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Sources of the stochastic regulation of climate

Managing uncertainty in an uncertain world

(This presentation draws on Mackey, R., 2007)

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1. Abstract

Douglass North emphasised that our capacity to deal with uncertainty effectively is essential to our succeeding in a non-ergodic world*. He explained that an ergodic phenomenon has a underlying structure so stable we can develop theory that can be ap-plied time after time, consistently. In contrast, the world with which we are concerned is continually changing: it is continually novel. According to Douglass North the main responsibility of governments in managing the impact of the potentially catastrophic events that arise in a non-ergodic world is to mange society's response to them so as to enable the society to adapt to them as efficiently as possible. It is crucial, there-fore, that the methodologies used to understand the exceedingly complex, perhaps intrinsically random, phenomena measured in the time series of natural climate and geophysical phenomena, inform governments as accurately as possible of the future uncertainty of the likely pattern of development indicated by the time series. Classical time series analysis (that features, for example, in the reports of the Intergovernmen-tal Panel on Climate Change) necessarily underestimates future uncertainty, whereas a stochastic approach using scaling methodologies estimates future uncertainty more accurately. Variations in the quantity, intensity and distribution over the Earth of so-lar output, including electromagnetic radiation, matter and the Sun's electromagnetic field, (including the impact of cosmic rays modulated by solar activity), the variable gravitation force the Sun exerts on the Earth, the Moon and the Moon and the Earth as a system, with total solar activity modulated by gravitational interaction between the Sun and the solar system, and interactions between these processes is hypothesised to be main source of the stochastic regulation of the climate. Interaction between the totality of solar influence and the major atmospheric/oceanic oscillations is a key way in which the stochastic regulation proceeds. The presentation examines these themes by reference to time series analysis of river flow and sunspot data, concluding with an outline of the strategic policy advice that scientists might present to the Australian Government, having regard to the relationship between Australia's episodes of flood, drought and bushfire on the one hand, and global atmospheric oscillations, oceanic variables and the Sun's variable activity on the other.

2. Our world is non-ergodic

- Ergodic….

  Stable underlying structure

  Theory of structure can be settled, results OK time after time

- Non-ergodic….

  Underlying structure unstable: continually changing, continually novel

  Potentially catastrophic events inevitable

  Theory of structures inherently partial

- A new philosophic outlook, probabilistic metaphysics, may be helpful to our understanding of a non-ergodic world (see definitions).

(Source: Douglass North)

3. Non-ergodic climate dynamics: Sources

- Gravitational interaction – Variable solar activity and solar system: The sun is engaged in continual motion but never recedes far from the common center of gravity of all the planets (Newton, 1687).
- Gravitational interaction – Planetary orbits effect the Sun's variable asphericities and the Sun's variable asphericities effect the planetary orbits: phase synchronisation is statistically significant (Palus et al., 2007)
- Variable solar activity…..
    - Electromagnetic radiation
    - Matter
    - Electromagnetic field (+ modulation of cosmic rays)
    - Gravitational field (includes impact on the Earth, the Moon, and the Earth/Moon system)
- Interaction of all variable solar activities
    - Non-linear mutual amplification
- Interaction of variable solar activities and global atmospheric and oceanic oscillations

Periodicity and randomness are a feature of each of all of the above.

4. The Solar Epitrochoid

- Solar inertial motion modulates the solar dynamo
- Solar inertial motion has epitrochoid-shaped orbit
- Solar activity high in near circular shaped component of epitrochoid
- Solar activity low during Sun’s retrograde loop-the-loop component
- Sun’s motion in retrograde loop since 1996 continuing until 2040
- The Earth cools during the retrograde loop and warms during the near circular shaped component

