WATER RESOURCES RESEARCH, VOL. 33, NO. 9, PAGES 2165–2177, SEPTEMBER 1997

A parametric rule for planning and management of multiple-reservoir systems

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Abstract. A parametric rule for multireservoir system operation is formulated and tested. It is a generalization of the well-known space rule of simultaneously accounting for various system operating goals, in addition to the standard goal of avoiding unnecessary spills, including avoiding leakage losses, avoiding conveyance problems, taking into account the impacts of the reservoir system topology, and assuring satisfaction of secondary uses. Theoretical values of the rule's parameters for each one of these isolated goals are derived. In practice, parameters are evaluated to optimize one or more objective functions selected by the user. The rule is embedded in a simulation model so that optimization requires repeated simulations of the system operation with specific values of the parameters each time. The rule is tested on the case of the multireservoir water supply system of the city of Athens, Greece, which is driven by all of the operating goals listed above. Two problems at the system design level are tackled. First, the total release from the system is maximized for a selected level of failure probability. Second, the annual operating cost is minimized for given levels of water demand and failure probability. A detailed simulation model is used in the case study. Sensitivity analysis of the rule's parameters revealed a subset of insensitive parameters that allowed for rule simplification. Finally, the rule is validated through comparison with a number of heuristic rules also applied to the test case.

1. Introduction

Planning and management of multiple reservoir systems have been and continue to be the subject of numerous research works. This attention is due to the essential benefits arising from reservoir system operation (e.g., hydropower) in combination with the reduction of natural risks (e.g., flood control). The problem of reservoir planning and/or management is most often stated as an optimal control problem. Its solution is not an easy task because of the large number of variables involved, the nonlinearity of the system dynamics, the stochastic nature of future inflows, and other uncertainties of the system (e.g., leakage from reservoirs).

Stochastic dynamic programming (SDP) has been repeatedly used by many researchers to study the problem [e.g., *Su and Deininger*, 1972, 1974; *Askew*, 1974a, b; *Sniedovich*, 1979; 1980a, b; *Bras et al.*, 1983; *Stedinger et al.*, 1984]. SDP could be satisfactory if it did not require excessive amounts of computer time and storage. To increase the efficiency of the solution algorithm, some researchers have treated the inflows' stochasticity in an analytic way without state-space discretization and then applied efficient deterministic optimization methods [*Wasimi and Kitanidis*, 1983; *Loaiciga and Mariño*, 1985; *Georgakakos and Marks*, 1987]. For example, *Georgakakos and Marks* [1987] represented the reservoir system dynamics in a state-space form and proposed an extension of stochastic control theory, which they termed extended linear quadratic Gaussian (ELQG). In this way these authors obtained a very efficient algorithm at the expense of an accurate representation of the stochastic structure of inflows (i.e., only Gaussian independent inflows were considered). In later studies [*Georgakakos*, 1989] the problem of the representation of the stochastic structure of inflows was effectively tackled. Other researchers continued their studies in the direction of stochastic dynamic programming with the purpose of remedying its deficiencies. Efficient interpolation schemes for dynamic programming (DP) algorithms are discussed by *Johnson et al.* [1993]. The problem of errors resulting from the state-space discretization in discrete dynamic programming was tackled [*Kitanidis and Foufoula-Georgiou*, 1987; *Foufoula-Georgiou and Kitanidis*, 1988; *Foufoula-Georgiou*, 1991]. These authors proposed gradient dynamic programming, which is based on an interpolation scheme of the cost-to-go function at each stage and reduces significantly the error due to discretization.

In spite of the large number of optimization techniques available in the literature, simulation models still remain the primary tool for reservoir planning and management studies in practice. The reason is that simulation models allow a more detailed and faithful representation of the system studied than optimization techniques do [*Loucks and Sigvaldason*, 1982]. Moreover, they can be easily combined with synthetically generated streamflow sequences [*Young*, 1967; *Loucks et al.*, 1981, p. 277]. The main drawback of simulation is that unlike optimization, it requires prior specification of the system operating policy. To remedy this problem, *Young* [1967] combined the use of synthetically generated annual inflows into a single reservoir with deterministic dynamic programming and inferred simple parametric rules for the operating policy using regression techniques. Other researches have employed optimization methods within simulation models [*Evenson and Moseley*, 1970; *Sigvaldason*, 1976; *Ginn and Houck*, 1989; *Johnson et al.*, 1991; *Tejada-Guibert et al.*, 1993]. *Tejada-Guibert et al.* [1993] compared two alternative approaches for defining the operation

Paper number 97WR01034.
0043-1397/97/97WR-01034$09.00

policy of a multireservoir system: (1) interpolation in policy tables derived through SDP and (2) the use of SDP-derived value functions within simulation to optimize the operating policy each time a decision is sought; they found this second approach clearly superior. *Johnson et al.* [1991] used a simulation model, which includes heuristic operating rules that are optimized within the simulation for each period of operation. The optimization tries to drive the real storages as close to the target storages as possible.

Operators of reservoir systems have long used heuristic rules that define desired storage and release targets. The well-known space rule [*Bower et al.*, 1962] defines storage targets so that the empty space in each reservoir is proportional to the expected inflow; this rule is applicable to parallel reservoirs for water supply purposes. The NYC rule, used for the water supply of New York City, defines storage targets so that the probability of spill from each reservoir will be equal for all reservoirs [*Clark*, 1950, 1956]. *Johnson et al.* [1991] showed how heuristic operating policies, including the space rule, can be effectively used in optimization models.

The aim of this work is to propose and test a parametric operating rule for a system of reservoirs. The parameters of the rule are estimated by optimization, using simulation to evaluate the objective function value for each trial set of parameter values. The rule is a generalization of, and is motivated by, the space rule to simultaneously account for various goals: (1) avoiding unnecessary spills, (2) avoiding leakage losses, (3) avoiding conveyance problems, (4) taking into account the impacts associated with the reservoir system topology, and (5) assuring satisfaction of secondary uses. These goals are achieved through parameterizing the rule and then optimizing its parameters. For each parameter set a series of simulations of the system operation allows system objectives to be evaluated and constraints to be satisfied. Parameters are optimized outside the simulation, or else, for each set of parameter values in the optimization a simulation is performed. The three-reservoir system used for the water supply of the greater Athens area, Greece, is selected as a test case. For validation purposes the operating rule is compared with a number of heuristic rules.

The paper is organized in four sections. In section 2 we present the proposed parametric operating rule; we derive theoretical values for its parameters in five special cases; we discuss other theoretical issues raised; and then we describe the optimization and the simulation model used. In section 3 we analyze an application of this rule for a real-world reservoir system, and we assess the capabilities of the proposed rule in comparison with heuristic operating rules. Section 4 summarizes the proposals and tests made and presents the final conclusions.

2. Parametric Rule

2.1. Description of the Rule

A system of N reservoirs is assumed for which an operating policy is sought. The policy is focused on consumptive water uses such as water supply for domestic and industrial use and irrigation. Other uses such as hydropower generation, recreation, or navigation are assumed absent or of secondary importance in this study. Our approach, however, can easily accommodate such nonconsumptive uses. The reservoirs are connected in series or in parallel to form a network with any topology. Water is withdrawn from all of them to meet a common downstream target release D (equal to the water demand). The continuity equation for each reservoir i is given for a certain time period by

$$S_i = \mathrm{BS}_i + Q_i - R_i - L_i - \mathrm{SP}_i \tag{1}$$

where BS_i is the beginning-of-period storage for reservoir i (known); S_i is the end-of-period storage, which is unknown; Q_i is the inflow; R_i is the total release from the reservoir; L_i is the total loss due to evaporation and leakages, and SP_i is the reservoir spill. Reference to time interval is omitted for convenience.

Let V denote the total storage in the system at the end of the time period of interest. In the simple case of one reservoir, V is completely determined by (1), in which case we omit the subscript i and replace S with V. The operation of a system of N reservoirs is much more complicated as, this time, the state of the system is described by N variables S_i, satisfying

$$\sum_{i=1}^{N} S_i = V \tag{2}$$

Assuming that the target release is fulfilled and the inflows, losses, and spills from all reservoirs are estimated in some manner, the total end-of-period storage of the system is given by

$$V = \sum_{i=1}^{N} (\mathrm{BS}_i + Q_i - L_i - \mathrm{SP}_i) - D \tag{3}$$

Thereafter the problem is to determine the releases from all reservoirs such that their sum equals D. Equivalently, the problem is to distribute the total volume V into the N reservoirs such that (3) is satisfied. This can be done in numerous ways, as the problem has several degrees of freedom. We call a specific way to perform this distribution an operating rule. To avoid ambiguity, we express the operating rules by means of some quantities S_i^*, which stand for the target storage for the reservoir i at the end of the period. The real storage S_i is generally different from the target storage S_i^* because of the physical constraints that were not considered in the determination of S_i^*. We propose to distribute V according to the following rule:

$$S_i^* = a_i + b_i V \tag{4}$$

where a_i and b_i, $i \in \{1, \cdots, N\}$, are unknown parameters.

