The Encyclopedia of Water

Reliability Concepts in Reservoir Design (SW-776)

Demetris Koutsoyiannis
Department of Water Resources, Faculty of Civil Engineering, National Technical University, Athens, Heroon Polytechneiou 5, GR-157 80 Zographou, Greece
dk@hydro.ntua.gr

Introduction

The reliability of a system is defined to be the probability that a system will perform the required function for a specified period of time under stated conditions (Chow et al., 1988, p. 434). Reliability is the complement of probability of failure (or risk), i.e. the probability that the “loading” will exceed the “capacity”. Denoting α the reliability, β the probability of failure and $P[\omega]$ the probability of an event ω, the mathematical expression of this definition is

$$a := P[L(t) < C(t);\ t \in \Pi] =: 1 - \beta \qquad (1)$$

where $L(t)$ and $C(t)$ represent the loading and capacity, respectively, at time t, within a certain time period Π (e.g. a year). Failure of a system can be classified as *structural failure* and *performance failure* (Tung, 1996, p. 7.3). Structural failure involves damage of the structure or facility, hindering its ability to function as desired in the future, whereas performance failure does not necessarily involve structural damage but rather inability of the system to perform as desired at some time within the period of interest, which results in temporary unfavorable consequences.

Reservoir dynamics

A reservoir’s function is to regulate natural inflows, which vary irregularly, to provide outflows at a more regular rate that is determined by water demand for one or more uses (water supply, irrigation, hydropower), temporarily storing the surplus, when inflows exceed outflows. The reservoir dynamics are more conveniently expressed in discrete rather than continuous time. The quantities that are necessary to describe dynamics are the following:

Storage S_t. More precisely known as *active storage,* it is the volume of water stored, at time t, above the minimum level, which is determined either technically (i.e. as the level of the lowest valve of off-take) or legally by a decree imposing rules for a reservoir's operation. Active storage S_t ranges between zero and a maximum value c imposed by the reservoir size, which corresponds to the level of the spillway crest (or some specified level above it in case that sluice gates are constructed over the spillway). During floods, excess water is routed through the spillway, which causes temporary storage above the normal limit c. This is known as flood control storage. Water storage below the minimum level is known as *dead* or *inactive storage* and it serves two main purposes: It provides volume for sediment accumulation and environmental protection, as it protects the habitat of the reservoir during dry periods by hindering complete emptying. Associated to the last function is also the conservation of the quality of landscape. This article is focused on the design of the active storage of a reservoir; some notes on the additional storage zones are contained in the last section of the article.

Net inflow X_t. It is the algebraic sum of cumulative inflows to the reservoir from time $t - 1$ to time t, minus the losses during the same time period. Inflows include runoff from the catchment upstream of the reservoir (typically, the main component of inflows), rainfall to the surface area of the reservoir and, possibly, water artificially conveyed from other sources (e.g. inter-basin transfers through tunnels or pipe-lines). Losses include evaporation from the surface area of the reservoir and possibly seepage to groundwater and leakage under or through the dam.

Water demand, δ_t. It is the sum of all water requirements for the different water uses served by the reservoir for the time period $(t - 1, t)$. The demand may vary with time (e.g. due to seasonal agricultural demand or due to some rule, usually based on the quantity of water in the reservoir).

Release, R_t. Also known as draft, withdrawal or outflow, it is the actual amount of water taken from the reservoir in an attempt to satisfy water demand during the time period $(t - 1, t)$. When there is a sufficient amount of water in the reservoir, R_t equals demand δ_t; otherwise $R_t < \delta_t$.

Spill, W_t. It is the excess water that, during times of floods and simultaneously high reservoir storage, cannot be stored in the reservoir due to the upper reservoir storage limit c.

