Workshop on Nonstationarity, Hydrologic Frequency Analysis, and Water Management

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Hurst-Kolmogorov dynamics and uncertainty

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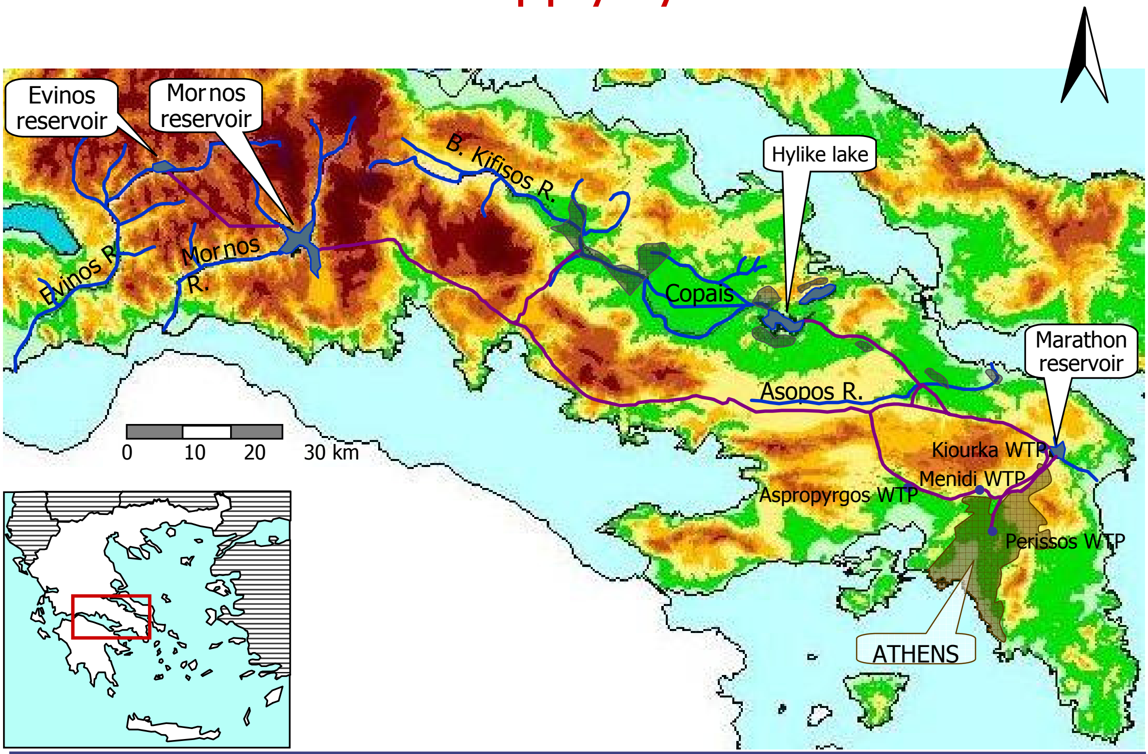
Presentation available online: http://www.itia.ntua.gr/en/docinfo/944/

1. Problem motivating the study

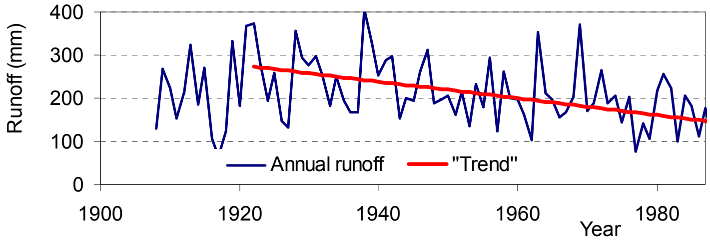
The Athens water supply system



Back in 1990s – Some worries…

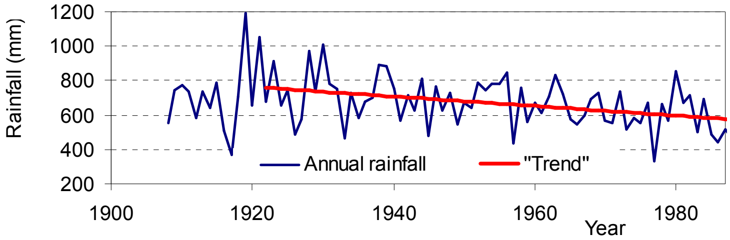
The historical time series of Boeoticos Kephisos runoff (Hydrological years 1907/08-1986/87)

A multi-year «trend» is observed



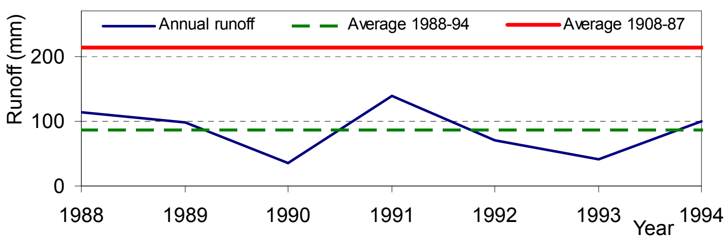
A similar «trend» in the rainfall time series

Explains the «trend» in runoff



Next was a shocking drought

Intense and persistent: Mean flow less than half compared to historical average, duration 7 years



Back in 1990s – Additional worries…

Some understood that water might be needed for the Athens Olympic Games (then in preparation)

ATHENS 2004

2. Mottos motivating the presentation

Motto 1: From *Science Magazine*

POLICY FORUM

CLIMATE CHANGE

Stationarity Is Dead: Whither Water Management?

P. C. D. Milly,[1*] Julio Betancourt,[2] Malin Falkenmark,[3] Robert M. Hirsch,[4] Zbigniew W. Kundzewicz,[5] Dennis P. Lettenmaier,[6] Ronald J. Stouffer[7]

Climate change undermines a basic assumption that historically has facilitated management of water supplies, demands, and risks.

Systems for management of water throughout the developed world have been designed and operated under the assumption of stationarity. Stationarity—the idea that natural systems fluctuate within an unchanging envelope of variability—is a foundational concept that permeates training and practice in water-resource engineering. It implies that any variable (e.g., annual streamflow or annual flood peak) has a time-invariant (or 1-year–periodic) probability density function (pdf), whose properties can be estimated from the instrument record. Under stationarity, pdf estimation errors are acknowledged, but have been assumed to be reducible by additional observations, more efficient estimators, or regional or paleohydrologic data. The pdfs, in turn, are used to evaluate and manage risks to water supplies, waterworks, and floodplains; annual global invest-

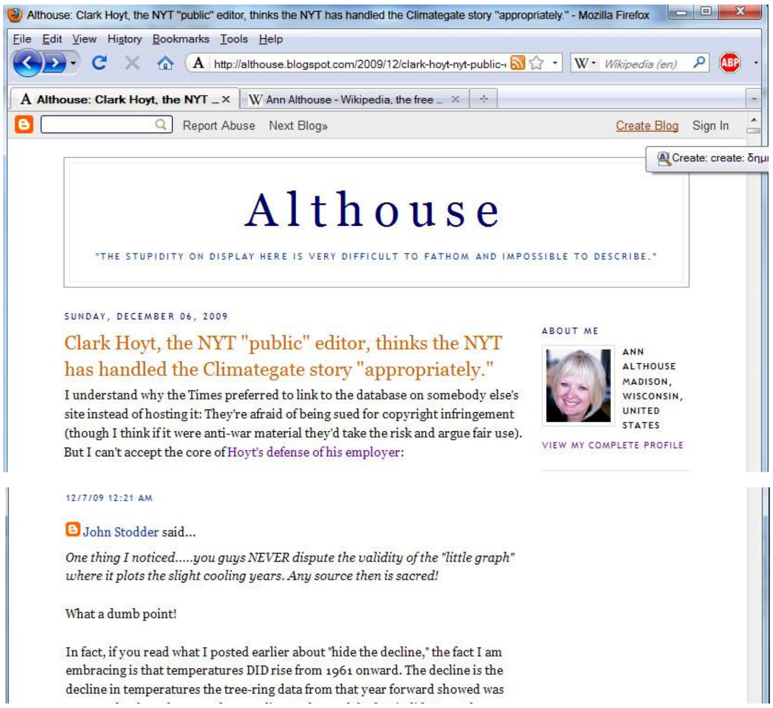
An uncertain future challenges water planners.

that has emerged from climate models (see figure, p. 574).

Why now? That anthropogenic climate change affects the water cycle (*9*) and water supply (*10*) is not a new finding. Nevertheless, sensible objections to discarding stationarity have been raised. For a time, hydroclimate had not demonstrably exited the envelope of natural variability and/or the effective range of optimally operated infrastructure (*11*, *12*). Accounting for the substantial uncertainties of climatic parameters estimated from short records (*13*) effectively hedged against small climate changes. Additionally, climate projections were not considered credible (*12*, *14*).

Recent developments have led us to the opinion that the time has come to move beyond the wait-and-see approach. Projections of runoff changes are bolstered by the recently demonstrated retrodictive skill of cli-

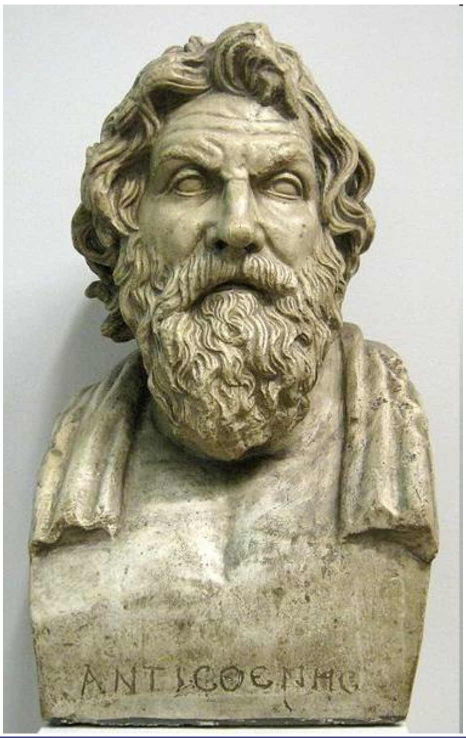
Motto 2: From a blog



"**Hydrologists**' work is used by **engineers** to plan large-scale projects designed to last **many decades**. **They can't play with models, especially models that so plainly diverge from reality**."