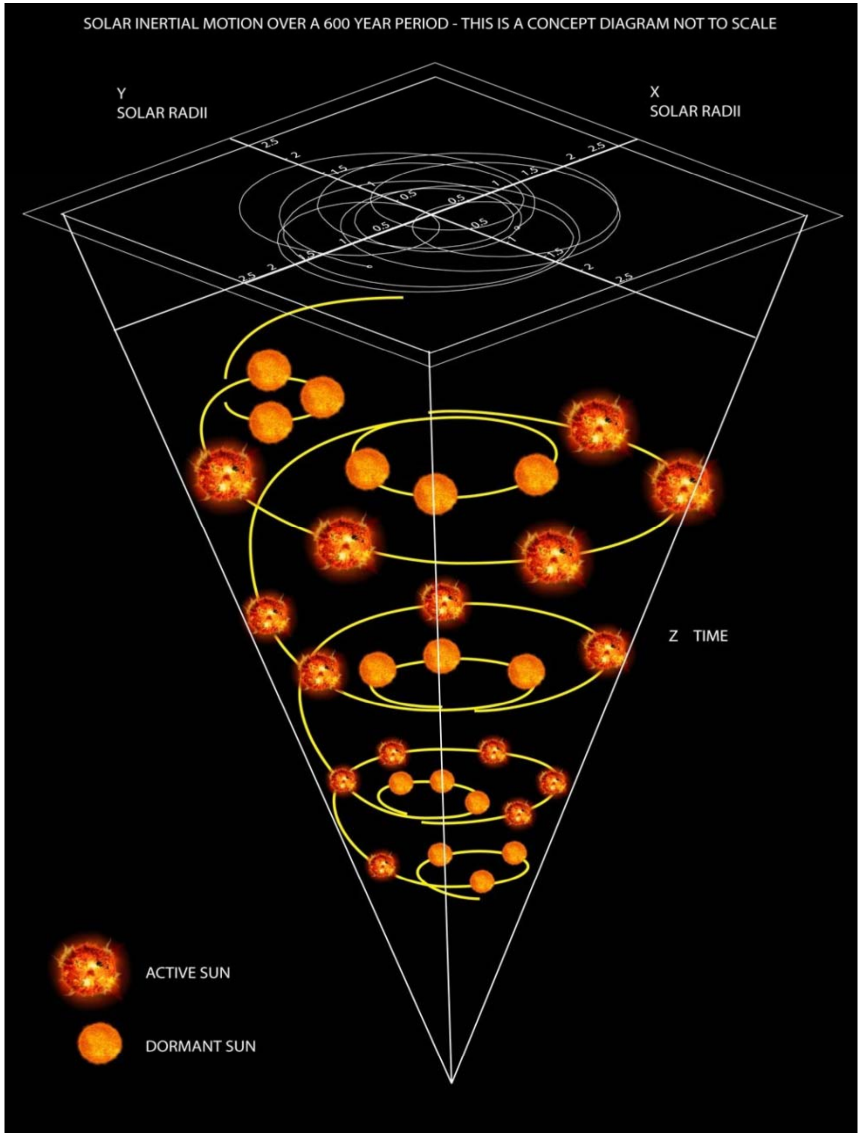

Figure 1. The solar epitrochoid (illustrated and produced by Daniel Brunato, University of Canberra, 2006)

5. Solar dynamo modulated by solar orbital motion: Possible processes

- Sun’s variable torque
- Precessional effect
- Solar orbital non-inertial Coriolis force
- Resonant effect of planets’ orbits
- Horizontal solar tides
- Superposition of planetary tides
- Spin –orbit coupling
- Phase synchronisation
- Resonant amplification between solar and climate periodicities: significant
- Solar cycle impact on key atmospheric (e.g. El Nino/Southern Oscillation (ENSO), Quasi-Biennial Oscillation (QBO), Pacific Decadal Oscillation (PDO), Interdecadal Pacific Oscillation (IPO), North Atlantic Oscillation (NAO), Atlantic Multidecadal Oscillation (AMO), Indian Ocean Dipole (IOD), Artic Oscillation (AO), and the northern and southern polar vortexes) , oceanic oscillations is significant
- lunisolar tides: significant impact
- Geomagnetic field: significant impact
- Global electric circuit: significant impact
- Significant non-linear interaction between and within processes

