Optimal Operation of a Run-of-River Small Hydropower Plant with Two Hydro-Turbines

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Abstract—The operation of a small hydroelectric power plant (HPS) with two hydro turbines of different types and power is usually done following a hierarchical rule, which is not necessarily the most efficient. Alternatively, other synergetic rules have been proposed that improve the delivered energy. In this paper, the operation of the two turbines is systematized by examining all possible operation combinations of the turbines, depending on the incoming water flow, its distribution (in the case of operation of both hydro turbines, at the optimal power mode) and the formation of a suitable lookup table for the optimal operation of an HPS. The implementation of the method is easily achieved using a quadratic equation efficiency-flow curve. In this way, the total efficiency of the two-turbine system is optimized.

Keywords— turbine efficiency curve, optimal operation, small hydropower plant.

I. INTRODUCTION

SMALL hydroelectric projects are renewable energy sources, whose stochastic behaviour is determined by the water flow at the intake [1], as the vast majority of them are run-of-river, without a reservoir [2]. Since 1990, there has been an extensive research activity around the optimal design, operation and efficiency of these projects, with particular emphasis on issues such as the optimal power size of the hydroelectric plant [3-4], the development of economic and energy evaluation indicators [4-5], the optimal overall plant design [4, 6], the design, operation and efficiency improvement of hydro-turbines [4, 7-9], the effect of the water flow as well as of the general hydrological behaviour of the catchment area, on the efficiency of the hydroelectric plant [10-11]. In the last five years, research has been combined with the utilization of data from other disciplines (e.g., use of geo-information systems and climate data [12]), in solving problems related to the liberalization of the electricity market, such as short-term power forecasting with stochastic models using prior power and precipitation [13] and/or supplies [14], the medium-term power forecast, based on the climatic data of the region [15] etc.

However, there is a number of issues that need improvement, such as the operation of a hydroelectric plant with two hydro-turbines. In particular, the typical mode of operation is the so-called "hierarchical" one, in which the main hydro-turbine (usually the one of larger power capacity) operates, whilst the second one (usually the small one) works in addition, with the remaining water not utilized by the main turbine [16-17]. Alternatively, the "synergetic" mode has been proposed, where intuitively it has been suggested to extend the operation of the second hydro-turbine in certain flow ranges, at the expense of the main hydro turbine, which, however, for the

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same flow fluctuations presents smaller efficiency fluctuations, so as to achieve a greater generated-power overall [2, 4, 17].

In this paper, a method of optimal operation of the two turbines is proposed, by considering all possible combinations of operation of the turbines depending on the level of flow. When it is possible to operate both turbines, the flow is distributed according to the optimal energy mode, using the derivative, and the power generation mode with the maximum value is selected. This results in the formation of a suitable lookup table for the optimal operation of an HPS, with which the hydroelectric plant can be adjusted in advance, so as to produce the maximum hydroelectric power in each case. The only requirement to achieve an easy mathematical solution is to use an analytical (more specifically, quadratic) equation efficiency-flow curve for the turbine. In section II, the two existing methods and the proposed new one are analyzed, while in section III, the necessary mathematical background are given, for the optimal operation of two hydro-turbines, as long as their operation is permissible, with regards to the available flow. In section IV, the proposed method is applied and compared with the previous ones, in a variety of case studies (e.g., Francis hydro-turbines of the same or different nominal flow, Francis/Pelton combination of different nominal power), proving, in all cases, that the power generated by the hydroelectric plant, is improved.

II. Modes of Operation of a Two-turbine Hydroelectric Power Plant

A hydroelectric station is given, consisting of two hydro-turbines, the main "I" and the secondary "II", which are supplied through the same penstock of a practically constant available gross head. The electrical powers produced from the hydro-turbines "I" P_I and "II" P_{II} are given respectively by:

$$P_I\left(q_I\right)=\eta_{gen}\cdot\eta_{tr}\ \ \eta_I\left(q_I\right)\cdot\rho\cdot g\cdot q_I\cdot H\left(q_I+q_{II}\right) \tag{1}$$

$$P_{II}\left(q_{II}\right)=\eta_{gen}\cdot\eta_{tr}\ \ \eta_{II}\left(q_{II}\right)\cdot\rho\cdot g\cdot q_{II}\cdot H\left(q_I+q_{II}\right) \tag{2}$$

where g is the gravitational acceleration (= 9.81m/s^2), ρ is the water density (= 999.7 kg/m^3), η_{gen}, η_{tr} are the degrees of efficiency of the generator and the transformer respectively (practically they are considered as having constant values, compared to the changes of the efficiency degrees of the hydro-turbines), q_I, q_{II} are the water flows utilized by the respective hydro-turbines, H is the available net head (which is given by the initial available gross head H_{gross} reduced by the hydraulic losses h_{loss}), and η_I, η_{II} are the degrees of efficiency, expressed as quadratic equations within the operating range of flow of hydro-turbines:

$$\eta_I\left(q_I\right)=\begin{cases} a_I\cdot\left(\dfrac{q_I}{q_{nom-I}}\right)^2+b_I\cdot\dfrac{q_I}{q_{nom-I}}+c_I & \begin{array}{c} q_{\min-I}\le q_I\le q_{\max\ I} \\ \text{or } q_{\max-I}<q_I \\ \xrightarrow[\text{no flood}]{\text{on condition}} q_I:=q_{\max-I}\end{array} \\ 0 & q_I<q_{\min-I} \text{ or flood}\end{cases} \tag{3}$$