John Stodder in http://althouse.blogspot.com/2009/12/clark-hoyt-nyt-public-editor-thinks-nyt.html

Motto 3: From classical sources



«Αρχή σοφίας, ονομάτων επίσκεψις» (Αντισθένης)

"The start of wisdom is the visit (study) of names" (Antisthenes)

Antisthenes (c. 445-c. 365 BC), pupil of Socrates, founder of Cynic philosophy; image from wikipedia

3. Visiting names: stationarity and nonstationarity

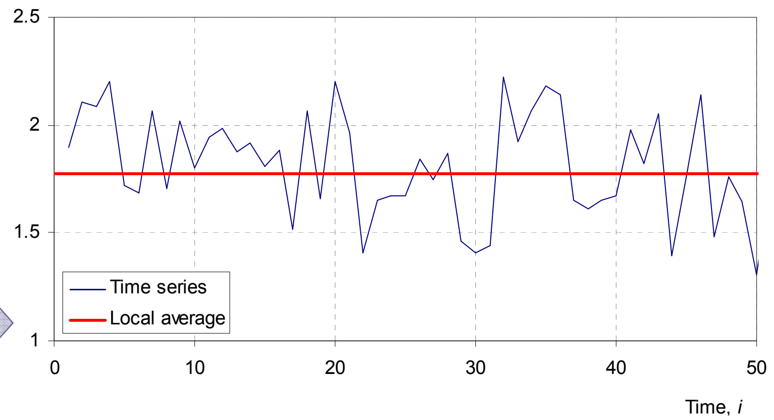
Finding invariant properties is essential in science

- Newton's first law: Position changes but **velocity is constant** (in absence of an external force)
  - $u = \mathrm{d}x/\mathrm{d}t = \mathrm{ct}$
    A huge departure from the Aristotelian view that bodies tend to rest
- Newton's second law: On presence of a constant force, the velocity changes but the **acceleration is constant**
  - $a = \mathrm{d}u/\mathrm{d}t = F/m = \mathrm{ct}$
  - For the weight W of a body $a = g = W/m = \mathrm{ct}$
- Newton's law of gravitation: The weight W (the attractive force exerted by a mass M) is not constant but inversely proportional to the square of distance; thus other **constants** emerge, i.e.,
  - $a\, r^2 = -G\, M = \mathrm{ct}$
  - $(\mathrm{d}\theta/\mathrm{d}t)\, r^2 = \mathrm{ct}$ (angular momentum per unit mass; θ = angle)

The stationarity concept: Seeking invariant properties in complex systems

- Complex natural systems are impossible to describe in full detail and predict their future evolution in detail and with precision
- The great scientific achievement is the materialization of macroscopic descriptions that need not model the details
- Essentially this is done using probability theory (laws of large numbers, central limit theorem, principle of maximum entropy)
- Related concepts are: stochastic process, statistical parameters, stationarity, ergodicity

Example 1:
50 terms of a synthetic time series (to be discussed later)



What is stationarity and nonstationarity?

Stationary Processes

A stochastic process $\mathbf{x}(t)$ is called *strict-sense stationary* (abbreviated SSS) if its statistical properties are invariant to a shift of the origin. This means that the processes $\mathbf{x}(t)$ and $\mathbf{x}(t+c)$ have the same statistics for any c.

WIDE SENSE. A stochastic process $\mathbf{x}(t)$ is called *wide-sense stationary* (abbreviated WSS) if its mean is constant

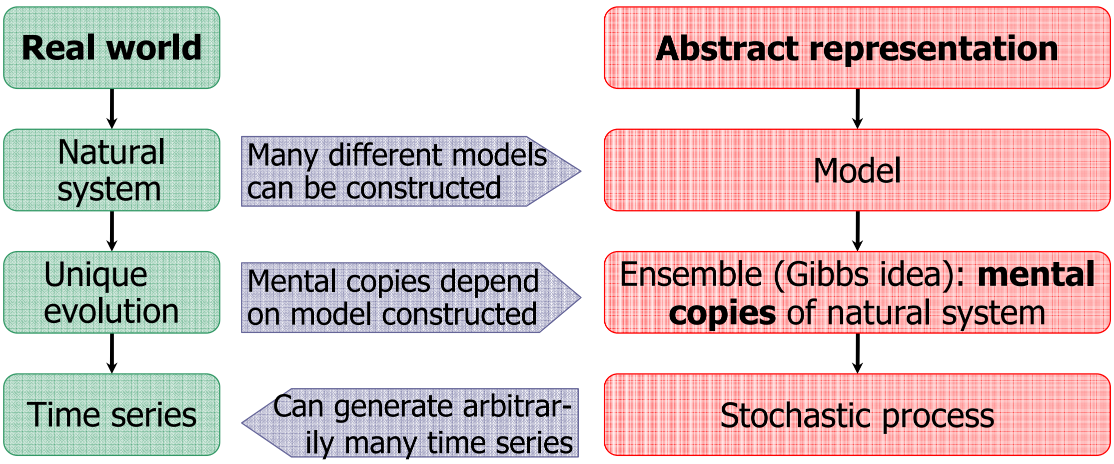
$$E\{\mathbf{x}(t)\} = \eta \qquad (10\text{-}41)$$

and its autocorrelation depends only on $\tau = t_1 - t_2$:

$$E\{\mathbf{x}(t+\tau)\mathbf{x}^*(t)\} = R(\tau) \qquad (10\text{-}42)$$

- Definitions copied from Papoulis (1991).
- Note 1: Definition of **stationarity** applies to a **stochastic process**
- Note 2: Processes that are not stationary are called **nonstationarity**; some of their statistical properties are **deterministic** functions of time

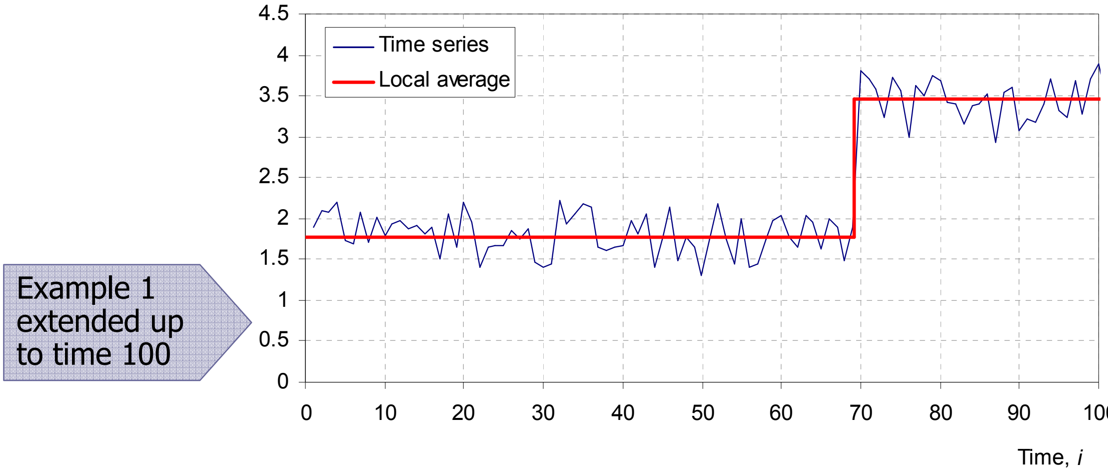
Some notes about stationarity and nonstationarity



Stationarity and nonstationarity apply here

Important consequences:
E.g. nonstationarity can hardly be dead

Does this example say that "stationarity is dead"?

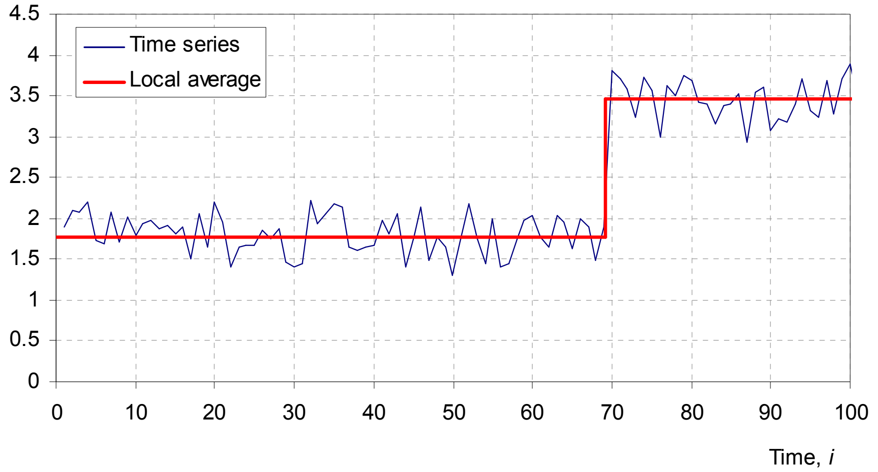


Mean *m* (red line) of time series (blue line) is:

$m = 1.8$ for $i < 70$

$m = 3.5$ for $i \geq 70$

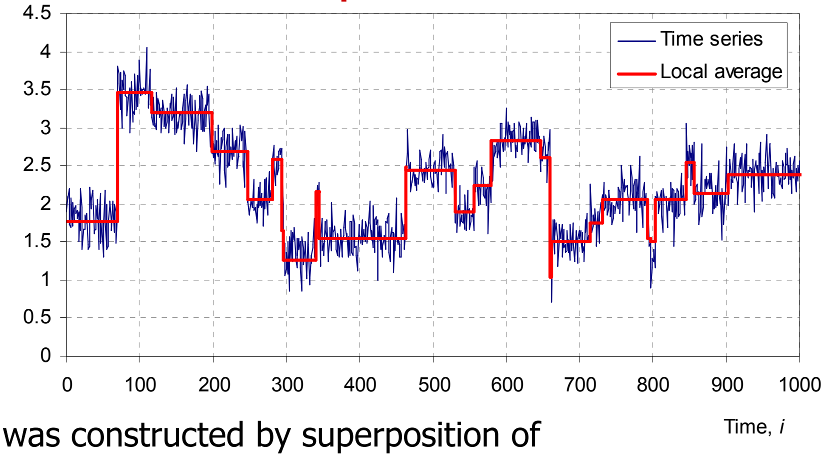
Reformulation of question: Does the red line reflect a **deterministic** function?