There exist $2N$ parameters for a system of N reservoirs. We note that because of (2) we have two constraints on the parameters, i.e.,

$$\sum_{i=1}^{N} a_i = 0 \qquad \sum_{i=1}^{N} b_i = 1 \tag{5}$$

and thus the number of unknown parameters is finally $2(N - 1)$. It will be shown in the next subsection that the rule specified by (4) is a generalization of the well-known space rule.

Having defined the operating rule in the linear form of (4) with parameters a_i and b_i obeying (5), we have introduced a convenient parameterization of the problem. This raises important issues regarding the validity of the rule proposed. These are related to (1) the ability of equation (4) to take into account various policies that result from different concerns about the system, (2) the need for further mathematical development of the rule to take into account physical constraints of the system, and (3) parameter issues such as whether the linearity of (4) is appropriate and whether the number of parameters is sufficient. These issues are discussed in the following subsections.

2.2. Justification of the Rule's Form

In this subsection we study five particular operating policies, which result from different concerns about the system properties and objectives. In each case we deal with one isolated objective of the system such as the minimization of spills or losses. To be able to obtain the operating rule for each case as an analytical solution, based on a theoretical objective function, we do not consider all physical constraints of the system at this stage. At a later stage we will incorporate the physical constraints in the rule. The five cases examined do not exhaust all possible concerns about the reservoir system operation, but they are indicative of the form such policies can take. As we will see, in all cases the result is the linear rule (4) with the particular values of coefficients a_i and b_i dependent on the main concern chosen. This justifies the linear form of (4) as a generalization of various operating rules.

2.2.1. Restricting spills. Assume that the primary concern is to avoid unnecessary spills from one reservoir while others still have empty space. This rule is appropriate for the refill cycle of the reservoirs or, equivalently, for the wet season. Spills are more likely to be avoided when more empty space is left for the reservoirs with larger expected cumulated inflows up to the end of their refill cycle. It has been shown [*Sand*, 1984; *Johnson et al*., 1991] that the minimum expected value of the total spills of the system corresponds to the case in which the probability of spill is the same for each reservoir, i.e.,

$$\mathrm{prob}(\mathrm{CQ}_i \geq K_i - S_i) = \mathrm{const} \qquad \forall\ i \tag{6}$$

where CQ_i is the cumulative inflow to reservoir i from the end of the current period to the end of the refill cycle, K_i is the storage capacity of reservoir i, and prob( ) denotes probability. *Johnson et al*. [1991] showed that under the assumption that the distribution of $\mathrm{CQ}_i/E[\mathrm{CQ}_i]$ (with $E[\quad]$ denoting expectation) is the same for each reservoir i, (6) results in

$$\frac{K_i - S_i^*}{E[\mathrm{CQ}_i]} = \frac{\left(\sum_{j=1}^{N} K_j - V\right)}{\left(\sum_{j=1}^{N} E[\mathrm{CQ}_j]\right)} \tag{7}$$

This is the well-known space rule, which consists in keeping equal for all reservoirs the ratio of the empty space to the expected cumulative inflow for the rest of the refill cycle. Equation (7) can be rewritten in the form of (4) for each i with values of parameters

$$a_i = K_i - b_i \sum_{j=1}^{N} K_j \qquad b_i = \frac{E[\mathrm{CQ}_i]}{\sum_{j=1}^{N} E[\mathrm{CQ}_j]} \tag{8}$$

If the reservoirs are all located in a region with the same climatic regime, the ratios b_i of (8) do not vary significantly from one month to another as demonstrated in Figure 5 for the case study. Thus quantities a_i and b_i of (8) can be considered as time invariant.

Furthermore, the assumption that the distribution of $\mathrm{CQ}_i/E[\mathrm{CQ}_i]$ is the same for all reservoirs is not obligatory to get the linear rule (4), as different assumptions can result in the same equation. For example, if all CQ_i have Gaussian distributions, one can easily obtain that (6) results again in (4) with a_i and b_i given by equations slightly different from (8).

2.2.2. Restricting losses. Very often the leakages from reservoirs are not negligible, especially if the reservoirs are natural lakes on a karstic background. It is also likely that evaporation losses are of main concern, especially if we consider natural shallow lakes. Thus let us assume that the losses due to leakage and evaporation are of much more importance when compared with spills. The losses due to leakage are commonly a function of water surface elevation, and those due to evaporation are a function of the surface area of the reservoir. Given the reservoir storage-elevation and area-elevation relationships, we can express the total losses of this kind as a function of storage, i.e.,

$$L_i = l_i(S_i) \tag{9}$$

If our concern is to minimize losses, using algebra and some rather general assumptions (functions $l_i(S_i)$ increasing and concave, which holds for almost any reservoir; see Appendix A1[1]), we find that the most efficient rule is the one that stores all water V at the reservoir m whose value $l_m(V)$ is the minimum among those of other reservoirs $l_i(V)$. Mathematically, this is expressed again by the linear equation (4) but with coefficients $a_i = 0$ for all i, $b_m = 1$ for the specific reservoir m whose value $l_m(V)$ is the minimum among all other $l_i(V)$, and $b_i = 0$ for all other i (except for $i = m$).

2.2.3. Ensuring conveyance. A third rule will be considered for periods with low system storage. In such periods the main concern is not avoiding reservoir spill but making withdrawals so as not to drive one or more reservoirs empty while demand cannot be satisfied from the remaining reservoirs because of limited conveyance capacity. In such a case it is straightforward that the optimal distribution is such that the storage in each reservoir is proportional to the conveyance capacity of the relative aqueduct. This rule is expressed by the same linear rule (4) but with coefficients

$$a_i = 0, \qquad b_i = \frac{C_i}{\sum_{j=1}^{N} C_j} \tag{10}$$

for all i, where C_i is the conveyance capacity of the aqueduct through which the release from reservoir i is made.

2.2.4. Taking into account the impacts of topology. In the above cases all reservoirs were assumed implicitly to be topologically equivalent; that is, each of them is located at a different river or branch of river, and they are all connected by separate aqueducts with the consumption location. However, in many cases there appear to be differences in the topology of the reservoir system that may affect greatly the operating rule. Let us consider, for example, the case where the reservoirs

[1]Appendices are available on microfiche. Order from American Geophysical Union, 2000 Florida Avenue, N.W., Washington, DC 20009. Document 97WR01034M; $2.50. Payment must accompany order.

form a cascade along the same river. In such a case the spills of all reservoirs but the most downstream one are not a loss for the system. Moreover, for energy-saving reasons (e.g., minimization of pumping) it may be a gain for the system to store the water as far upstream as possible. In addition it is always possible to move the water from upstream to downstream if necessary, while the opposite needs pumping. Thus a good operating rule for such a case would be to keep the water at the most upstream reservoir (if feasible), leaving the downstream reservoirs empty. Mathematically, this is expressed by the same linear rule (4) with coefficients $a_i = 0$ for all i, $b_m = 1$ for the most upstream reservoir m, and $b_i = 0$ for all other i (except for $i = m$).

2.2.5. Assuring satisfaction of secondary water uses. In many cases, apart from the main water use, there are some secondary water uses in the neighborhood of each reservoir (e.g., irrigation, satisfaction of environmental demands, etc.). In such cases we want to avoid situations where some reservoirs are almost empty, while others are almost full. Thus we can set a rule that stores the water proportionally to cumulative local water demand for consumptive use CLD_i in order to balance the satisfaction of all local uses. This leads again to the linear rule (4) with

$$a_i = 0, \qquad b_i = \frac{E[\mathrm{CLD}_i]}{\sum_{j=1}^{N} E[\mathrm{CLD}_j]} \qquad \forall\ i \tag{11}$$

We have seen that in each of the above simple situations the operation rule has always the linear form (4) with parameters a_i and b_i given by different simple equations for each case. In real-world situations we have to deal with more than one such concern (or goal) simultaneously. In these situations we can keep the formalism and parameterization of the linear rule, but the parameters a_i and b_i are no longer determined by simple equations such as the above because the objective function is not simple enough to be treated analytically. The parameterization of the rule allows for estimation of parameters using simulation via sampling and search procedures [*Loucks et al.*, 1981, p. 65]. Before we proceed to the description of the models for simulation and optimization it is necessary to incorporate physical constraints into the linear rule in order for it to be operational for real-world situations.