6. Non-ergodic climate dynamics: Results (i)

- Major, often catastrophic, changes to cultures and societies globally since dawn of history (Fagan, 1999, 2000, 2004)
- Sudden (< 40 years) sea level changes (2m) during last 1,000 years (Baker, 2005)
- Increased solar activity has warmed the Earth over last 150 years (Solanki et al., (2003 to 2005)) cited in Mackey, R. (2007))
- Earth's temperature, especially ocean, periodicities inherit solar activity periodicities
- Increased solar activity has increased amounts of CO2 and water vapour in atmosphere
- Solar/climate relationship varies with latitude and longitude: a global solar/climate uniform measure is therefore misleading
- Solar activity impact is non-linear and stochastic
- Solar activity has decreased since 1992
- Cooling of oceans between 2003 and 2005 reported in 2006 (Lyman et al, 2006)
- Increased frequencies and intensities of ENSO since 1997

7. Non-ergodic climate dynamics: Results (ii)

NASA Panel solar cycle 24 prediction
to be announced on April 25

- Solar Cycle 24 began on July 23, 2006
- Solar Cycle 24 expected to be similar to Solar Cycle 14, the weakest in the last 150 yrs
- Amplitude of solar cycles 24, 25, 26 (2007 to 2040) expected to diminish
- A little ice age expected to begin during current solar cycle, becoming similar to 1784 to 1823 and 1877 to 1913
- Climate expected to warm again from 2040 onwards

8. Non-ergodic climate dynamics: Australia

- Solar activity largely regulates Australia's climate

  Since ENSO (modulated by IPO, IOD, and the southern polar vortex) largely regulates Australia’s climate, specifically, the cycles of drought, flood and fire and solar activity largely regulates ENSO and other atmospheric oscillations

- Solar cycle/climate relationship established by the Bureau of Meteorology (BoM) in 1920s

  BoM (correctly) attributed the catastrophic Federation Drought (1900 - 14) to Solar Cycle 14

- Australia’s devastating phase of drought and fire likely to continue if Solar Cycle 24 turns out to be similar to Solar Cycle 14

9. Intrinsic Randomness

- If a system, such as Poincare's homoclinic tangle, is chaotic, and future outcomes of the system uncertain, the system may be equivalent to a random system.
- Non-ergodic climate dynamics involves the following possibly chaotic systems which also contain intrinsic randomness and periodicity:

  Solar system

  Solar dynamo

  Sunspot cycle

  Sun and the solar system interaction

  Atmospheric/ocean oscillations

  Solar activity and atmospheric/oceanic oscillation interaction

10. Periodicity and randomness in climate-geophysical time series

Objective: To predict the future path of a dynamic non-ergodic climate system characterised by intrinsic randomness and a type of periodicity (ie Hurst) which increases dramatically the range and intensity of climate variability.

- Hydrological and other geophysical time series show fluctuations over many time scales: the trends observed in relatively short time scales, such as 10 or 100 years, are but elements of fluctuations over longer time scales: and there is no new thing under the sun. (The Book of Ecclesiastes, Chapter One verse 9. King James Bible).
- Classical statistics, applied to hydrology and climatology, describe only a portion of natural uncertainty, and underestimates seriously all relevant risks. (Koutsoyiannis (2000, 2002, 2003, 2005a, 2005b, 2005c).
- Do not assume that the variables are normally distributed independent random variables.
- Apply the Principle of Maximum Entropy to real world variables, subject to the constraints of intrinsic randomness and a type of periodicity (Koutsoyiannis (2000, 2002, 2003, 2005a, 2005b, 2005c).
- The Principle of Maximum Entropy is equivalent to the Principle of Stationary Action, an elementary principle of Physics.
- Huang et al. (1998) stressed the use of appropriate analytic methodologies to reveal clearly any nonlinear relationships (perhaps containing intrinsic fluctuations) when analysing time series of natural phenomena. Cohn and Lins (2005) noted: *with respect to temperature data, there is overwhelming evidence that the planet has warmed during the past century. But could this warming be due to natural dynamics? Given what we know about the complexity, long-term persistence, and nonlinearity of the climate system, it seems the answer might be yes.*

11. Adaptive Efficiency

- Adaptive rather than allocative efficiency is the key to long run growth.
- Successful political/economic systems have evolved flexible institutional structures that can survive the shocks and changes that are a part of successful evolution.
- Society's effectiveness in creating institutions that are productive, stable, fair, broadly accepted and flexible enough to respond to social, political, economic, and environmental crises.