The reservoir dynamics are easily expressed by means of the mass conservation, or equivalently, water balance equation. Considering that S_t is limited between 0 and c the water balance equation is easily formulated as

$$S_t = \max[0, \min(S_{t-1} + X_t - \delta_t, c)] \tag{2}$$

In addition, the release is determined as

$$R_t = \min(S_{t-1} + X_t, \delta_t) \tag{3}$$

and the spill as

$$W_t = S_{t-1} - S_t + X_t - R_t = \max[0, S_{t-1} + X_t - \delta_t - c)] \tag{4}$$

Equations (2)-(4) apply when the inflow and withdrawal occur at constant rates throughout the period (t, $t - 1$) – this could be called the "steady" model. A simple modification to the equations allows for the case where the inflow (or withdrawal) is highly seasonal so can (in the limit) be modeled as a sudden occurrence; this could be called the "sudden" model. These two models (called "simultaneous" and "staggered" by Pegram (1980)) bound all the behavior observed in real reservoirs.

Definition of reliability applied to reservoir

Now, the above stated general definition of reliability (also known as dependability; e.g. Raudkivi, 1979, p. 312) can be applied to a reservoir. It is observed that the failure of a reservoir's function is a performance failure, i.e., a failure to meet the water demand. At time t the loading is the water demand δ_t and the capacity is the sum of S_{t-1} (storage at time $t - 1$) and X_t (inflow from time $t - 1$ to t). Thus, application of (1) yields

$$a = 1 - \beta = P[\delta_t < S_{t-1} + X_t] \tag{5}$$

Considering (2) and (3), the following equivalent and more convenient expressions are found

$$a = P[S_t > 0], \quad \beta = P[S_t = 0] \tag{6}$$

and

$$a = P[R_t = \delta_t], \quad \beta = P[R_t < \delta_t] \tag{7}$$

In this context, the demand δ_t is regarded as a known quantity at any time instant t. All other involved quantities, namely S_t, X_t, R_t, and W_t, are regarded as random variables. Given the storage capacity c, the demand δ_t and the probability distribution and autocorrelation functions of the input X_t, in theory, the probability distributions of output variables S_t, R_t, and W_t can be determined in terms of that of X_t; this, however, is not an easy task due to the nonlinearity of the dynamics expressed in equations (2)-(4). Theoretically, once the distribution function of S_t or R_t has been determined, the reliability α is directly obtained from (6) or (7), respectively. However, although the theoretically based calculations unavoidably involve the complete knowledge of the distribution function of S_t and R_t, the reliability α can be determined in an alternative, much simpler, manner. That is, under the assumptions of stationarity and ergodicity, a can be estimated from a historical (for an existing reservoir) or synthesized (via simulation) time series of storage s_t or release r_t with adequate length n. (Here lower case symbols were used for values of the random variables S_t and R_t.) Specifically, the estimate of a based on (6) is

$$\alpha = \frac{1}{n} \sum_{t=1}^{n} [1 - U(-s_t)] \tag{8}$$

where $U(x)$ is the Heaviside's unit step function, with $U(x) = 1$ for $x \geq 0$ and $U(x) = 0$ for $x < 0$. Correspondingly, the estimate of a based on (7) is

$$\alpha = \frac{1}{n} \sum_{t=1}^{n} U(r_t - \delta_t)] \tag{9}$$

The purpose of the sum in (8) (or (9)) is to count the periods where storage is not zero (or release r_t equals the demand δ_t). Thus, reliability is expressed as the proportion of time steps in which the system performs as desired. For this reason, α has also been termed *time-based* reliability. Here it must be observed that although (5)-(7) are all mathematically equivalent to each other, as are (8) and (9), when applied to historical time series they may result in different estimates. For example, a city's water supply may not be allowed to empty, as restrictions on releases are applied before this situation is reached. In such a case, (8) may result in the erroneous estimation $\alpha = 1$, whereas (9) will estimate the reliability correctly if the desired demand, before restrictions, is entered into the calculations. Thus, (9) is preferable when dealing with historical time series but in mathematical simulations both are equivalent; moreover, application of (8) is faster as it does not require simulation of releases at all (only (2) needs to be applied).

