An entropic-stochastic representation of rainfall intermittency: The origin of clustering and persistence

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Appendix (Electronic Data Supplement)

An alternative technique to determine elements of vectors of state probabilities

The following technique, additional to the quasi-Markov technique described in section 8, and referred to as the proportionality technique, is based on the following intuitive equations relating the conditional probabilities in a non-Markovian manner:

$$\frac{\pi_{0|0\mathrm{x}1}}{\pi_{0|0\mathrm{x}0}} = \zeta = \frac{\pi_{1|1\mathrm{x}0}}{\pi_{1|1\mathrm{x}1}} \quad \text{(A.1)}$$

where ζ is a constant, independent on x, which, as explained in section 7 is an abbreviation of the index permutation $j_1 \dots j_{q-2}$. At each order q, the constant ζ is determined directly as $\zeta = \pi_{0|00\dots01} / \pi_{0|00\dots00}$ (where all items in ellipses are zero). Both terms of this ratio are known for given $p^{(k)}$, as described in section 8. The intuitive rationale behind this assumption can be described considering a sequence of four time intervals. If it is less likely to have a dry interval following two dry and one wet interval than it is for having a dry interval following three dry intervals (i.e. $\pi_{0|001} < \pi_{0|000}$ or $\zeta < 1$), then it is equally more likely to have a dry interval following one dry and two wet intervals than it is for having a dry interval following a dry, a wet and another dry interval (i.e. $\pi_{0|011} < \pi_{0|010}$). A similar logic is behind the second part of (A.1). It can be easily shown that (A.1) results in the following equations determining all conditional probabilities contained in vector $\boldsymbol{\pi}_q$ in terms of unconditional probabilities contained in vector $\mathbf{p}_{q-1}$:

$$\pi_{0|0\mathrm{x}0} = \frac{p_{00\mathrm{x}}}{p_{0\mathrm{x}0} + \zeta p_{0\mathrm{x}1}}, \; \pi_{0|0\mathrm{x}1} = \frac{\zeta p_{00\mathrm{x}}}{p_{0\mathrm{x}0} + \zeta p_{0\mathrm{x}1}}, \; \pi_{0|1\mathrm{x}0} = 1 - \frac{\zeta p_{11\mathrm{x}}}{\zeta p_{1\mathrm{x}0} + p_{1\mathrm{x}1}}, \; \pi_{0|1\mathrm{x}1} = \frac{1}{\zeta} - \frac{p_{11\mathrm{x}}}{\zeta p_{1\mathrm{x}0} + p_{1\mathrm{x}1}} \quad \text{(A.2)}$$

A comparison of the quasi-Markov (section 8) and proportionality assumptions is given in Figure A1, which is related to the hypothetical case 5 of Figure 4. For comparison, Figure A1 includes also plots for case 4 (Markov).