2.3. Further Development of the Rule and Parameter Issues

In introducing (4) we have ignored the physical constraints, which demand that the storage cannot be negative nor can it exceed the reservoir capacity. To correct this inconsistency, we modify (4) so that

$$S_i'^* = \begin{cases} 0 & a_i + b_iV < 0 \\ a_i + b_iV & 0 \le a_i + b_iV \le K_i \\ K_i & a_i + b_iV > K_i \end{cases} \tag{12}$$

However, this creates another inconsistency as the quantities $S_i'^*$ defined by (12) may no longer add up to V. Several adjustment procedures can be used, the most refined being the transformation of straight lines of (4) into broken lines. Here we adopt another procedure that is computationally simpler. We distribute the departure $V - \sum_{j=1}^{N} S_j'^*$ proportionally to the quantity $S_i'^*\ (1 - S_i'^*/K_i)$ so that $(S_i'^* = 0)$ maps to $(S_i''^* = 0)$ and $(S_i'^* = K_i)$ maps to $(S_i''^* = K_i)$. In this way the adjustment procedure does not affect the cases where the reservoir i was found by (12) to be either empty or full. Thus we get the final target storage $S_i''^*$ by

$$S_i''^* = S_i'^* + \frac{S_i'^*(1 - S_i'^*/K_i)}{\sum_{j=1}^{N} S_j'^*(1 - S_j'^*/K_j)} \left(V - \sum_{j=1}^{N} S_j'^* \right)$$
$$= S_i'^*[1 + \phi(1 - S_i'^*/K_i)] \tag{13}$$

with

$$\phi := \frac{V - \sum_{j=1}^{N} S_j'^*}{\sum_{j=1}^{N} S_j'^*(1 - S_j'^*/K_j)} \tag{14}$$

We note that under certain circumstances (e.g., for ϕ lying outside of the interval $[-1, 1]$), (13) may lead to values of $S_i''^*$ that still violate the physical constraints. These circumstances are described in detail in the Appendix A2 along with an iterative algorithm to obtain $S_i''^*$ such that $0 \le S_i''^* \le K_i$ in all cases. We emphasize that the final operating rule, expressed by means of $S_i''^*$, is completely determined from the initial parameters a_i and b_i. An example of an initial rule expressed in terms of S_i^* along with its corresponding final rule expressed in terms of $S_i''^*$ are given in Figure 6 for the case study described in section 3.

Having introduced the full mathematical description of the rule proposed, several issues concerning the rule's parameters are raised: (1) Is the linear form (4) of the rule adequate, or do we need a more complicated nonlinear form? (2) Is the number of parameters in the rule (two parameters per reservoir) adequate, or do we need more or fewer parameters? (3) Do we need to introduce a seasonal variation of the parameters?

It is difficult to answer these questions in a strict mathematical sense. However, we will attempt to give some detailed but rather intuitive answers. The answer to question 1 is threefold. First, as we have shown in section 2.2, the linear form is justified for several simple cases. Second, the operational form of the rule is not strictly linear since the corrections (12) and (13) introduce strong nonlinearity as demonstrated in the example of Figure 6, where the final target storages and their initial values are compared. The initial linear form is, in fact, used as an efficient way to parameterize the problem using two parameters for each reservoir. Third, the physical constraints of a reservoir system strongly modify the form of any initial rule no matter which this specific form is. Different initial rules thus have very similar final operational forms. To demonstrate that numerically, we can experiment using a quadratic rule instead of the linear, i.e.,

$$S_i^* = a_i' + b_i'V + c_i'V^2 \tag{15}$$

where a_i', b_i', and c_i' are parameters for each reservoir i. Experimenting with different sets of parameters a_i and b_i of (4), we can find a parameter set of this linear rule such that the final rules (after introducing corrections for constraints) of both the linear and quadratic form are very close to each other. A comparison of the two rules (linear and quadratic) is illustrated in Appendix A3 for the quadratic rule with the highest possible curvature, where the final forms are almost indistinguishable (the overall root-mean-square error, normalized by the respective reservoir capacity, is less than 0.1%).

The above discussion already gives some indication of the adequacy of the number of parameters (question 2): the use of three parameters per reservoir instead of two essentially makes no difference. We could also consider reducing the number of parameters to one parameter per reservoir, thus formulating the rule as a homogeneous line of the form $S_i^* = b_i V$. To test this, we approximated a quadratic and a linear nonhomogeneous rule with a linear homogeneous rule (see Appendix A3). In both cases we obtained approximations of the final operational rules with overall root-mean-square error less than 10%, although the initial rules differed by as much as 100%. This suggests that the rule may be satisfactory for practical applications even in its reduced homogeneous form. However, to develop a clearer idea of the adequacy of the number of parameters, we must assess the sensitivity of the objective function to some parameters. As it will be shown in the section 3, in our test case we started by using two parameters per reservoir (a_i and b_i) and found that the optimum of the objective function was practically insensitive to a_i, which indicates that one parameter per reservoir suffices. This, however, cannot be transferred to any reservoir system without prior investigation.

Question 3 concerns another form of nonlinearity that can be introduced through seasonal variation of the parameters. First, we note that in systems consisting of reservoirs with very high capacities that perform overyear regulation, there is no reason to consider target storages dependent on the season, as the overyear variation of storage is more important than the within-the-year variation. In systems with smaller capacities it seems reasonable to have the target storages dependent on the season. However, the parametric rule implicitly contains such a dependence of the target storages on V. This is particularly true for reservoirs with considerable drawdown in the dry season. In such cases V takes large values only in the wet season. We note, though, that intermediate values of V are normally attained twice a year: once during the refill period and once during the drawdown period. It may be beneficial to distribute among the reservoirs the same total volume V in a different way in each of the two periods. This means that the use of two parameter sets for the rule, one for the refill and one for the drawdown period, may be advantageous. For simplicity the parameters a_i and b_i are considered in this study as time invariant and constant for each reservoir. However, the approach proposed can be directly modified to include two parameter sets, but this will require more computations because of the doubling of the number of parameters.

2.4. Optimization Model

As described above our proposal in this paper is to consider the coefficients a_i and b_i of the operating rule as unknown parameters and to determine them by optimization. Their values are optimized in the following way:

1. A simulation model of the reservoir system operation is built together with a multivariate stochastic model of the system's inflows. A long series of synthetic inflows is generated and is passed into the simulation model to evaluate the objective function of the optimization model described in point (2), below.

2. An optimization method is used to determine a_i and b_i. At each evaluation of the objective function one or more simulations of the system operation (depending on the constraints of the optimization) for the whole operation period are performed. Thus, in our case, simulation is embedded in the optimization algorithm.

To formulate the objective function of the optimization model, we consider two typical problems. In the first problem the objective is to maximize the target release of the system for a given reliability level. For example, this is the objective in the first three simple cases examined in section 2.2, which can be represented by a common objective function. Mathematically, this is expressed by

$$\max D = f_1(\mathbf{a}, \mathbf{b}) \tag{16}$$

where $\mathbf{a} = (a_1, \cdots, a_N)^T$ and $\mathbf{b} = (b_1, \cdots, b_N)^T$. The constraint for this optimization is related to a total reliability measure that the system should have, i.e.,

$$\text{prob}\left(\sum_{i=1}^{N} R_i = D\right) = \alpha \tag{17}$$

where α is the reliability level. For example, if $\alpha = 0.95$, the above equation means that in a simulated period of 2000 years the total release equals the target release D during 1900 years (95%), whereas we allow 100 years (5%) where the target release is not completely satisfied. The failure probability α' corresponds to the case of partial satisfaction of the demand and $\alpha' = 1 - \alpha$. Apparently, failure occurs in cases where release targets are not physically achievable.

In the second problem our concern is the cost of conveyance (or the profit, in cases of energy production). This is, for example, our concern for the fourth case examined in section 2.2. In this problem we can formulate the objective function as

$$\min E\left[\sum_{i=1}^{N} c_i(R_i)\right] = f_2(\mathbf{a}, \mathbf{b}) \tag{18}$$

where $c_i(R_i)$ is the cost paid for conveying the quantity R_i to the consumption location (negative in cases of energy production) and expectation is taken over the releases. Equation (17) still remains a constraint for (18).