The stationarity assumption that was inherent in the above analysis is satisfactory when the time step is a year (either calendar or hydrological). However, the annual time step is usually too large and hides the variation of both inflows and demand within a year, which may result in a failure some time within the year that is recovered in the end of the year. Therefore, a smaller time step (e.g. monthly) is usually chosen, so that one year corresponds to $k > 1$ (e.g. 12) time steps. At this finer time scale all processes depend on the time step in a periodic manner, that is, they are cyclostationary. Yet the reliability and failure probability are usually expressed at the annual scale, in which stationarity is redeemed. To shift from the finer scale to the annual scale, the rule adopted is that a failure occurring in one or more finer-scale time steps is regarded as a failure for the year. With this rule, (6) becomes

$$a' = P\Big[\bigcap_{i=1}^{k} (S_{t-i} > 0)\Big], \beta' = P\Big[\bigcup_{i=1}^{k} S_{t-i} = 0\Big] \tag{10}$$

where to distinguish from the time-based reliability the symbols a' and β' were used instead of α and β, whereas the symbol '$\bigcap$' indicates that all of the following events should occur simultaneously and '$\bigcup$' indicates that any of the following events should occur. In a similar manner, (7) becomes

$$a' = P\Big[\bigcap_{i=1}^{k} (R_{t-i} = \delta_{t-i})\Big], \beta' = P\Big[\bigcup_{i=1}^{k} R_{t-i} < \delta_{t-i}\Big] \tag{11}$$

or alternatively

$$a' = P\Big[\sum_{i=1}^{k} R_{t-i} = \sum_{i=1}^{k} \delta_{t-i}\Big], \beta' = P\Big[\sum_{i=1}^{k} R_{t-i} < \sum_{i=1}^{k} \delta_{t-i}\Big] \tag{12}$$

Likewise, (8) and (9) become

$$a' = \frac{k}{n} \sum_{p=1}^{n/k} \min\{[1 - U(-s_t)];\ t = k\,(p-1) + 1, \ldots, k\,p\} \tag{13}$$

$$a' = \frac{k}{n} \sum_{p=1}^{n/k} \min\{U(r_t - \delta_t)\ ;\ t = k\,(p-1) + 1, \ldots, k\,p\} \tag{14}$$

respectively. The sums in (13) and (14) count the number of years in which no failure has occurred. Apparently, a' and β' provide information on the occurrence of a failure within a year and not on the time period the failure lasted. Therefore, they have been known as *occurrence-based* reliability and failure probability, respectively. An overall indication of the time period that failures last within an average year can be obtained by applying equation (8) or (9) and estimating β, the time-based probability of failure.

Apart from the occurrence-based and time-based reliability, an additional reliability measure has been often used, which is not expressed in terms of probability (and thus, literally does not comply with the general definition of reliability). This is the so-called *volumetric* or *quantity-based* reliability, expressed as the ratio of the average release to demand, i.e.,

$$a_{\mathrm{V}} = 1 - \beta_{\mathrm{V}} = E\left[\sum_{i=1}^{k} R_t\right] \Big/ \sum_{i=1}^{k} \delta_t \tag{15}$$

Given that a failure occurring in a year does not extend over the whole year, and, in addition, the release during the failure is not necessarily zero but some positive quantity smaller than demand, it is easily concluded that

$$a' \le \alpha \le a_{\mathrm{V}} \tag{16}$$

Among the three measures of reliability, the most important and most frequently used is the severest, i.e., the occurrence-based reliability a'. Another means for expressing virtually the same concept is the return period or recurrence interval of emptiness, T. This is the mean time between two consecutive empty states of the reservoir and it is none other than the reciprocal of the probability of failure β' (Pegram, 1980), i.e.,

$$T := 1 / \beta' = 1 / (1 - a') \tag{17}$$

which is expressed in years (given that a' and β' are expressed on annual time scale). The concept of return period of emptiness of a reservoir is similar to that typically used for design floods. The difference is that in design floods, failure is the exceedance of the magnitude of design flood, whereas in a reservoir failure is the emptying of the reservoir. Typical design values of reliability and return period for reservoir design are a' = 99% (T = 100 years) for municipal water supply reservoirs, a' = 70-85% (T = 3.3-6.7 years) for irrigation reservoirs in subhumid climates and a' = 80-95% (T = 5-20 years) for irrigation reservoirs in arid climates (Raudkivi, 1979. p. 313).