$$\eta_{II}\left(q_{II}\right)=\begin{cases} a_{II}\cdot\left(\dfrac{q_{II}}{q_{nom-II}}\right)^2+b_{II}\cdot\dfrac{q_{II}}{q_{nom-II}}+c_{II} & \begin{array}{c} q_{\min-II}\le q_{II}\quad q_{\max\ II} \\ \text{or } q_{\max-II}<q_{II} \\ \xrightarrow[\text{no flood}]{\text{on condition}} q_{II}:=q_{\max-II}\end{array} \\ 0 & q_{II}<q_{\min-II} \text{ or flood}\end{cases} \tag{4}$$

where q_{min-I} is the minimum permissible operating flow of hydro-turbine "I"(e.g., 15% for Pelton, 50% for Francis), q_{max-I} is the maximum permissible operating flow of hydro turbine "I" (e.g., 115 % for Pelton or Francis), q_{nom-I} is the nominal operating flow of hydro-turbine "I", a_I, b_I, c_I are the respective coefficients of the quadratic equation, while the respective parameters of hydro-turbine "II", q_{min-II}, q_{max-II}, q_{nom-II}, a_{II}, b_{II}, c_{II}, are similarly defined.

From the river there is an available flow q_{in} (having subtracted the residual (environmental) flow from the total one), out of which q_d is utilized (depending on the mode of operation), which is equal to:

$$q_d=q_I+q_{II} \tag{5}$$

The flow q_d enters the hydraulic system and causes hydraulic losses. These are the sum of linear (i.e., friction) losses, estimated by the Darcy-Weisbach formula, and local losses that are due to geometrical changes, etc.:

$$h_{loss}\left(u_d\right)=f\cdot\frac{L}{d_{in}}\cdot\frac{u_d^2}{2\cdot g}+z_{tot}\cdot\frac{u_d^2}{2\cdot g} \tag{6}$$

where:

$$u_d=\frac{4\cdot q_d}{\pi\cdot d_{in}^2} \tag{7}$$

$$\mathrm{Re}=\frac{u_d\cdot d_{in}}{v} \tag{8}$$

$$\frac{1}{\sqrt{f}}=-2\cdot\log_{10}\left(\frac{e}{3.71\cdot d_{in}}+\frac{2.51}{\mathrm{Re}\cdot\sqrt{f}}\right) \tag{9}$$

where u_d is the velocity in a pipeline under pressure, of a circular cross-section of d_{in} radius, pipe roughness e, Re the Reynolds number, v the water kinematic viscosity (= $1.14\cdot10^{-6}$ m^2/s), f the friction coefficient (e.g., estimated by the Colebrook-White formula), determined iteratively, z_{tot} is the coefficient of overall local losses, as given by technical manuals [16].

The mode of flow distribution to two turbines, is hereinafter analysed.

A. Hierarchical Method

In the hierarchical method, the available flow q_{in} is first distributed to the main hydro-turbine "I" and the remainder to hydro-turbine "II". Without loss of generality, it holds that: $q_{min-I}\geq q_{min-II}$, $q_{max-I}\geq q_{max-II}$, $q_{nom-I}\geq q_{nom-II}$. Therefore, its typical operation mode, by [16-17], is:

- If q_{in} is lower than the minimum operating flow $\min\{q_{min\text{-}I}, q_{min\text{-}II}\}=q_{min\text{-}II}$, then no hydro-turbine operates and the usable flow q_d is 0.
- If q_{in} is between $q_{min\text{-}II}$ and $q_{min\text{-}I}$, then hydro-turbine "II" is operating and the usable flow q_d is equal to q_{in}.
- If q_{in} is between $q_{min\text{-}I}$ and $q_{max\text{-}I}$, then hydro-turbine "I" is working and the usable flow q_d is equal to q_{in}.
- If q_{in} is between $q_{max\text{-}I}$ and $q_{max\text{-}I}+q_{min\text{-}II}$, then hydro-turbine "I" operates at the maximum flow of $q_{max\text{-}I}$, hydro turbine "II" does not operate due to insufficient flow, the usable flow q_d is equal to $q_{max\text{-}I}$ and an unused flow, equal to $q_{in}-q_{max\text{-}I}$, remains.
- If q_{in} is between $q_{max\text{-}I}+q_{min\text{-}II}$ and $q_{max\text{-}I}+q_{max\text{-}II}$, then hydro-turbine "I" operates at the maximum flow of $q_{max\text{-}I}$, hydro-turbine "II" operates with the remaining flow equal to $q_{in}-q_{max\text{-}I}$ and the usable flow q_d is equal to q_{in}.
- If q_{in} is between $q_{max\text{-}I}+q_{max\text{-}II}$ and smaller values, which correspond to flood phenomena, then both hydro-turbines "I" and "II" operate, at the maximum $q_{max\text{-}I}$ and $q_{max\text{-}II}$ flows respectively, the usable flow q_d is equal to $q_{max\text{-}I}+ q_{max\text{-}II}$ and the non-usable flow is equal to $q_{in}- q_{max\text{-}I}- q_{max\text{-}II}$.
- If q_{in} is greater than the value corresponding to flood flow, then the two hydro-turbines interrupt their operation, for safety reasons, and the usable flow q_d is 0.

In this method, no particular calculation is made, in terms of energy efficiency. The main hydro-turbine "I" simply precedes, on the basis that the larger hydro-turbine yields a greater amount of power.

