



The scaling properties in the distribution of hydrological variables as a result of the maximum entropy principle

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It is well known that the principle of maximum entropy (ME), when applied to the probability distribution of a random variable with known mean and variance, results in the normal distribution. If the variable is non-negative, as happens with hydrological variables such as rainfall and runoff, the same principle results in the truncated normal distribution. Mathematically, this distribution can have a coefficient of variation ranging from zero to unity, with the upper bound corresponding to the exponential distribution. At fine time scales, rainfall and runoff have coefficients of variations higher than one, so the classical entropy approach, constrained by known mean and variance, is not applicable. However, a generalization of entropy (specifically the use of the concept of nonextensive entropy) allows the application of the ME principle even in such cases and results in power-type distributions, which for low probabilities of exceedence have scaling properties. Thus, the ME principle can be used to infer the type of the distribution of a hydrological variable, i.e. whether it has scaling properties or not, and to quantify the scaling exponent using a simple indicator such as the coefficient of variation. This theoretical framework is validated with several real-world examples concerning rainfall, runoff and temperature data at several time scales. Given that entropy is a measure of uncertainty, the applicability of the ME principle to the distribution of hydrological variables emphasizes the dominance of uncertainty in hydrological processes.