Traditional reservoir design procedures

Most hydraulic structures, e.g. flood protection works, drainage networks, etc., whose load is randomly varying, have been designed on a probabilistic basis, adopting a certain reliability level or, equivalently, a certain return period for the design flood. Traditionally, however, this has not been the case in reservoir design, which has rarely been based on a sound probabilistic basis. This is witnessed even from the terminology traditionally used. For example, the use of the term *firm yield* implies a non-probabilistic, or failure-free concept. Specifically, firm yield of a reservoir has been defined to be the draft or withdrawal that lowers the water content in a reservoir from a full condition to its minimum allowable level just once during the critical historical drought. (McMahon, 1993, p. 27.8; Chow et al., 1988, p. 534). It has been characterized as essentially the no-failure yield (McMahon, 1993, p. 27.8). In a probabilistic context, however, any draft has a non zero probability of failure (unless the demand is less than a hypothetical lower bound of the inflow distribution, which can be plausible only for perennial streams; this is unusual).

Several procedures have been widely used in reservoir design, which are rather deterministic and not consistent with the reliability concept. The most common has been *mass curve analysis* and its variations. A mass curve is a plot of cumulative inflow volumes (typically based on historical discharge records) as a function of time. Using this plot the firm yield, as well as the required reservoir storage to attain this firm yield, can be determined graphically. In addition, the method can determine the required storage for a smaller target release. This graphical method was developed one hundred and twenty years ago (Rippl, 1883) and has been widely used until now, although criticized (e.g. Schultz, 1976) for providing no information on the probability of failure and for the fact that the reservoir capacity determined by this method increases with the arbitrary length of available observed inflow data. As shown by Feller (1951) this increase is asymptotically proportional to the square root of the length of record.

A first variation of the method is its application with synthetic, rather than observed, data (e.g. Schultz, 1976). This eliminates the drawback of the arbitrary length of record and also provides some measure of uncertainty by applying the same procedure with different generated synthetic series. However, this kind of description of uncertainty is not consistent with a rational definition of reliability (e.g. that of the previous section).

A more theoretical flavor for the method has been given by the so-called *range analysis*, commenced with the work of Hurst (1951; see also Kottegoda, 1980, p. 184). Range is essentially the algebraic difference of the maximum and minimum departures of the mass curve from the straight line that joins its starting and ending points. The range concept has greatly contributed to the understanding and description of the so-called Hurst phenomenon (see the entry SW-434 – Hydrologic Persistence and the Hurst Phenomenon) in hydrology, climatology and other geophysical sciences. Applied to a reservoir, the range represents the required storage of a reservoir operating without any spill or other loss and providing a constant outflow equal to the mean flow. Obviously, this is an oversimplification of a real reservoir. On the other hand, the range concept involves complexity in estimation, and simpler and more efficient methods have been proposed which can be used instead of range analysis (Koutsoyiannis, 2002).

An additional design method is the so-called *sequent-peak analysis* (Mays and Tung, 1992, p. 274; Mays, 2001, p. 400). Essentially, it is a tabulated version of mass-curve analysis and can also incorporate in the calculations, apart from runoff, the effect of precipitation, evaporation and leakage. The method does not involve the reliability concept, nor does it consider spills from the reservoir.

Simplified reliability-based procedures for reservoir design

As already mentioned, analytical determination of reliability in the general case of a reservoir fed by inflows with seasonality and arbitrary probability distribution and autocorrelation functions, is a very difficult, if not impossible, task (Pegram et al., 1980). Therefore, existing analyses have been based on several simplifications. However, the results of such analyses are very useful, at least for the initial stage of reservoir design. The typical simplifying assumptions are to:

- neglect secondary inflows (precipitation) and losses (evaporation, leakage);
- neglect seasonality by the adoption of an annual time step;
- neglect autocorrelation and assume that inflows are independent in time;
- use a specific distribution function for inflows, typically two-parameter such as normal, lognormal or gamma.

The objective of such probability-based theoretical analyses is to determine the relation of the following three quantities:

- reservoir size c; it is usually standardized as $\kappa := c / \sigma$, where σ is the standard deviation of annual net inflow X_t;
- demand δ, which is assumed constant for all years; it is usually standardized as $\varepsilon := (\mu - \delta) / \sigma$, where μ is the mean of annual net inflow X_t; ε has been termed the standardized inflow (Hurst, 1951; McMahon, 1993, p. 27.7) or the *drift* (Pegram, 1980);
- probability, expressed either as reliability α, probability of failure β, or return period T; because of the annual time step used, the time-based reliability (equations (6)-(7)) is identical to the occurrence-based reliability (equations (10)-(12)).