<u>An adaptively efficient society:</u>

- copes with the novelty and uncertainty of a non-ergodic world by the maintenance of institutions which enable trial and error and experimentation.
- enables effective societal learning resulting in the elimination of unsuccessful solutions and the retention of successful ones.

<u>The main responsibility of governments in managing society’s response to the impact of the potentially catastrophic events of a non-ergodic world is to enable society to adapt to them as efficiently as possible.</u>

12. Australia’s climate: Patterns and uncertainties

- ENSO, interacting with other atmospheric & oceanic oscillations, largely regulates Australia’s climate and the regular episodes of flood, drought and bushfire;
- These regular episodes are a natural, expected and predictable feature of Australia’s climate.
- Solar activity (electromagnetic radiation & force, matter, and gravity) influences ENSO and the other major atmospheric/oceanic oscillations;
- The solar system most likely modulates solar activity;
- Governments cannot influence these key drivers of Australia’s climate, but have a significant role in managing Australia’s response to them.
- The strategic use of knowledge about Australia’s regular episodes of flood, drought and bushfire can improve the allocative, productive and adaptive efficiency of Australia’s management of water.
- Recognition of the scale of the uncertainties can improve the design and management of water systems making them safer, more efficient and more effective (Koutsoyiannis, 2004).

13. Advice to the Australian Government (i)

- Strategic policies to better manage Australia’s water resources should include:

  Just, transparent system of tradeable water property rights.

  Regulated markets in which the price of water responds to supply and demand.

  Adaptation of the water resource system used to manage the water supply of Athens to the management of Australia’s water resource systems.

- Features of the Athenian water resource system (See Koutsoyiannis, 2006, 2007):

  Prediction of future water supply based on knowledge that regular episodes of drought and flood are as much part of the future as they have been a normal part of the past;

  Stochastic simulation and forecasting models of hydrological processes used because climate records, especially the hydrologic ones, are too short.

  an adaptive method for the release of water that takes into account the near past and near likely future on both the supply and demand.

14. Advice to the Australian Government (ii)

- Given that regular episodes of drought, bushfire and flood characterise Australia's climate and are predictable in approximate terms, reshape Australia's approach to natural resource management so that this feature is the cornerstone of policy, not an after thought.
- Australia cannot be drought, flood or fire proofed.
- Policies that acknowledge Australia's regular episodes of drought, bushfire and flood can improve significantly the productivity of Australian agriculture, reducing the waste and hardship that accompanies current policies that ignore these episodes.
- Improve significantly the productivity and efficiency of Australia's use of water.
- Periodically evaluate the effectiveness of water policies having regard to allocative, productive and adaptive efficiency.
- Invest substantially in research that improves the understanding and predictability of the regular episodes of drought, flood and fire.
- Brian Fagan (1999, 2000, 2004) has shown that over the past 5,000 years catastrophic climate change have destroyed several governments, societies and civilisations that could not adapt efficiently to them.
- These catastrophic climate changes were most likely produced by the Sun's variable activity
- Beware of predictions presented as certainties!!

15. Definitions and notes (i)

Chaotic systems

Chaotic systems have two distinctive features. The *first* is to do with the system's relationship with its environment. A chaotic system is one that shows extreme sensitivity to any minimal change in the system's dynamics arising internally or from external impact, (i.e. perturbation) however slight. The *second* is to do with processes initiated by any impact, internal or external. In a chaotic system, the process of the system in response to any impact, internal or external, is non-linear. In addition, if the time series describing the process of the system's response to an impact converges within a practical time frame, it does so to a strange attractor. (Note 1).

As a result, after a certain time has elapsed, the future states of chaotic systems cannot be predicted any better than chance. In other words, after a certain time, future states of the system are entirely unpredictable and the phenomena is indistinguishable from phenomena generated by a random process. Sometimes the time frame at which such unpredictability arises can be very short, in the case of weather just a few hours; in the case of climate maybe several months. In the case of the solar system, it can be several millions of years.