B. Synergetic Method

In the synergetic method, the objective is to better utilize the available flow across specific ranges, by tuning the precedence between the two hydro-turbines, in order to optimize the power produced at the, each time, available flow q_{in}. Without loss of generality, it holds that: $q_{min\text{-}I} \geq q_{min\text{-}II}$, $q_{max\text{-}I} \geq q_{max\text{-}II}$, $q_{nom\text{-}I} \geq q_{nom\text{-}II}$. Therefore, its typical operation mode, by [2, 4, 17], is:

- If q_{in} is lower than the minimum operating flow $\min\{q_{min\text{-}I}, q_{min\text{-}II}\}=q_{min\text{-}II}$, then no hydro-turbine operates and the usable flow q_d is 0.
- If q_{in} is between $q_{min\text{-}II}$ and $q_{min\text{-}I}$, then hydro-turbine "II" is operating and the usable flow q_d is equal to q_{in}.
- If q_{in} is between $q_{min\text{-}I}$ and $q_{max\text{-}II}$, then whichever of the two hydro-turbines produces more power, according to (1) and (2), is selected and only that one operates. The usable flow q_d is equal to q_{in}. At this point, this method differs from the hierarchical, achieving better or equal energy efficiency.
- If q_{in} is between $q_{max\text{-}II}$ and $q_{max\text{-}I}$, then hydro-turbine "I" is operating and the usable flow q_d is equal to q_{in}.
- If q_{in} is between $q_{max\text{-}I}$ and $q_{max\text{-}I}+q_{min\text{-}II}$, then both hydro-turbines operate in their optimal mode. According to [17], this mode means that hydro-turbine "II" operates at its maximum efficiency (meaning at a flow $q_{max\text{-}II}$, from the operating range of water flows), while hydro-turbine "I" operates with the remainder $q_{in}-q_{max\text{-}II}$, provided that it is greater than $q_{min\text{-}I}$. The usable flow q_d is equal to q_{in}. At this point, this differs from the hierarchical method, as more water is utilized and in addition the smaller hydro-turbine "II" operates with a very high efficiency. At the same time, the larger hydro turbine "I" operates with a varying efficiency, which yet presents limited fluctuations, since the power varies within a limited flow range from ($q_{max\text{-}I} -q_{max\text{-}II}$) to ($q_{max\text{-}I}+q_{min\text{-}II}-q_{max\text{-}II}$). In [2, 4] the term "optimal mode of operation" is not clarified.
- If q_{in} is between $q_{max\text{-}I}+q_{min\text{-}II}$ and $q_{max\text{-}I}+ q_{max\text{-}II}$, then both hydro-turbines operate in their optimal mode, as previously analysed according to [17], i.e., hydro-turbine "II", operates at its maximum efficiency and hydro-turbine "I" operates with the remainder $q_{in}-q_{max\text{-}II}$, provided that it is greater than $q_{min\text{-}I}$. The usable flow q_d is equal to q_{in}. At this point, this differs from the hierarchical method, since the smaller hydro-turbine "II" operates with a very high efficiency, while the larger hydro-turbine "I" operates with a varying efficiency, which changes within the flow range limits from ($q_{max\text{-}I}+q_{min\text{-}II} -q_{max\text{-}II}$) to $q_{max\text{-}I}$.
- If q_{in} is between $q_{max\text{-}I}+q_{max\text{-}II}$ and smaller values, which correspond to flood phenomena, then both hydro-turbines "I" and "II" operate, at the maximum $q_{max\text{-}I}$ and $q_{max\text{-}II}$ flows respectively, the usable flow q_d is equal to $q_{max\text{-}I}+ q_{max\text{-}II}$ and the non-usable flow is equal to $q_{in}- q_{max\text{-}I}- q_{max\text{-}II}$.
- If q_{in} is greater than the value corresponding to flood flow, then no hydro-turbine operates and the usable flow q_d is 0.

In this method, particular calculations are made, in terms of energy efficiency, increasing the amount of power produced intuitively, having understood, as engineers, the synergetic mode in which the two hydro-turbines operate.

C. Proposed / Optimized Method

In this method, all possible combinations of operation are considered, for each available flow q_{in} and the one with the highest power production is selected. In particular, for the case of the operation of two hydro-turbines, there are $2^2=4$ possibilities:

- If q_{in} is lower than the minimum operating flow $\min\{q_{min\text{-}I}, q_{min\text{-}II}\}=q_{min\text{-}II}$, or a flood phenomenon occurs, then no hydro-turbine operates and the usable flow q_d is 0.
- If q_{in} is greater than $q_{min\text{-}II}$ and lower than that of a flood effect, then only hydro-turbine "II" can operate. In this case, if the available flow is lower than or equal to $q_{max\text{-}II}$, then the usable flow q_d is equal to q_{in}, otherwise, it is equal to $q_{max\text{-}II}$. Furthermore, the usable flow q_d is equal to the flow of hydro-turbine "II" q_{II} and the produced power is calculated through (2), let be $P_{II}(q_{in})$, assuming that the usable flow of hydro-turbine "I" q_I, is equal to 0.
- If q_{in} is greater than $q_{min\text{-}I}$ and lower than that of the flood effect, then only hydro-turbine "I" can operate. In this case, if the available flow is lower than or equal to $q_{max\text{-}I}$, then the usable flow q_d is equal to q_{in}, otherwise it is equal to $q_{max\text{-}I}$. Furthermore, the usable flow q_d, is equal to the flow of hydro-turbine "I" q_I, and the produced power is calculated through (1), let be $P_I(q_{in})$, assuming that the usable flow of

hydro-turbine "II" q_{II} , is equal to 0.