The first among the probabilistic approaches used in such analyses is the discretization of reservoir storage into several zones, each representing a certain state, and the use of a Markov chain model to represent transitions from state to state (Moran; 1959; Zsuffa and Gálai, 1987; see also Kottegoda, 1980, p. 264).

A second method is stochastic (Monte Carlo) simulation, in which a long synthetic series of inflows is generated from the appropriate distribution function and then transformed into a series of storage values using equation (2); reliability is then easily determined from the storage time series using (8). Gould (1960) using this method was able to propose a reservoir size-yield-reliability formula, fitted to 240 sets of Monte Carlo simulations for various combinations of demand, reservoir size and skewness of gamma distributed inflows. This is

$$(\varepsilon + 0.15)\ [\kappa + d_1(\alpha, \gamma)] = d_2(\alpha, \gamma) \tag{18}$$

where d_1 and d_2 are coefficients depending on reliability α and skewness γ, and are given by nomographs (see also Raudkivi, 1979. p. 323). McMahon and Mein (1986) adapted this formula to more explicitly indicate the reliability; this can be estimated from the standardized normal variate z_α corresponding to α, using equation

$$z_\alpha = 2\sqrt{\varepsilon\ (\kappa + d(\alpha)\ \sigma/\mu)} \tag{19}$$

where d is a coefficient depending on reliability α, and is given by a table (see also McMahon, 1993, p. 27.14).

A more accurate and rigorous theoretical methodology to estimate the reservoir size-yield-reliability relationship has been developed by Pegram (1980). This has been based on finite-difference and integral equations, which employ the reservoir dynamics equation (2) in a probabilistic context to determine the return period of emptiness. Pegram applied his methodology, for normal, lognormal and discrete inputs both independent and serially correlated. His results, when compared to those of the Gould method (equations (18)-(19)) indicate that the latter underestimates the reservoir size required to attain a certain reliability level. Using Pegram's results for normally distributed inflows, which were verified and expanded here with extended simulations, the following approximate relationship has been established:

$$\ln(T - 1) = 2\ (\varepsilon + 0.25)\ (\kappa + 0.5)^{0.8} \tag{20}$$

This is valid for $T > 2$ ($\alpha > 0.5$) and can be alternatively written as

$$\ln(T - 1) = -\ln(1/\alpha - 1) = (2/\sigma^{1.8})\ (\mu + 0.25\sigma - \delta)\ (c + 0.5\sigma)^{0.8} \tag{21}$$

For known mean μ and standard deviation σ of inflows, (21) can directly yield either the reliability α for known reservoir size and demand, or the reservoir size (c) for given demand and reliability, or the demand (δ) that can be met with a given reliability for known reservoir size. Equation (20) is graphically depicted in Figure 1 in comparison with Pegram's exact results. This equation can be suggested for preliminary estimations, but it should be applied with caution for the reasons explained in the next section.

