The Scaling Model of Storm Hyetograph Versus Typical Stochastic Rainfall Event Models

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Session HSA4/NPl.03
"*Scaling vs. non-scaling methods in rainfall modelling*"

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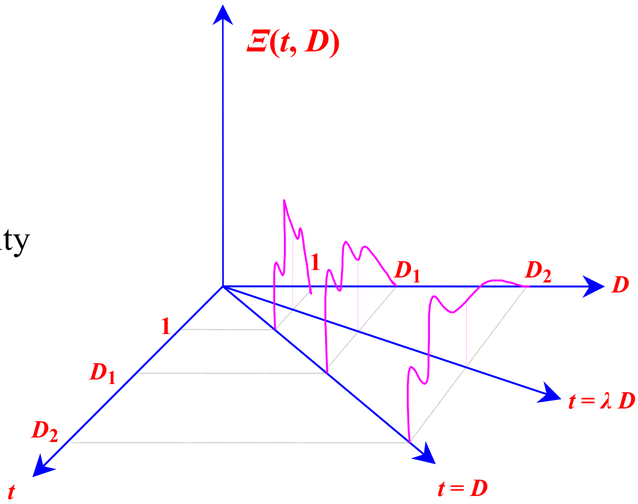
Topics of the Presentation

- ★ **Synopsis of the Scaling Model of Storm Hyetograph**
- ★ **Scaling Model performance evaluation**
- ★ **Some applications of the Scaling Model**
- ★ **Synopsis of the Bartlett-Lewis (BL) models**
- ★ **Comparison of the models**
- ★ **Conclusions**

Synopsis of the Scaling Model of Storm Hyetograph
General Structure



Main hypothesis

$$\{\Xi(t, D)\} \overset{d}{=} \{\lambda^{-\kappa}\, \Xi(\lambda\, t, \lambda\, D)\}$$

where

$\Xi()$: instantaneous rainfall intensity
D : duration of the event
t : time ($0 \le t \le D$)
κ : scaling exponent

Secondary hypothesis
(weak stationarity, within the event)

$$E[\Xi(t, D)] = c_1\, D^{\kappa},$$

$$E[\Xi(t, D), \Xi(t + \tau, D)] = \psi(\tau / D)\, D^{2\kappa} \qquad \text{where} \quad \psi(\tau / D) = \alpha \left[\left(\frac{\tau}{D}\right)^{-\beta} - \zeta\right]$$

and c_1, α, β, ζ parameters ($c_1 > 0$, $\alpha > 0$, $0 < \beta < 1$, $\zeta < 1$)

Synopsis of the Scaling Model of Storm Hyetograph
Statistics of Total and Incremental Depth

Total depth, H

$E[H] = c_1\, D^{1+\kappa}$
$\mathrm{Var}[H] = c_2\, D^{2(1+\kappa)}$

Incremental depth, X, for a time interval $\Delta = \delta\, D$

$$E[X_i] = c_1\, D^{1+\kappa}\, \delta$$

$$\mathrm{Var}[X_i] = D^{2(1+\kappa)}\, \delta^2\, \frac{(c_1^2 + c_2)(\delta^{-\beta} - \varphi) - c_1^2\,(1 - \varphi)}{1 - \varphi}$$

$$\mathrm{Cov}[X_i, X_{i+m}] = D^{2(1+\kappa)}\, \delta^2\, \frac{(c_1^2 + c_2)[\delta^{-\beta} f(m, \beta) - \varphi] - c_1^2\,(1 - \varphi)}{1 - \varphi}$$

where

$$c_2 = \frac{\alpha\,(1 - \varphi)}{(1 - \beta)\,(1 - \beta/2)}$$

$$\varphi = \zeta\,(1 - \beta)\,(1 - \beta / 2)$$

$$f(m, \beta) = \begin{cases} \frac{1}{2}[(m-1)^{2-\beta} + (m+1)^{2-\beta}] - m^{2-\beta}, & m > 0 \\ 0, & m = 0 \end{cases}$$