In some cases the unpredictability of a chaotic system may be the result of sensitivity to initial conditions. This is a special case of extreme sensitivity to slight changes in the system's dynamics. (Note 2). It occurs when the perturbation under consideration is characterised as the initial condition of the system. In this case, the smallest difference in initial conditions results in vastly different behaviour of the system after a relatively short time. As with the impact of perturbation, if the time series describing the behaviour of the system converges within a practical time frame, it does so to a strange attractor.

Chaotic systems may be deterministic or non-deterministic systems, depending on the nature of the systems of equations that describe them. Deterministic systems can be described by systems of non-probabilistic differential equations. Non-deterministic systems are those where the equations that describe them are probabilistic.

There are many systems that cannot yet (or ever) be described by any sort of equations.

Homoclinic tangles

Newton's equations, which describe the behaviour of the solar system in terms of gravity, are deterministic. Descartes, fifty years before Newton, introduced the idea of a clockwork Universe. In his *Principia* Newton rejected the Cartesian view of the Universe, especially the idea of a clockwork Universe. He regarded the *Principia* as proof of the existence of God. Newton also knew that his account of the tides in the *Principia* was flawed. He knew that the interactive effects of the Sun, the Earth and the Moon resulted in irregular and chaotic motion. He tried several different ways to compute tables of longitude from his mathematical theory, but they all failed. (Note 3). He knew that the Universe did not work like clockwork. He rejected the concept of a clockwork universe entirely. Newton was absolutely confident that without the continual intervention of God, the Universe would descend into total chaos.

There are good reasons to conclude that Newton's awareness of these shortcomings in his *Principia* may have been enough for him to give up his scientific work and turn to other interests.

The French scientists and philosophers continued with the clockwork metaphor after Newton, even though he had clearly rejected Cartesian thinking. The most celebrated pronouncement of the refined clockwork Universe came from LaPlace who believed that he had proven once and for all that the solar system worked like clock work and was stable. In 1815 he stated: (Note 4).

Assume an intelligence that at a given moment knows all the forces that animate nature as well as the momentary positions of all things of which the Universe consists, and further that it is sufficiently powerful to perform a calculation based on these data. It would then include in the same formulation the motions of the largest bodies in the Universe and those of the smallest atoms. To it, nothing would be uncertain. Both future and past would be present before its eyes."

We now know that this idea is just plain false.

16. Definitions and notes (ii)

Homoclinic tangles (continued)

Henri Poincare showed in 1892 that Newton's equations for three bodies acting under Newton's gravitational equations have chaotic solutions as well as the familiar regular, non-chaotic ones. This meant that the solar system was, in some real sense, unstable, a view at odds to the consensus world view of scientists at the time.

Henri Poincare's proof that the solar system was unstable in 1892 was the unexpected result of entering a competition to prove that it was stable, correcting a flaw in Laplace's mathematics. More surprising, was that Poincare had submitted a paper that he and the judges believed provided the long awaited proof of the stability of the solar system. But his proof contained fallacious, slipshod reasoning. Only after he was awarded the prize, and the scientific journal containing his prize-winning, but fatally flawed essay, printed, was the mistake in his reasoning discovered. Poincare took six weeks of intensive work to correct the error. The corrected reasoning changed everything. It proved that the solar system was unstable. Poincare had to pay for the reprinting of the journal, the cost of which exceeded the value of his monetary prize. The Swedish organizers, and Poincare, covered up the blunder and the modern theory of dynamical systems was announced to the world. By accident, and very much against the scientific consensus of the day, Poincare laid the foundations of the mathematical theory of chaotic systems.

Poincare was astounded at what he found. He had a deep visual intuition at what was happening, calling the phenomena homoclinic tangles. He wrote: (Note 5).

Therefore, between two arbitrary intersection points of two curves, there is an infinity of other points belonging to the first category, and an infinity of other points belonging to the second category.