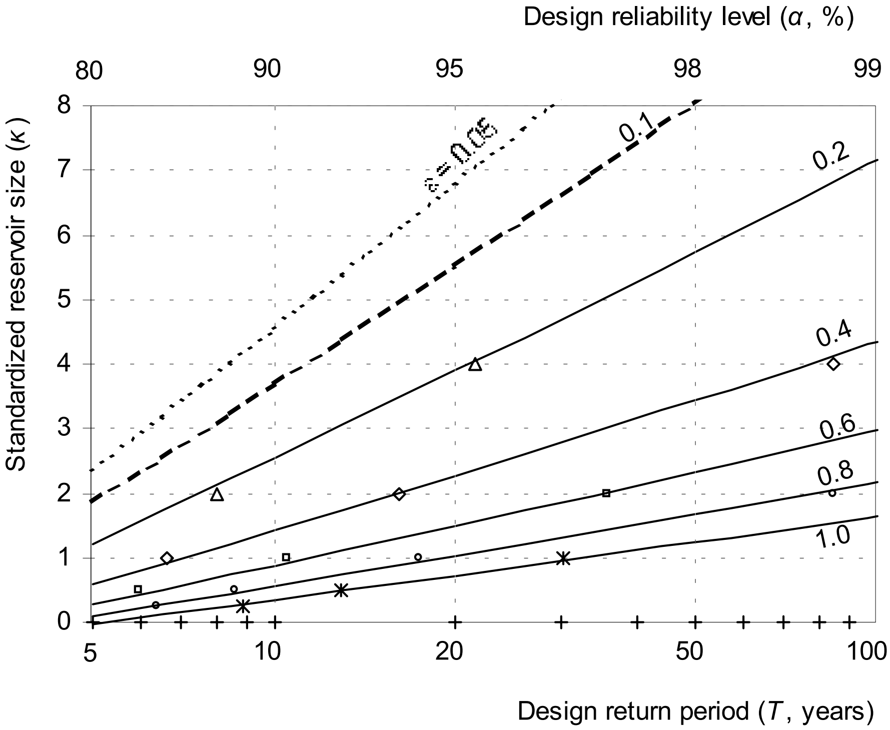

Figure 1 A simple representation of reservoir size-yield-reliability: Standardized reservoir size ($\kappa = K / \sigma$) required to achieve a certain drift ($\varepsilon = (\mu - \delta)/\sigma$) with a certain reliability level, for independent inputs normally distributed. Lines are constructed from equation (20) whereas plotted points are theoretical results by Pegram (1980) for $\varepsilon = 0.2$ (triangles), 0.4 (diamonds), 0.6 (squares), 0.8 (circles) and 1.0 (stars).

Effects of inflow characteristics to reservoir size

As explained earlier, simplified design procedures such as the one using equation (20), are based on a number of abridging assumptions about the inflows. Significant differences may appear if these assumptions are not valid. More specifically, what may cause significant departures from (20) are the hydrologic persistence, especially the long-term one, and the seasonal distribution of inflow and demand. Less significant differences are caused by the skewness of inflows and secondary inflows and losses.

The effect of hydrologic persistence is demonstrated in Figure 2, which depicts the standardized reservoir size (κ) required to achieve two combinations of drift and reliability (ε = 0.8, α = 98% and ε = 0.2, α = 90%) versus the lag-one autocorrelation coefficient, ρ. Two cases of hydrologic persistence have been examined, short-term and long-term. In the case of short-term persistence the inflows were assumed to follow the autoregressive process of order 1 (AR(1) or Markov), whereas in the case of long-term persistence they were assumed to follow the fractional Gaussian noise (FGN) process with Hurst exponent $H = \ln(2 + 2\rho) / \ln 4$. Obviously, the effect of persistence is very significant, especially in the case of long-term persistence and high demand (low drift ε). Thus, the required reservoir size for $\varepsilon = 0.2$ and α = 90% is $c = 2.4\ \sigma$ when $\rho = 0$ ($H = 0.5$) and becomes 4 times larger ($c = 9.6\ \sigma$) when $\rho = 0.4$ (H = 0.74).

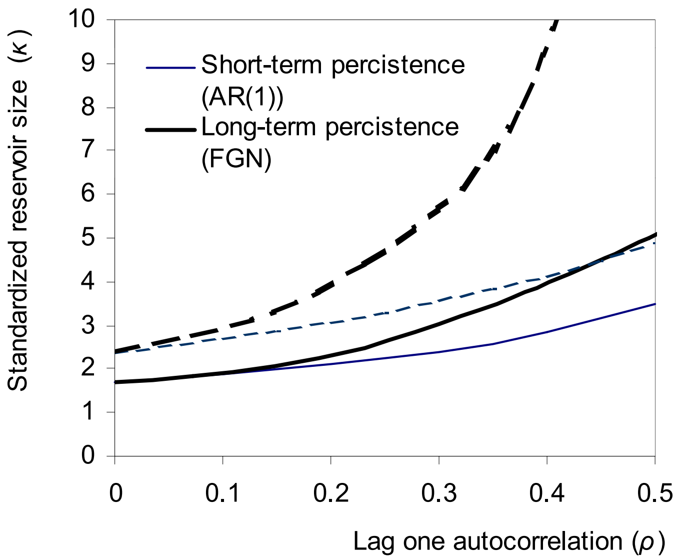

Figure 2 Effect of short-term and long-term hydrologic persistence of inflows to the standardized reservoir size (κ) required to achieve a drift $\varepsilon = 0.8$ with a reliability level α = 98% (continuous lines) and a drift $\varepsilon = 0.2$ with a reliability level α = 90% (dashed lines). Results are obtained by simulation.

