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Optimal Operation of a Run-of-River Small Hydropower Plant with Two Hydro-Turbines

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August 22 – 24, Marathon Beach, Athens

7th Internal Conference on Mathematics and Computers in Sciences and Industry

Since 1990, there has been an extensive research activity around the optimal design, operation and efficiency of these projects, with particular emphasis on issues such as:

1. the optimal power size of the hydroelectric plant,
2. the development of economic and energy evaluation indicators,
3. the optimal overall plant design,
4. the design, operation and efficiency improvement of hydro-turbines,
5. the effect of the water flow as well as of the general hydrological behavior of the catchment area, on the efficiency of the hydroelectric plant

In the last five years, research has been combined with the utilization of data from other disciplines. Some examples are:

1. Using of geo-information systems and climate data,
2. Solving problems related to the liberalization of the electricity market, such as short-term power forecasting with stochastic models using prior power and precipitation
3. The medium-term power forecast, based on the climatic data of the region

Several studies for **Optimal Operation** of a Run-of-River Small Hydropower Plant with Two Hydro-Turbines

1. **Hierarchical**, in which the main hydro-turbine (usually the one of larger power capacity) operates, whilst the second one (usually the small one) works in addition, with the remaining water not utilized by the main turbine.

2. **Synergetic mode**: where intuitively it has been suggested to extend the operation of the second hydro-turbine in certain flow ranges, at the expense of the main hydro turbine, which, however, for the same flow fluctuations presents smaller efficiency fluctuations, so as to achieve a greater generated-power overall. In this method, particular calculations are made, in terms of energy efficiency, increasing the amount of power produced intuitively, having understood, as engineers, the synergetic mode in which the two hydro-turbines operate.

Proposal on **Optimal Operation** of a Run-of-River Small Hydropower Plant with Two Hydro-Turbines

Considering all possible combinations of operation of the turbines depending on the level of flow. When it is possible to operate both turbines, the flow is distributed according to the optimal energy mode, using the derivative, and the power generation mode with the maximum value is selected.

The only requirement to achieve an easy mathematical solution is to use **an analytical (more specifically, quadratic) equation efficiency-flow curve for the turbine.**

A hydroelectric station is given, consisting of two hydro-turbines, **the main "I"** and **the secondary "II",** which are supplied through the same penstock of a practically constant available gross head. The electrical powers produced from the hydro-turbines "**I**" $\boldsymbol{P_I}$ and **"II"** $\boldsymbol{P_{II}}$ are given respectively by the below equations:

$$P_I\left(q_I\right) = \eta_{gen} \cdot \eta_{tr} \cdot \eta_I\left(q_I\right) \cdot \rho \cdot g \cdot q_I \cdot H\left(q_I + q_{II}\right) \qquad (1)$$

$$P_{II}\left(q_{II}\right) = \eta_{gen} \cdot \eta_{tr} \cdot \eta_{II}\left(q_{II}\right) \cdot \rho \cdot g \cdot q_{II} \cdot H\left(q_I + q_{II}\right) \qquad (2)$$

Where:

g is the gravitational acceleration (=9.81m/s^2),
ρ is the water density (=999.7 kg/m^3),
η_{gen}, η_{tr} are the degrees of efficiency of the generator and the transformer respectively (practically they are considered as having constant values, compared to the changes of the efficiency degrees of the hydro-turbines,
q_I, q_{II} are the water flows utilized by the respective hydro-turbines,
H is the available net head

η_I, η_{II} are the degrees of efficiency, expressed **as quadratic equations within the operating range of flow of hydro-turbines** and are given from the below equations:

On Condition there is no flood

$$\eta_I(q_I) = \begin{cases} a_I \cdot \left(\frac{q_I}{q_{nom-I}}\right)^2 + b_I \cdot \frac{q_I}{q_{nom-I}} + c_I & q_{min-I} \le q_I \le q_{max-I} \\ & \text{or } q_{max-I} < q_I \ \underline{\text{no flood}} \\ & q_I := q_{max-I} \\ 0 & q_I < q_{min-I} \text{ or flood} \end{cases} \quad (3)$$

On Condition there is no flood

$$\eta_{II}(q_{II}) = \begin{cases} a_{II} \cdot \left(\frac{q_{II}}{q_{nom-II}}\right)^2 + b_{II} \cdot \frac{q_{II}}{q_{nom-II}} + c_{II} & q_{min-II} \le q_{II} \le q_{max-II} \\ & \text{or } q_{max-II} < q_{II} \ \underline{\text{no flood}} \\ & q_{II} := q_{max-II} \\ 0 & q_{II} < q_{min-II} \text{ or flood} \end{cases} \quad (4)$$

Where q_{min-I} is the minimum permissible operating flow of hydro-turbine “I”(e.g., 15% for Pelton, 50% for Francis), q_{max-I} is the maximum permissible operating flow of hydro turbine “I” (e.g., 115 % for Pelton or Francis), q_{nom-I} is the nominal operating flow of hydro-turbine “I”, a_I, b_I, c_I are the respective coefficients of the quadratic equation, while the respective parameters of hydro-turbine “II”, q_{min-II}, q_{max-II}, q_{nom-II}, a_{II}, b_{II}, c_{II}, are similarly defined.

From the river there is an available flow q_{in} (having subtracted the residual (environmental) flow from the total one), out of which q_d is utilized (depending on the mode of operation), which is equal to:

$$q_d = q_I + q_{II} \quad (5)$$

I. *Proposed / Optimized Method*

In this method, all possible combinations of operation are considered, for each available flow q_{in} and the one with the highest power production is selected. In particular, for the case of the operation of two hydro-turbines, there are 2^2=**4** possibilities:

1. If q_{in} is lower than the minimum operating flow min{q_{min-I}, q_{min-II}}=q_{min-II}, or a flood phenomenon occurs, then no hydro-turbine operates and the usable flow q_d is 0.

