 0$ for the differenced
 363 process. Actually, all available data belong to a period in which [CO₂] exhibits a monotonic
 364 increasing trend (as also verified by the fact that all values of $\Delta \ln[\text{CO}_2]$ in Figure 8 and Figure 9
 365 are positive, while stationarity entails a zero mean of the differenced process). Had the available
 366 data base been broader, both positive and negative trends could appear. Indeed, a broader view
 367 of the [CO₂] process, based on palaeoclimatic data (Figure 4) would justify a stationarity
 368 assumption.



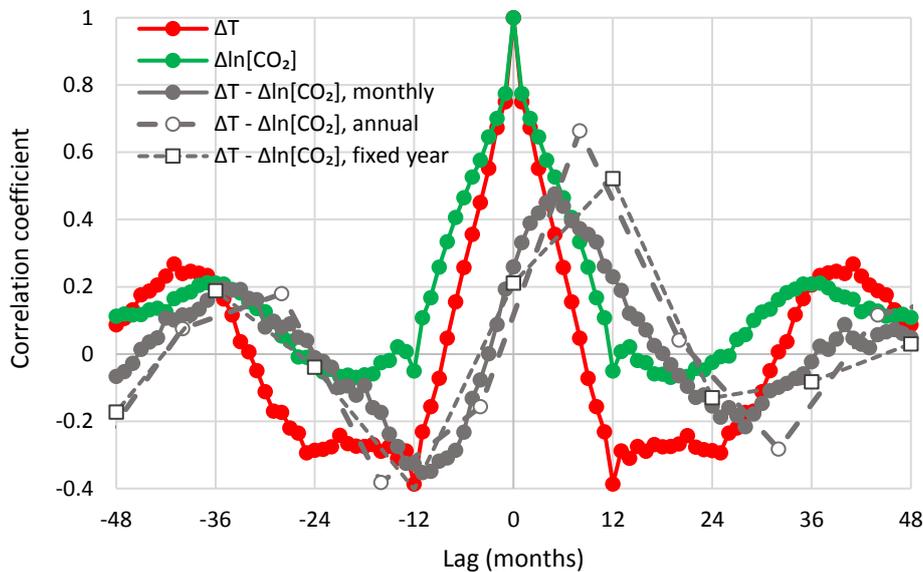
369

370 Figure 10. Empirical climacograms of the indicated differenced time series; the characteristic slopes
 371 corresponding to values of the Hurst parameter $H = 1/2$ (large-scale randomness), 0 (full antipersistence)
 372 and 1 (full persistence) are also plotted (note, $H = 1 - \text{slope}/2$),

373 A preliminary qualitative observation from graphical inspection of Figure 8 and Figure 9,
 374 suggests that the temperature change very often precedes and the CO₂ change follows—in the
 375 same direction. We note, though, that temperature changes alternate in sign while CO₂ changes
 376 are always positive.

377 A quantitative analysis, based on the methodology in section 3.2, requires the study of
 378 lagged cross-correlations of the two processes. Figure 11 shows the cross-correlogram between
 379 UAH temperature and Mauna Loa CO₂ concentration; the autocorrelograms of the two processes
 380 are also plotted for comparison. The fact that the cross-correlogram does not have values
 381 consistently close to zero at any of the semi-axes eliminates the possibility of an exclusive
 382 (unidirectional) causality and suggests consistency with “hen-or-egg” causality.

383 The maximum cross correlation of the monthly series is 0.47 and appears at a positive lag,
 384 $\eta_1 = 5$ months, thus suggesting temperature, rather than CO₂, as dominant cause. Similar are the
 385 graphs of the other combinations of temperature and CO₂ data sets, which are shown in
 386 Appendix C. In all cases η_1 is positive, ranging from 5 to 11 months.



387

388 Figure 11. Auto- and cross-correlograms of the differenced time series of UAH temperature and Mauna
 389 Loa CO₂ concentration.