Synopsis of the Scaling Model of Storm Hyetograph
Parameter Estimation

κ, scaling exponent; c_1, mean value parameter — Estimated by least squares from $E[H] = c_1\, D^{1+\kappa}$

c_2, variance parameter — Estimated from $c_2 = \mathrm{Var}[H] \,/\, D^{2(1+\kappa)}$

β, φ — correlation decay parameters — Estimated by least squares from $\dfrac{\delta^{-\beta} - \varphi}{1 - \varphi} = \dfrac{E[X^2]}{E^2[X]}\,\dfrac{E^2[H]}{E[H^2]}$

Alternatively: Simultaneous estimation of c_2, β, φ by minimising the fitting error in Var[H], Var[X] and $\rho_X(1)$.

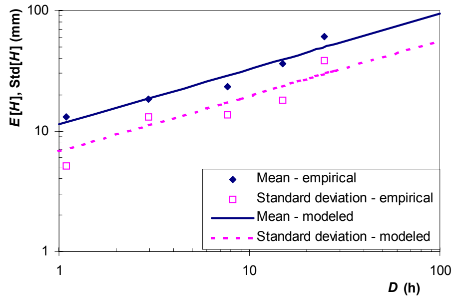
Scaling Model Performance Evaluation
Data Sets

Location	Country	Event type	Season	Record period (yr)	Number of events
Aliakmon	N. Greece	All	April	13	89
Reno (areal)	N. Italy	HD >1 mm	All year	2	149
Evinos	C. Greece	HD > 7 mm or DD > 25 mm	Oct-Apr	20	200
Evinos	C. Greece	HD > 7 mm or DD > 25 mm	May-Sep	20	93
Ortona	Florida-USA	HD > 1 mm	All year	2	430
Parrish	Florida-USA	All	All year	18	1035
AMTSP Zografou	Athens-Greece	HD > 5 mm or DD > 15 mm	All year	5	81

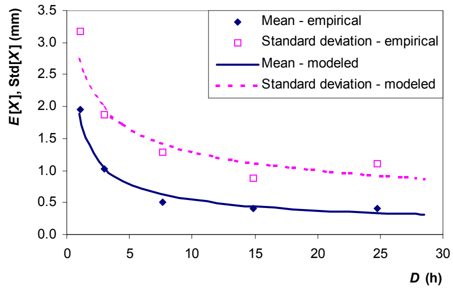
Scaling Model Performance Evaluation

Zografou, 10-min data

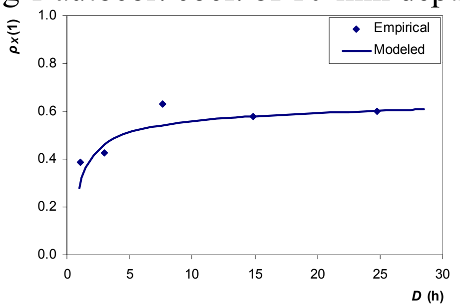
Mean and st. deviation of total depth



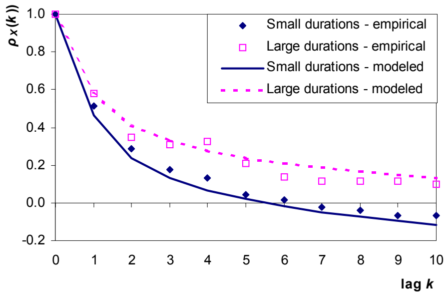
Mean and st. deviation of 10-min depth



Lag 1 autocor. coef. of 10-min depth



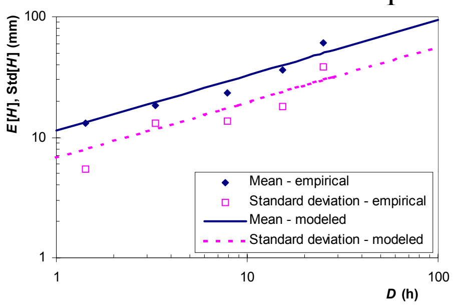
Autocorrelation function of 10-min depth