2. If q_{in} is greater than q_{min-II} and lower than that of a flood effect, then only hydro-turbine "II" can operate. In this case, if the available flow is lower than or equal to q_{max-II}, then the usable flow q_d is equal to q_{in}, otherwise, it is equal to q_{max-II}. Furthermore, the usable flow q_d is equal to the flow of hydro-turbine "II" q_{II} and the produced power is calculated through (2), let be $P_{II}(q_{in})$, assuming that the usable flow of hydro-turbine "I" q_I, is equal to 0.

3. If q_{in} is greater than q_{min-I} and lower than that of the flood effect, then only hydro-turbine "I" can operate. In this case, if the available flow is lower than or equal to q_{max-I}, then the usable flow q_d is equal to q_{in}, otherwise it is equal to q_{max-I}. Furthermore, the usable flow q_d, is equal to the flow of hydro-turbine "I" q_I, and the produced power is calculated through (1), let be $P_I(q_{in})$, assuming that the usable flow of hydro-turbine "II" q_{II} , is equal to 0.

4. If q_{in} is greater than $(q_{min-I}+q_{min-II})$ and lower than that of the flood effect, then both hydro-turbines can operate. In this case, if the available flow is lower than or equal to $(q_{max-I}+q_{max-II})$, then the usable flow q_d is equal to q_{in} and the optimal flow distribution will be done by solving the problem:

$$\max\left(P_I\left(q_I\right)+P_{II}\left(q_{II}\right)\right) \quad \text{on condition that} \quad q_{in}=q_I+q_{II} \qquad (6)$$

II. Case Study

For the application of any operating method, **the efficiency curve of each turbine** is initially approximated, in the **form of a quadratic flow function, using the least squares method**. In the case of a Francis hydro-turbine, the following curve results, with a correlation coefficient R^2 equal to 0.9874, for a flow q from 50% to 115% of the respective nominal flow q_{nom}:

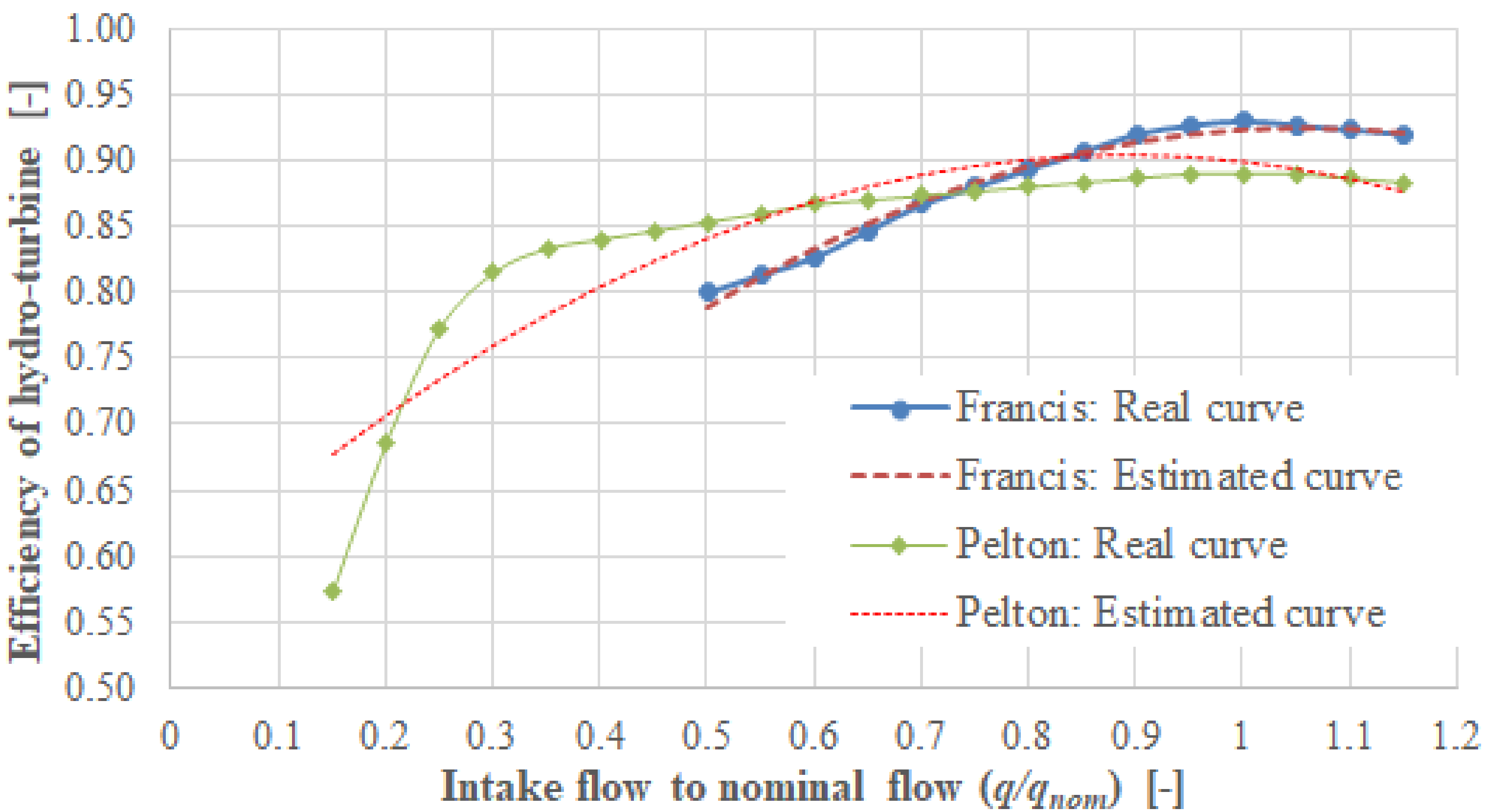
$$\eta_{Francis}(q) = -0.4403 \cdot \left(\frac{q}{q_{nom}}\right)^2 + 0.9302 \cdot \frac{q}{q_{nom}} + 0.4339 \qquad (7)$$