390 To perform similar analyses on the annual scale, we fixed the specification of a year for
 391 temperature for the period July-June, as already mentioned, and then slid the initial month
 392 specifying the beginning of a year for CO₂ concentration so as to find a specification that
 393 maximizes the cross-correlation at the annual scale. In Figure 11, maximization occurs when the
 394 year specification is February-January (of the next year), i.e., if the lag is 8 months. The
 395 maximum cross-correlation is 0.66. If we keep the specification of the year for CO₂ concentration
 396 same as in temperature (July-June), then maximization occurs at lag one year (12 months) and
 397 the maximum cross correlation is 0.52. Table1 summarizes the results for all combinations
 398 examined. The lags are always positive. They have varied between 8 and 14 months for a sliding
 399 window specification and are 12 months, for the fixed window specification. Most interestingly,
 400 the opposite phase in the annual cycle of CO₂ concentration in the South Pole, with respect to the
 401 other three sites, does not produce any noteworthy difference in the shape of the cross-
 402 correlogram and the time lags maximizing the cross-correlations.

403 Table1. Maximum cross-correlation coefficient (MCCC) and corresponding time lag in months. The annual
 404 window for temperature is July-June, while for CO₂ it is either different (sliding), determined so as to
 405 maximize MCCC, or same (fixed).

Temperature series	-	CO ₂	Monthly time series		Annual time series – sliding annual window		Annual time series – fixed annual window	
			MCCC	Lag	MCCC	Lag	MCCC	Lag
UAH – Mauna Loa			0.47	5	0.66	8	0.52	12
UAH – Barrow			0.31	11	0.70	14	0.59	12
UAH – South Pole			0.37	6	0.54	10	0.38	12
UAH – Global			0.47	6	0.60	11	0.60	12
CRUTEM4 – Mauna Loa			0.31	5	0.55	10	0.52	12
CRUTEM4 – Global			0.33	9	0.55	12	0.55	12

406

407 5 Physical interpretation

408 The omnipresence of positive lags on both monthly and annual time scales reduces the
 409 likelihood that our results are statistical artefacts. Still, our results require physical
 410 interpretation which we seek in the natural process of soil respiration.

411 Soil respiration, R_s , defined to be the flux of microbially and plant-respired CO₂, clearly
 412 increases with temperature. It is known to have increased in the recent years (Bond-Lamberty
 413 and Thomson, 2010; IPCC, 2013). Observational data of R_s (e.g. Makita et al., 2018) show that
 414 process intensity increases with temperature. Rate of chemical reactions, metabolic rate, as well
 415 as microorganism activity, generally increase with temperature. This has been known for more
 416 than 70 years and is routinely used in engineering design (Pomeroy and Bowlus, 1946).

417 The latest report of IPCC (IPCC, 2013, Fig. 6.1) gives quantification of the mass balance of
 418 the carbon cycle in the atmosphere, representative of the recent years. The soil respiration,
 419 assumed to be the sum of respiration (plants) and decay (microbes) is 113.7 Gt C/year (IPCC
 420 gives a value of 118.7 including fire, which, along with biomass burning, is estimated to 5 by
 421 Green and Byrne, 2004).

422 We can expect that the sea respiration would have increased too. Also, the photosynthesis
 423 must have been increased as in the 21st century the earth has been greening, mostly due to CO₂

424 fertilization effects (Zhu et al., 2016) and human land-use management (Chen et al., 2019).
425 Specifically, satellite data show a net increase in leaf area of 2.3% per decade (Chen et al., 2019).
426 The sums of carbon outflows from the atmosphere (terrestrial and maritime photosynthesis as
427 well as maritime absorption) amount to 203 Gt C / year. The carbon inflows to the atmosphere
428 amount to 207.4 Gt C / year and include natural terrestrial processes (respiration, decay, fire,
429 freshwater outgassing as well as volcanism and weathering), natural maritime processes
430 (respiration) as well as anthropogenic processes. The latter comprise human CO₂ emissions
431 related to fossil fuels and cement production as well as land-use change, and amount to 7.7 Gt C
432 / year and 1.1 Gt C / year, respectively. The change in carbon fluxes due to natural processes is
433 likely to exceed the change due to anthropogenic CO₂ emissions, even though the latter are
434 generally regarded as responsible for the imbalance of carbon in the atmosphere.