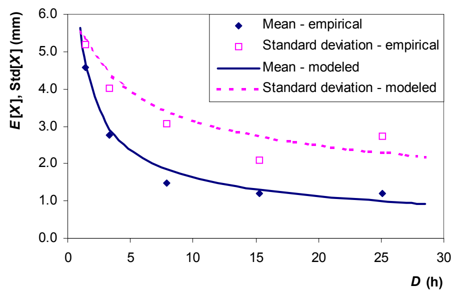
Scaling Model Performance Evaluation

Zografou, 30-min data (parameters from 10-min data)

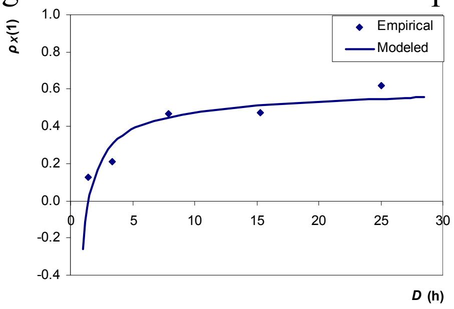
Mean and st. deviation of total depth



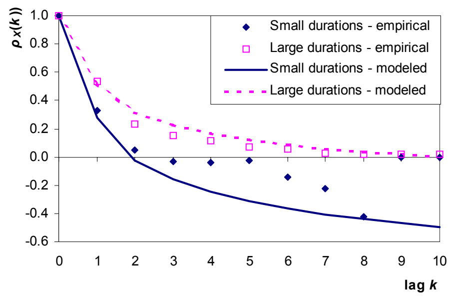
Mean and st. deviation of 30-min depth



Lag 1 autocor. coef. of 30-min depth



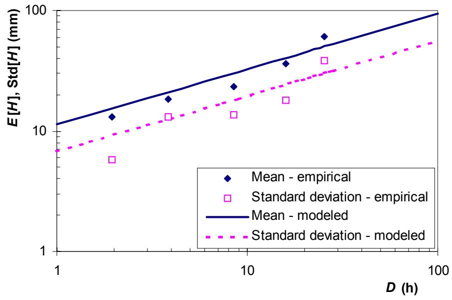
Autocorrelation function of 30-min depth



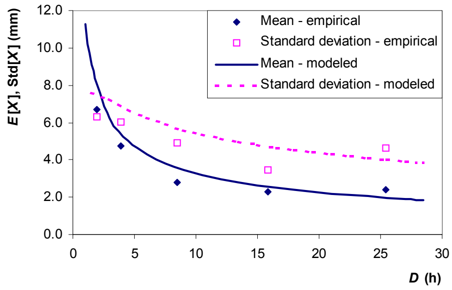
Scaling Model Performance Evaluation

Zografou, hourly data (parameters from 10-min data)

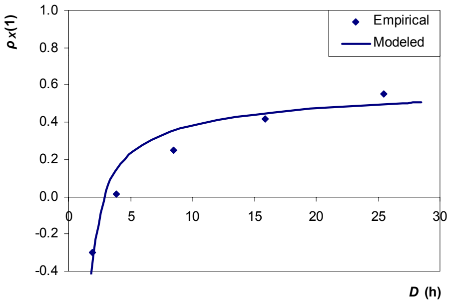
Mean and st. deviation of total depth



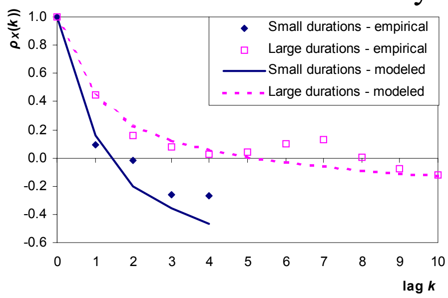
Mean and st. deviation of hourly depth



Lag 1 autocor. coef. of hourly depth



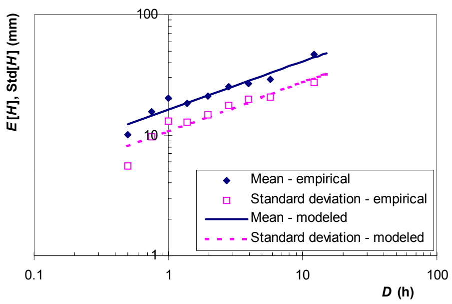
Autocorrelation function of hourly depth