Other concerns of the system may lead to different objective functions (single or multivariate) or to different constraints. In this paper we consider only the above two problems with single-objective optimizations having the form of equations (16) and (18).

2.5. Simulation of the System Operation

As we have seen previously, the optimization process involves a certain number of simulations of the system operation. In each simulation, trial values of the parameters a_i and b_i are used. At each time period of simulation the following computations are performed:

1. The end-of-period storage in the system V is estimated from (3).

2. The target storages S_i^* are obtained from (4). Then, these are corrected according to (12) and (13) to give the final values of the target storages $S_i''^*$.

3. The releases from each reservoir are determined so as to meet the target storages $S_i''^*$ while also satisfying

$$0 \le R_i \le C_i \tag{19}$$

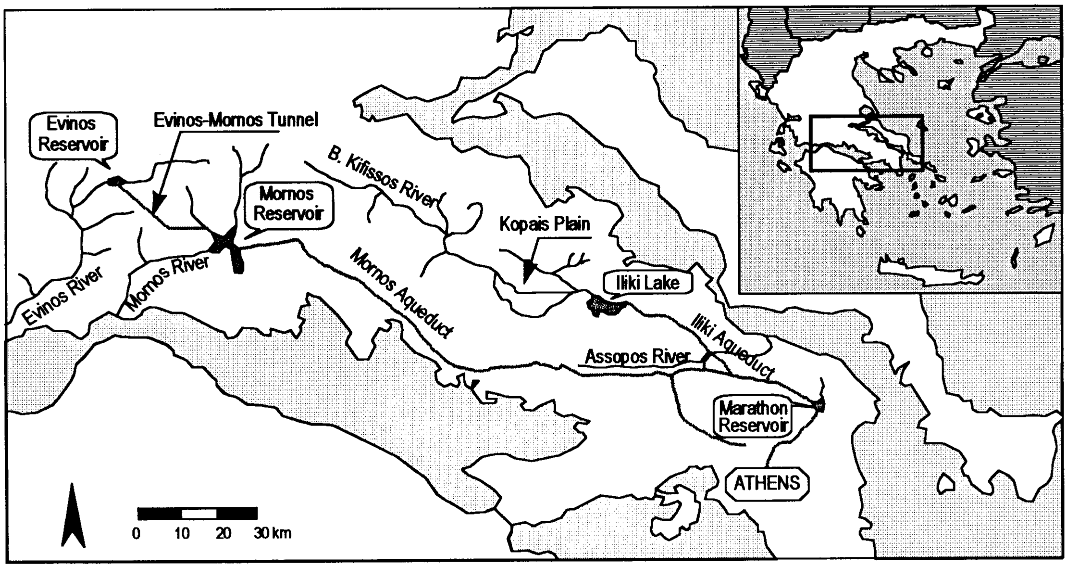

Figure 1. Layout of the Athens water supply system.

In case the releases R_i are outside the limits set by (19) they are set equal to these limits, and the remainder from the total target release is redistributed among the remaining reservoirs.

4. The spill from each reservoir i is given by

$$\mathrm{SP}_i = \max\{0, (\mathrm{BS}_i + Q_i - R_i - L_i - K_i)\} \qquad (20)$$

In some cases this procedure may require an iteration. Initially, to estimate V in step (1), spills are assumed zero. If nonzero spills are derived from (20), V is reevaluated on the basis of those spills, and the whole procedure is repeated again. Finally, the simulation model may include other equations that determine leakages and safety storages. Examples are discussed in the next section in the presentation of the case study.

3. Case Study

3.1. The Reservoir System for Water Supply of the Greater Athens Area and Its Simulation

The reservoir system of greater Athens is used to supply water mainly for domestic and industrial use to the metropolitan area of Athens. It comprises two main reservoirs (Figure 1): (1) the Mornos Reservoir with an active storage capacity of 643 hm^3 and (2) the natural Lake Iliki with a storage capacity of 587 hm^3. A small reservoir near Athens, the Marathon Reservoir, with a storage capacity of 41 hm^3, is also part of the system. This reservoir is considered full all the time for emergency situations. Major water transfer works are (1) the Mornos Aqueduct, some 200 km long, which carries water from the Mornos Reservoir to Athens and comprises a number of different hydraulic works, for example, 70 km of tunnels, and (2) the Iliki Aqueduct from Iliki to the Marathon Reservoir, which is some 60 km long. The growing water demand and the system's vulnerability to drought during the severe drought of 1989–1990, which was followed by 6 years with low flows except for 1990–1991 [*Nalbantis et al.*, 1994], led public authorities to decide to construct a new reservoir (Evinos) with a dam at the site of Aghios Dimitrios on the Evinos River just west of the Mornos River Basin. Water from the new reservoir will be diverted to the neighboring Mornos Reservoir and from there to Athens via the Mornos Aqueduct. The storage capacity of the reservoir is small (104 hm^3) as compared to that of the Mornos Reservoir. On the other hand, inflows to the new reservoir are of a magnitude comparable to that of the inflows to the Mornos Reservoir. As a result, the Mornos Reservoir will be the main storage work for the Evinos River flows as well. A map with the reservoir system is given in Figure 1, while a schematic layout is sketched in Figure 2, where, also, the technical characteristics of the system are annotated. Mean values, standard deviations, and lag-one autocorrelation coefficients for monthly inflows to the three main reservoirs (Evinos, Mornos, and Iliki) of the system are given in Table 1.

Water from the western part of the system (Evinos and Mornos reservoirs) flows to Athens via gravity. Contrary to this, water from Lake Iliki has to be pumped. Another important feature of the system is that Lake Iliki lies on a karstic geologic formation that causes significant leakages. These depend strongly on the water surface elevation of the lake and may equal half of the annual inflow for high elevations. Analysis of historical data established two distinct leakage-elevation relationships: a first one for the dry period (April through September) and a second one for the wet period (October through March). The relationship for the dry period is given by

$$L_L = 0.01242Z^2 - 0.999Z + 17.46 + e \qquad (21)$$

where L_L is the monthly leakage in cubic hectometers and Z is the water elevation of the lake in meters. For the wet period the following relationship was found

$$L_L = 0.01242Z^2 - 0.999Z + 22.16 + e \qquad (22)$$

In both (21) and (22) a random term e is added to account for discrepancies from the deterministic $L_L - Z$ relationship. For this term, $E[e] = 0$, while its standard deviation is $\sigma_e = 2.64$ hm^3 for the dry period and $\sigma_e = 5.96$ hm^3 for the wet period. These two statistics are used to produce simulated values of leakages through random generation of e that are added to the deterministic part in (21) and (22) during simulation.

The Mornos Reservoir leakages are concentrated in a limited area of the reservoir and are rather small compared to