435 **6 Conclusion**

436 Relationships between atmospheric concentration of carbon dioxide and the global temperature
437 are widely recognized and a common knowledge is that increasing CO₂ concentration plays the
438 major role in enhancement of the greenhouse effect and contributes to global warming.

439 While the fact that these two variables are tightly connected is beyond doubt, the
440 direction of the causal relationship needs to be studied further. The purpose of this study is to
441 complement the conventional and established theory that increased CO₂ concentration due to
442 anthropogenic emissions cause increase of temperature, by considering the concept of reverse
443 causality. The problem is obviously more complex than that of exclusive roles of cause and
444 effect, classifying it as a “hen-or-egg” (“ὄρνις ἢ ᾠόν”) causality problem, where it is not always
445 clear which of two interrelated events is the cause and which the effect. Since increased
446 temperature causes increase in CO₂ concentration, hence we propose the formulation of the
447 entire process in terms of a “hen-or-egg” causality.

448 We examine the relationship of global temperature and atmospheric carbon dioxide
449 concentration using the most reliable global data that are available—the data gathered from
450 several sources, covering the common time interval 1980-2019, available at the monthly time
451 step.

452 The results of the study support the hypothesis that both causality directions exist, with
453 the latter ($T \rightarrow \text{CO}_2$) being the dominant, despite the fact that the former ($\text{CO}_2 \rightarrow T$) prevails in
454 public, as well as in scientific, perception. Indeed, our results show that changes in CO₂ follow
455 changes in T by about six months on monthly scale, or about one year on annual scale.

456 The opposite causality direction opens a nurturing interpretation question. We attempted
457 to interpret this mechanism by noting the possibility of a positive feedback loop: increase of soil
458 respiration leads to increasing CO₂ emission accompanying the temperature rise. We pose the
459 challenging scientific question of interpretation for further studies whose results would, no
460 doubt, find their way to professional literature. In our opinion, scientists of the 21st century
461 should have been familiar with unanswered scientific questions, as well as with the idea that
462 complex systems resist simplistic explanations.

463 **Appendix A. Some notes on the averaged differenced process**

464 The cumulative process of the differenced process $\tilde{x}_{t,v}$ will be:

$$\begin{aligned}\tilde{X}_{\kappa,\nu} &:= \tilde{x}_{1,\nu} + \tilde{x}_{2,\nu} + \cdots + \tilde{x}_{\kappa,\nu} = \underline{x}_{1+\nu} - \underline{x}_1 + \underline{x}_{2+\eta} - \underline{x}_2 + \cdots + \underline{x}_{\kappa+\nu} - \underline{x}_\kappa \\ &= \underline{X}_{\kappa+\nu} - \underline{X}_\nu - \underline{X}_\kappa\end{aligned}\quad (\text{A1})$$

465 Note that for $\eta = 1$ this simplifies to

$$\tilde{X}_{\kappa,1} = \underline{X}_{\kappa+1} - \underline{X}_1 - \underline{X}_\kappa = \underline{x}_{\kappa+1} - \underline{x}_1 = \tilde{x}_{\kappa,1} =: \tilde{x}_\kappa \quad (\text{A2})$$

466 Following equation (7), the average differenced process at discrete time scale $\kappa = \eta$ will be:

$$\tilde{x}_\tau^{(\kappa)} = \frac{\tilde{X}_{\tau\kappa,\kappa} - \tilde{X}_{(\tau-1)\kappa,\kappa}}{\kappa} = \frac{(X_{\tau\kappa+\kappa} - X_\kappa - X_{\tau\kappa}) - (X_{(\tau-1)\kappa+\kappa} - X_\kappa - X_{(\tau-1)\kappa})}{\kappa} \quad (\text{A3})$$