- If q_{in} is greater than ($q_{min-I}+q_{min-II}$) and lower than that of the flood effect, then both hydro-turbines can operate. In this case, if the available flow is lower than or equal to ($q_{max-I}+q_{max-II}$), then the usable flow q_d is equal to q_{in} and the optimal flow distribution will be done by solving the problem:

$$\max\left(P_I\left(q_I\right)+P_{II}\left(q_{II}\right)\right) \text{ on condition that } q_{in}=q_I+q_{II} \tag{10}$$

The maximization problem is analysed in section III. Otherwise, if the available flow is greater than ($q_{max-I}+q_{max-II}$), the usable flow is equal to ($q_{max-I}+q_{max-II}$) and each hydro-turbine is loaded with its maximum flow. From this combined flow distribution, the produced power by the hydroelectric plant is calculated, let be $P_{I+II}(q_{in})$.

From the combinations above, for each available flow q_{in}, the one among $P_I(q_{in})$, $P_{II}(q_{in})$, $P_{I+II}(q_{in})$ which yields the highest power is selected, as long as its corresponding operation is permissible. In this way, a lookup table for the optimal operation of an HPS is formed, maximizing the production of electricity in any case.

III. Optimum Flow Distribution to Two Hydro-Turbines, to Maximize Power Produced

The problem posed in (10) is reiterated, to be solved by replacing the flow of hydro-turbine "II" q_{II} by q_{in}-q_I as follows:

$$\max\left(P_{I+II}\left(q_I\right)\right)=\max\left(P_I\left(q_I\right)+P_{II}\left(q_{in}-q_I\right)\right) \tag{11}$$

Substituting (1) and (2) into (11), for the total power produced function $P_{I+II}(q_I)$, it follows that:

$$P_{I+II}\left(q_I\right)=\begin{cases}\eta_{gen}\cdot\eta_{tr}\cdot\eta_I\left(q_I\right)\cdot\rho\cdot g\cdot q_I\cdot H\left(q_{in}\right)+\\ \eta_{gen}\cdot\eta_{tr}\cdot\eta_{II}\left(q_{in}-q_I\right)\cdot\rho\cdot g\cdot\left(q_{in}-q_I\right)\cdot H\left(q_{in}\right)\end{cases} \tag{12}$$

To maximize the total power produced $P_{I+II}(q_I)$, it must hold that:

$$\left.\frac{dP_{I+II}\left(q_I\right)}{dq_I}\right|_{q_I=q_{Iopt}}=0\Rightarrow$$

$$\left\{\begin{aligned}&\left.\frac{d\eta_I\left(q_I\right)}{dq_I}\right|_{q_I=q_{Iopt}}\cdot\eta_{gen}\cdot\eta_{tr}\cdot\rho\cdot g\cdot q_{Iopt}\cdot H\left(q_{in}\right)\\&+\eta_{gen}\cdot\eta_{tr}\cdot\eta_I\left(q_{Iopt}\right)\cdot\rho\cdot g\cdot H\left(q_{in}\right)+\\&\left.\frac{d\eta_{II}\left(q_{in}-q_{Iopt}\right)}{dq_I}\right|_{q_I=q_{Iopt}}\cdot\eta_{gen}\cdot\eta_{tr}\cdot\rho\cdot g\cdot\left(q_{in}-q_{Iopt}\right)\cdot H\left(q_{in}\right)+\\&-\eta_{gen}\cdot\eta_{tr}\cdot\eta_{II}\left(q_{in}-q_{Iopt}\right)\cdot\rho\cdot g\cdot H\left(q_{in}\right)\end{aligned}\right\}=0\Rightarrow$$

$$\left\{\begin{aligned}&\left.\frac{d\eta_I\left(q_I\right)}{dq_I}\right|_{q_I=q_{Iopt}}\cdot q_{Iopt}+\eta_I\left(q_{Iopt}\right)+\\&\left.\frac{d\eta_{II}\left(q_{in}-q_{Iopt}\right)}{dq_I}\right|_{q_I=q_{Iopt}}\cdot\left(q_{in}-q_{Iopt}\right)-\eta_{II}\left(q_{in}-q_{Iopt}\right)\end{aligned}\right\}=0 \tag{13}$$

on condition that:

$$\left.\frac{d^2P_{I+II}\left(q_I\right)}{d^2q_I}\right|_{q_I=q_{Iopt}}<0\Rightarrow$$

$$\left\{\begin{aligned}&\left.\frac{d^2\eta_I\left(q_I\right)}{d^2q_I}\right|_{q_I=q_{Iopt}}\cdot\eta_{gen}\cdot\eta_{tr}\cdot\rho\cdot g\cdot q_{Iopt}\cdot H\left(q_{in}\right)+\\&2\cdot\left.\frac{d\eta_I\left(q_I\right)}{dq_I}\right|_{q_I=q_{Iopt}}\cdot\eta_{gen}\cdot\eta_{tr}\cdot\rho\cdot g\cdot H\left(q_{in}\right)+\\&\left.\frac{d^2\eta_{II}\left(q_{in}-q_{Iopt}\right)}{d^2q_I}\right|_{q_I=q_{Iopt}}\cdot\eta_{gen}\cdot\eta_{tr}\cdot\rho\cdot g\cdot\left(q_{in}-q_{Iopt}\right)\cdot H\left(q_{in}\right)\\&-2\cdot\left.\frac{d\eta_{II}\left(q_{in}-q_{Iopt}\right)}{dq_I}\right|_{q_I=q_{Iopt}}\cdot\eta_{gen}\cdot\eta_{tr}\cdot\rho\cdot g\cdot H\left(q_{in}\right)\end{aligned}\right\}<0\Rightarrow$$