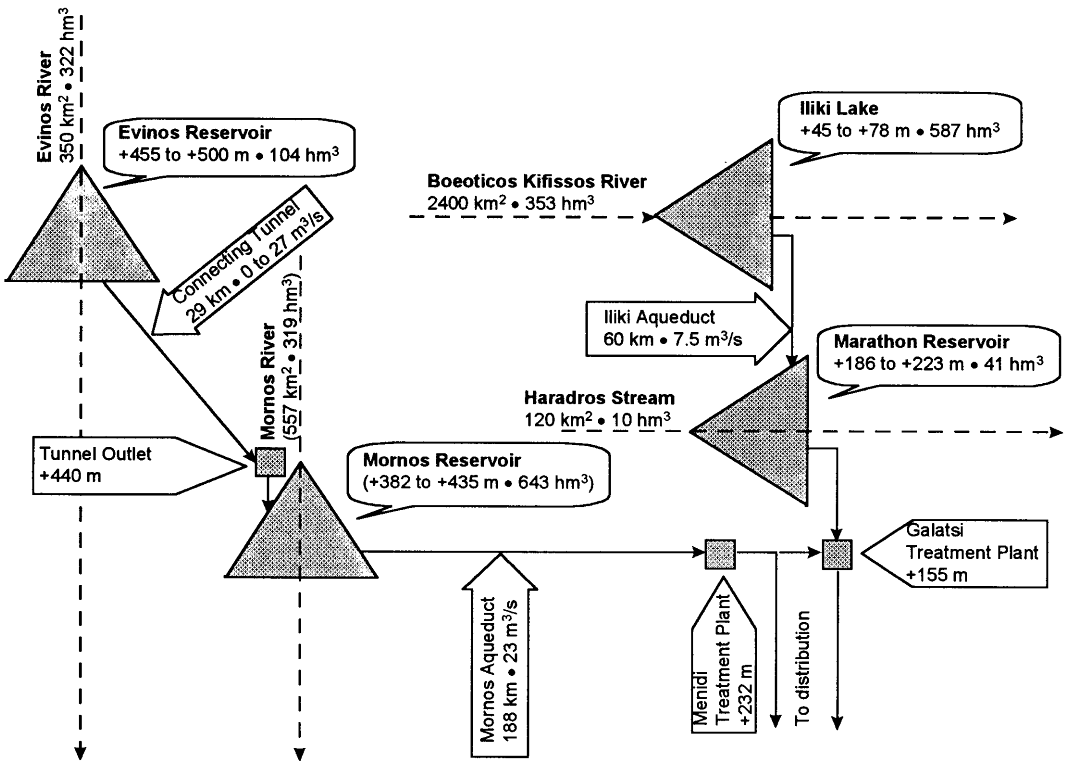

Figure 2. Schematic representation of the Athens water supply system. Characteristic data of the system are annotated: for rivers, the watershed area and the mean annual reservoir inflow; for reservoirs, the minimum and maximum water level and the active storage capacity; for aqueducts, the length and conveyance capacity; and for other components, the characteristic water levels.

those of Lake Iliki. They are effectively modeled via the following linear relationship:

$$L_L = 22.865 \times 10^{-3}(Z - 384.2) \qquad Z \geq 384.2 \text{ m} \tag{23}$$

Apart from water supply to the greater Athens area, the system provides water for irrigation of the Kopais Plain in the Boeotia district. This secondary water use is fixed by decree at 50 hm^3/yr but may be reduced in case of water shortages in the water supply of Athens.

In the simulation model of the system operation an arrangement has been made for keeping safety storages in case of possible damages to the system aqueducts. For the case of

Table 1. Mean Values, Standard Deviations, and Lag-One Autocorrelation Coefficients of Monthly Inflows to the Reservoirs of the Athens Water Supply System

	Evinos			Mornos			Iliki§		
	m*	s†	r‡	m	s	r	m	s	r
October	7.2	6.5	0.32	12.2	13.9	0.16	20.1	10.1	0.59
November	30.4	23.5	0.17	31.0	22.9	0.32	25.3	9.4	0.65
December	60.0	47.1	0.49	48.8	28.1	0.01	44.3	37.5	0.46
January	48.3	34.6	0.19	51.9	32.3	0.25	52.5	28.5	0.60
February	56.4	32.0	0.75	48.1	25.8	0.31	53.1	20.7	0.59
March	47.8	27.1	0.0	39.9	14.3	0.23	63.3	18.3	0.26
April	34.0	12.2	0.29	33.4	9.7	0.73	40.4	21.5	0.78
May	18.5	7.1	0.60	24.1	10.5	0.78	18.9	14.5	0.80
June	8.2	3.1	0.73	13.5	5.9	0.48	3.8	5.4	0.45
July	4.7	1.5	0.81	6.4	3.8	0.0	0.4	1.0	0.54
August	3.1	0.8	0.68	5.3	3.1	0.20	1.3	2.6	0.47
September	2.9	0.9	0.11	4.8	2.9	0.75	9.9	6.8	0.56
Year	321.5	111.2	0.17	319.1	77.9	0.03	333.4	115.8	0.0

Where the record periods are defined as follows: Evinos, 1961–1963 and 1970–1988; Mornos, 1951–1956, 1963–1968, and 1979–1988; and Illiki, 1960–1964, 1968–1976, and 1977–1988.

*Mean value, in cubic hectometers.

†Standard deviation, in cubic hectometers.

‡Lag-one autocorrelation coefficient.

§Inflow from B. Kifissos River (not including inflow from Iliki's own basin)

Table 2. Monthly Water Demand Distribution Coefficients d_j ($j = 1, 2, \cdots, 12$) for the Athens Water Supply System

	Water Supply of Athens, %	Irrigation of Kopais Plain, %
October	8.75	0.00
November	7.75	0.00
December	7.75	0.00
January	7.17	0.00
February	6.58	0.00
March	7.42	0.00
April	7.58	2.58
May	8.67	7.17
June	9.33	17.58
July	10.08	39.84
August	9.75	32.83
September	9.17	0.00
Annual sum	100.00	100.00

damage to the Mornos Aqueduct a sufficient volume of water is always kept in Lake Iliki to satisfy water demand of Athens and irrigation of the Kopais Plain for six months to come. Minimum inflow to Lake Iliki as well as the storage in the Marathon Reservoir are considered to contribute to safety storage. Owing to the large dead volume of the Mornos Reservoir (119 hm^3), which can be pumped in emergency situations, and to the absence of local water uses from that reservoir, no such safety concern was necessary for the case of damage to the Iliki aqueduct.

The annual target release D is expressed in cubic hectometers per year. In the calculations this is first transformed into a monthly mean value $D_m (= D/12)$, which, in turn, is distributed throughout the months of the year via the water demand distribution coefficients

$$d_j = \frac{D_j}{D_m} \tag{24}$$

where d_j and D_j are the water demand distribution coefficient and the target release for month j ($j = 1, 2, 3, \cdots, 12$), respectively. Water demand distribution coefficients for both the water supply of the greater Athens area and the irrigation of Kopais Plain are given in Table 2.

3.2. Brief Review of the Model Used for Synthetic Inflow Generation

A multivariate stochastic model was used for generation of inflows. The model generates the runoff of the three basins and the concurrent rainfall depths at the three reservoirs simultaneously. In addition, it generates the evaporation depths from the three reservoirs simultaneously but with no reference to the concurrent rainfall and runoff. These generations result in equivalent water depths, while the corresponding volume quantities are determined during the system simulation, given the variation of the reservoir areas. For each of the two cases (concurrent rainfall and runoff, and evaporation) we start with the generation of annual quantities, which is performed by a multivariate AR(1) model. Then these quantities are disaggregated into monthly depths as the monthly step was proven sufficient for the system simulation. The disaggregation is performed using the Dynamic Disaggregation Model (DDM) [*Koutsoyiannis*, 1992]. This model preserves the first three marginal moments of the lower level (monthly) variables, the lag-one autocorrelation coefficients, and the lag-zero cross-correlation coefficients. We note that the test hydrologic system for the development of DDM was the same system as the present application, and thus the interested reader is referred to *Koutsoyiannis* [1992] for a detailed description of the model and its performance.

3.3. Results

The proposed method was applied in two real-world problems related to the water supply system of greater Athens. In the first problem (problem 1) the ultimate development of the system is studied. Specifically, the maximum possible system release is sought by taking no account of the operating cost (i.e., for pumping). In the second problem (problem 2) the system operation is studied for a level of development lower than the ultimate, considering this time the related economic aspects. Specifically, a target release level is assumed to be less than the maximum that is estimated in problem 1, and the minimization of the operating cost is sought.

In problem 1 the total target release from the system, D, is maximized for a selected level of failure probability. The objective function to be maximized is given by (16), while constraint (17) must also be satisfied. The adopted level of the probability of failure for the water supply system of greater Athens is $\alpha' = 1\%$ [*Koutsoyiannis and Xanthopoulos*, 1990], a value that provides a high level of security. So during the optimization process, the point $(\mathbf{a}, \mathbf{b})$ in the parameter space, which yields the maximum target release for $\alpha' = 1\%$ is sought. However, the simulation of the system operation for a specific set of parameter values yields α' for a given water demand D. To avoid an excessive number of simulations with large computing times, we followed a procedure with two steps. In step 1 a level of target release D is selected and parameters are estimated that minimize the probability of failure or, otherwise, maximize the system reliability

$$\max \text{prob}\left(\sum_{i=1}^{N} R_i = D\right) = f_1'(\mathbf{a}, \mathbf{b}) \tag{25}$$

with the constraint

$$D = \text{const} \tag{26}$$

Step 2 simply involves finding a target release that gives the desired level of reliability with the parameter values already estimated in step 1. The basic hypothesis behind this two-step optimization lies in the fact that (16) and (17) can be interchanged as far as their role as objective function and constraint is concerned. This is reasonable when the assumed level of target release in step 1 does not differ significantly from the target release estimated in step 2, a condition that must be checked a posteriori.