467 which, noting that in the rightmost part the two terms \underline{X}_κ cancel each other and by virtue of (7),
468 simplifies to:

$$\tilde{x}_\tau^{(\kappa)} = \underline{x}_{\tau+1}^{(\kappa)} - \underline{x}_\tau^{(\kappa)} = \tilde{x}_{\tau,1}^{(\kappa)} \quad (\text{A4})$$

469 In other words, the average differenced process equals the differenced average process in case
470 that the differencing time step η has chosen equal to the averaging time scale κ . For $\kappa = \eta = 1$ we
471 have $\tilde{x}_\tau^{(1)} \equiv \tilde{x}_{\tau,1} \equiv \tilde{x}_\tau$.

472 **Appendix B. Some notes on (unidirectional) causal systems**

473 In a unidirectional causal system in continuous time t , in which the process $\underline{x}(t)$ is the cause of
474 $\underline{y}(t)$, an equation of the form:

$$\underline{y}(t) = \int_0^\infty \alpha(s) \underline{x}(t-s) ds \quad (\text{B1})$$

475 should hold (Papoulis, 1991), where $\alpha(t)$ is the impulse response function. The causality
476 condition is:

$$\alpha(t) = 0 \text{ for } t < 0 \quad (\text{B2})$$

477 Here we consider systems with positive dependence, in which $\alpha(t) \geq 0$ for $t \geq 0$, which possibly
478 are also excited by another process $\underline{v}(t)$, independent of $\underline{x}(t)$. Working in discrete time we
479 write:

$$\underline{y}_\tau = \sum_{j=0}^{\infty} \alpha_j \underline{x}_{\tau-j} + \underline{v}_\tau \quad (\text{B3})$$

480 Assuming (without loss of generality) zero means for all processes, multiplying by $\underline{x}_{\tau-\eta}$, taking
481 expected values and denoting the cross-covariance function as $c_{xy}[\eta] := E[\underline{x}_{\tau-\eta} \underline{y}_\tau]$ and the
482 autocovariance function as $c_x[\eta] := E[\underline{x}_{\tau-\eta} \underline{x}_\tau]$ we find:

$$c_{xy}[\eta] = \sum_{j=0}^{\infty} \alpha_j c_x[\eta-j] \quad (\text{B4})$$

483 For $\eta > 0$, using the property that $c_x[\eta]$ is an even function ($c_x[\eta] = c_x[-\eta]$) we get:

$$c_{xy}[\eta] = \sum_{j=0}^{\infty} \alpha_j c_x[j - \eta] = \sum_{j=0}^{\eta-1} \alpha_j c_x[\eta - j] + \sum_{j=\eta}^{\infty} \alpha_j c_x[j - \eta] \quad (\text{B5})$$

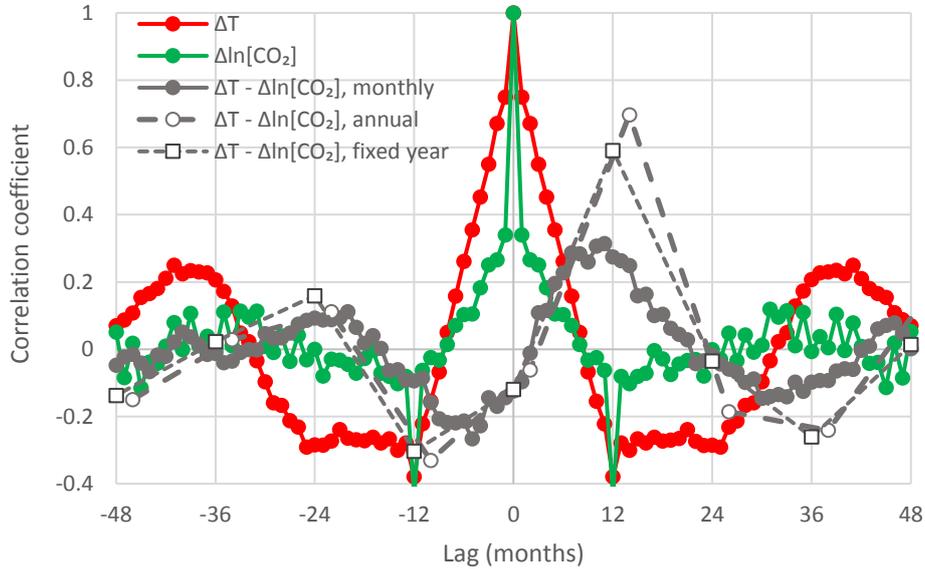
484 and for the negative part:

$$c_{xy}[-\eta] = \sum_{j=0}^{\infty} \alpha_j c_x[j + \eta] \quad (\text{B6})$$

485 With an intuitive reasoning, assuming that the autocovariance function is decreasing
 486 ($c_x[j'] < c_x[j]$ for $j' > j$), as usually happens in natural processes, we may see that the rightmost
 487 term of equations (B5) and (B6) should be decreasing functions of η (as for $j' > j$ it will be
 488 $c_x[j' - \eta] < c_x[j - \eta]$ and $c_x[j' + \eta] < c_x[j + \eta]$). However, the term $\sum_{j=0}^{\eta-1} \alpha_j c_x[\eta - j]$ of
 489 equation (B5), is not decreasing. Therefore, it should attain a maximum value at some positive
 490 lag $\eta = \eta_1$. Thus, a positive maximizing lag, $\eta = \eta_1 > 0$, is a necessary condition for causality
 491 directional from \underline{x}_τ to \underline{y}_τ . Conversely, the condition that the maximizing lag is negative is a
 492 sufficient condition to exclude the causality direction exclusively from \underline{x}_τ to \underline{y}_τ .

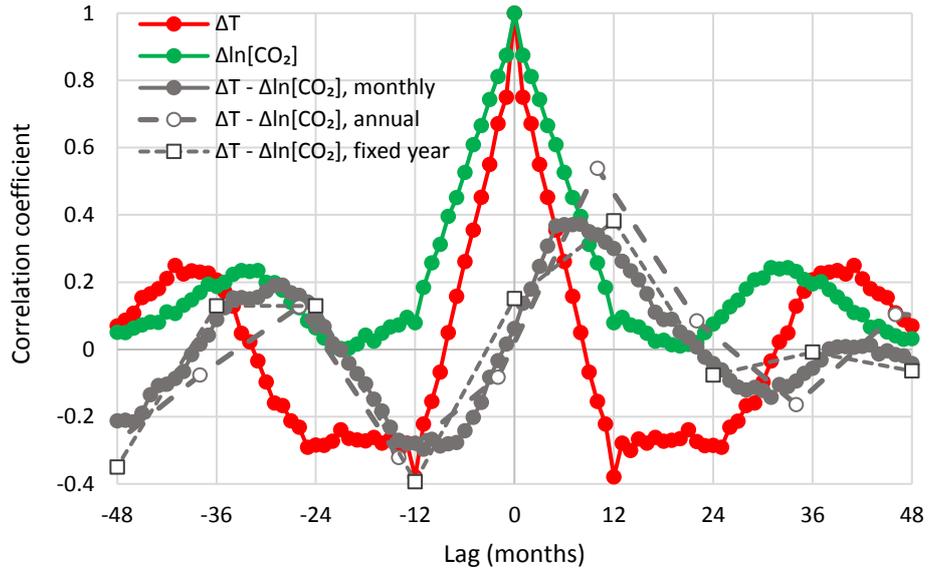
493 All above arguments remain valid if we standardize (divide) by the product of standard
 494 deviations of the processes \underline{x}_τ and \underline{y}_τ , and thus we can replace cross-covariances $c_{xy}[\eta]$ with
 495 cross-correlations $r_{xy}[\eta]$ (or, in the case of differenced processes, $r_{\hat{x}\hat{y}}[v, \eta]$).