$$\left\{\begin{aligned}&\left.\frac{d^2\eta_I\left(q_I\right)}{d^2q_I}\right|_{q_I=q_{Iopt}}\cdot q_{Iopt}+2\cdot\left.\frac{d\eta_I\left(q_I\right)}{dq_I}\right|_{q_I=q_{Iopt}}+\\&\left.\frac{d^2\eta_{II}\left(q_{in}-q_{Iopt}\right)}{d^2q_I}\right|_{q_I=q_{Iopt}}\cdot\left(q_{in}-q_{Iopt}\right)\\&-2\cdot\left.\frac{d\eta_{II}\left(q_{in}-q_{Iopt}\right)}{dq_I}\right|_{q_I=q_{Iopt}}\end{aligned}\right\}<0 \tag{14}$$

In the case of a quadratic efficiency-flow curve, (3) and (4) yield the following derivatives, regarding non-zero areas:

$$\frac{d\eta_I\left(q_I\right)}{dq_I}=\frac{2\cdot a_I}{q_{nom-I}^{\;2}}\cdot q_I+\frac{b_I}{q_{nom-I}} \tag{15}$$

$$\frac{d^2\eta_I\left(q_I\right)}{d^2q_I}=\frac{2\cdot a_I}{q_{nom-I}^{\;2}} \tag{16}$$

$$\eta_{II}\left(q_I\right)=a_{II}\cdot\left(\frac{q_{in}-q_I}{q_{nom-II}}\right)^2+b_{II}\cdot\frac{q_{in}-q_I}{q_{nom-II}}+c_{II} \tag{17}$$

$$\frac{d\eta_{II}\left(q_I\right)}{dq_I}=\frac{2\cdot a_{II}}{q_{nom-II}^{\;2}}\cdot q_I-\frac{b_{II}}{q_{nom-II}}-\frac{2\cdot a_{II}\cdot q_{in}}{q_{nom-II}^{\;2}} \tag{18}$$

$$\frac{d^2\eta_{II}\left(q_I\right)}{d^2q_I}=\frac{2\cdot a_{II}}{q_{nom-II}^{\;2}} \tag{19}$$

By substituting (3), (15), (17), (18) into (13), the following quadratic equation results:

$$A \cdot q_I^2 + B \cdot q_I + C = 0 \quad (20)$$

where:

$$A = 3 \cdot \left(\frac{a_I}{q_{nom-I}{}^2} - \frac{a_{II}}{q_{nom-II}{}^2} \right) \quad (21)$$

$$B = 2 \cdot \left(\frac{b_I}{q_{nom-I}} + \frac{b_{II}}{q_{nom-II}} \quad \frac{3 \cdot a_{II}}{q_{nom-II}{}^2} \cdot q_{in} \right) \quad (22)$$

$$C = c_I - c_{II} \quad \frac{2 \cdot b_{II}}{q_{nom-II}} - \frac{3 \cdot a_{II}}{q_{nom-II}{}^2} \cdot q_{in}{}^2 \quad (23)$$

Respectively, solving (20), yields two possible results:

$$q_{I-opt-1} = \frac{-B + \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \quad (24)$$

$$q_{I-opt-2} = \frac{-B - \sqrt{B^2 - 4 \cdot A \cdot C}}{2 \cdot A} \quad (25)$$

on condition that:

$$B^2 - 4 \cdot A \cdot C \geq 0 \; , \quad A \neq 0 \qquad q_{\min-I} \leq q_{I-opt-1}, q_{I-opt-2} \leq q_{\max-I} \quad (26)$$

From those two solutions, it is examined which one satisfies (14), in combination with (15), (16), (18), (19). Alternatively, in a simpler manner, the total power of the hydroelectric plant is calculated through (12) and the one of the highest value is selected.

In special cases, individual equations are solved, such as:

$$\text{If} \quad A = 0 \quad \Rightarrow q_{I-opt} = -\frac{C}{B} \quad (27)$$

$$\text{If} \quad B^2 - 4 \cdot A \cdot C < 0 \quad \Rightarrow q_{I-opt} \approx -\frac{B}{A} \quad (28)$$

$$\text{If} \quad q_{\min-I} > q_{I-opt-1} \quad \Rightarrow q_{I-opt-1} = q_{\min-I} \quad (29)$$

$$\text{If} \quad q_{\max-I} < q_{I-opt-1} \quad \Rightarrow q_{I-opt-1} = q_{\max-I} \quad (30)$$

$$\text{If} \quad q_{\min-I} > q_{I-opt-2} \quad \Rightarrow q_{I-opt-2} = q_{\min-I} \quad (31)$$

$$\text{If} \quad q_{\max-I} < q_{I-opt-2} \quad \Rightarrow q_{I-opt-2} = q_{\max-I} \quad (32)$$

Consequently, having calculated the flow rate of hydro-turbine “I”, q_I, the flow of hydro-turbine “II”, q_{II}, is calculated (as q_{in}-q_I), while simultaneously satisfying its respective flow limits (between the values q_{min-II} and q_{max-II}), since from the start, the available flow q_{in}, is greater than (q_{min-I}+q_{min-II}) and lower than or equal to (q_{max-I}+q_{max-II}).