When we try to represent the figure formed by these two curves and their infinity of intersections, each of which corresponds to a doubly asymptotic solution, these intersections form a type of trellis, net, tissue, or grid with infinitely serrated mesh. Neither of the two curves must ever cut across itself again, but must bend or fold back upon itself in a very complex manner in order to cut across all of the meshes in the grid an infinite number of times.

The complexity of this figure will be striking, and I shall not even try to draw it. Nothing is more suitable for providing us with an idea of the complex nature of the three-body problem and of all the problems of dynamics in general, where there is no uniform integral and where the Bohlin series are divergent.

Poincare laid the foundations of the modern qualitative theory of differential equations. This, in turn, evolved into dynamical systems theory. It is from this that the mathematics of chaotic systems has developed. Poincare's findings were contrary to the consensus of the day about how the solar system worked. They were largely ignored by the mainstream scientific community, even though Poincare was universally acknowledged as the leading mathematical scientist of the time.

Poincare was deeply impressed by what he learnt about sensitive dependence on initial conditions in 1892. Sixteen years later in 1908 Poincare stated the following in a public lecture: (Note 6)

A very small cause which escapes our notice determines a considerable effect that we cannot fail to see, and then we say that the effect is due to chance. If we knew exactly the laws of nature and the situation of the universe at the initial moment, we could predict exactly the situation of that same universe at a succeeding moment. But even if it were the case that the natural laws had no longer any secret for us, we could still only know the initial situation approximately. If that enabled us to predict the succeeding situation with the same approximation, that is all we require, and we should say that the phenomenon had been predicted, that it is governed by laws. But it is not always so; it may happen that small differences in the initial conditions produce very great ones in the final phenomenon. A small error in the former will produce an enormous error in the latter. Prediction becomes impossible, and we have the fortuitous phenomenon.

17. Definitions and notes (iii)

Randomness

A process can be considered to be random if it can be demonstrated that there is no system of equations that can generate it other than a function which is a random variable. A random variable is a measurable (in the sense of the mathematical theory of integration) function defined on an unknown sample space that takes real numbers as values.

The distribution of the random variable is the probability distribution that says for any real number, the probability that the random variable will have a value equal to or greater than that real number. In a climate or hydrological situation, physical variables such as rainfall, the volume of a dam, river flow, temperature, or atmospheric pressure may be considered as random variables. In a physics situation, the position and momentum of an elementary particle such as a photon or an electron may be considered as random variables. (Note 7). A process generated by a random variable has no structure, no mathematical pattern.

Ergodic and Non-ergodic

The concept "ergodic" is used metaphorically to refer to uniformity or sameness in which any one arrangement of the elements of a system are as likely, or as unlikely, as any other. At an aggregate, or macroscopic, level this means that an ergodic system is characterised by uniformity and predictability. The patterns that will characterise an ergodic system most of the time are those macroscopic ones satisfied by very large numbers of microscopic ones. An ergodic system would be characterised by monotony at the macroscopic level.

Conversely, a non-ergodic system will be characterised by novelty, surprise, unpredictable patterns, spontaneous order, creativity, patterns self similar across a wide, perhaps indefinite, range of scales. Sometimes improbable structures may be common place. Non-ergodic systems would exhibit the rich complexity and diversity of the world we experience. A non-ergodic system would be characterised by creativity and novelty at the macroscopic level.

The Principle of Stationary Action

An object with a specified mass that starts from a fixed initial position and, after a time, moves to a final position, will follow the path that results in the action, a function of energy over time, having a stationary value, that is, it has a minimum, or a maximum or saddle point, value. (Note 8). This Principle can be shown to be equivalent to the Principle of Least Action, or Least Time, according to which an object, whether an elementary particle, a planet, a star or a galaxy, will take the path that requires the least energy or equivalently, least time, to complete. It has also be known as the Principle of Insufficient Reason. (Note 9).

Principle of Maximum Entropy

The entropy of a set of mutually exclusive events is maximum when they are equi-probable. It is then equal to the natural logarithm of the number of events. The Principle of Maximum Entropy is used to infer unknown probabilities from known information.

According to the Principle, the probability distribution is assigned to the random variable that maximises the entropy of the set, subject to some conditions, expressed as constraints, which incorporate the information already available in this variable's time series.