To quantify the effect of seasonal variation of inflow and demand, it is observed that in the worst case, the total annual inflow comes before the beginning of withdrawal (the "sudden" model). The reservoir dynamics in (2) assume that inflow and withdrawal are distributed evenly during a year (the "steady" model). If inflow precedes withdrawal, (2) should be modified to read $S_t = \max[0, \min(S_{t-1} + X_t, c) - \delta_t]$. It is then easily determined that an extra storage equal to δ is required in addition to that estimated from (21). This worst case however is not very realistic, except in arid regions. In real world cases, the extra storage capacity required (in addition to that estimated from (21)) is a percentage of δ. This can be as high as 50% for water supply reservoirs and 80% for irrigation reservoirs in semiarid regions.

The effect of the skewness of inflows is demonstrated in Figure 3, which depicts the standardized reservoir size (κ) required to achieve three combinations of drift and reliability (ε = 0.8, α = 98%; ε = 0.2, α = 90%; and ε = 0.2, α = 98%) versus the coefficient of skewness, C_s. It can be observed that the effect of skewness is not significant; for low draft (high drift ε) this effect can be beneficial (lowering of required storage) but for high draft the required storage is practically insensitive to skewness.

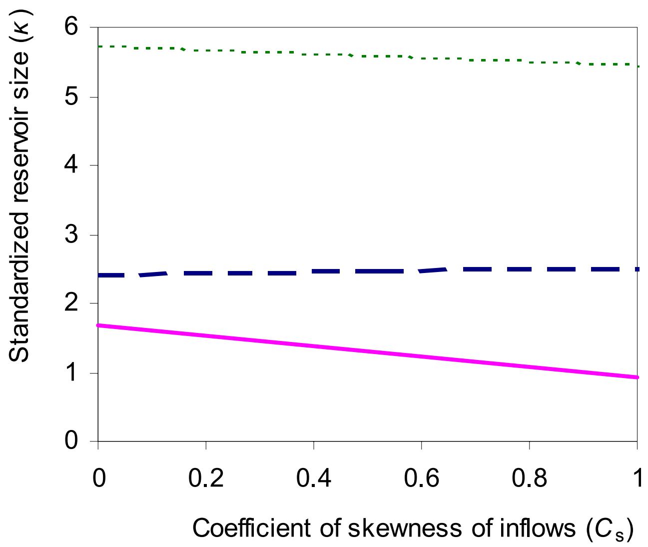

Figure 3 Effect of the skewness of inflows to the standardized reservoir size (κ) required to achieve a drift ε = 0.8 with a reliability level α = 98% (continuous line), a drift ε = 0.2 with a reliability level α = 90% (dashed line), and a drift ε = 0.2 with a reliability level α = 98% (dotted line). Results are obtained by simulation using independent two-parameter gamma distributed inflows.

Generalized simulation procedure for reliability-based reservoir design

The simplest general procedure to estimate the reservoir size-yield-reliability relationship in an accurate and detailed manner, and for any arbitrary inflow characteristics is stochastic (Monte Carlo) simulation. A simplified simulation procedure is outlined as follows:

1. Generate a series of inflows X_t at an appropriate timescale (e.g. monthly) using an appropriate stochastic model (e.g. Koutsoyiannis, 2000, 2001; see also the entry SW-913 – Stochastic Simulation in Water Resources).
2. Assume a reservoir size c.
3. Calculate a series of reservoir storages using (2).
4. Estimate the reliability using (13).
5. Repeat steps 3-4 for different reservoir sizes.

This procedure is very simple to execute even in a tabulated form on a spreadsheet.