- If the red line is a deterministic function of time: → **nonstationarity**
- If the red line is a random function (realization of a stationary stochastic process) → **stationarity**

Answer of the last question: the red line is a realization of a stochastic process

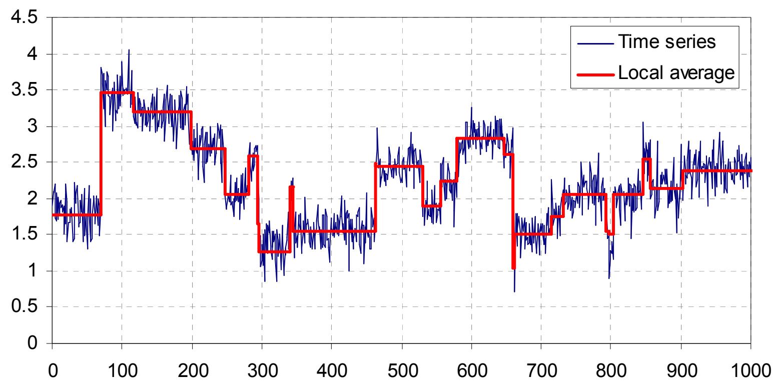
Example 1 extended up to time 1000



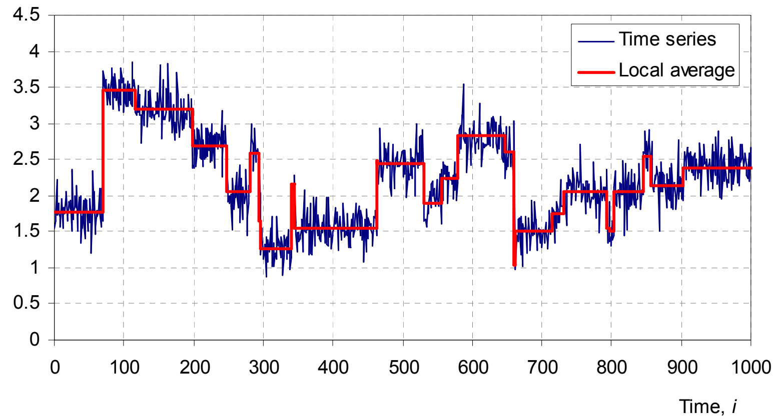
- The time series was constructed by superposition of
  - A stochastic process with values $\underline{m}_j \sim N(2, 0.5)$ each lasting a period $\underline{T}_j$ exponentially distributed with $E[\underline{T}_j] = 50$ (red line);
  - White noise $N(0, 0.2)$.
- Nothing in the model is nonstationary
- The process of our example is **stationary**

The big difference of **nonstationarity** and stationarity (1)

The initial time series



A mental copy generated by a **nonstationary** model (assuming the red line is a deterministic function)

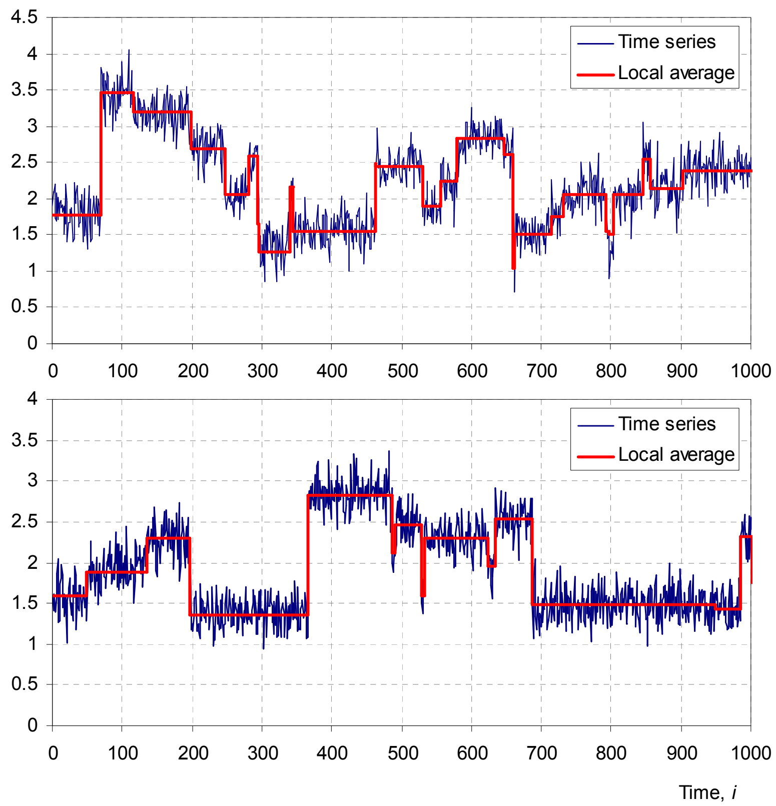


Unexplained variance (differences between blue and red line): $0.2^2 = 0.04$

The big difference of nonstationarity and **stationarity** (2)

The initial time series

A mental copy generated by a **stationary** model (assuming the red line is a stationary stochastic process)



Unexplained variance (the "undecomposed" time series): 0.38 (~10 times greater)

Caution in using the term "nonstationarity"

- **Stationary** is not synonymous to **static**
- **Nonstationary** is not synonymous to **changing**
- In a nonstationary process the change is described by a deterministic function
- A deterministic description should be constructed by deduction (the Aristoteleian **apodeixis**), not by induction (direct use of data)
- To claim nonstationarity, we must :
    1. Establish a causative relationship
    2. Construct a quantitative model describing the change as a deterministic function of time
    3. Ensure applicability of the deterministic model in future time
- Nonstationarity reduces uncertainty!!! (because it explains part of variability)
- Unjustified/inappropriate claim of nonstationarity results in underestimation of variability, uncertainty and risk!!!

Do climate models enable a nonstationary approach?

- Do general circulation models (GCMs) provide credible deterministic predictions of the future climate evolution?
- Do GCMs provide good predictions, at least for temperature (and somewhat less good for precipitation)?
- Do GCMs provide good predictions at least for global and continental scales (and, after downscaling, for local scales)?
- Do GCMs provide good predictions for the distant future (albeit less good for the nearer future, e.g. for the next 10-20 years—or for the next season or year)?
- Is climate predicable in deterministic terms?

A related Aesop's fable: The Braggart

Αυτοῦ γάρ καὶ ῾Ρόδος καὶ πήδημα

Hic Rhodus, hic saltus!

Rhodes is right here: make the jump.

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Ἀνὴρ κομπαστής

Ἀνήρ πένταθλος ἐπὶ ἀνανδρίᾳ ἑκάστοτε ὑπὸ τῶν πολιτῶν ὀνειδιζόμενος, ἀποδημήσας ποτὲ καὶ μετὰ χρόνον ἐπανελθών, ἀλαζονευόμενος ἔλεγεν ὡς πολλὰ καὶ ἐν ἄλλαις πόλεσιν ἀνδραγαθήσας, ἐν τῇ Ῥόδῳ τοιοῦτον ἥλατο πήδημα ὡς μηδένα τῶν Ὀλυμπιονικῶν ἐφικέσθαι· καὶ τούτου μάρτυρας ἔφασκε παρέξεσθαι τοὺς παρατετυχηκότας, ἂν ἄρα ποτὲ ἐπιδημήσωσι. Τῶν δὲ παρόντων τις ὑποτυχὼν ἔφη πρὸς αὐτόν· «Ἀλλ', ὦ οὗτος, εἰ τοῦτο ἀληθές ἐστι, οὐδὲν δεῖ σοι μαρτύρων· αὐτοῦ γὰρ καὶ Ῥόδος καὶ πήδημα.»

Ὁ λόγος δηλοῖ ὅτι ὧν πρόχειρος ἡ δι' ἔργων πεῖρα, περὶ τούτων πᾶς λόγος περιττός ἐστι.

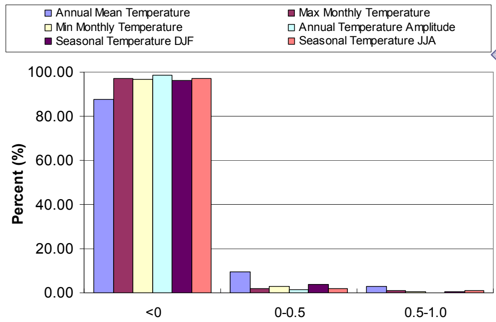
A man who practised the pentathlon, but whom his fellow-citizens continually reproached for his unmanliness, went off one day to foreign parts. After some time he returned, and he went around boasting of having accomplished many extraordinary feats in various countries, but above all of having made such a jump when he was in Rhodes that not even an athlete crowned at the Olympic Games could possibly equal it. And he added that he would produce as witnesses of his exploit people who had actually seen it, if ever they came to his country. Then one of the bystanders spoke out: 'But if this is true, my friend, you have no need of witnesses: **Rhodes is right here - make the jump**.'

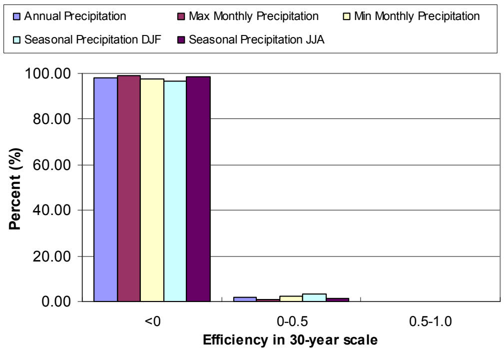
The fable shows that as long as one can prove something by doing, speculation is superfluous.

Aesop (Αίσωπος; 620-560 BC), A slave renowned for his fables

English translation adapted from Temple *et al.*, 1998.