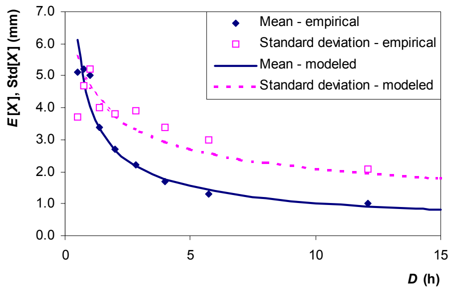
Scaling Model Performance Evaluation

Parrish, 15-min data

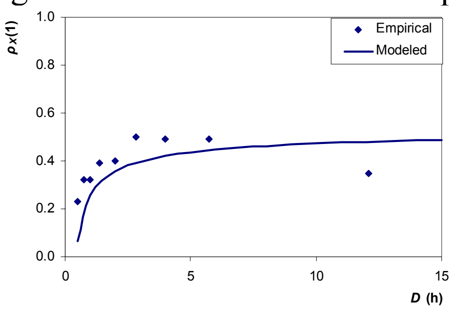
Mean and st. deviation of total depth



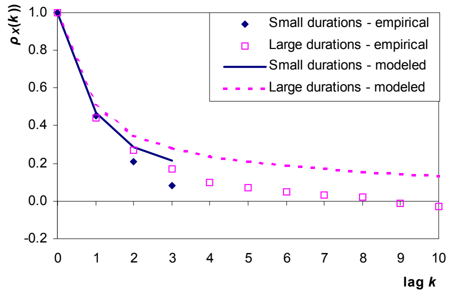
Mean and st. deviation of 15-min depth



Lag 1 autocor. coef. of 15-min depth



Autocorrelation function of 15-min depth



D. Koutsoyiannis and N. Mamassis, The Scaling Model versus typical stochastic rainfall models

Some Applications of the Scaling Model

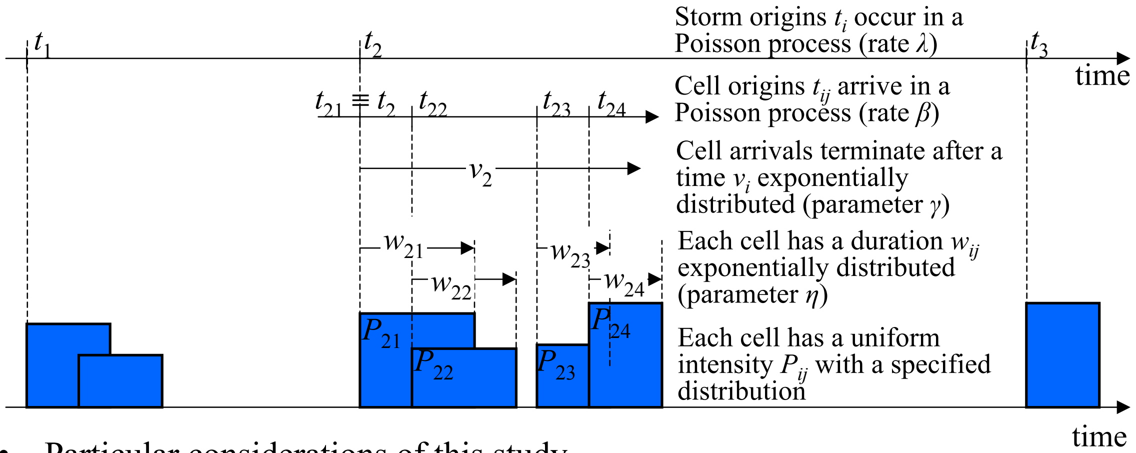
★ Stochastic rainfall forecasting by conditional simulation (approximately known duration and total depth) [Mamassis, N., D. Koutsoyiannis, and E. Foufoula-Georgiou,, XIX General Assembly of European Geophysical Society, Grenoble, Annales Geophysicae, Vol. 12, 1994]