All simulations are based on a synthetic data set for a period of 2000 hydrological years. Nine hydrological variables are simulated, i.e., three reservoirs (Evinos, Mornos, and Iliki) $\times$ three variables (runoff, precipitation, and evaporation).

The main focus of this work is to explore the features of the approach associated with the parameterization of the proposed reservoir system operating rule and not to establish an efficient optimization algorithm. Our purpose is served better by using the uniform grid method of parameter optimization already described in classical texts [*Loucks et al.*, 1981, pp. 65–68]. In this study the method is applied in the form of successive steps or iterations with grids that are nested to each other and become progressively finer. In this method the objective function is evaluated via simulation at all the grid points of the parameter space that satisfy constraints (5). The algorithm starts by dividing the interval of variation $\mathbf{P}_1(p)$ of each parameter p with an interval divider δ_1 to obtain the initial (coarsest) grid. Then, we construct a second grid with finer resolution by taking a smaller interval $\mathbf{P}_2(p)$ of each parameter p only in the vicinity of the optimum and dividing it by a divider δ_2. This is considered as the first iteration. The algorithm proceeds in this way for a number of iterations M until convergence to one or more optima. Note that simulation runs are performed for $M + 1$ grids. The convergence criterion depends on the objective function to be optimized. For (25), of step 1, iterations are stopped when maximum difference between failure probability values within a grid drops below the critical value $\varepsilon_1 = 0.002$. This is chosen as a small multiple of 0.0005, which is the minimum probability level that can be calculated for a period of simulation of 2000 years.

In our study the parameter set is six-dimensional, i.e., (a_1, a_2, a_3, b_1, b_2, b_3), where indexes 1, 2, and 3 correspond to Evinos, Mornos, and Iliki, respectively, but owing to (5), this is reduced to a four-dimensional problem. Preliminary tests showed little sensitivity to parameters a_i ($i = 1, 2, 3$). One example is given in Table 3 for a particular set of parameters $\mathbf{b} = (0.20, 0.80, 0.00)$ and $D = 700$ hm³. This table shows that results are insensitive to parameters $\mathbf{a}$, and the rule proposed was initially overparameterized, at least for our case study.

Given the results of the sensitivity analysis and the discussion of the number of parameters presented in section 2.3, we opted to proceed to the optimization of parameters b_i ($i = 1, 2, 3$) by selecting constant values for a_i, i.e., $a_i = 0$ ($i = 1, 2, 3$), or equivalently, to use the homogeneous instead of the complete linear rule. In this case the parameter space is initially three-dimensional with $0 \le b_i \le 1$ ($i = 1, 2, 3$) and is restricted to a two-dimensional parameter space given that (5) holds. The results are presented in Table 4. In Figure 3 we depict the results of the initial (coarsest) grid in contours with equal probability of failure for the space of parameters b_i that is mapped to an equilateral triangle. We observe that (1) the probability of failure follows a rather smooth and continuously curved surface; (2) this surface is not symmetrical with respect to the sides of the triangle, which is explained by the different conditions of the three reservoirs; (3) the lowest values of the surface correspond to $b_3 = 0$, which is explained by the high leakages of Lake Iliki (the zero value means that we withdraw water from Iliki as much as possible); (4) there is a flat area with minimum probability (equal to 1.4%) rather than a single point; and (5) further investigation of this area is needed for the selection of the final parameter set.

After three iterations we obtained the final grid. The flat area already detected in the initial grid was proved larger, and no probability less than 1.4% appears. The flat area is advantageous as it provides flexibility: any point with $\alpha' = 1.4\%$ could be chosen. The selection of the final parameter set was based on engineering criteria. We have chosen the point with the lowest value of b_1, which corresponds to conveying as much water as possible from the Evinos to the Mornos Reservoir. The idea behind this is to store water as close to Athens as possible for safety reasons. Thus the final parameters set is $(\mathbf{a}, \mathbf{b}) = [(0, 0, 0)^T, (0.08, 0.88, 0.04)^T]$. In Table 4 we depict the main characteristics of the optimization process.

The optimization process for problem 1 is completed with

Table 3. Sensitivity Analysis of the Failure Probability α' of the Athens Water Supply System to Parameters α_i ($i = 1, 2, 3$) for Step 1 of the Optimization Process

Test	Parameters α_i: Evinos	Mornos	Iliki	α', %
1	0	−500	500	1.40
2	0	−400	400	1.40
3	0	−300	300	1.40
4	0	−200	200	1.40
5	0	−100	100	1.40
6	0	0	0	1.40
7	100	−400	300	1.40
8	100	−300	200	1.40
9	100	−200	100	1.40
10	100	−100	0	1.40

In the case of maximization of the expected annual total release from the system (Problem 1). The annual target release is 700 hm³. Parameters b_i are held constant: $b_1 = 0.20$ for Evinos, $b_2 = 0.80$ for Mornos and $b_3 = 0.00$ for Iliki.

Table 4. Summary of Results of the Optimization Process for Problem 1 (Step 1) and Problem 2.

	Problem 1 (Step 1)	Problem 2
Mean annual target release, hm³	700	600
Number of iterations, M	3	6
Initial interval for b_i	[0, 1]	[0, 1]
Interval divider δ_j ($j = 1, \cdots, M$)	2*	2
Critical value for stopping	0.002	0.005
Final failure probability, %	1.40	1.00
Mean annual abstraction from Lake Iliki $E[R_3]$, hm³	182	104

*For all iterations except the first, where $\delta_1 = 5$.

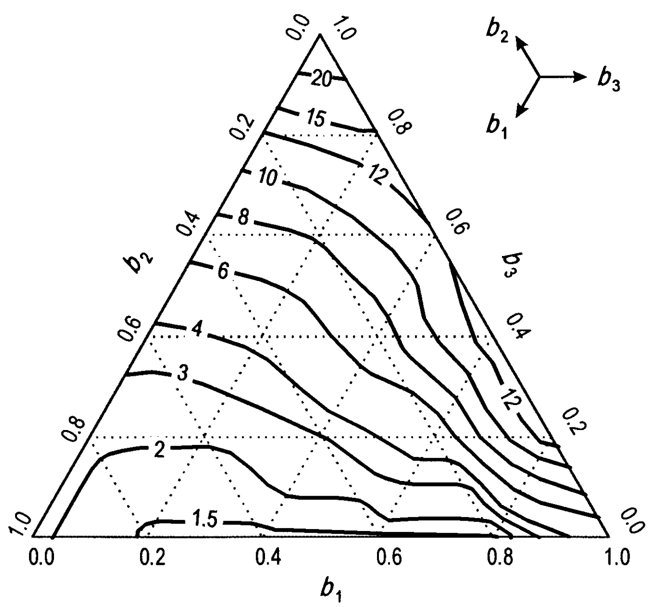

Figure 3. Contours of equal probability of failure α' (%) of the Athens water supply system for the first (coarsest) grid of step 1 of the optimization process of problem 1. The annual target release is 700 hm³. Parameters a_i are zero for all reservoirs.

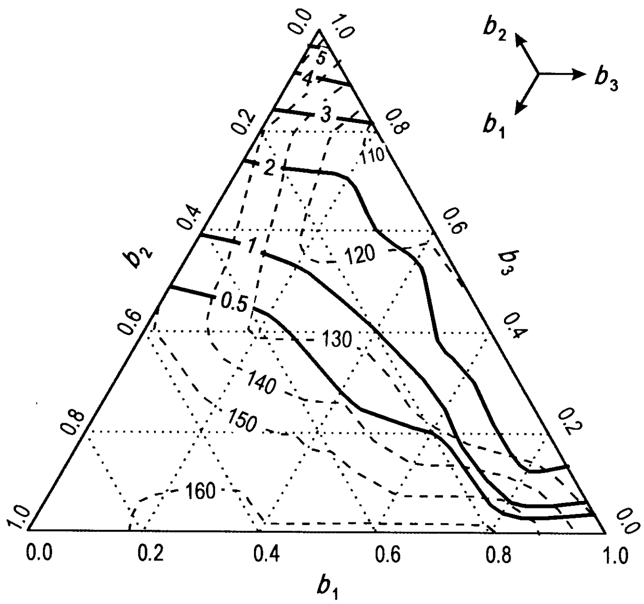

Figure 4. Contours of equal probability of failure α' (%) of the Athens water supply system (solid lines) and of equal mean annual abstraction from Lake Iliki $E[R_3]$ in cubic hectometers (dashed lines) for the first (coarsest) grid of the optimization process in problem 2. The annual target release is 600 hm^3. Parameters a_i are zero for all reservoirs.

step 2 of the overall procedure. The final maximum target release for $\alpha' = 1.0\%$ is estimated at 690 hm^3/yr, a value very close to that of step 1 (700 hm^3).