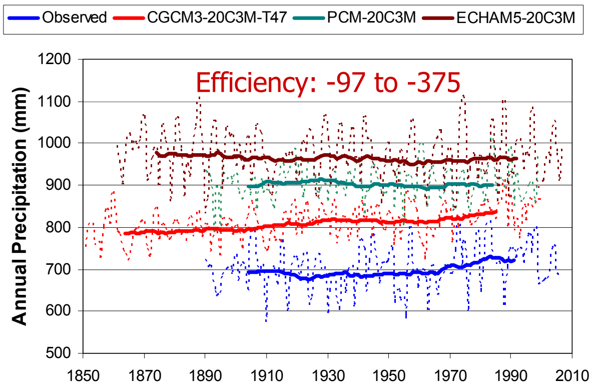
"Hic Rhodus" (i.e. 20th century), "hic saltus" (i.e. skill to reproduce reality)





Comparison of 3 IPCC TAR and 3 IPCC AR4 climate models with historical series of more than 100 years length in 55 stations worldwide

Comparison of 3 IPCC AR4 climate models with reality in sub-continental scale (contiguous USA)

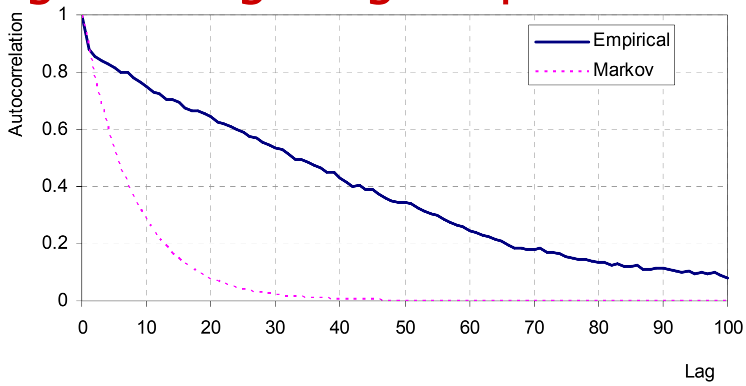


Source: Anagnostopoulos, *et al.* (2009)
See also Koutsoyiannis *et al.* (2008).

4. Change under stationarity and the Hurst-Kolmogorov dynamics

Change is tightly linked to dependence and long-term change to long-range dependence

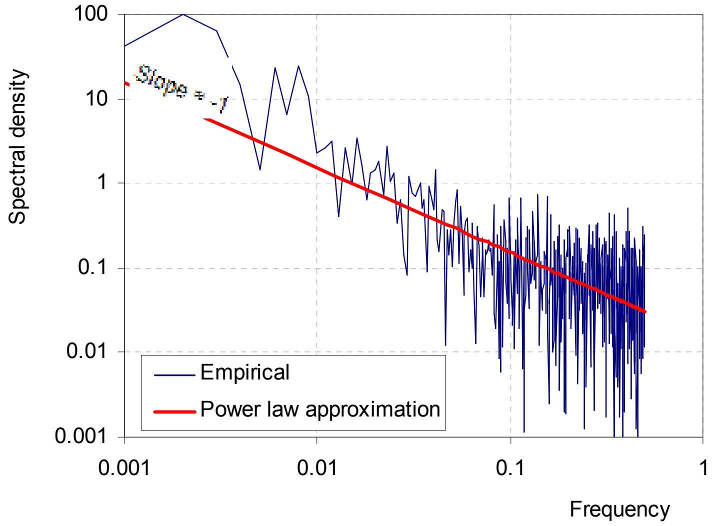
Autocorrelogram of 1000 items of our example time series in comparison to that of a Markov process



- The typical autocorrelogram (autocorrelation vs. lag) is meaningful only for stationary processes
- Here the autocorrelogram suggests long-range dependence (to be contrasted with Markovian, short-range dependence)
- This dependence should not be interpreted as "long memory"; it is a result of "long-term change"
- This has been first pointed out by Klemes (1974)

Change and frequency: The power spectrum

- The power spectrum is the inverse finite Fourier transform of the autocorrelogram
- Again it is meaningful only for stationary processes
- The large values of spectral density for small frequencies (large periods or scale lengths) indicates dominance of the long-term variability



- The slope in a double logarithmic plot (here ~-1) is an indicator of the long-range dependence (or long-term persistence)—but its estimation is not accurate due to rough shape

Change and scale: The climacogram

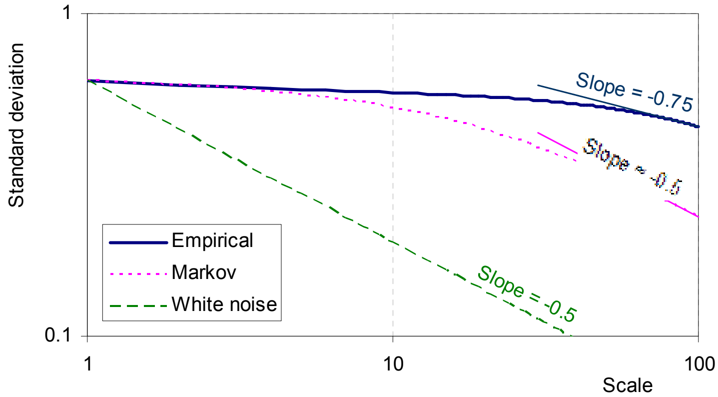
- This is simply a plot of standard deviation $\sigma^{(k)}$ at scale k vs. scale k; $\sigma^{(k)}$ can be calculated directly from the time averaged process

$$\underline{x}_i^{(k)} := \frac{1}{k} \sum_{l=(i-1)k}^{ik} \underline{x}_l$$

- It is a transformation of the autocorrelogram ρ_j (where j is lag), i.e.,

$$\sigma^{(k)} = \frac{\sigma}{\sqrt{k}} \sqrt{\alpha_k}, \quad \alpha_k = 1 + 2\sum_{j=1}^{k-1}\left(1 - \frac{j}{k}\right)\rho_j \leftrightarrow \rho_j = \frac{j+1}{2}\alpha_{j+1} - j\alpha_j + \frac{j-1}{2}\alpha_{j-1}$$

- The asymptotic slope (high k) in a logarithmic plot is a characteristic defining the so-called Hurst coefficient: $H = 1 + \text{slope}$
- H values in the interval (0.5, 1) indicate long-range dependence



The Hurst-Kolmogorov (HK) process and its multi-scale stochastic properties

- Example 1 admits (irregular) fluctuations at two characteristic time scales: $k_1 = 1$ and $k_2 = E[\underline{T}_j] = 50$
- Assuming additional scales of fluctuation, k_3, k_4 … (although practically, three time scales of fluctuation suffice—Koutsoyiannis, 2002), we may construct a Hurst-Kolmogorov process, which has very simple properties

Properties of the HK process	At an arbitrary observation scale $k = 1$ (e.g. annual)	At any scale k
Standard deviation	$\sigma \equiv \sigma^{(1)}$	$\sigma^{(k)} = k^{H-1}\sigma$ (can serve as a definition of the HK process; H is the Hurst coefficient; $0.5 < H < 1$)
Autocorrelation function (for lag j)	$\rho_j \equiv \rho_j^{(1)} = \rho_j^{(k)} \approx H(2H-1)\,\lvert j\rvert^{2H-2}$	
Power spectrum (for frequency ω)	$s(\omega) \equiv s^{(1)}(\omega) \approx 4(1-H)\,\sigma^2\,(2\omega)^{1-2H}$	$s^{(k)}(\omega) \approx 4(1-H)\,\sigma^2\,k^{2H-2}\,(2\omega)^{1-2H}$

All equations are power laws of scale k, lag j, frequency ω

Fluctuations at multiple temporal or spatial scales are common in Nature

- **Example 2:** turbulence in a hydraulic jump
- The energy associated with each scale increases with scale length (e.g. without the macroturbulence of the hydraulic jump, the energy loss due to molecular motion and microturbulence would be much lower)

Photo from Chanson (2007)

Molecular motion (not visible) + microturbulence (visible)

Molecular motion (not visible) + microturbulence (visible) + macroturbulence (manifest)

A historical note: Hurst & Kolmogorov

The recognition that real world processes behave differently from an ideal roulette wheel (where the differences mainly rely on long excursions of local averages from the global mean) is due to Hurst and Kolmogorov (see Koutsoyiannis and Cohn, 2008)

AMERICAN SOCIETY OF CIVIL ENGINEERS
Founded November 5, 1852
TRANSACTIONS

Paper No. 2447

LONG-TERM STORAGE CAPACITY OF RESERVOIRS

By H. E. Hurst[1]

With Discussion by Ven Te Chow, Henri Milleret, Louis M. Laushey, and H. E. Hurst.

Synopsis

A solution of the problem of determining the reservoir storage required on a given stream, to guarantee a given draft, is presented in this paper. For example, if a long-time record of annual total discharges from the stream is available, the storage required to yield the average flow, each year, is obtained by

Hurst (1951) studied numerous geophysical time series and observed that: "*Although in random events groups of high or low values do occur, their tendency to occur in natural events is greater. This is the main difference between natural and random events.*"

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1940. Volume XXVI, № 2

MATHEMATIK

WIENERSCHE SPIRALEN UND EINIGE ANDERE INTERESSANTE KURVEN IM HILBERTSCHEN RAUM

Von A. N. KOLMOGOROFF, Mitglied der Akademie

Wir werden hier einige Sonderfälle von Kurven betrachten, denen meine vorhergehende Note «Kurven im Hilbertschen Raum, die gegenüber einer einparametrigen Gruppe von Bewegungen invariant sind» (1) gewidmet ist.

Unter einer Ähnlichkeitstransformation im Hilbertschen Raum H werden wir eine beliebige ...

... $y \neq x$ der Punkte, die auf derselben Kurve liegen, übergeht.

Satz 6. *Die Funktion* $B_\xi(\tau_1, \tau_2)$, *die der Funktion* $\xi(t)$ *der Klasse* $\mathfrak{A}$ *entspricht, kann in der Form*

$$B_\xi(\tau_1, \tau_2) = c\left[|\tau_1|^\gamma + |\tau_2|^\gamma - |\tau_1 - \tau_2|^\gamma\right]$$

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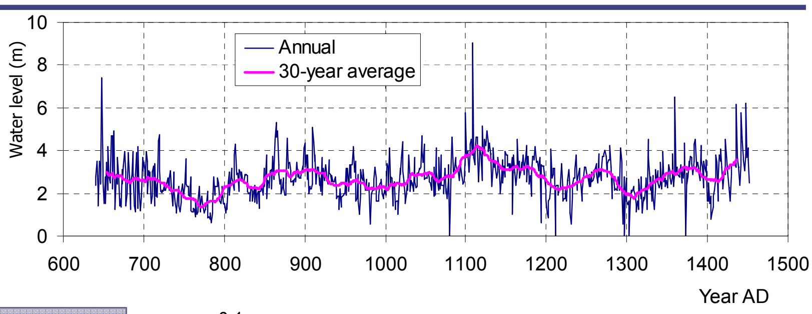
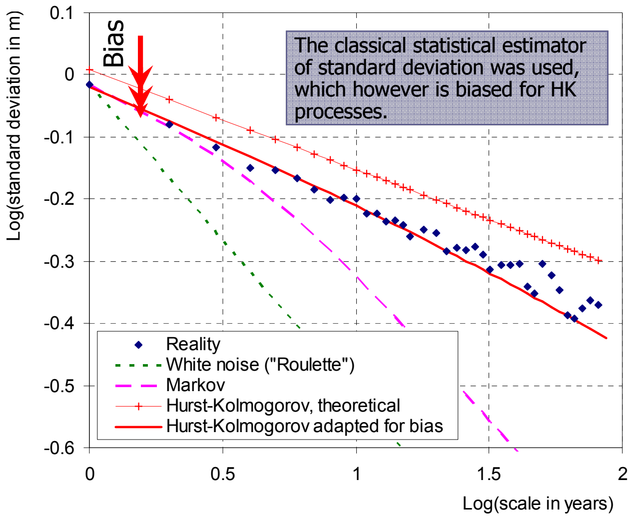
Kolmogorov (1940) studied the stochastic process that describes this behaviour 10 years earlier than Hurst.