★ Continuous simulation of rainfall and comparison of simulated with historical series. Notably this comparison is not exhausted to typical statistical descriptors but includes also descriptors used in chaos literature such and correlation dimension and correlation integral. The results show a very satisfactory agreement between generated and historical series. [Koutsoyiannis, D., and D. Pachakis, Journal of Geophysical Research-Atmospheres, 101(D21), 1996]

★ Generation of synthetic storms (coupled with disaggragation techniques) for a given duration and total depth, extracted from IDF curves. The model generates an ensemble of hyetographs by stochastically disaggregating the total depth to incremental depths. [Koutsoyiannis, D., and D. Zarris, Presentation at the XXIV EGS General Assembly, Session HSA4.03, 1999]

Synopsis of the Bartlett-Lewis (BL) Models

BL Point Process (Rodriguez-Iturbe Et Al., 1987, 1988)



- Particular considerations of this study
  - Focus on the storm event only (not in continuous time)
  - Neglecting of the possibility of overlapping of storms ($\lambda << \beta, \gamma, \eta$)
  - Neglecting of nonstationarities due to the origin and the termination effect
- With these assumptions the structure of the storm event depends on the parameters β and η, and the distribution of P_{ij} (not affected by λ and γ)

Synopsis of the Bartlett-Lewis (BL) Models

Used Versions of the BL Model and Corresponding Equations

Equations derived for the Original model but for the interior of the event

$$E[X_i] = \kappa\, E[P]\, \varDelta, \qquad \mathrm{Var}[X_i] = \frac{2\,\kappa\, E[P^2]}{\eta^2}\,(\eta\, \varDelta - 1 + \mathrm{e}^{-\eta\, \varDelta}),$$

$$\mathrm{Cov}[X_i, X_{i+m}] = \frac{\kappa\, E[P^2]}{\eta^2}\,(1 - \mathrm{e}^{-\eta\, \varDelta})^2\, \mathrm{e}^{-\eta\,(m-1)\, \varDelta} \qquad (m > 1)$$

where $\kappa = \beta / \eta$ and X = the incremental rainfall depth for time step $\varDelta$.

Equations derived for the Modified model: random parameter η (gamma distributed with shape parameter α and scale parameter ν) and constant parameter κ

$$E[X_i] = \kappa\, E[P]\, \varDelta, \qquad \mathrm{Var}[X_i] = \frac{2\,\kappa\, E[P^2]\, \nu^2}{(\alpha-1)(\alpha-2)}\,\left(\varphi_1^{\alpha-2} + \frac{\alpha-2}{\varphi_1} - \alpha + 1\right),$$

$$\mathrm{Cov}[X_i, X_{i+m}] = \frac{\kappa\, E[P^2]\, \nu^2}{(\alpha-1)(\alpha-2)}\left\{\varphi_{m+1}^{\alpha-2} + \varphi_{m-1}^{\alpha-2} + \varphi_m^{\alpha-2}\left[(\alpha-2)\left(\frac{\varphi_m}{\varphi_{m-1}} + \frac{\varphi_m}{\varphi_{m+1}}\right) - 2\,(\alpha-1)\right]\right\} \qquad (m > 1)$$

where $\varphi_m = \dfrac{\nu}{\nu + m\, \varDelta}$

Synopsis of the Bartlett-Lewis (BL) Models

Additional Versions of the BL Model

Additional Version 1

As in the Modified Model (random parameter η) but assuming mean cell duration $1 / \eta$ proportional to the (known) D

- Assumption equivalent to scaling of all parameters in time
- In agreement with the remark of Rodriguez-Iturbe et al. (1988) that *cells with longer durations tend to last longer and to have longer interarrival times between cells*
- κ remains constant (as in the Modified Model)
- Equations as in the Original Model, but with $\eta = \eta_0\, D^{-1}$

Additional Version 2

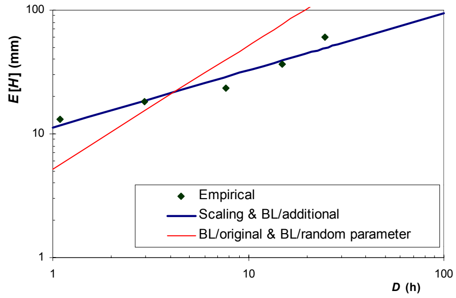
Generalisation of Additional Version 1, assuming that both η and β depend on D in a power law. Equations as in the Original Model, but with