Problem 2 involves minimizing operating costs for a given level of target release and a level of system reliability. The problem is formulated so as to optimize the objective function (18) with the constraints (17) and (26). Water from the western part of the system (the Evinos-Mornos subsystem) flows to Athens via gravity, while water from Iliki is pumped. Consequently, the operating cost of the Evinos and Mornos works can be neglected if compared to the cost from Iliki. Furthermore, the cost of pumping is a linear function of withdrawals R_3 from Iliki. So the objective function (18) becomes

$$\min E[R_3] = f_2(\mathbf{a}, \mathbf{b}) \tag{27}$$

As in problem 1, the uniform grid method is applied with parameters $a_i = 0$ ($i = 1, 2, 3$) and parameters b_i satisfying (5). The procedure here tries, for a given target release D, to get a solution that is closer to satisfying constraint (17) while at the same time optimizing f_2 in (27).

The results are presented in Table 4. The values of the objective function for the initial (coarsest) grid are also shown in Figure 4, where we have drawn contours of equal probability of failure and of equal mean annual abstraction from Lake Iliki. We observe that the general shape of the surface of probability is quite similar to that of Figure 3 and has its minimum values in the same region (although the absolute values of probability are different in the two figures). Figure 4 allows us to localize the area where the contour with probability of failure 1% passes, i.e., where the constraint (17) is valid. Guided by this we constructed a finer grid and so on. The criterion to stop the iteration was to obtain improvements of the objective function that are less than a certain critical value ε_2 in relative terms. In our case $\varepsilon_2 = 0.005$. Table 4 summarizes the results. The final set of optimal parameters is $(\mathbf{a}, \mathbf{b}) = [(0, 0, 0)^T, (0.106, 0.291, 0.603)^T]$. The value of the objective function is $E[R_3] = 104$ hm^3. Note that this value is 78 hm^3 lower than the corresponding value for problem 1 (182 hm^3). We can also easily notice that the optimal parameter set of problem 2 is clearly different from that of problem 1.

To validate the rule proposed, we compared the above results with those obtained by heuristic rules with no parameters to be optimized. These are (1) the well-known space rule expressed by (8), (2) the leakage rule as described in section 2.2, and (3) the conveyance rule given by (10). We have tested all three rules applied throughout the year as well as combinations of them applied separately for the dry and wet season, as shown in Table 6 (except for three combinations that had no meaning).

The comparison is performed only for Problem 1 since in this problem we can find the maximum target release from the system that corresponds to a failure probability equal to 1%. The application of the above heuristic rules to Problem 2 is not possible because, in that case, there is no degree of freedom: once the target release is fixed the failure probability is also fixed and cannot be made equal to its desired level (1% in our case).

For each one of the three basic heuristic rules we estimated the values of the parameters in equation (4). First, the parameter values for the space rule are estimated. From Figure 5 we conclude that ratios $E[\mathrm{CQ}_i]/\Sigma_{j=1}^{N} E[\mathrm{CQ}_j]$ are nearly constant for all months with mean values 0.313, 0.297, and 0.390 for Evinos, Mornos, and Iliki, respectively. With these values we obtain from (8) the values of $(\mathbf{a}, \mathbf{b})$ shown in Table 5. The graphical representation of the space rule is given in Figure 6, in comparison with the optimized rules of problems 1 and 2. The parameter sets for all other heuristic rules, determined from the corresponding equations of section 2.2, as well as those obtained by optimizing the parametric rule for problems 1 and 2, are shown also in Table 5. We observe that (1) in all rules the parameters a_i are zero except for the space rule, (2) the parameter b_3 for Lake Iliki optimized for problem 1 (parametric rule) takes a value similar to that of the leakage rule, and (3) the parameters b_i for the Evinos and Mornos reser-

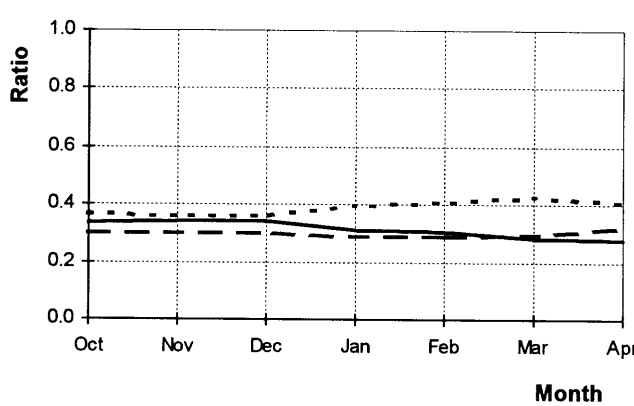

Figure 5. Ratios of cumulative monthly inflows into each one of the three reservoirs to the system cumulative monthly inflows (b_i in (8)). Cumulative inflows are considered up to the end of the refill cycle. Displayed values for the months of the refill cycle (October to April) are averages for the common period (for all reservoirs) of data availability (1979–1980 to 1987–1988). Continuous, dashed, and dotted lines correspond to the Evinos, Mornos, and Iliki reservoirs, respectively.

Table 5. Parameter Values for Various Heuristic Operating Rules and the Optimized Proposed Rule

	Evinos		Mornos		Iliki	
Rule	α_1, hm^3	b_1	a_2, hm^3	b_2	a_3, hm^3	b_3
Space	−315	0.313	247	0.297	68	0.390
Leakage	0	1	0	0	0	0
Conveyance	0	0.377	0	0.377	0	0.246
Parametric, problem 1	0	0.080	0	0.880	0	0.040
Parametric, problem 2	0	0.106	0	0.291	0	0.603

voirs optimized for problem 1 are not well approximated by any one of the heuristic constant-parameter rules.

In Table 6 we depict the annual target release corresponding to the 1% failure probability for each one of the rules tested. These results allow us to make the following observations and interpretations. First, the space rule, applied throughout the year (case 1), gives a total annual release of 620 hm^3, which is 70 hm^3 less than that obtained by our method. This is expected since the avoidance of spills results in storing water mainly in the Mornos and Iliki reservoirs, thus leading to high leakage losses especially from Iliki. Second, the introduction of the