496 Appendix C. Graphical depictions of cross-correlograms for different 497 combinations of temperature and carbon dioxide data



498

499 Figure C1. Auto- and cross-correlograms of the differenced time series of UAH temperature and Barrow
 500 CO₂ concentration.

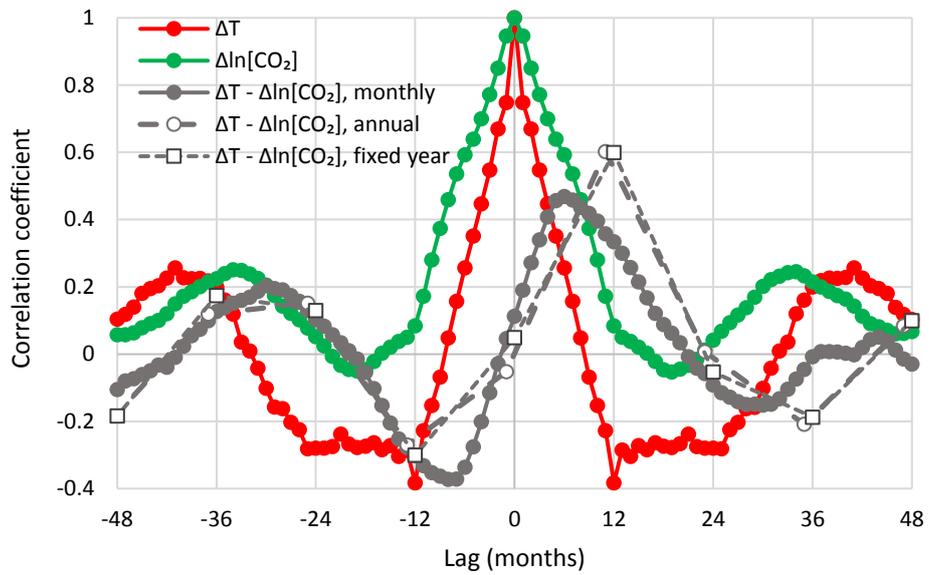


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Figure C2. Auto- and cross-correlograms of the differenced time series of UAH temperature and South Pole CO₂ concentration.