IV. Case Study

A. General Remarks

For the application of any operating method, the efficiency curve of each turbine (given by the manufacturers' data [16]), is initially approximated, in the form of a quadratic flow function, using the least squares method. In the case of a Francis hydro-turbine, the following curve results, with a correlation coefficient R^2 equal to 0.9874, for a flow q from 50% to 115% of the respective nominal flow q_{nom}:

$$\eta_{Francis}(q) = -0.4403 \cdot \left(\frac{q}{q_{nom}} \right)^2 + 0.9302 \cdot \frac{q}{q_{nom}} + 0.4339 \quad (33)$$

In the case of a Pelton hydro-turbine, the following curve results, with a correlation coefficient R^2 equal to 0.8137, for a flow q from 15% to 115% of the respective nominal flow q_{nom}:

$$\eta_{Pelton}(q) = -0.4147 \cdot \left(\frac{q}{q_{nom}} \right)^2 + 0.7395 \cdot \frac{q}{q_{nom}} + 0.5751 \quad (34)$$

Fig. 1, summarizes the above.

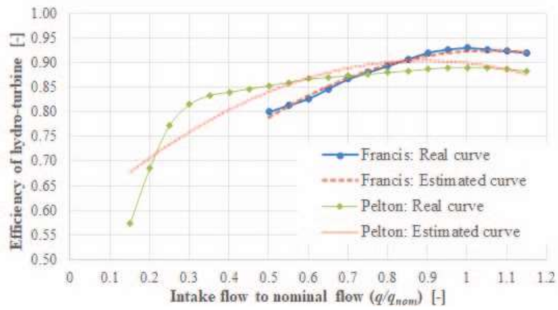


Fig. 1. Francis and Pelton turbine efficiency, with respect to flow: real curve (manufacturer's data), estimated curve (quadratic equation/least-squares method).

The typical characteristics of the hydroelectric plant under study are the following:

- the initial available gross head H_{gross}, is equal to 150 m,
- the internal circular cross-section d_{in} of the penstock, of nominal diameter D1400, is equal to 1404.92 mm, with a roughness e equal to 0.1 mm,
- the coefficient of local losses z_{tot} (due to the existence of bends, valves, contractions/expansions etc.) is equal to 4,
- η_{gen}, η_{tr}, the efficiency degrees of the generator and the transformer, are equal to 96.5% and 99% respectively.

The following hydro-turbine scenarios are indicatively examined, as listed in Table I.

Then, for each scenario, the hierarchical, the synergetic and the proposed method are examined, where the comparison of the methods is made for different available flows, from Q_{min} =0 to Q_{max} =6.60 m^3/s with a step dQ=0.01 m^3/s. The benefit in using method ‘’a’’, over method ‘’b’’, is quantified through the power difference P_a-P_b and, in aggregate, through the mean power difference, which is calculated as follows:

$$P_{a-b} = \frac{1}{n_{pop}} \cdot \sum_{i=1}^{n_{pop}} \left(P_{a,i} - P_{b,i} \right) \quad (35)$$

where the total number of terms is equal to:

$$n_{pop} = \left\lceil \frac{Q_{\max} - Q_{\min}}{dQ} \right\rceil + 1 \quad (36)$$

where the 1st term corresponds to flow Q_{min} and the n_{pop}^{th} term to Q_{max}. Essentially, this would be the improvement in average power produced, if the flow duration curve presented a uniform probability density distribution, between Q_{min} and Q_{max}. In the present case, the following power differences are identified:

- difference in power produced in the synergetic method, with respect to the hierarchical $P_{syn-hier}$,
- difference in power produced in the proposed method, with respect to the hierarchical $P_{prop-hier}$,
- difference in power produced in the proposed method, with respect to synergetic $P_{prop-syn}$.

TABLE I. BASIC SCENARIOS FOR THE CONFIGURATION OF A TWO-TURBINE HYDROELECTRIC PLANT

Scenario	Turbine I		Turbine II	
	Kind of turbine	q_{nom-I} [m³/s]	*Kind of turbine*	q_{nom-II} [m³/s]
A	*Francis*	4.552	*Francis*	0.616
B	*Francis*	2.584	*Francis*	2.584
C	*Francis*	4.552	*Pelton*	0.616

B. Scenario "A"

In the present case, it is found that the synergetic method gives better results, than the hierarchical, within the flow range [5.24, 5.94] in m^3/s, while in other ranges there is no difference. This is due to the different flow distribution (and by extension to the distribution of produced power), between the two hydro-turbines in operation. Accordingly, the proposed method gives better results than the hierarchical and synergetic methods within the flow range [5.12, 5.94] in m^3/s, while in the other ranges there is no difference. Especially, in the proposed method, the operation of the two hydro-turbines is optimally adjusted within the flow range [5.12, 5.23] in m^3/s, while the other two utilize only hydro-turbine ''I''. Within the flow range [5.24, 5.94] in m^3/s, the differences between the synergetic and proposed methods are smaller, because both hydro-turbines are operating, where the synergetic gives close flow settings to those of the proposed method. The aforementioned is evident in Fig. 2, which shows the power produced by applying each method, and in Fig. 3, where the differences in power produced are shown. Collectively, the respective average power differences are recorded in Table 2, where the improvement with either the synergetic or the proposed method, over the hierarchical, is more significant, with 11.1 and 11.6 kW respectively. On the contrary, the improvement with the proposed method compared to the synergetic is small (only 0.5 kW).