Example 3: Annual minimum water levels of the Nile



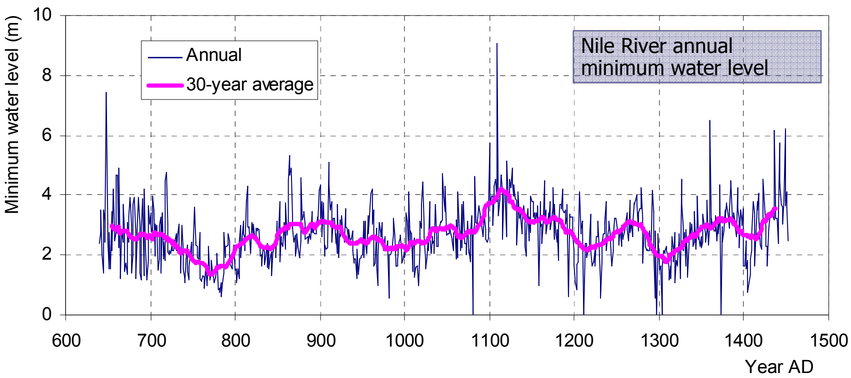
Roda Nilometer



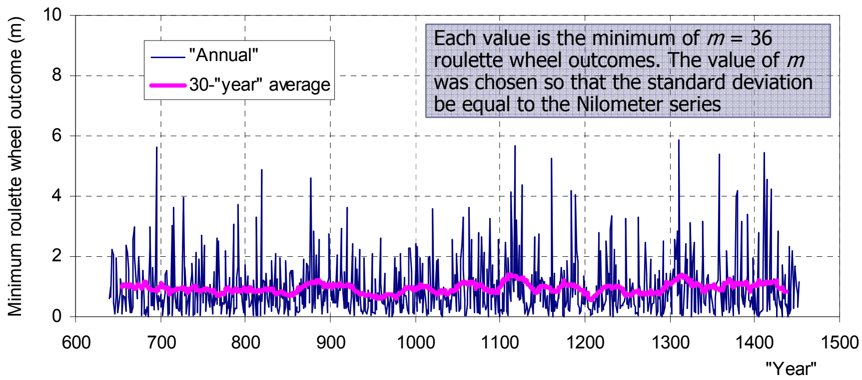
- The longest available instrumental hydroclimatic data set (813 years).
- Hurst coefficient $H = 0.84$.
- The same H is estimated from the simultaneous record of maximum water levels and from the modern record (131 years) of the Nile flows at Aswan.

Real-world processes vs. simplified random processes

Real-world or "Hurst-Kolmogorov" process

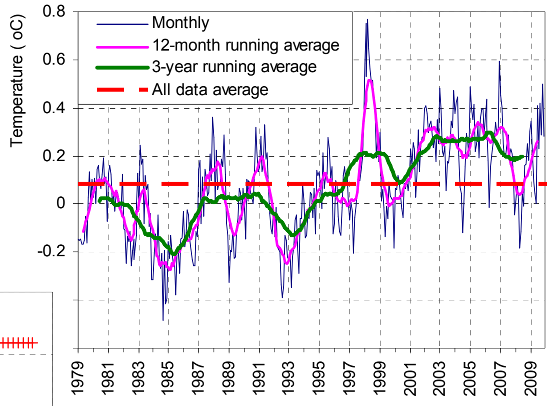


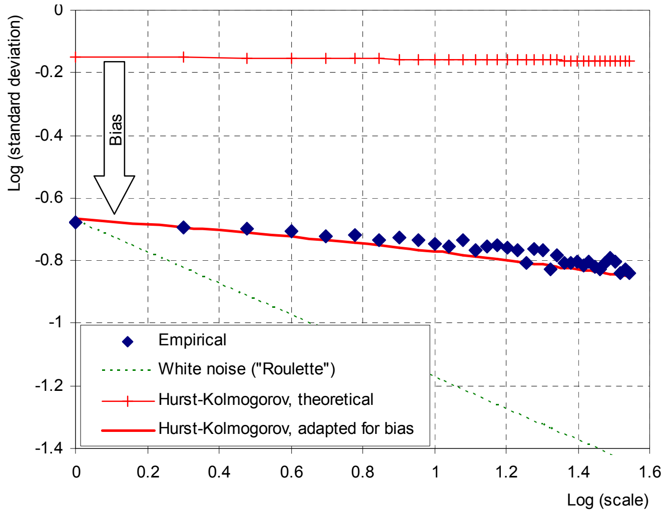
Example 3:
A "roulette" process



Example 4: The lower tropospheric temperature

Suggests an HK behaviour with a very high Hurst coefficient: $H = 0.99$





Data: 1979-2009, from http://vortex.nsstc.uah.edu/public/msu/t2lt/tltglhmam_5.2

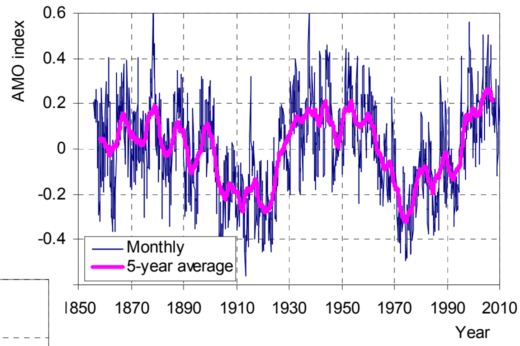
Impacts to statistical estimation: Hurst-Kolmogorov statistics (HKS) vs. classical statistics (CS)

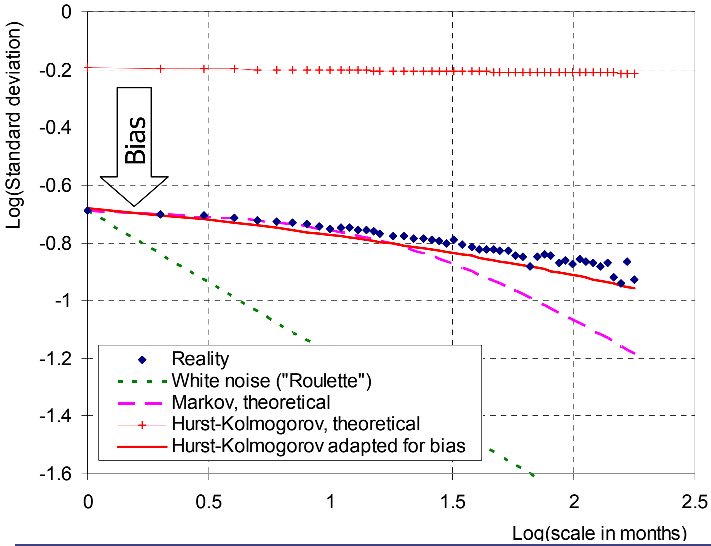
True values →	Mean, μ	Standard deviation, σ	Autocorrelation ρ_l for lag l
Standard estimator	$\underline{\bar{x}} := \frac{1}{n}\sum_{i=1}^{n} \underline{x}_i$	$\underline{s} := \sqrt{\frac{1}{n-1}}\sqrt{\sum_{i=1}^{n}(\underline{x}_i - \underline{\bar{x}})^2}$	$\underline{r}_l := \frac{1}{(n-1)\underline{s}^2} \cdot \sum_{i=1}^{n-l}(\underline{x}_i - \underline{\bar{x}})(\underline{x}_{i+l} - \underline{\bar{x}})$
Relative bias of estimation, CS	0	≈ 0	≈ 0
Relative bias of estimation, HKS	0	$\approx \sqrt{1 - \frac{1}{n'}} - 1 \approx -\frac{1}{2n'}$ (–22%)	$\approx -\frac{1/\rho_l - 1}{n' - 1}$ (–79%)
Standard deviation of estimator, CS	$\frac{\sigma}{\sqrt{n}}$ (0.1)	$\approx \frac{\sigma}{\sqrt{2(n-1)}}$ (0.071)	
Standard deviation of estimator, HKS	$\frac{\sigma}{\sqrt{n'}}$ (0.63)	$\approx \frac{\sigma\sqrt{(0.1\,n + 0.8)^{\lambda(H)}(1 - n^{2H-2})}}{\sqrt{2(n-1)}}$ where $\lambda(H) := 0.088\,(4H^2 - 1)^2$ (0.093)	

Note: $n' := n^{2-2H}$ is the "equivalent" or "effective" sample size: a sample with size n' in CS results in the same uncertainty of the mean as a sample with size n in HKS.

The numbers in parentheses are numerical examples for $n = 100$, $\sigma = 1$, $H = 0.90$ and $l = 10$, so that $n' = 2.5$.