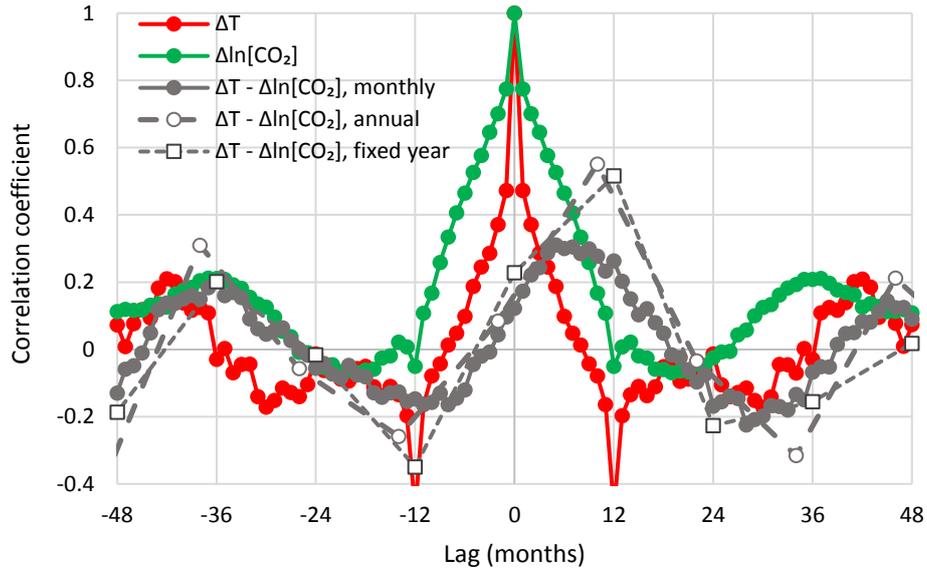


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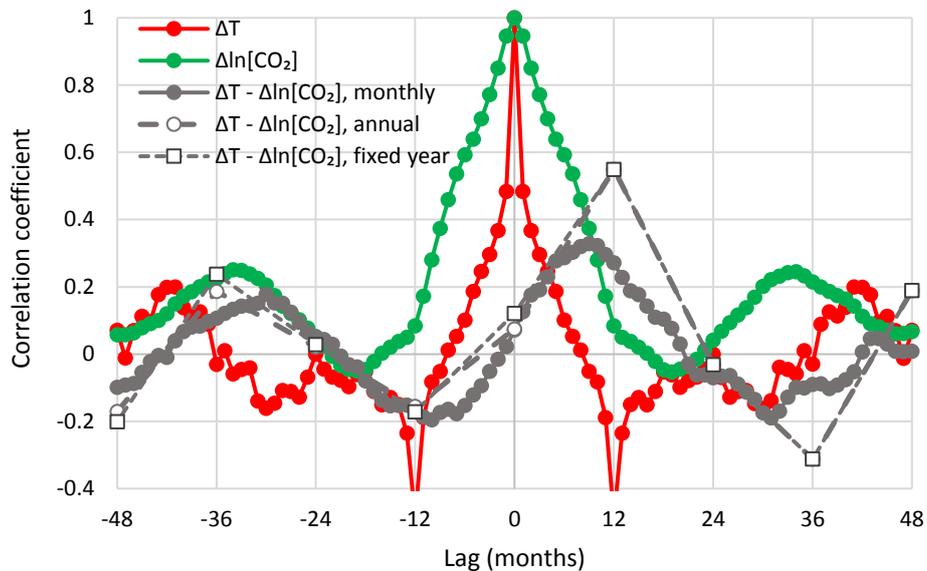
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Figure C3. Auto- and cross-correlograms of the differenced time series of UAH temperature and global CO₂ concentration.



507

508 Figure C4. Auto- and cross-correlograms of the differenced time series of CRUTEM4 temperature and
 509 Mauna Loa CO₂ concentration.



510

511 Figure C5. Auto- and cross-correlograms of the differenced time series of CRUTEM4 temperature and
 512 global CO₂ concentration.

513 Data availability

514 The two temperature time series and the Mauna Loa CO₂ time series are readily available on
 515 monthly scale from <http://climexp.knmi.nl>. All NOAA CO₂ data are available from
 516 https://www.esrl.noaa.gov/gmd/ccgg/trends/gl_trend.html. The CO₂ data of Mauna Loa were
 517 retrieved from http://climexp.knmi.nl/data/imaunaloa_f.dat while the original measurements
 518 are in <https://www.esrl.noaa.gov/gmd/dv/iadv/graph.php?code=MLO>. The Barrow series is
 519 available (in irregular step) in <https://www.esrl.noaa.gov/gmd/dv/iadv/graph.php?code=BRW>,
 520 and the South Pole series in <https://www.esrl.noaa.gov/gmd/dv/data/index.php?site=SPO>. All
 521 these data were accessed (using the “Download data” link in the above sites) in June 2020. The

522 global CO₂ series is accessed at https://www.esrl.noaa.gov/gmd/ccgg/trends/gl_data.html, of
523 which the “Globally averaged marine surface monthly mean data” are used here.

524 The palaeoclimatic data of Vostok CO₂ were retrieved from [http://cdiac.ess-](http://cdiac.ess-dive.lbl.gov/ftp/trends/co2/vostok.icecore.co2)
525 [dive.lbl.gov/ftp/trends/co2/vostok.icecore.co2](http://cdiac.ess-dive.lbl.gov/ftp/trends/co2/vostok.icecore.co2) (dated Jan. 2003, accessed Sep. 2018) and the
526 temperature data from [http://cdiac.ess-dive.lbl.gov/ftp/trends/temp/vostok/vostok.1999-](http://cdiac.ess-dive.lbl.gov/ftp/trends/temp/vostok/vostok.1999-temp.dat)
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