The Principle was first formulated by Jaynes in 1957. His formulation was that we should have an exhaustive set of mutually exclusive hypothesis (this is the hypothesis space) that would predict, say, the next value in a time series. We should then assign a probability distribution to that set which maximises the entropy of the hypothesis set, subject to constraints that express properties we wish the distribution to have, but are not sufficient to determine it. By this procedure we get a probability distribution for the hypothesis space, not the probability of a particular hypothesis. It does not require the numerical values of any probabilities of particular hypotheses. It assigns those numerical values directly out of the information, as expressed by our choice of hypothesis space and constraints.

The Principle of Maximum Entropy can be shown to be equivalent to the Principle of Stationary Action.

18. Definitions and notes (iv)

Principle of Maximum Entropy (continued)

Entropy also means disorder and uncertainty. It can mean the quantity of information required to specify a particular microstate of a system. An increase in entropy is an increase in uncertainty. A decrease in entropy is an increase in information. Entropy can also be a measure of the temporal disorder in a stochastic process. Entropy can be a measure of complexity and relates to the computability of a system or process as in Kolmogorov-Chaitin complexity (i.e. algorithmic complexity or entropy). A process or system with maximum algorithmic complexity is not computable. It does not conform to any set of rules or computational procedure. In this sense entropy can be shown to be related to the incompleteness or inconsistency of system of reasoning or logic.

Entropy is also related to “compressibility”, i.e. the percentage of variance explained by the optimal model of the data. Something can be compressed if there exists some sort of correlation structure linking the various elements of a system, such correlations meaning that the information in one component of the system is implicit in another part.

Application of the Principle of Maximum Entropy to climate and hydrological phenomena results in the Hurst phenomena, in which periods of high rainfall cluster together as do periods of low rainfall. The result is that episodes of similar climate phenomena, such as episodes of plenty and/or flooding and episodes of scarcity and/or drought and bushfire arise from this clustering. The phenomenon of clustering on many time scales is the rule rather the exception throughout the natural world.

A practical and universal consequence of the Principle of Maximum Entropy, and its equivalent, the Principle of Stationary Action, is that nature behaves in a manner that makes uncertainty as high as possible.

Probabilistic metaphysics

In a book of this name, Patrick Suppes (1984) outlined a philosophic perspective which is based on the idea that the world and rationality are intrinsically probabilistic. The fundamental laws of natural phenomena are essentially probabilistic. To the extent that there are valid causal laws, those laws will be probabilistic. Other features of this perspective include pluralism of knowledge, the absence of the convergence of scientific theory to some bounded fixed result. (Suppes, Patrick. Probabilistic Metaphysics. Basil Blackwell 1984).

Angular momentum

Angular momentum is the momentum of rotation. An object's angular momentum will not change unless it is acted upon by an outside force. Furthermore, as a consequence of the principle of the conservation of energy, the angular momentum of a closed system, such as the solar system, is conserved

The Sun’s asphericities

The shape, or figure, of the Earth is portrayed as the Geoid, a global ellipsoid, the surface of which varies slightly over time in relation to distributions of mass and angular velocity that vary over time. Similarly the shape, or figure, of the Sun can be portrayed as the helioid. The asphericities of the Sun refer to shape distortions of the helioid. These include variations in the Sun’s diameter not only over time, but also in relation to points on the surface of the Sun. The net effect is that the surface of the Sun resembles a walnut whose exterior lumpiness is variably distributed over the Sun’s surface and which also varies over time. The asphericities also includes the Sun’s variable oblateness, as the poles of the Sun sometimes flatten and sometimes rise. The Sun’s asphericities are considered to vary internally with the Sun’s internal structures showing variable asphericities, not necessarily synchronised over time and location within the Sun. (See Rozelot, P., (Ed). The Sun’s Surface and Subsurface: Investigating Shape and Irradiance. Springer 2003 and papers on the website of Sophie Pireaux:

http://www.obs-azur.fr/gemini/pagesperso/pireaux/curriculum_051219.html

19. Definitions and notes (v)

Allocative, productive and adaptive efficiency

Productive Efficiency: One production system has higher productive efficiency than another if it produces the most output with least input; or has the best mix of inputs for a given output; or has the best output mix for a given input; or has best combination of input and output mix, where "best" means least cost. In making this comparison it is necessary to have good measures of the quantity of outputs produced by both. It is also necessary that all of the output produced by both is measured satisfactorily and that there are good measures of the cost and type of inputs used by both production systems.