Example 5: The Monthly Atlantic Multidecadal Oscillation (AMO) Index

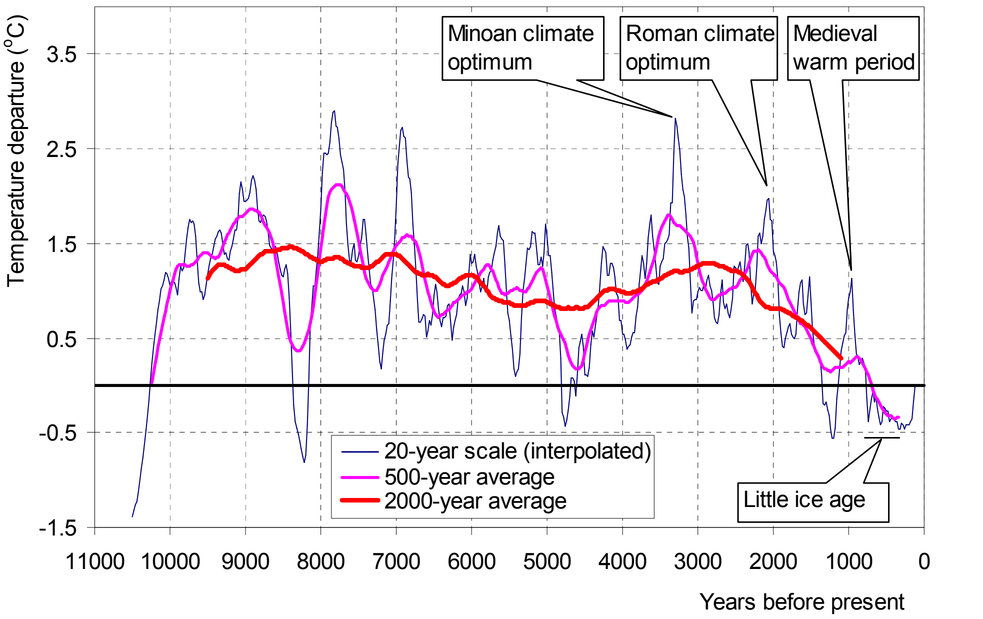




Suggests an HK behaviour with a very high Hurst coefficient: $H = 0.99$

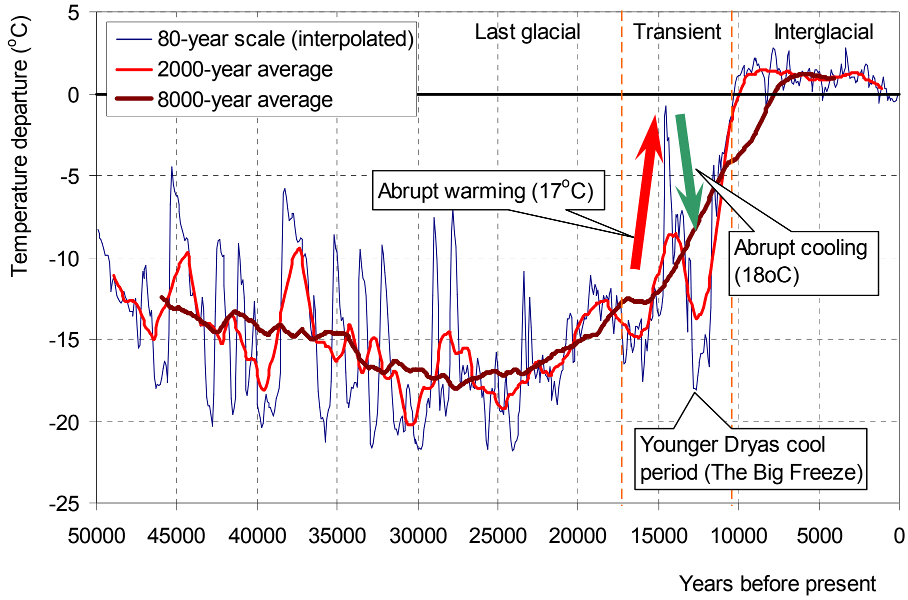
Data: 1856-2009, from NOAA, http://www.esrl.noaa.gov/psd/data/timeseries/AMO/

Example 6: The Greenland temperature proxy during the Holocene



Reconstructed from the GISP2 Ice Core (Alley, 2000, 2004). Data from: ftp.ncdc.noaa.gov/pub/data/paleo/icecore/greenland/summit/gisp2/isotopes/gisp2_temp_accum_alley2000.txt

Example 6 (cont.): The Greenland temperature proxy on multi-millennial time scales

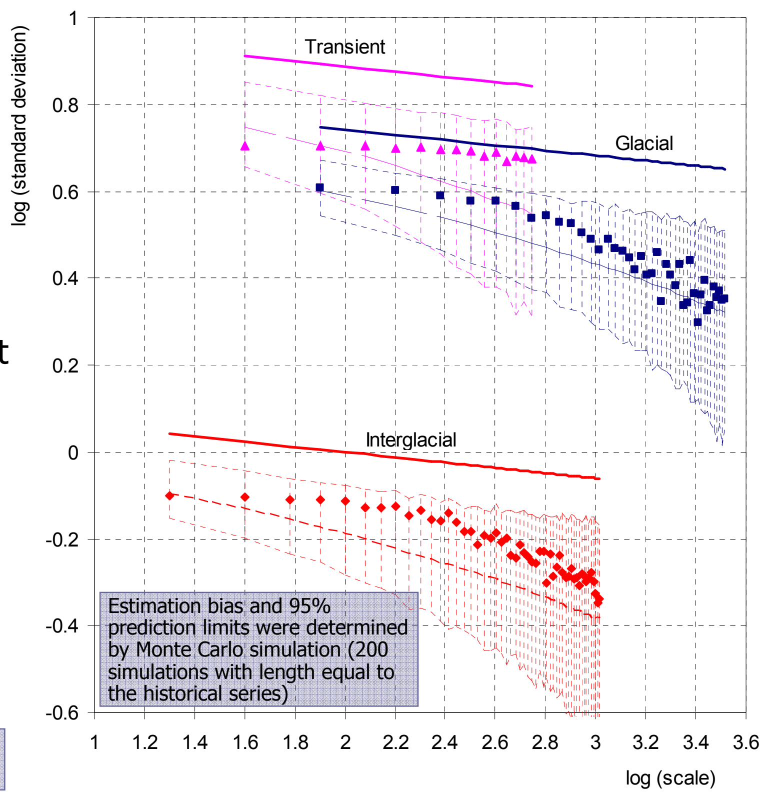


Reconstructed from the GISP2 Ice Core (Alley, 2000, 2004). Data from: ftp.ncdc.noaa.gov/pub/data/paleo/icecore/greenland/summit/gisp2/isotopes/gisp2_temp_accum_alley2000.txt

Example 6 (cont.): The Greenland temperature proxy on all scales

All three periods suggest an HK behaviour with a very high Hurst coefficient

Here an $H \approx 0.94$ was used for all three periods, assuming different standard deviation in each one



Estimation bias and 95% prediction limits were determined by Monte Carlo simulation (200 simulations with length equal to the historical series)

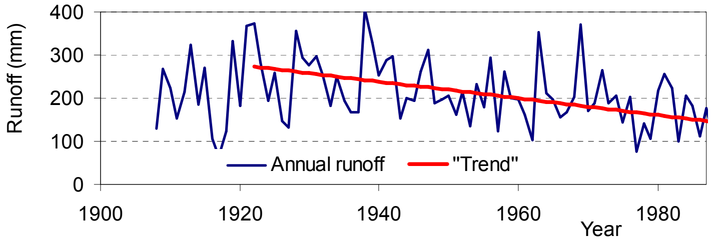
Reproduced from Koutsoyiannis *et al.* (2009)

5. Implications in engineering design and water resources management

Example 7: Back in the Athens water supply system

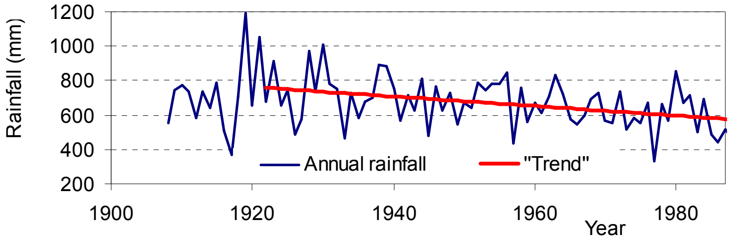
The historical time series of Boeoticos Kephisos runoff (Hydrological years 1907/08-1986/87)

A multi-year «trend» is observed



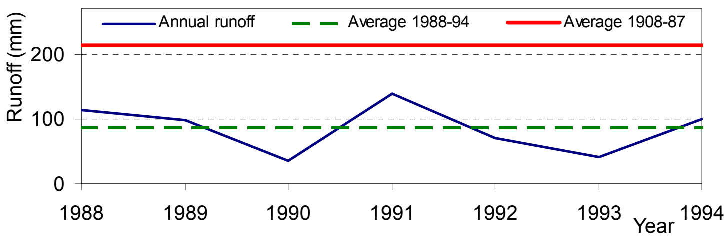
A similar «trend» in the rainfall time series

Explains the «trend» in runoff



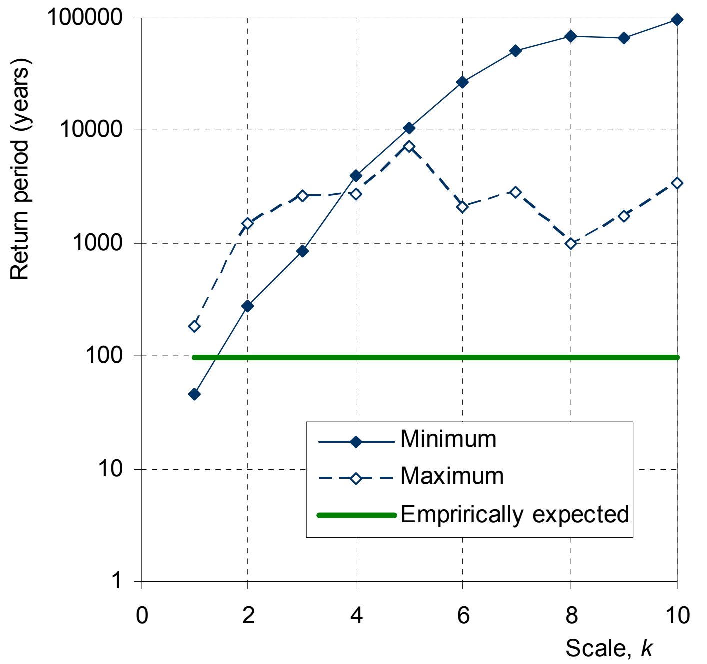
The next years were dry

Intense and persistent drought: Mean flow less than half of the historical average, duration 7 years



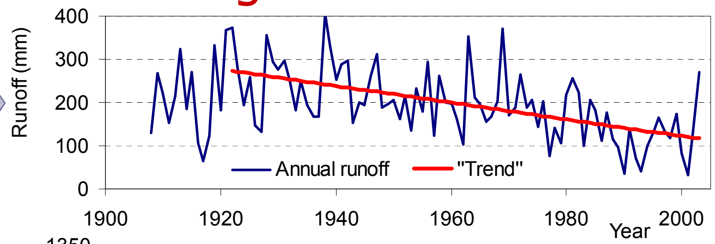
Classical statistics: Return period of the persistent drought