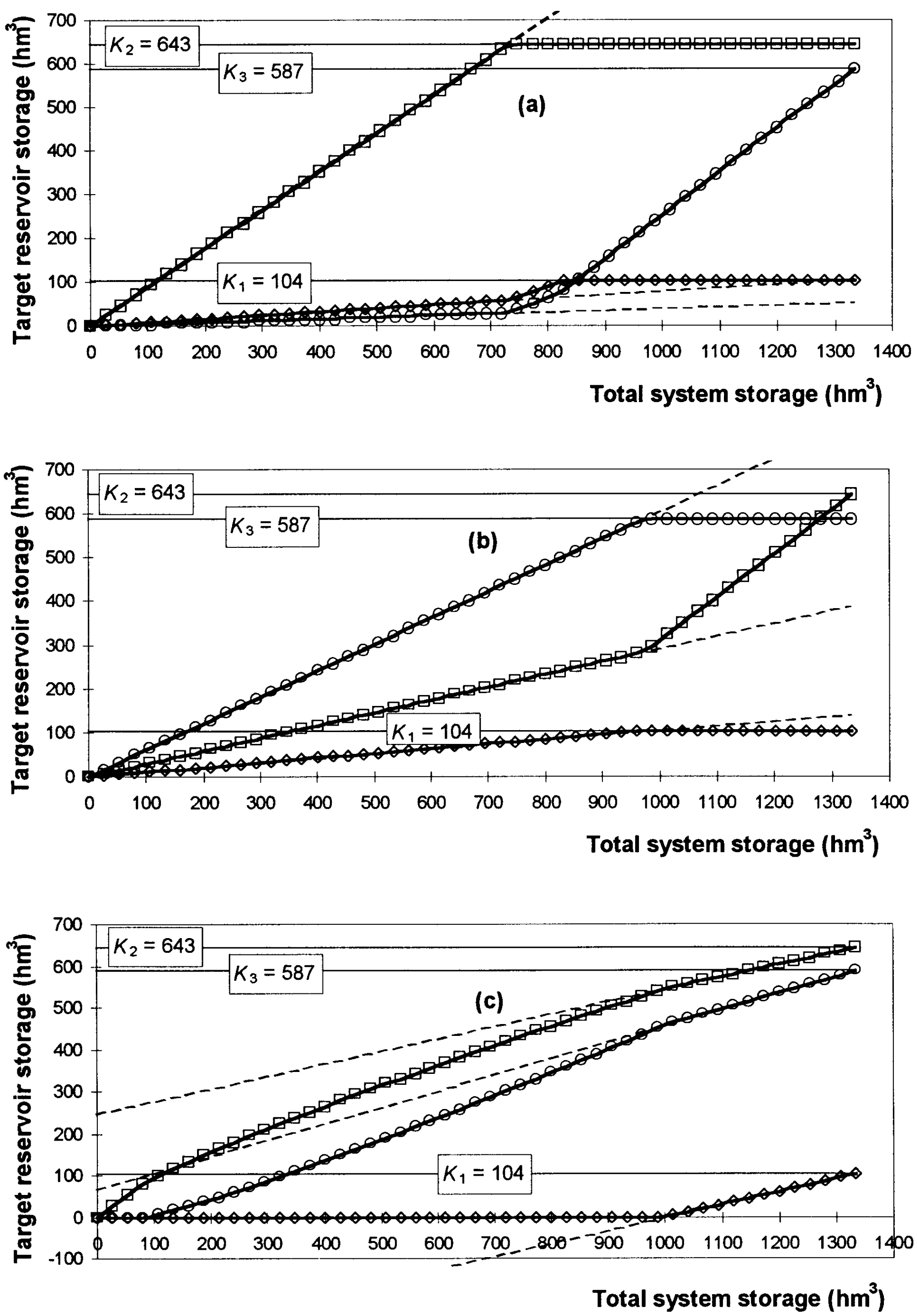

Figure 6. Graphical representation of operating rules for (a) the final parameter set of problem 1, (b) the final parameter set of problem 2, and (c) the parameter set of the space rule. Solid lines with rhombi, squares, and circles correspond to reservoirs 1, 2, and 3 (Evinos, Mornos, and Iliki), respectively, and represent the adjusted rule (equation (13)). Dashed lines represent the initial linear rule (equation (4)).

Table 6. Annual Target Release Satisfied With 1% Failure Probability for Various Heuristic Operating Rules and the Optimized Proposed Rule (problem 1)

Case	Rule: Throughout the Year	Rule: Wet Season	Rule: Dry Season	Annual Target Release, hm^3
1	S			620
2		S	L	620
3		S	C	628
4	L			633
5		L	C	635
6	C			652
7	P			690

The rules are applied throughout the year or by season.
S is the space rule; L is the leakage rule; C is the conveyance rule; and P is the parametric rule proposed.

leakage rule in the dry season while the space rule is still used in the wet season (case 2) does not improve the results. In this case the leakage rule tries to store all water of the dry season in the Evinos Reservoir, while in the previous wet period this was almost emptied by the space rule to keep empty space for the significant inflows from the Evinos basin. Because of the very low inflows in the dry season, no sensitivity to the introduction of the leakage rule is revealed. Third, the introduction of the conveyance rule in the dry season while the space rule is still used in the wet season (case 3) gives a small improvement of 8 hm^3 with regard to the previous case. We note that the conveyance rule tries to store more water in the Evinos-Mornos subsystem, thus producing a beneficial result. Fourth, the leakage rule used throughout the year (case 4) performs better than the space rule and the combination of the latter with the leakage rule. In this case the leakage rule tries keep the Evinos Reservoir full for both seasons. In the wet season this is perfectly possible because of high inflows but is done at the expense of a significant risk of spillage. However, the Mornos reservoir is left relatively empty, although it does not leak significantly (as compared to Iliki). Fifth, the leakage rule used throughout the year (case 4) has a slightly better performance in comparison with the space rule combined with the conveyance rule (case 3). Again, here the introduction of the leakage rule in the wet season proved beneficial. Sixth, the performance of the leakage rule when combined with the conveyance rule in the dry season (case 5) improved very slightly in comparison with the leakage rule throughout the whole year (case 4). We notice the same beneficial result of the use of the conveyance rule in the dry season, although the improvement is minor. Seventh, the use of the conveyance rule throughout the year (case 6) has the maximum performance of all the other rules tested (cases 1–5). As said before, this rule tries to store more water in the Evinos-Mornos subsystem, which happens to have large conveyance capacity. Coincidentally, the same subsystem has also lower leakage losses. As a result, the two effects are combined to improve the performance, but this is undoubtedly a fortuitous situation.

Comparing the results from all six rules or combinations thereof (cases 1–6) with those of the parametric rule proposed (case 7), we observe that in all cases the parametric rule gives significantly better results. We note that our parametric rule tries to store water mainly in the Mornos Reservoir, leaving small quantities to the other two reservoirs; this behavior is not encountered by any of the heuristic rules tested.

As mentioned above, the heuristic rules without parameters subject to optimization are not suitable for problems such as problem 2 examined here. However, for illustrative purposes we give results only for the space rule in this case. Simulations with this rule and a level of annual target release of 600 hm^3 gave a probability of failure equal to 0.6% and a mean annual release from Iliki of 127 hm^3.

4. Summary and Conclusions

A parametric rule for multireservoir system operation is formulated and tested. It can be considered a generalization of the well-known space rule, which aims at avoiding unnecessary spills in one reservoir while others still have empty space. The proposed rule is much more general in the sense that in addition to the spill-avoidance objective, it simultaneously accounts for various other system operating goals: avoiding leakage losses, avoiding conveyance problems, taking into account the impacts of the reservoir system topology, and assuring satisfaction of downstream secondary uses. The rule is parameterized so that it contains two parameters for each reservoir. Theoretical values of the parameters are derived for each one of the above isolated goals. Since many real-world problems involve more than one of these goals, parameters are evaluated numerically to optimize one or more objective functions that are selected by the user. The rule drives a simulation model of the reservoir system, which is embedded in a scheme that optimizes the rule's parameters.

The parametric rule proposed is tested on the case of the water supply system of the city of Athens, Greece, comprising three main reservoirs on three separate water basins. Its complexity and idiosyncrasies make the system ideal as a test system, since many of the operating goals examined theoretically appear in this case study. Two problems are tackled in this case study. First, the ultimate development of the system is considered, and the total release from the system is maximized for a selected level of system reliability. Second, an intermediate development of the system is sought, and the pumping cost is minimized for a given reliability and a given level of target release less than that obtained in the first problem. A detailed simulation model on a monthly timescale has been used in the analyses. This included a generation model of synthetic annual hydrological data and a model for disaggregation of annual values into monthly values. Also, it included models describing system losses such as leakages and evaporation. The system's operating details such as the maintenance of safety storages were also taken into consideration. It appears that the parametric rule proposed has proven satisfactory in tackling the problem of finding the capabilities of a reservoir system on a long-term basis. Through its parameterization it effectively accommodates various system operating goals into a single-objective function. Insensitivity to a subset of the parameters was revealed in the case study, which allowed further simplification of the rule and restriction of the dimension of the parameter space to half the initial value.

Finally, the rule proposed is validated through comparison with other heuristic rules that satisfy specific goals (avoidance of spills, leakage losses, and conveyance problems). In all cases the proposed parametric rule was superior in its performance. Of course, storage and release trajectories obtained are not "optimal" in the absolute mathematical sense as the trajectories must comply with a simple parametric relation. Nevertheless, once optimized, the proposed rule is very simple, mathematically, to apply even for a nonexpert user and is therefore recommended for situations with long-term studies of reservoir systems.

Acknowledgments. The research leading to this paper was partly performed within the framework of the project Evaluation and Management of the Water Resources of Sterea Hellas, project 8976701, funded by the Greek Ministry of Environment, Regional Planning and Public Works, Directorate of Water Supply and Sewage. We wish to thank the directors I. Leontaritis and T. Bakopoulos and the staff of this directorate for the support of this research. We also thank the personnel of the Athens Water Supply and Sewage Corporation and the Public Power Corporation for providing the necessary data for the case study and N. Mamassis and C. Anifanti for assisting in the preparing and criticizing the historic data records. The project manager, T. S. Xanthopoulos, is warmly thanked for his support throughout this work. We are grateful to the reviewers K. Ponnambalam, S. A. Johnson, and J. R. Stedinger for their constructive comments on an initial manuscript submitted in April 1995. We also thank the Associate Editor, S. A. Johnson, and an anonymous reviewer for their comments on the final version of the paper.

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(Received April 11, 1996; revised March 10, 1997;
accepted April 4, 1997.)