In the case of a Pelton hydro-turbine, the following curve results, with a correlation coefficient R^2 equal to 0.8137, for a flow q from 15% to 115% of the respective nominal flow q_{nom} :

$$\eta_{Pelton}(q) = -0.4147 \cdot \left(\frac{q}{q_{nom}}\right)^2 + 0.7395 \cdot \frac{q}{q_{nom}} + 0.5751 \qquad (8)$$

Francis and Pelton turbine efficiency, with respect to flow

- Real curve (*manufacturer's data*)
- Estimated curve (*quadratic equation/least-squares method*)



The typical characteristics of the hydroelectric plant

- the initial available gross head H_{gross}, is equal to 150 m,

- the internal circular cross-section d_{in} of the penstock, of nominal diameter D1400, is equal to 1404.92 mm, with a roughness e equal to 0.1 mm,

- the coefficient of local losses z_{tot} (due to the existence of bends, valves, contractions/expansions etc.) is equal to 4,

- η_{gen}, η_{tr}, the efficiency degrees of the generator and the transformer, are equal to 96.5% and 99% respectively.

Basic scenarios for the configuration of a Two – Turbine hydroelectric plant

	Turbine I		Turbine II	
Scenario	*Kind of turbine*	q_{nom-I} [m^3/s]	*Kind of turbine*	q_{nom-II} [m^3/s]
A	*Francis*	4.552	*Francis*	0.616
B	*Francis*	2.584	*Francis*	2.584
C	*Francis*	4.552	*Pelton*	0.616

For each scenario, the **hierarchical**, the **synergetic** and the **proposed** method are examined, where the comparison of the methods is made for different available flows, from Q_{min} =0 to Q_{max} =6.60 m^3/s with a step dQ=0.01 m^3/s. The benefit in using method ‘’a”, over method ‘’b”, is quantified through the power difference P_a-P_b and, in aggregate, through the mean power difference, which is calculated as follows:

$$P_{a-b} = \frac{1}{n_{pop}} \cdot \sum_{i=1}^{n_{pop}} \left(P_{a,i} - P_{b,i} \right) \qquad (9)$$

Where the total number of terms is equal to :

$$n_{pop} = \left\lceil \frac{Q_{\max} - Q_{\min}}{dQ} \right\rceil + 1 \qquad (10)$$

Where1st term corresponds to flow Q_{min} and the $n_{pop}{}^{th}$ term to Q_{max}..

Basic scenarios for the configuration of a Two – Turbine hydroelectric plant

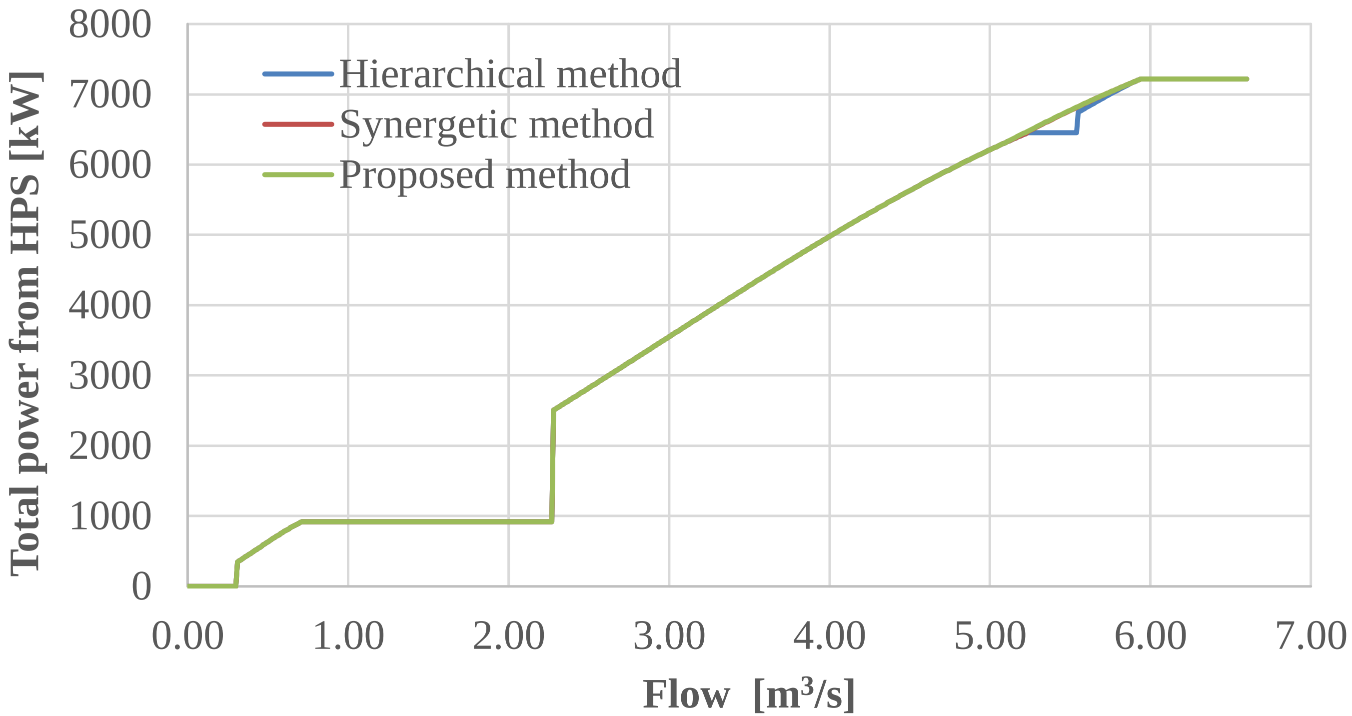
In the present case, the following power differences are identified:

- difference in power produced in the synergetic method, with respect to the hierarchical $P_{syn\text{-}hier}$,
- difference in power produced in the proposed method, with respect to the hierarchical $P_{prop\text{-}hier}$,
- difference in power produced in the proposed method, with respect to synergetic $P_{prop\text{-}syn}$.