- At the annual scale, the drought was a record minimum but with typical magnitude
- Aggregated at larger scales, it appeared something extraordinary
- Similar behaviour was observed for maxima et aggregate scales



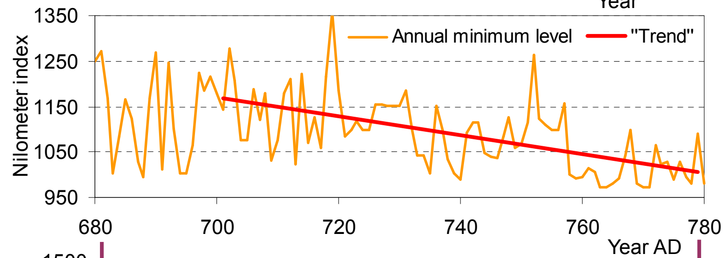
Comparisons with even longer series

The complete historical time series of Boeoticos Kephisos runoff



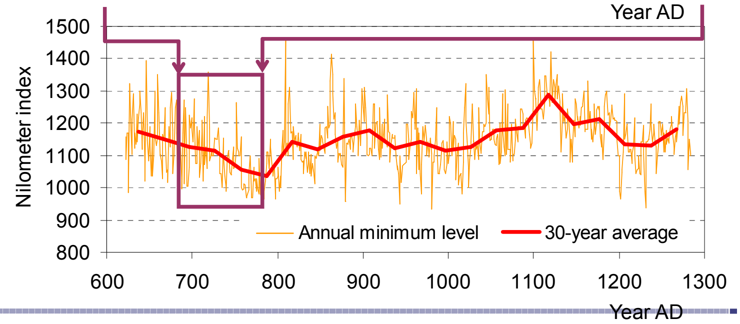
A part of the Nilometer series (the minimum annual water level in the Nile River, in cm)

A similar «trend»

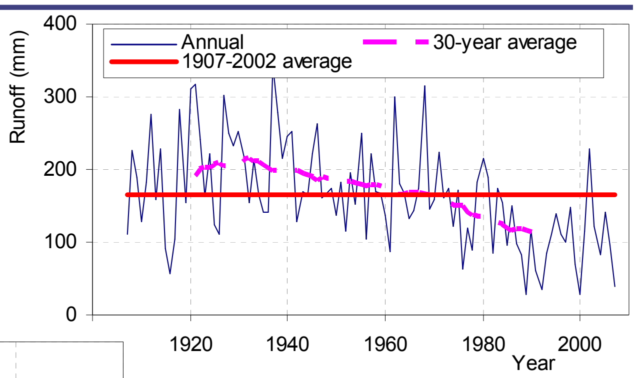


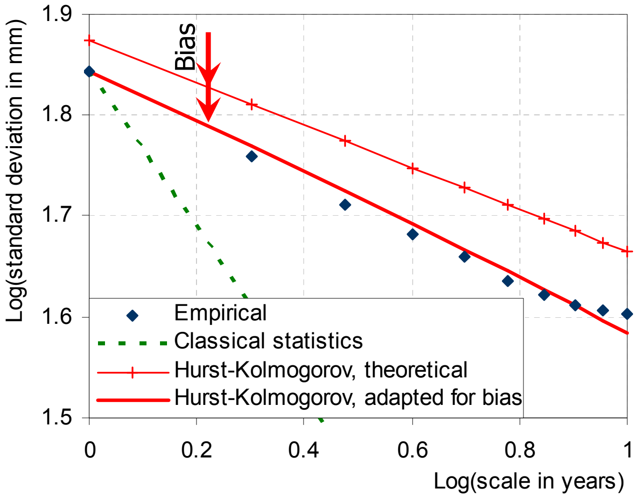
The complete Nilometer series

Upward and downward fluctuations on all scales



Hurst-Kolmogorov modelling of the Boeoticos Kephisos hydrological processes

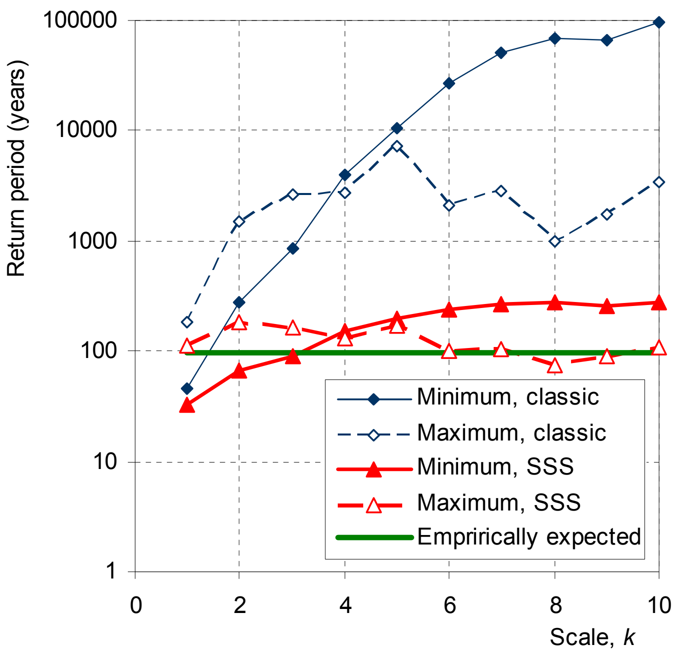




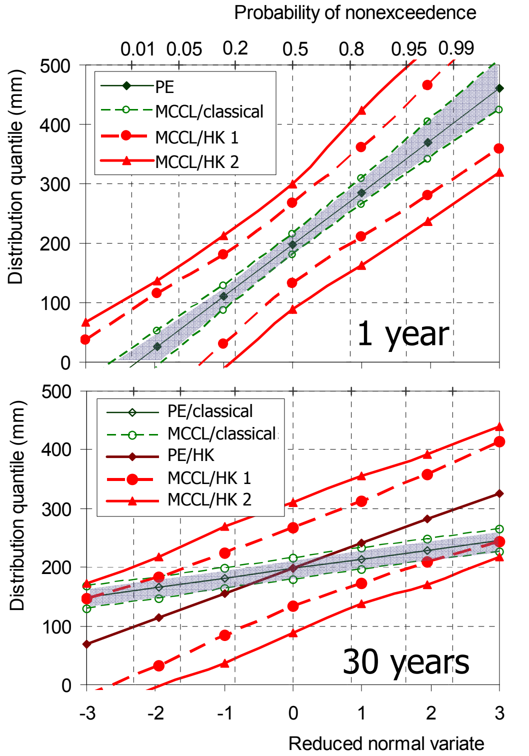
Suggests an HK behaviour with Hurst coefficient $H = 0.79$ in runoff (also $H > 0.5$ in temperature and precipitation)

Back to return period of the persistent drought

- The persistent drought is not extraordinary; it is a natural and expected behaviour
- Also, the trend is a natural and usual behaviour (Another "**naturally trendy**" process)



Perception and quantification of uncertainty with HK statistics



Boeoticos Kephisos River runoff (close to Athens, Greece); *H* = 0.84; from Koutsoyiannis *et al.* (2007)

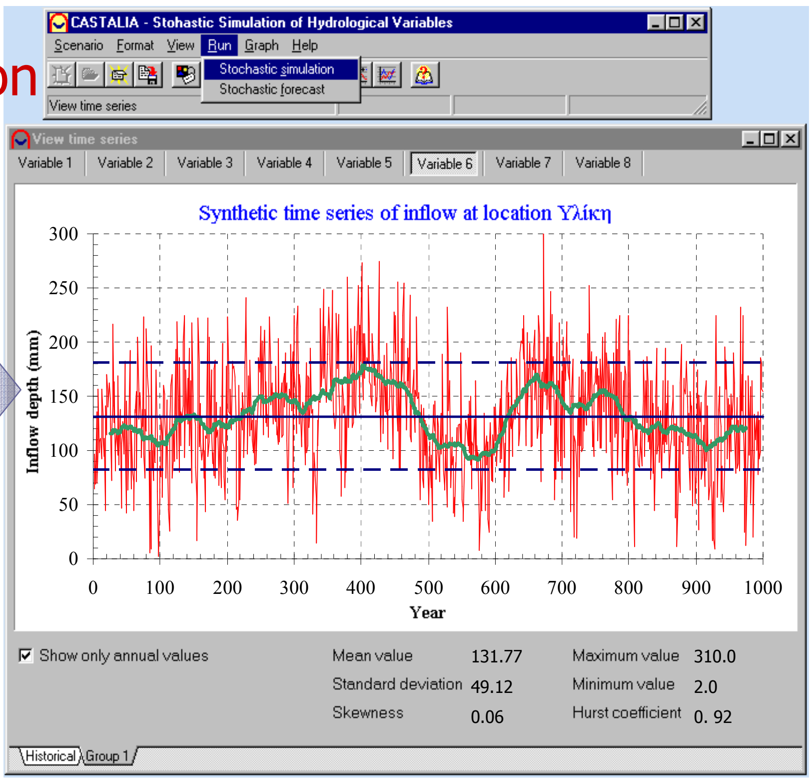
Statistical model	Total uncertainty in runoff (due to variability and parameter estimation) % of average	
	Annual scale	30-year scale
Classical	200	50
HK	270	200

Classical model (cf. common definition of climate)
Climate is what we expect
Weather is what we get

HK model
Weather is what we get … immediately
Climate is what we get
… if you keep expecting for a long time

Methodology implementation in the Athens water supply system

Castalia: Multivariate stochastic simulator for generalized HK processes

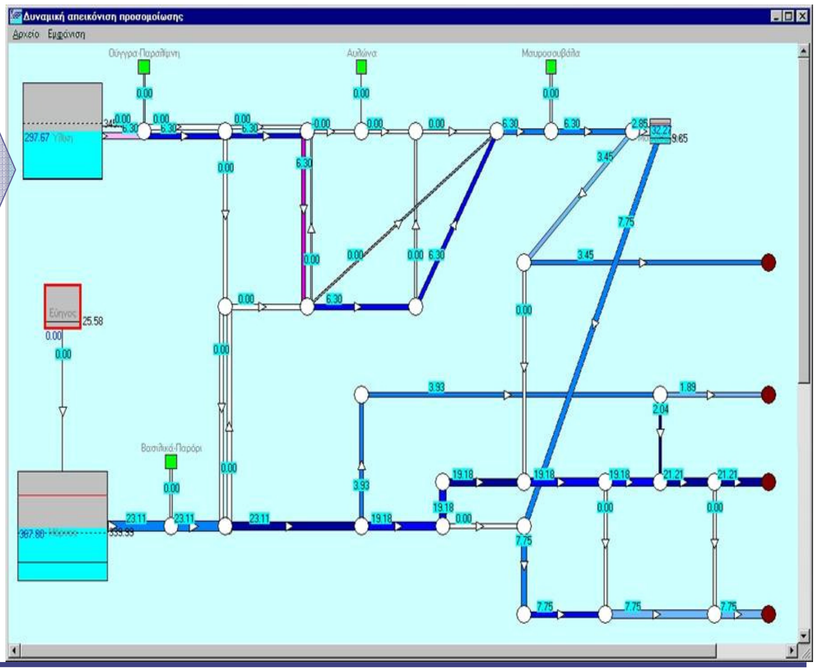


See theoretical and practical justification of the approach in Koutsoyiannis (2000, 2001) and Koutsoyiannis and Efstratiadis (2001)