- $\eta = \eta_0\, D^{\eta_1}$
- $\beta = \beta_0\, D^{\beta_1}$
- $\kappa = \kappa_0\, D^{\kappa_1}$ where $\kappa_0 = \beta_0 / \eta_0$ and $\kappa_1 = \beta_1 - \eta_1$

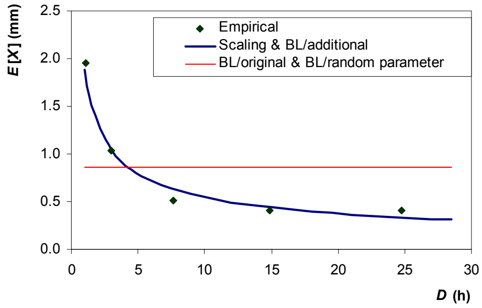
Comparison of the Models

Zografou, 10-min data

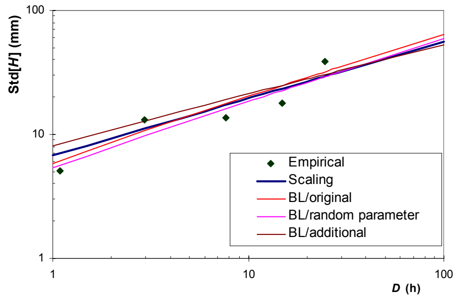
Mean of total depth



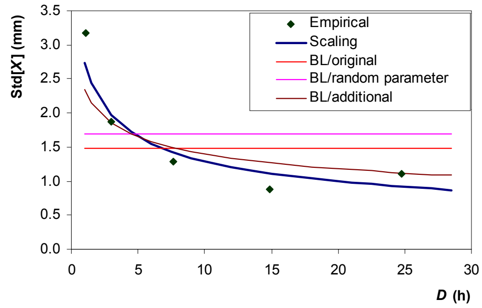
Mean of 10-min depth



St. deviation of total depth



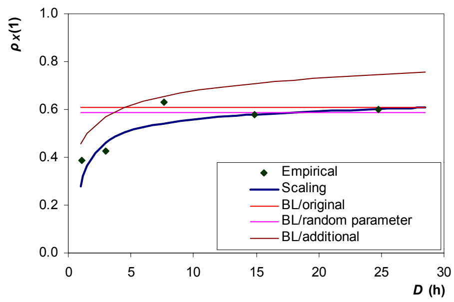
St. deviation of 10-min depth



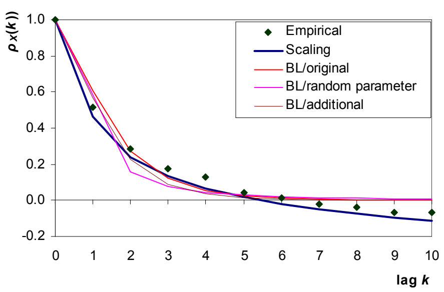
Comparison of the Models

Zografou, 10-min data (2)

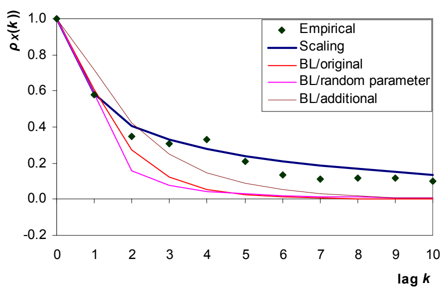
Lag 1 autocorrelation coefficient of 10-min depth



Autocorrelation function of 10-min depth (small durations)

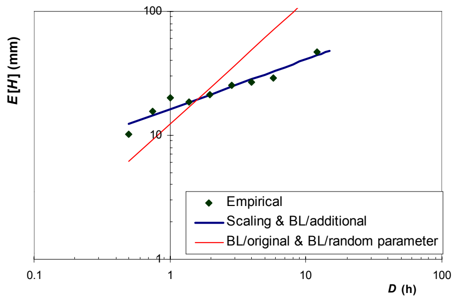


Autocorrelation function of 10-min depth (large durations)