Scenario A

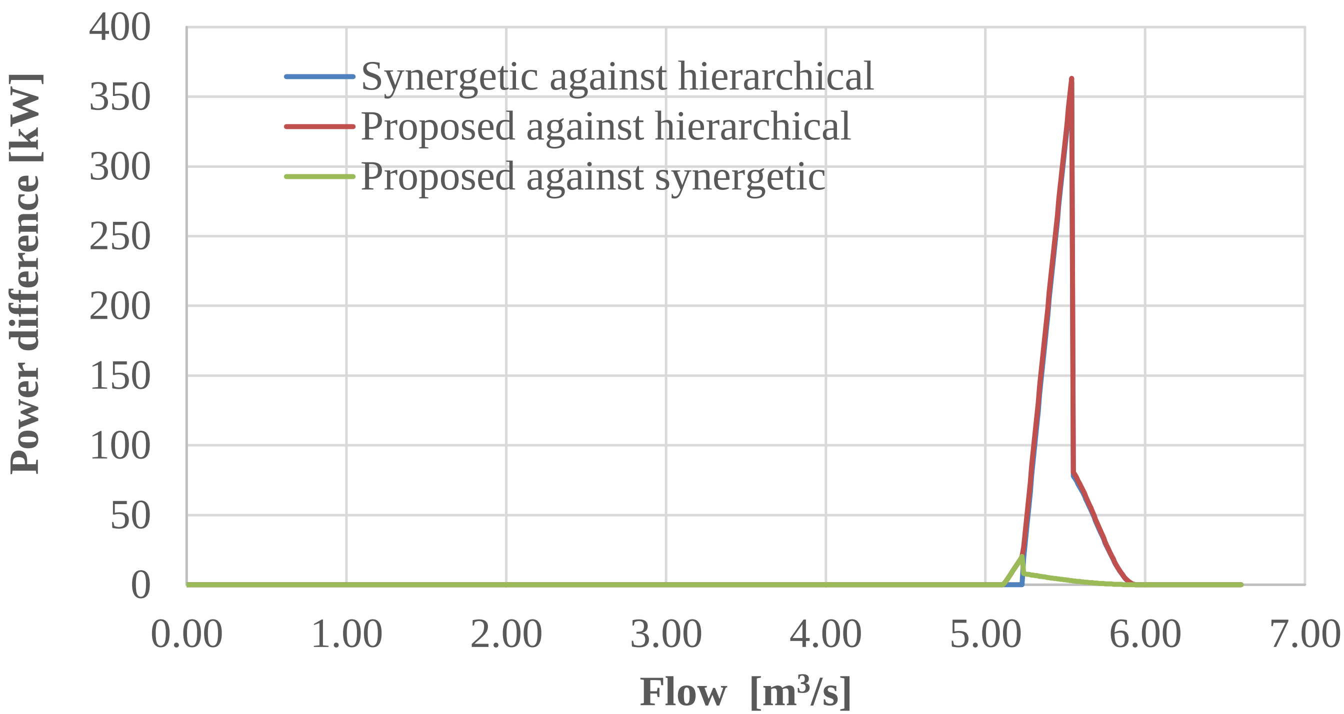
Hydro-turbine power production, of scenario "A", with respect to flow, for hierarchical, synergetic and proposed methods

	Turbine I		Turbine II	
Scenario	*Kind of turbine*	q_{nom-I} [m³/s]	*Kind of turbine*	q_{nom-II} [m³/s]
A	*Francis*	4.552	*Francis*	0.616