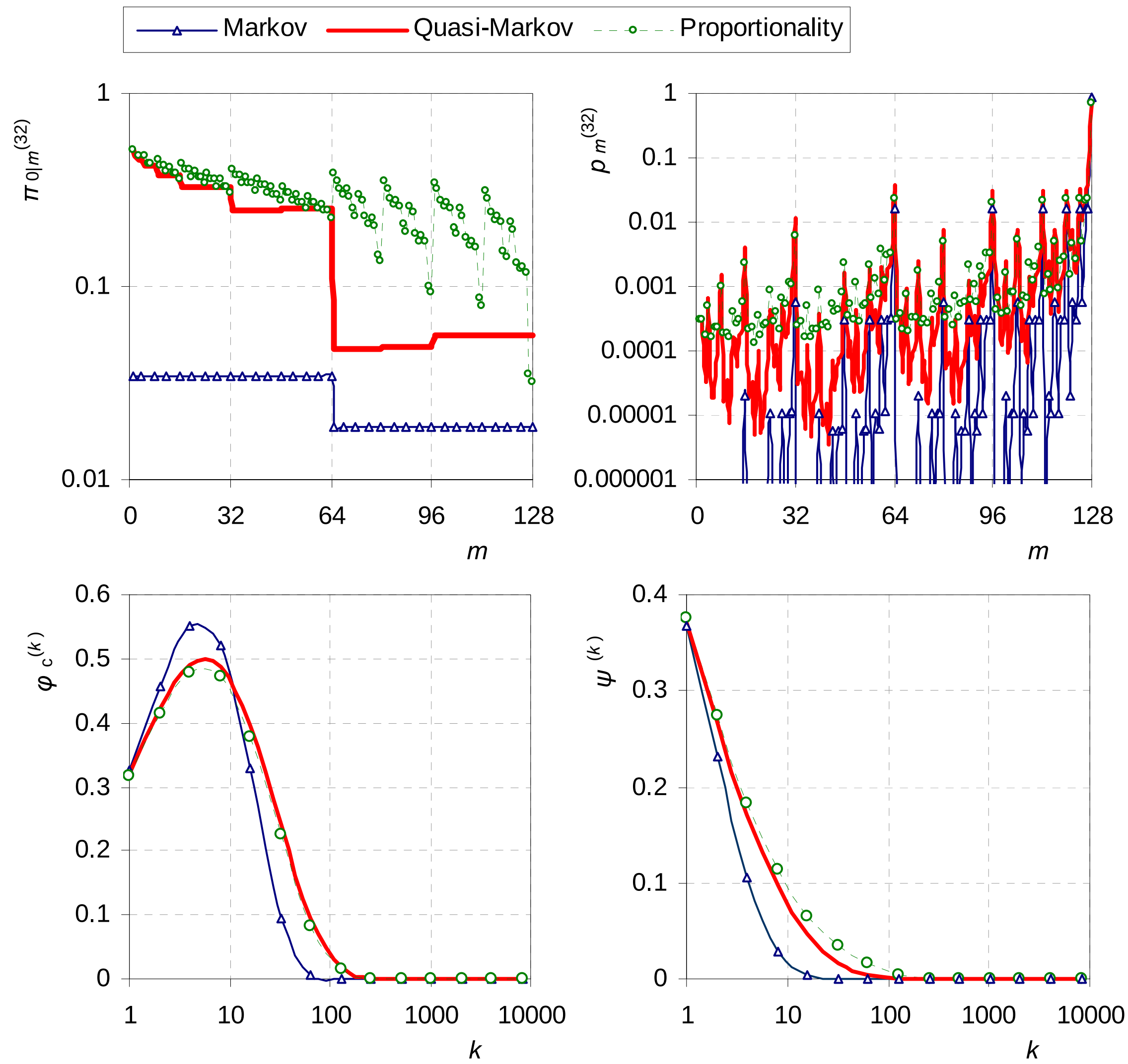

Figure A1 More detailed depiction of hypothetical case 5 of Figure 4 and comparison of results obtained by the quasi-Markov and proportionality assumptions: (Up) Transition probabilities to zero state ($\pi_{0|m}^{(32)}$) and state probabilities ($p_m^{(32)}$) for order 7 and scale 32 of each of the 2^7 = 128 possible states (where m = 1 denotes the state 0000000 and m = 128 denotes the state 1111111). (Down) Conditional entropy ($\varphi_c^{(k)}$) and information gain ($\psi^{(k)}$) versus scale k. For further comparison, case 4 (Markov) of Figure 4 is also plotted in all panels.

The plots of the 2^7 = 128 transition probabilities for scale 32 and order 7 (elements of $\boldsymbol{\pi}_7^{(32)}$) indicate very significant differences between the quasi-Markov and proportionality models. These differences become milder in the case of unconditional probabilities for the same order and scale (elements of $\mathbf{p}_7^{(32)}$) and almost disappear in the conditional entropy ($\varphi_c^{(k)}$) and information gain ($\psi^{(k)}$). On the other hand, the differences of both from the Markov model are very significant and this indicates that the deviation of probability dry $p^{(k)}$ (first element of $\mathbf{p}_q^{(k)}$) from the Markov case is more significant than is the Markovian or non-Markovian structure of the remaining probabilities (other elements of $\mathbf{p}_q^{(k)}$). In addition, it is observed in Figure A1 that the quasi-Markov assumption results in very slightly higher conditional entropy in comparison to the proportionality assumption. This happens for all scales and the same results were found for other similar examples as well in comparisons with other assumptions investigated. Therefore, the quasi-Markov structure seems to be more consistent with the ME approach.