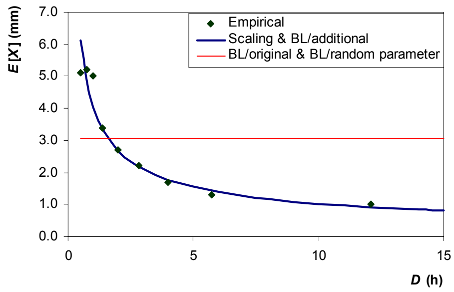


Comparison of the Models
Parrish, 15-min data

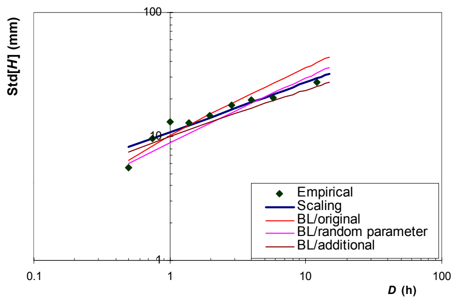
Mean of total depth



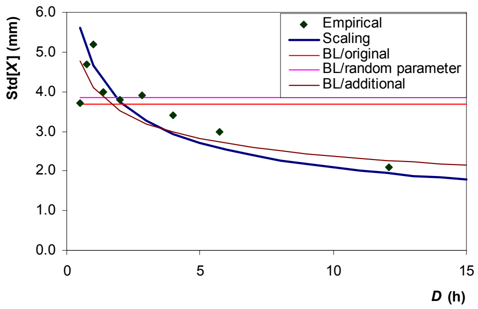
Mean of 15-min depth



St. deviation of total depth

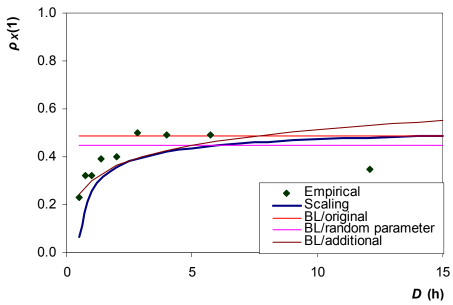


St. deviation of 15-min depth

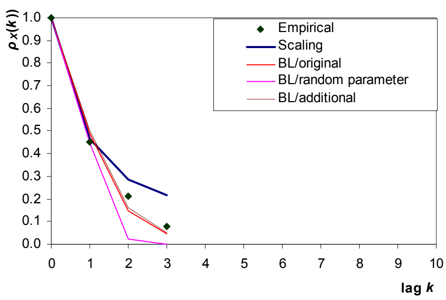


Comparison of the Models
Parrish, 15-min data (2)

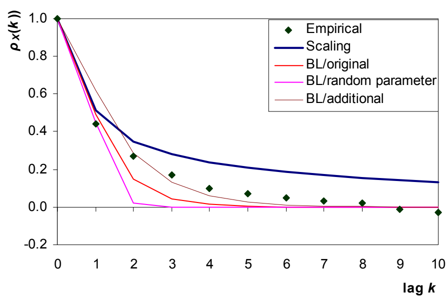
Lag 1 autocorrelation coeficient of 15-min depth



Autocorrelation function of 15-min depth (small durations)



Autocorrelation function of 15-min depth (large durations)



Conclusions

★ The Scaling Model of Storm Hyetograph is suitable for a variety of data sets regardless of season and rain type. The model can preserve characteristics such as

- ✦ the increase of the mean and standard deviation of total depth with duration
- ✦ the decrease of mean and standard deviation of the incremental depth with duration
- ✦ the increase of the lag 1 autocorrelation coefficient of the incremental depth with duration
- ✦ the decay of the autocorrellation function of the incremental depth

★ The comparison of the models shows that the Scaling Model can preserve better the internal structure of the rainfall events, than several versions of the Bartlett-Lewis model

★ The additional versions of the Bartlett-Lewis model, developed in this study, have an improved ability to capture some of the rainfall event characteristics, especially the standard deviation and the lag1 autocorrelation coefficient of the incremental depth