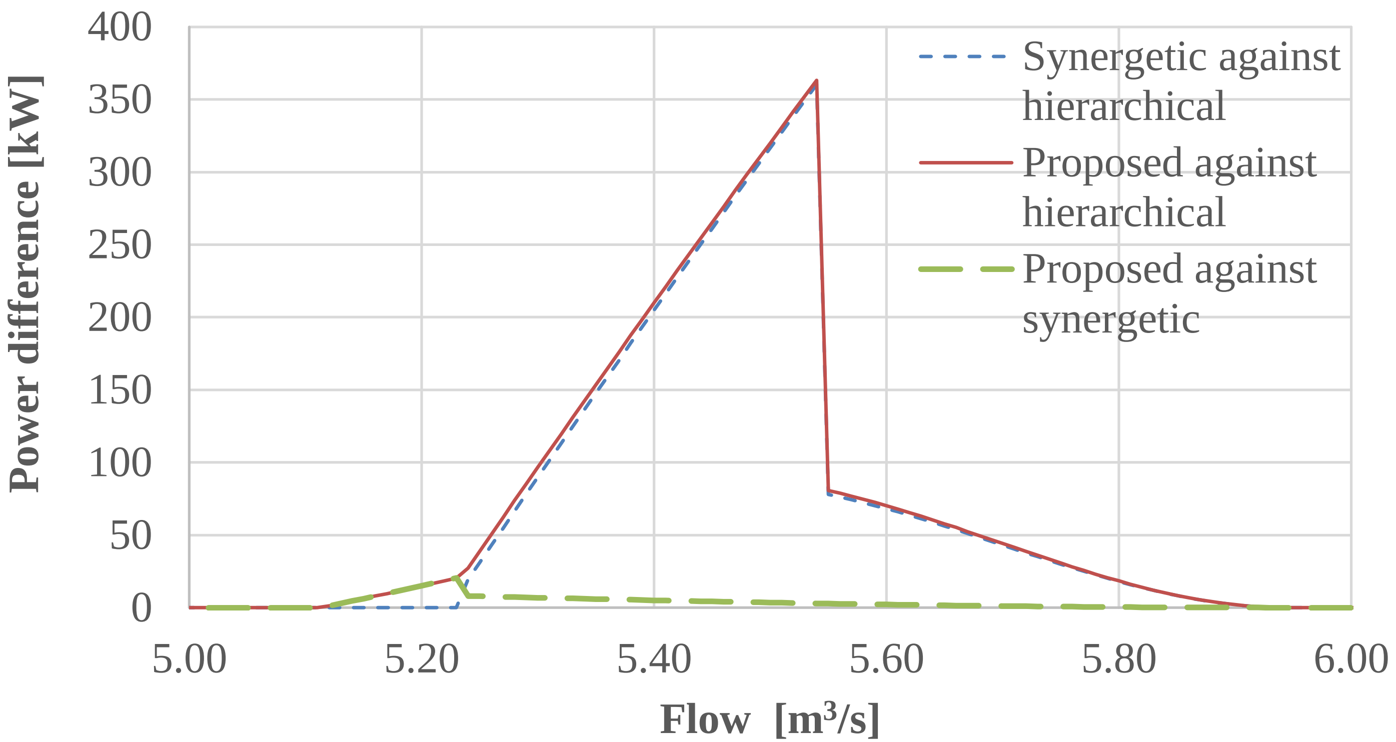
Scenario A

Difference in turbine power produced, of scenario "A", with respect to flow, comparing synergetic with hierarchical, proposed with hierarchical and proposed with synergetic methods: *full form*,



Scenario A

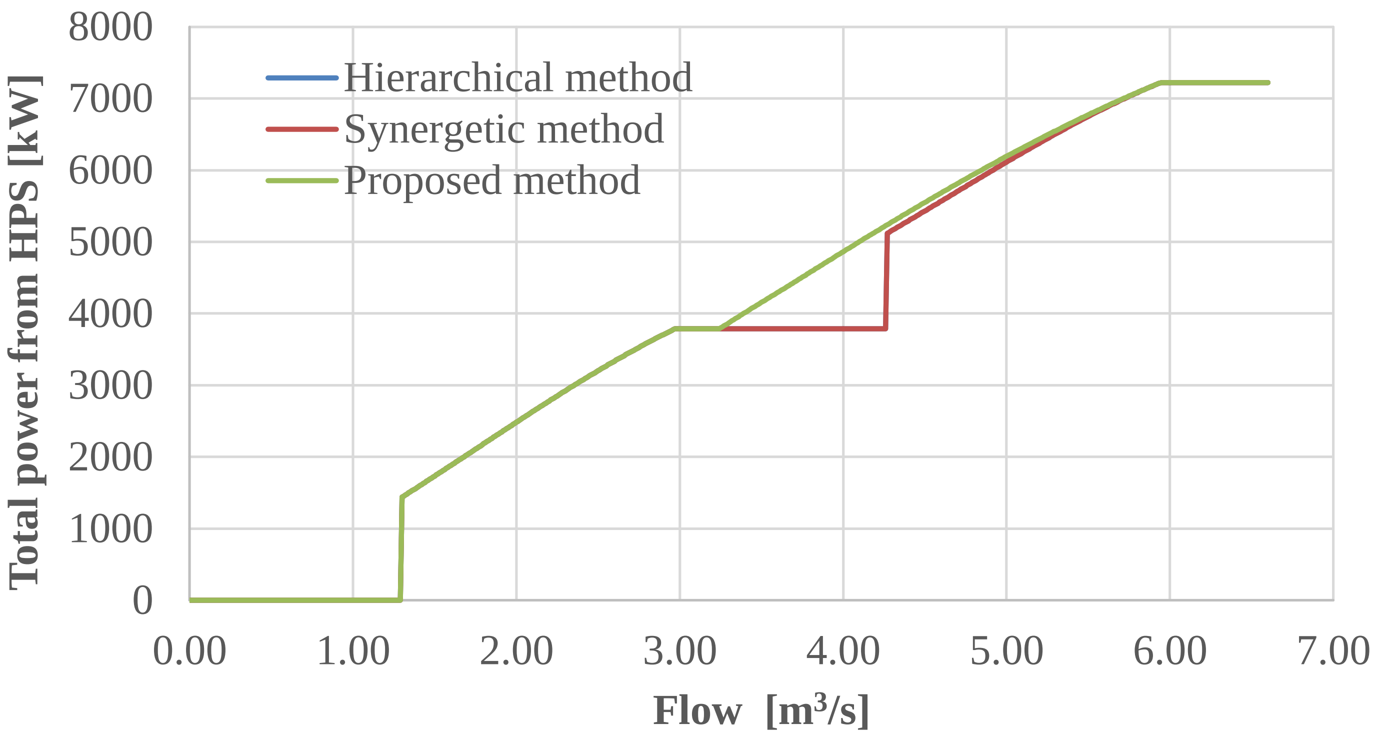
Difference in turbine power produced, of scenario “A”, with respect to flow, comparing synergetic with hierarchical, proposed with hierarchical and proposed with synergetic methods: *augmentation of non-zero values of power produced difference.*