Methodology implementation in the Athens water supply system (2)

Hydronomeas: A decision support system implementing a methodology termed **parameterization-simulation-optimization**

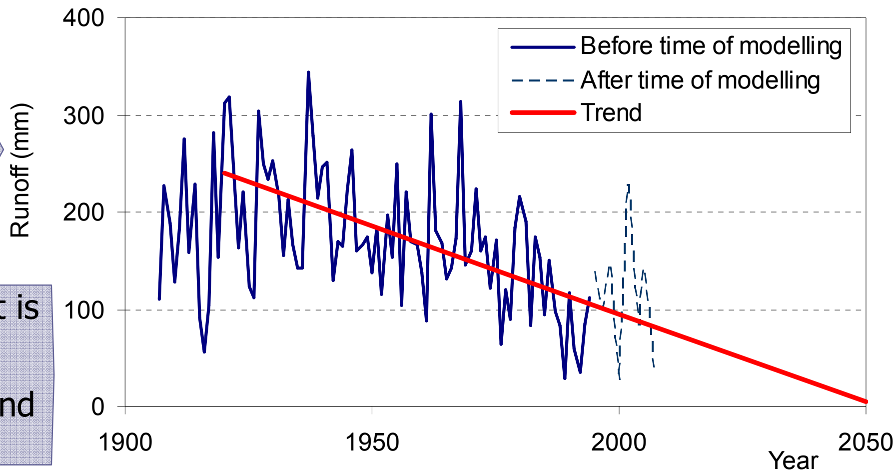
See theoretical and practical justification of the approach in Koutsoyiannis and Economou (2003); Koutsoyiannis *et al.* (2002, 2003); and Efstratiadis *et al.* (2004)



Alternative approach 1: Nonstationary, trend based

- The flows would disappear at about 2050…
- The trend reduces uncertainty (because it "explains" part of variability): The initial standard deviation of 70 mm decreases to 55 mm
- In contrast, in the HK approach the standard deviation increases to 75 mm

Boeoticos Kephisos runoff and projected trend



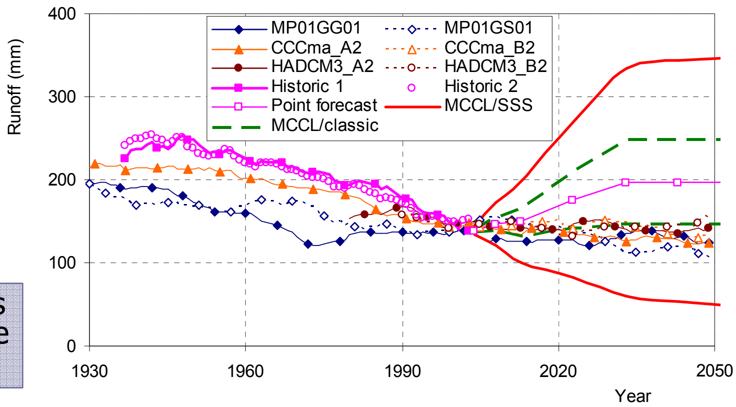
Conclusion: It is absurd to use such simplistic methods as trend projection

Alternative approach 2: Nonstationary, GCM based

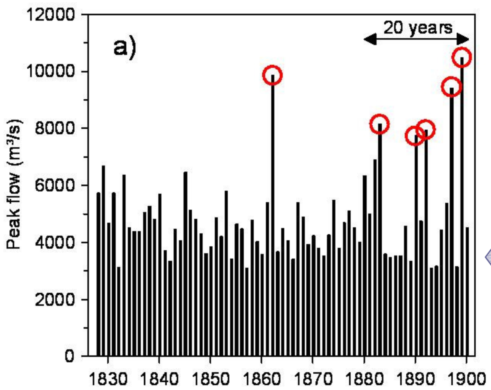
- Outputs from three GCMs for two scenarios were used
- The original GCM outputs (not shown) had no relation to reality (highly negative efficiencies at the annual time scale and above)
- After adaptations (also known as "downscaling") the GCM outputs improved with respect to reality (to about zero efficiencies at the annual time scale)
- For **the past**, despite adaptations, the proximity of models with reality is not satisfactory
- For **the future** the runoff obtained by adapted GCM outputs is too stable

Boeoticos Kephisos runoff produced with downscaled GCM outputs, superimposed to confidence zones produced with HK statistics under stationarity
(Koutsoyiannis *et al.*, 2007)

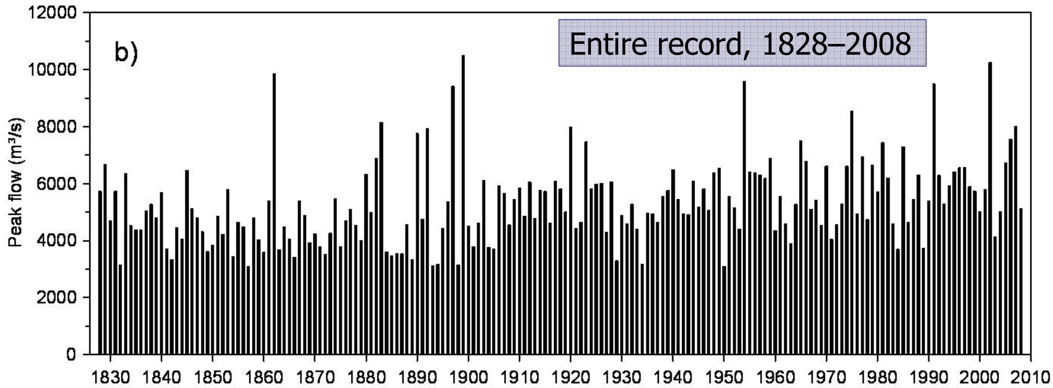
Conclusion: It is dangerous (too risky) to use GCM future projections



HK and extremes: Timing of flood peaks



Example 8: Annual maximum floods of the Danube at Vienna for 73 years (100 000 km^2 catchment area):
"Five of the six largest floods have occurred in the last two decades"
(Blöschl and Montanari, 2010)



HK and extremes: Distribution tails

Assumptions:

1. Probability density function of x_i (gamma—exponential tail):

   $f_i(x | \alpha_i) = \alpha_i^{\theta} x^{\theta - 1} \mathrm{e}^{-\alpha_i x} / \Gamma(\theta)$

2. The scale parameter α_i changes in time (e.g. due to overdecadal climatic fluctuation) with probability density function (gamma):

   $g(\alpha_i) = \beta^{\tau} \alpha_i^{\tau - 1} \mathrm{e}^{-\beta \alpha_i} / \Gamma(\tau)$

Result:

Unconditional density function of x:

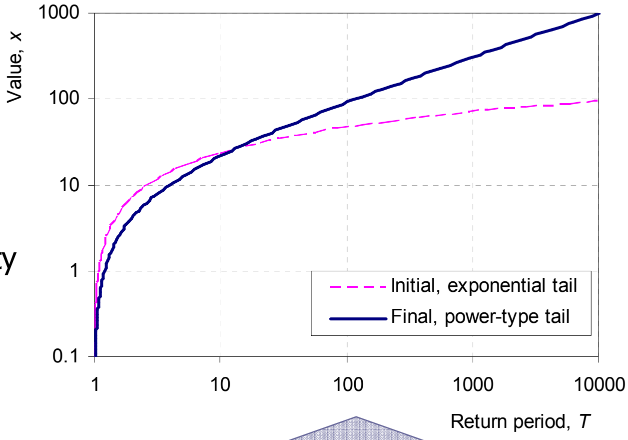
$f(x) = [1/ \beta B(\theta + \tau)]\ (y/\beta)^{\theta - 1}/(1 + y/\beta)^{\tau + \theta}$

$F(x) = [B_{x/(x + \beta)}(\theta, \tau) \,/\, B(\theta, \tau)]^n$

(Beta distribution of the second kind—power tail)

Conclusion:

Exponential distribution tails may become power type (Koutsoyiannis, 2004)



Example 9: Demonstration of the shift from exponential to power tail of distribution: gamma distribution with shape parameter $\theta = 1$ and scale parameter either constant $\alpha = 0.1$ (initial) or randomly varying following a gamma distribution with $\tau = 2$ and $\beta = 10$ (final); both have mean = 10

6. Final remarks

Advantages (or disadvantages?) of the "new" HK approach

- HK is old-fashioned—not "trendy" (despite admitting that natural processes are "naturally trendy"…)
  - Is as old as Kolmogorov (1940) and Hurst (1951)
  - Involves nothing like "artificial intelligence", "neural networks", "fuzzy logic", "chaotic attractors", "global circulation models", etc.
- HK is stationary—not nonstationary
  - Demonstrates how stationarity can coexist with change at all time scales
- HK is linear—not nonlinear
  - Deterministic dynamics need to be nonlinear to produce realistic trajectories—stochastic dynamics need not
- HK is simple, parsimonious, and inexpensive —not complicated, inflationary and expensive
- HK is honest—not deceitful
  - Does not hide uncertainty
  - Does not pretend to predict the distant future

Concluding remarks

- Change is Nature's style
- Change occurs at all time scales
- Change is not nonstationarity
- Hurst-Kolmogorov dynamics is the key to perceive multi-scale change and model the implied uncertainty and risk
- In general, the Hurst-Kolmogorov approach can incorporate deterministic descriptions of future changes, if available
- In absence of credible predictions of the future, Hurst-Kolmogorov dynamics admits stationarity

Long live stationarity!!!

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