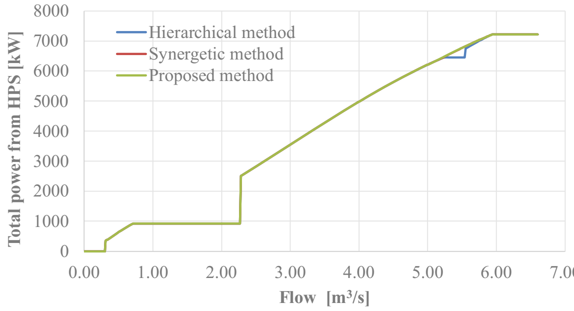


Fig. 2. Hydro-turbine power production, of scenario "A", with respect to flow, for hierarchical, synergetic and proposed methods.

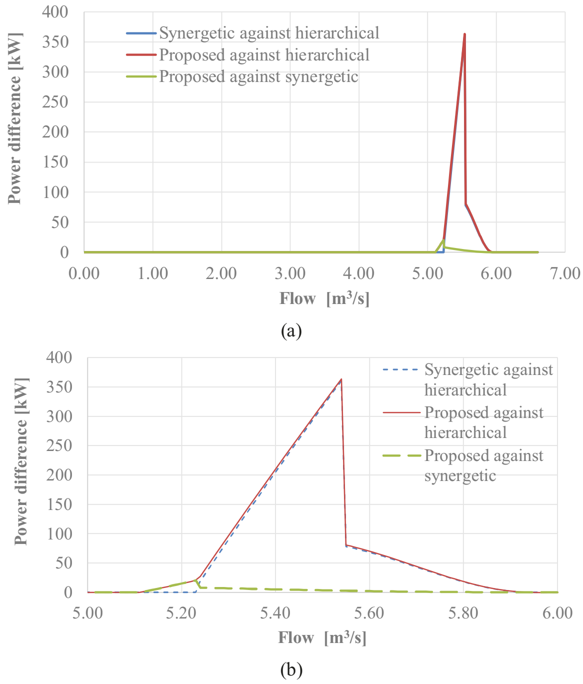


Fig. 3. Difference in turbine power produced, of scenario "A", with respect to flow, comparing synergetic with hierarchical, proposed with hierarchical and proposed with synergetic methods: (a) full form, (b) augmentation of non-zero values of power produced difference.

C. Scenario "B"

In the present case, of two identical hydro-turbines, it is found that the synergetic method gives the same results as the hierarchical, since the use of either hydro-turbine, within the flow ranges where the two methods differ, would yield the same results. On the contrary, the proposed method gives better results than the hierarchical and the synergetic, within the flow range [3.25, 5.94] in m^3/s, while in the other ranges there is no difference. Especially within the flow range [3.25, 4.26] in m^3/s, the operation of the two hydro-turbines is optimally adjusted, according to the proposed method, with equal flow distribution, while the other two methods use only one hydro-turbine in high power production mode. Within the flow range [4.27, 5.94] in m^3/s, the differences are limited, as in all methods the operation of both hydro-turbines is suggested (although with a different flow distribution). The aforementioned is evident in Fig. 4, which shows the power produced by applying each method, and in Fig. 5, where the differences in produced power are shown. The respective average power differences are listed in Table 2, where the improvement with the proposed method, compared to the hierarchical (or synergetic) method is quite significant, reaching 128.9 kW.

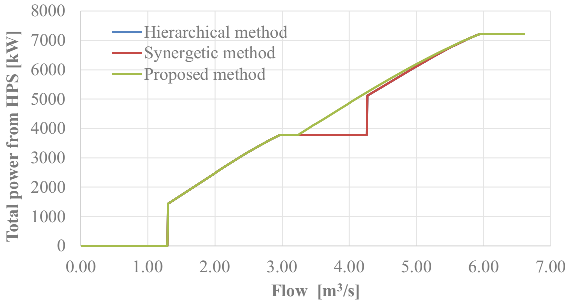


Fig. 4. Hydro-turbine power production, of scenario “B”, with respect to flow, for hierarchical, synergetic and proposed methods.

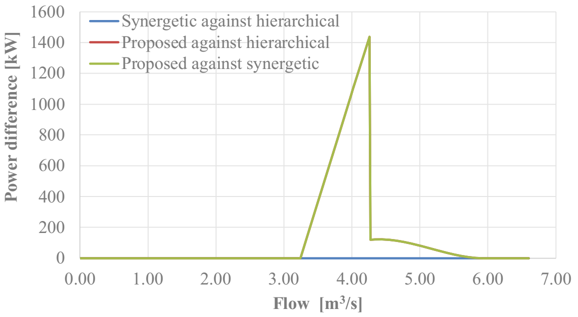


Fig. 5. Difference in turbine power produced, of scenario “B”, with respect to flow, comparing synergetic with hierarchical, proposed with hierarchical and proposed with synergetic methods.