Scenario B

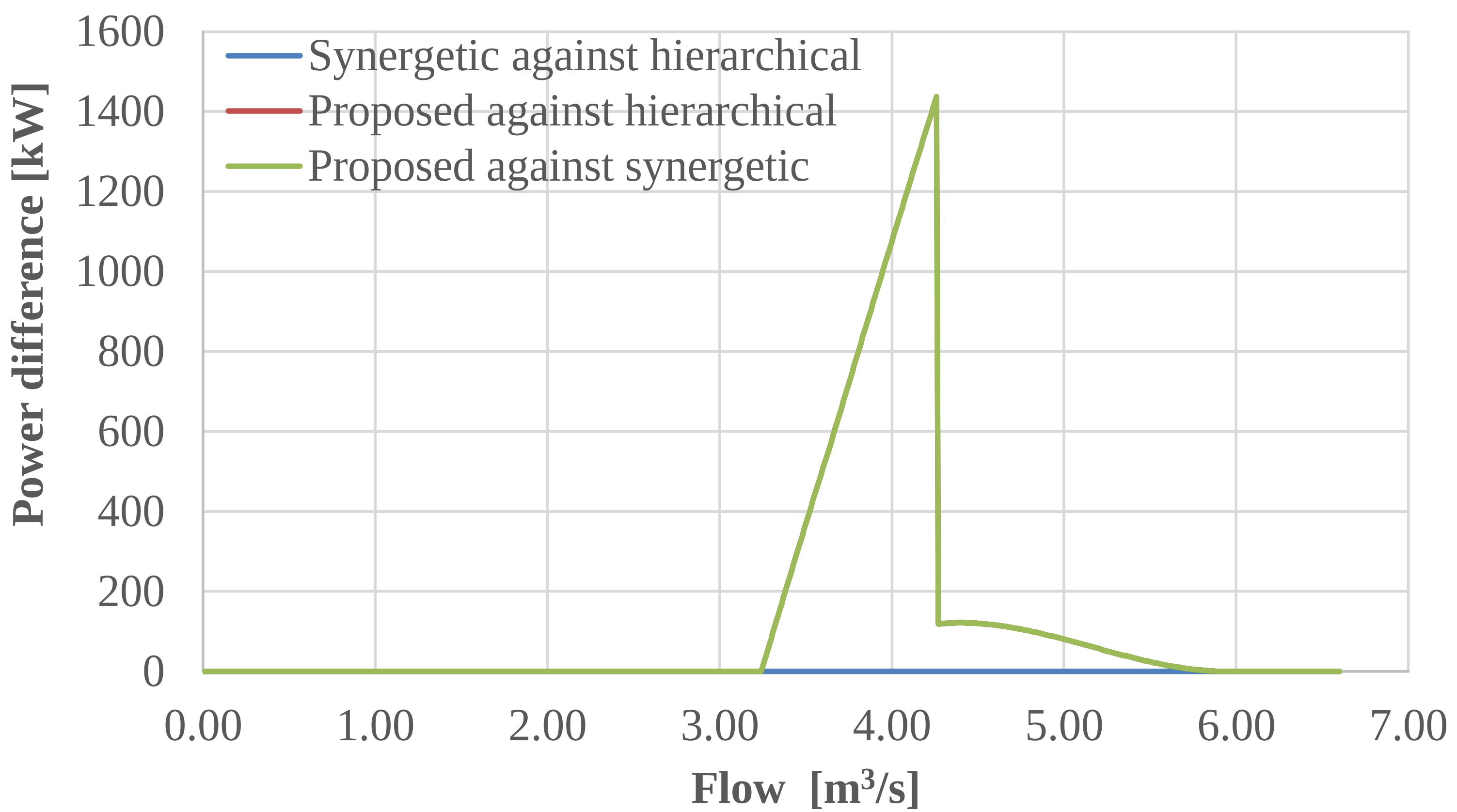
Hydro-turbine power production, of scenario “B”, with respect to flow, for hierarchical, synergetic and proposed methods

	Turbine I		Turbine II	
Scenario	*Kind of turbine*	$q_{nom\text{-}I}$ [m³/s]	*Kind of turbine*	$q_{nom\text{-}II}$ [m³/s]
B	*Francis*	2.584	*Francis*	2.584