Allocative Efficiency: A production system achieves maximum allocative efficiency when the price of its output equals the output's marginal. The allocative efficiency of a market is maximum when resources are optimally allocated so that the net benefits gained from their consumption equals the marginal cost of production. The realisation that transaction costs are a necessary element of market trades, thus putting a limit on allocative efficiency, lead to the establishment of institutional economics to which Douglass North has been a foundational contributor. In 1960 Ronald Coase demonstrated that transaction costs can never be zero highlighting the need to examine the circumstances under which the transaction process can be made maximally efficient and effective.

Adaptive Efficiency: The relative efficiency with which a society adapts to change

Note 1: A process is said to converge to a 'strange attractor' when it converges, not to a point (as does the process of compounding interest daily on an amount of money deposited in an bank account) or to a line that describes a regular geometrical shape (like the graph that describes a process like the ticking of an electronic clock in which there is stable, regular periodicity) but to a line that lies within a definite area but which describes a shape that defies conventional description. For example, the shape could resemble a pile of tangled thread (i.e. Poincare's homoclinic tangle), except that although it might intersect, or crossover, with itself in incredibly complicated tangles or braids, the line is never cut or torn; it could resemble a shape, whether regular or not, that repeats itself indefinitely on increasingly smaller scales (in which case the strange attractor is a fractal) or it could be a shape composed of spirals, whorls, apparently endless loops, or knots except that the line is never cut or torn.

Note 2: This generalisation arose from the examination of chaos in quantum systems. In contrast to classical systems, quantum systems cannot be characterised by extreme sensitivity to initial conditions. The way a quantum wave develops in time is determined by the associated energy levels. A mathematical consequence of the existence of energy levels is that quantum time development contains only periodic motions with definite frequencies. This is the opposite of classical chaos. However the generalisation from 'extreme sensitivity to initial conditions' to 'extreme sensitivity to slight changes in the system's dynamics' does carry over to quantum mechanics. The British physicist, Sir Michael Berry, introduced the name "Quantum Chaology" to refer to quantum phenomena characteristic of classically chaotic systems. For a further discussion see Haake, Fritz, *Quantum Signatures of Chaos* Second Revised and Enlarged Edition with 66 Figures Springer Germany Corrected Second Printing 2004, page 3 and Chapter 7.

Note 3: Kollerstom, Nicholas *Newton's Forgotten Lunar Theory: His Contribution to the Quest for Longtitude* Green Lion Press USA 2000.

Note 4: La Place , P. S. *A Philosophic Essay on Probabilities*. Dover, New York 1951. Page 4. Originally published in French in 1814 in France.

Note 5: Poincare, H., *New Methods of Celestial Mechanics Volume III Integral Invariants, Periodic Solutions of the Second Type, Doubly Asymptotic Solutions*. *Translation of Les Methodes Nouvelles de la Mecanique Celeste. Tome III Invariants integraux. Solutions periodices du deuxieme genre. Solutions doublement asymptotiques*. Dover Publications New York 1957. Translation by the National Aeronautics and Space Administration Washington DC 1967, pps 381 to 382.

Note 6: Poincare, H. *Science and Method*. New York: Dover Publications 1952, page 76. Originally published as *Science et Methode* in 1908.

Note 7: In this context the point of Heisenberg's uncertainty principle is that the distribution of position and momentum cannot be made arbitrarily sharp or peaked to a point, which would be the 'true' value.

Note 8: A stationary point of a function is a point where the function's value does not vary as the independent variable varies. Local maxima and minima are stationary points.

Note 9: See the website www.eftaylor.com. For an historical account see Jourdain, Philip E., *The Principle of Least Action.* The Open Court Publishing Company Chicago 1913.

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