D. *Scenario “C”*

In the present case, it is found that the synergetic method yields better results than the hierarchical, within the flow range [5.26, 5.65] in m^3/s and worse within the flow ranges [5.24, 5.25] and [5.66, 5.94] in m^3/s, while in the rest ranges there is no difference. This is due to the different flow distribution (and by extension to the distribution of produced power), between the two hydro-turbines in operation. Accordingly, the proposed method gives better results with respect to the hierarchical, within the flow range [5.17, 5.84] in m^3/s (while in the rest ranges there is no difference), as well as with the synergetic, within the flow range [5.17, 5.94] in m^3/s (while in the rest ranges there is no difference). Especially, within the flow range [5.17, 5.23] in m^3/s, the operation of the two hydro-turbines is optimally adjusted, according to the proposed method, while the other two methods only use hydro-turbine “I’’. Within the flow range [5.24, 5.84] in m^3/s, the differences between hierarchical and proposed method vary strongly, between 0 to 109 kW, since both hydro-turbines are operating. In particular, the hierarchical method stabilizes the flow to hydro turbine “I”, in contrast to the proposed method, which optimizes the distribution between the two. In the flow range [5.24, 5.94] in m^3/s, the differences between the synergetic and proposed method start at 40 kW and then drop, as the flow increases, because both hydro-turbines are operating, where the synergetic stabilizes the flow to hydro-turbine “II” and gives flow settings close to those of the proposed method (but not optimal). The aforementioned is evident in Fig. 6, which shows the power produced by applying each method, and in Fig. 7, where the differences in power produced are shown.

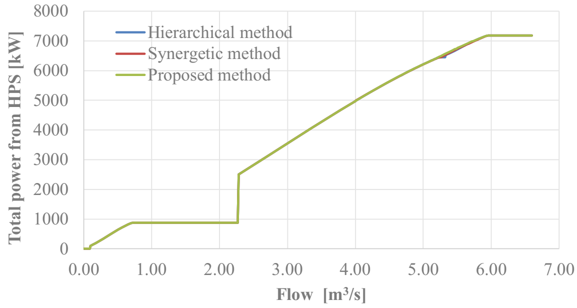
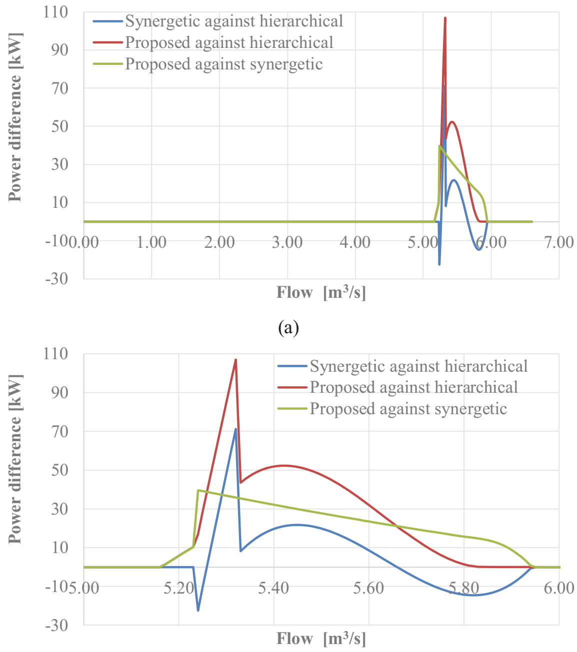


Fig. 6. Hydro-turbine power production, of scenario “C”, with respect to flow, for hierarchical, synergetic and proposed methods.



(a)

(b)

Fig. 7. Difference in turbine power produced, of scenario “C”, with respect to flow, comparing synergetic with hierarchical, proposed with hierarchical and proposed with synergetic methods: (a) full form, (b) augmentation of non-zero values of power produced difference

To sum up, the respective average power differences are listed in Table 2, where the improvement with the synergetic, compared to the hierarchical method is very small (only 0.66 kW), whilst the improvement with the proposed method, against the synergetic and hierarchical is limited to 3.28 and 2.62 kW respectively.

TABLE II. POWER PRODUCED AVERAGE DIFFERENCES OF SCENARIOS "A", "B" AND "C", COMPARING SYNERGETIC WITH HIERACHICAL, PROPOSED WITH HIERCHICAL, PROPOSED WITH SYNERGETIC METHODS

Scenario	$P_{syn\text{-}hier}$ [kW]	$P_{prop\text{-}hier}$ [kW]	$P_{prop\text{-}syn}$ [kW]
A	11.066	11.567	0.501
B	0.000	128.874	128.874
C	0.664	3.282	2.618

E. Comparison

From the comparison of the three operation methods, it is clear that in the three scenarios examined, the proposed approach is systematically advantageous, as can also be seen in the comprehensive data of Table 2 and Figs. 3, 5 and 7. Especially, in the case of identical hydro-turbines (scenario "B") the improvement is significant. In the remaining scenarios, there are specific high flow ranges, where both hydro-turbines operate and the proposed method slightly improves the total produced power of the hydroelectric plant.

V. CONCLUSIONS

In this paper, the method ensuring an optimal operation of the two turbines was proposed, by considering all possible combinations of operation of the turbines, depending on flow incoming. When it is possible to operate both hydro-turbines, the flow is distributed according to the optimal energy mode, with the appropriate use of a derivative (having expressed the efficiency curve as a quadratic equation of the flow) and from all combinations, the power production mode, achieving the highest value, is selected. This results in the formation of a suitable lookup table for the optimal operation of an HPS, with which the hydroelectric plant can be adjusted, in advance, to produce the maximum hydroelectric power in each case. The proposed method is compared with the hierarchical and synergetic methods (which have been analysed in relevant literature). Of course, the problem of the optimal design of the hydro-turbines (type of turbine, nominal power) is not addressed, based on technical and economic criteria, but only that of their optimal operation, given the configuration of the hydroelectric plant. For the three scenarios examined, it was found that the proposed method improves (or maintains) the power produced by the hydroelectric plant, when compared to the two pre-existing methods, especially in the case of using two identical hydro-turbines.

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