Scenario B

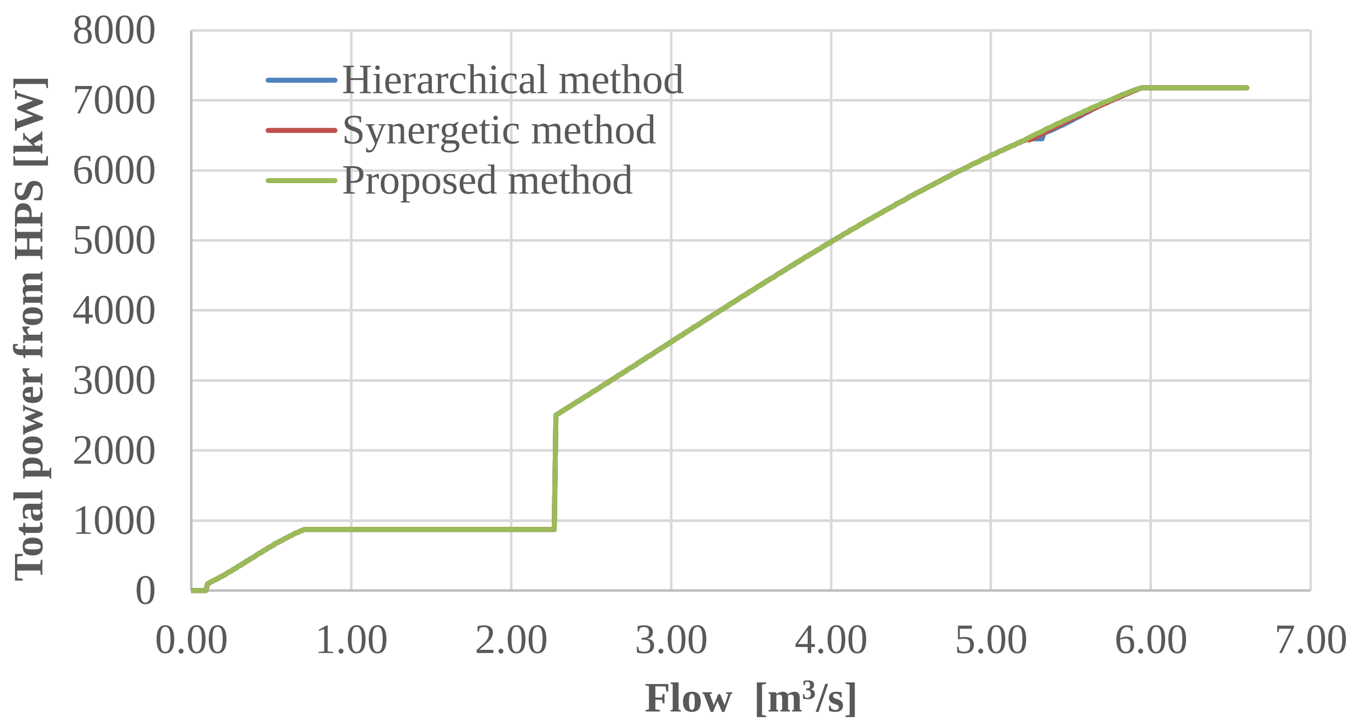
Difference in turbine power produced, of scenario “B”, with respect to flow, comparing synergetic with hierarchical, proposed with hierarchical and proposed with synergetic methods.



Scenario C

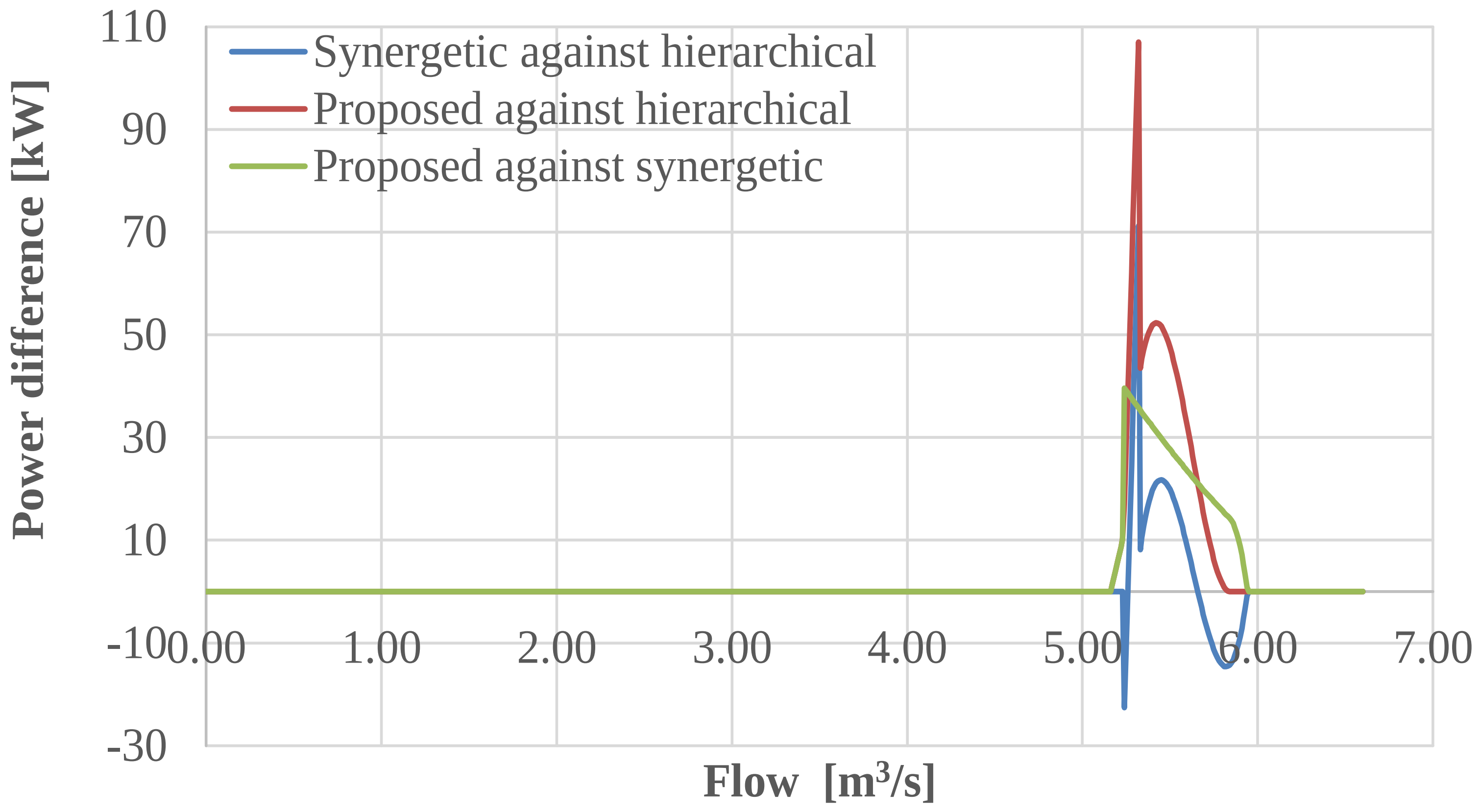
Hydro-turbine power production, of scenario "C", with respect to flow, for hierarchical, synergetic and proposed methods