The drawback of stochastic simulation is that it requires a vast simulation length to find accurate results. It can be shown that the required number of simulated time steps (e.g. months) to estimate the occurrence-based failure probability β' with an acceptable error $\pm\varepsilon\beta'$ and confidence γ is

$$n = k\,(z_{(1+\gamma)/2} / \varepsilon)^2\,(1 / \beta' - 1) \tag{22}$$

where k is the number of time steps per year and z_p is the p-quantile of the standard normal distribution. For instance, for $k = 12$, $\gamma = 95\%$ ($z_{(1+\gamma)/2} = 1.96$), $\varepsilon = 10\%$ and $\beta' = 0.01$ this yields $n = 456\,000$ months (38 000 years). Today, this is not a major problem as the required computer time for such a simulation length can be less than one second in a common PC (but not in a spreadsheet environment, which requires much more time).

The above-described simplified procedure can be extended to a detailed simulation procedure, which includes, in addition to runoff, the precipitation, evaporation and leakage of the reservoir. In this case, the level-area-volume and the level-leakage relationships are required to establish the functions $a(S)$ and $l(S)$ which yield the reservoir area a and the leakage l for any storage S (usually using interpolation from arrays of tabulated values). In this case, the runoff Q, has to be expressed in equivalent depth units, as are precipitation P and evaporation E. If f is the catchment area, the net inflow becomes

$$X_t = Q_t\,[f - a(S_{t-1})] + (P_t - E_t)\,a(S_{t-1}) - l(S_{t-1}) \tag{23}$$

where it was implicitly assumed that variations of the reservoir area and leakage within a time step are not large, so that $a(S_{t-1})$ and $l(S_{t-1})$ can be assumed as representative for the entire period within a time step. When the detailed simulation procedure is employed, three time series, instead of one, have to be synthesized. Obviously, Q_t and P_t are cross-correlated and therefore they cannot be generated independently of each other; a bivariate stochastic model is needed in this case. On the other hand, E_t can be generated independently of the other two series.

Obviously, the simulation procedure, either the simplified or the detailed one, can be directly applied with historical, rather than synthesized inputs. However, due to small record length, the accuracy of results in this case will not be satisfactory.

Extension of the simulation method, combined with optimization, for a multiple reservoir system has been discussed by Nalbantis and Koutsoyiannis (1997).

Design of additional storage zones of a reservoir

The design of the flood control storage is typically reliability-based but on a very different context from that described above for the active storage. A failure of the flood routing function of a reservoir is not a performance failure but a structural one: an overtopping of the dam due to a severe flood can result in collapse of the dam. Therefore much lower levels of probability of failure are adopted, of the order of 10^{-3}-10^{-6}. The typical steps here are (a) estimation of a design storm, based on statistical analysis of rainfall, for an appropriate return period (10^3-10^6 years); (b) estimation of the inflow hydrograph using an appropriate rainfall-runoff model; and (c) routing of this hydrograph through the spillway and estimation of the outflow hydrograph and the maximum water level. Implicit assumptions in the entire procedure, like the assumption that the reservoir is full at the beginning of the flood, decrease the risk further. It must be noted that several procedures have been proposed that are supposedly risk-free. These are based on the so-called probable maximum precipitation concept. However, it has been argued that a risk-free procedure is an illusion and the value of probable maximum precipitation can be exceeded with a certain probability (e.g. of the order of 10^{-5}; Koutsoyiannis, 1999).

The sizing of the dead storage has been typically based on the expected sediment accumulation in the reservoir for a certain design period (e.g. of the order of 10^2 years). Because of the large design period and the accumulation character of this process, the approach to this problem is very different. Expected values, rather than probabilities are involved in the calculations. The additional objectives that the dead storage serves, i.e. environmental protection (protection of the habitat of the reservoir during dry periods) and conservation of the quality of landscape, have not been given special attention, until now, and have not been considered in the design procedure. One would expect that, some years after construction, these additional objectives would not be served adequately because the water in the dead volume would be reduced due to sediment accumulation. Fortunately, however, this has not been the case: the implicit assumption that sediments will reach the bottom of reservoir near the dam is not verified. Thus, in large reservoirs, sediment accumulation occurs mainly in the active zone of reservoir (near the entrance of the river to the reservoir) and much less in the dead storage. The unfavorable consequence is the reduction of the active storage. These problems need to be further investigated in future reservoir designs.

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