	Turbine I		Turbine II	
Scenario	*Kind of turbine*	$q_{nom\text{-}I}$ [m^3/s]	*Kind of turbine*	$q_{nom\text{-}II}$ [m^3/s]
C	*Francis*	4.552	*Pelton*	0.616



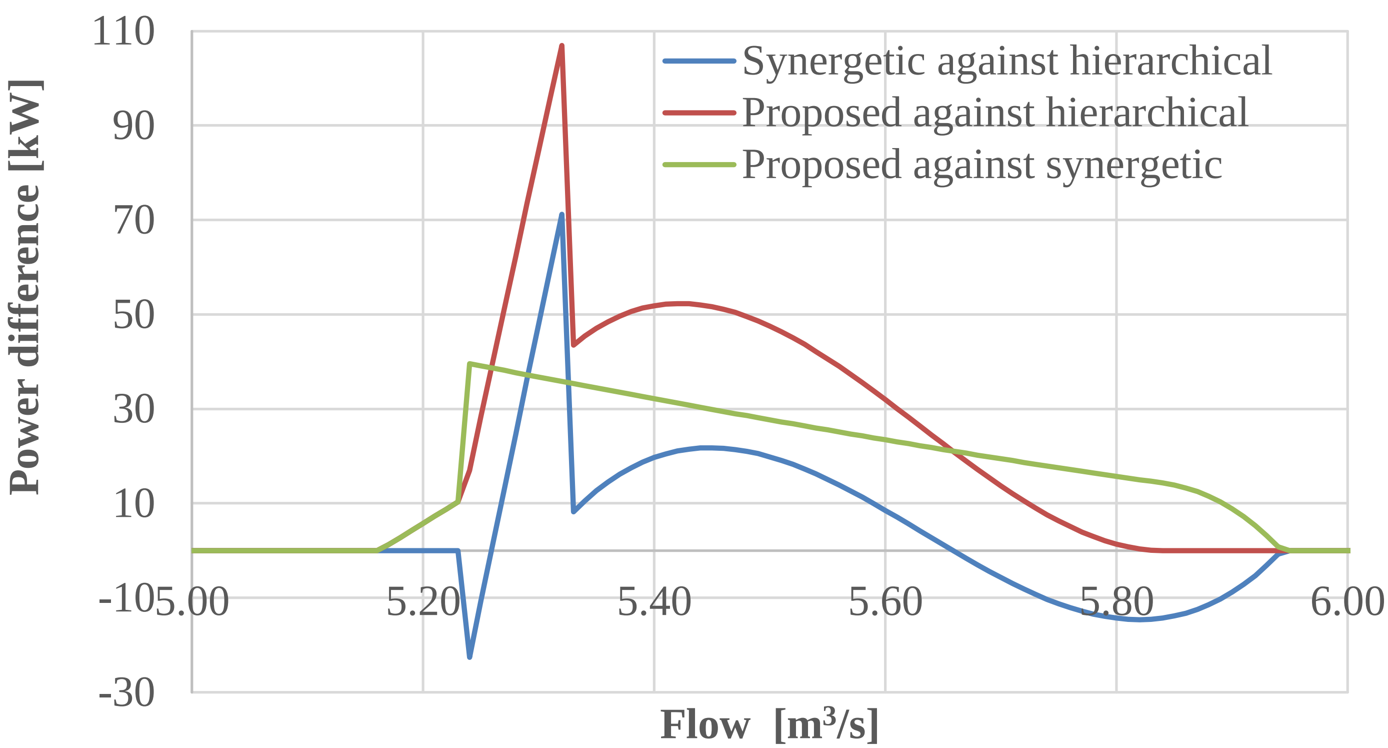
Scenario C

Difference in turbine power produced, of scenario "C", with respect to flow, comparing synergetic with hierarchical, proposed with hierarchical and proposed with synergetic methods: *full form*,



Scenario C

Difference in turbine power produced, of scenario “C”, with respect to flow, comparing synergetic with hierarchical, proposed with hierarchical and proposed with synergetic methods: *augmentation of non-zero values of power produced difference.*



Power produced average differences of scenarios “A”, “B”, “C”, comparing *Synergetic with Hierarchical, Proposed with Hierarchical, Proposed with Synergetic Methods*

Scenario	$P_{syn\text{-}hier}$ [kW]	$P_{prop\text{-}hier}$ [kW]	$P_{prop\text{-}syn}$ [kW]
A	11.066	11.567	0.501
B	0.000	128.874	128.874
C	0.664	3.282	2.618

From the comparison of the three operation methods, it is clear that in the three scenarios examined, the proposed approach is systematically advantageous, as can also be seen in the comprehensive data of above Table and Figures. Especially, in the case of identical hydro-turbines (scenario “B”) the improvement is significant. In the remaining scenarios, there are specific high flow ranges, where both hydro-turbines operate and the proposed method slightly improves the total produced power of the hydroelectric plant.

Future Steps / Studies

• Optimal Operation of a Run-of-River Small Hydropower Plant **with more than Two Hydro-Turbines**

• Optimal Operation of a Run-of-River Small Hydropower Plant **with different head gross**

• Optimal Operation of a Run-of-River Small Hydropower Plant with Two Hydro-Turbines **not only** with having expressed the efficiency curve as a **quadratic equation of the flow**

• Optimal Operation of a Run-of-River Small Hydropower Plant with Two Hydro-Turbines **target on financial analysis.**

Optimal Operation of a Run-of-River Small Hydropower Plant with Two Hydro-Turbines